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Research Article

Total Energy of Charged Black Holes in Einstein-Maxwell-Dilaton-Axion Theory

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We focus on the energy content (including matter and fields) of the Møller energy-momentum complex in the framework of Einstein-Maxwell-Dilaton-Axion (EMDA) theory using teleparallel gravity. We perform the required calculations for some specific charged black hole models, and we find that total energy distributions associated with asymptotically flat black holes are proportional to the gravitational mass. On the other hand, we see that the energy of the asymptotically nonflat black holes diverge in a limiting case.

1. Introduction

There are many interesting theories aiming to investigate gravitational effects: general relativity and teleparallel gravity. In these theories, calculating the energy-momentum distribution is an old and interesting problem. One can construct a teleparallel equivalent of the general relativity by assuming that curvature and torsion give the equivalent descriptions of the gravitational interactions.

In order to avoid the singularities in the general relativity and to give a general definition of energy momentum, Møller obtained a new expression by using the teleparallel gravity [1, 2]. Pellegrini and Plebanski found the Lagrangian formulation of the teleparallel theory, and this formalism was developed further by Møller [3, 4]. The gauge theory studied in detail by Hayashi [5] for the translational group is formulated by Hayashi and Nakano [6]. After that Hayashi [7] underlined the connection between this theory and the teleparallel theory. Later, Hayashi and Shirafuji [8] tried to unify these two theories. Hehl et al. [9] discussed a generalization of Einstein's gravitational theory with spin and torsion. In the near past Mikhail et al. [10] used the method of the superpotential in the case of the spherical symmetry for the Møller's energy-momentum expression in the teleparallel theory. By using the Møller's super potential method Shirafuji et al. [11, 12] found that energy is equivalent to

the gravitational mass. Maluf [13] showed that the energy is equivalent to the gravitational mass by constructing a connection between the gravitational energy related closely to the Sparling two-forms and the teleparallel equivalence of the general relativity. de Andrade et al. [14] obtained a teleparallel equivalence of the Kaluza-Klein theory in five dimensions. Respectively, Pereira et al. [15] and Zhang [16] found the tetrad and torsion fields for the axial symmetric Kerr spacetime and alternative Kerr spacetime by using the teleparallel theory. A new interpretation for torsion in connection with the gravitational interaction is given by Arcos and Pereira [17] as a review. It is pointed out that the teleparallel gravity successfully was applied to describe the rotation spin effect as another application of spin and axial torsion interaction [18]. About the spacetime torsion one can examine the work in [19]. Nashed [20] calculated total energy for some special situations of the most general spherical symmetric and nonsingular black hole solutions in the teleparallel gravity by using the Møller's super potential technique.

The study carried out by Vargas [21] for the Friedman-Robertson-Walker (FRW) universe possessing a cartesian geometry has been a starting point for the other studies done after that. Later, the energy densities for the Einstein, Bergman-Thomson, and Landau-Lifshitz energy momentum expressions written in cartesian coordinates are obtained in the teleparallel theory. Therefore the teleparallel equivalence of general relativistic case is obtained by Nashed [22] for some solutions of the models of the universe having charged and spherical symmetric solutions. Furthermore, the energy-momentum and angular momentum are calculated for two different fields by using energy-momentum tensor [23]. In the framework of Riemann's geometry it is found that the energy density in Kerr-Nut spacetime model is proportional to the gravitational mass by using Møller's energy-momentum densities calculated both in general relativity theory and teleparallel theory are equivalent. In this point of view it is concluded that the teleparallel theory is an alternative one for the general relativity.

This paper is organized as follows. In the next section we introduce Møller energymomentum complex in the teleparallel theory. In the Section 4, the total energy is found for the EMDA theory. Last section is devoted to discussion. Throughout this paper, Latin indices (i, j, k, ...) denote the vector numbers and Greek indices $(\mu, \nu, \sigma, ...)$ represent the vector components. All indices run from 0 to 3, and we use units in which G = 1 and c = 1.

2. The Møller Energy-Momentum Complex

The metric tensor can be written in the tetrad form:

$$g^{\mu\nu} = \eta^{ij} h_i^{\ \mu} h_j^{\ \nu}, \tag{2.1}$$

where η^{ij} is the Minkowski metric defined by $\text{Diag}\{-1,+1,+1,+1\}$. The torsion tensor in Møller's theory is

$$T^{\mu}{}_{\nu\lambda} = \Gamma^{\mu}{}_{\lambda\nu} - \Gamma^{\mu}{}_{\nu\lambda}, \tag{2.2}$$

and here $\Gamma^{\mu}_{\mu\nu}$ is Weitzenböck connection [25] given by

$$\Gamma^{\mu}{}_{\nu\lambda} = h_{a}{}^{\mu}\partial_{\lambda}h^{a}{}_{\nu}. \tag{2.3}$$

A general expression for an energy-momentum complex was found by Møller by using the method of infinitesimal transformations in the superpotential $\Im_{\beta}^{\mu\nu}$ form [10]:

$$M_{\beta}{}^{\mu} \equiv \sqrt{-g} \left(T_{\beta}{}^{\mu} + t_{\beta}{}^{\mu} \right) = \Im_{\beta;\nu}{}^{\mu\nu}.$$
 (2.4)

Here, $T_{\beta}{}^{\mu}$, $t_{\beta}{}^{\mu}$ energy-momentum tensor and energy-momentum pseudotensor arise from matter and gravitational field, respectively. $\mathfrak{F}_{\beta}{}^{\mu\nu}$ is given by

$$\Im_{\beta}^{\mu\nu} = \frac{\sqrt{-g}}{2\kappa} P_{\chi\rho\sigma}{}^{\tau\mu\nu} \left[\Phi^{\rho} g^{\sigma\chi} g_{\beta\tau} - \lambda g_{\tau\beta} \xi^{\chi\rho\sigma} - (1 - 2\lambda) g_{\tau\beta} \xi^{\sigma\rho\chi} \right].$$
(2.5)

Here λ is a free dimensionless parameter. Φ^{ρ} is defined by

$$\Phi_{\rho} = \xi^{\sigma}{}_{\rho\sigma}, \tag{2.6}$$

and $\xi_{\alpha\beta\mu} = h_{i\alpha} e^i_{\ \beta;\mu}$ is the con-torsion tensor. $P_{\chi\rho\sigma}{}^{\tau\mu\nu}$ is the tensor of the form

$$P_{\chi\rho\sigma}^{\tau\mu\nu} = \delta_{\chi}^{\tau} (\delta_{\rho}^{\mu}\delta_{\sigma}^{\nu} - \delta_{\sigma}^{\mu}\delta_{\rho}^{\nu}) + \delta_{\rho}^{\tau} (\delta_{\sigma}^{\mu}\delta_{\chi}^{\nu} - \delta_{\chi}^{\mu}\delta_{\sigma}^{\nu}) - \delta_{\sigma}^{\tau} (\delta_{\chi}^{\mu}\delta_{\rho}^{\nu} - \delta_{\rho}^{\mu}\delta_{\chi}^{\nu}).$$
(2.7)

The total energy is given by the surface integral below

$$E = \lim_{r \to \infty} \int_{r=\text{const.}} \mathfrak{S}_0^{0\alpha} \eta_\alpha dS, \qquad (2.8)$$

where *dS* is the surface element and η_{α} is the unit 3-vector normal to the surface.

3. Energy Contents of Charged Black Holes in EMDA Theory

In Einstein's frame, Einstein-Hilbert-Maxwell action coupled with a string in four dimensions is given [26]:

$$S = \int d^4x \frac{\sqrt{-g}}{2\kappa} \left[R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} \omega(\varphi) \partial_\sigma \zeta \partial^\sigma \zeta - \alpha(\varphi, \zeta) F_{\gamma\eta} F^{\gamma\eta} - \beta(\varphi, \zeta) F_{\gamma\eta} \widetilde{F}^{\gamma\eta} \right], \tag{3.1}$$

where κ is the four-dimensional gravitational coupling constant, R is the curvature scalar, and $F_{\mu\nu}$ is the field strength of the Maxwell field. Here φ , ζ are scalar and pseudoscalar fields, respectively. Additionally, α and β functions describe how the Maxwell field is coupled with φ and ζ . Also, the Maxwell field strength is defined as $\tilde{F}^{\gamma\eta} = (1/2)\epsilon^{\gamma\eta\epsilon\tau}F_{\epsilon\tau}$ (ϵ is the fourth Levi-Civita tensor).

Taking $\omega(\varphi) = e^{2a\varphi}$, $\alpha(\varphi) = e^{-a\varphi}$, and $\beta(\zeta) = b\zeta$ for the four dimensional Einstein-Maxwell theory the generalized action coupled with massless scalar dilaton φ and pseudoscalar axion ζ is given as

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} \left(R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} e^{2a\varphi} \partial_\sigma \zeta \partial^\sigma \zeta \right) - e^{-a\varphi} F_{\gamma\eta} F^{\gamma\eta} - b\zeta F_{\gamma\eta} \tilde{F}^{\gamma\eta} \right], \tag{3.2}$$

where *a* and *b* are two constant-free parameters.

The general metric for this theory is written as

$$ds^{2} = -G^{2}(r)dt^{2} + \frac{1}{G^{2}(r)}dr^{2} + F^{2}(r)\left(d\theta^{2} + \sin^{2}\theta d\phi\right).$$
(3.3)

For this line element, the metric tensor and its inverse are obtained as

$$g_{\mu\nu} = \operatorname{diag}\left\{-G^{2}(r), \frac{1}{G^{2}(r)}, F^{2}(r), F^{2}(r)\sin^{2}\theta\right\},$$

$$g^{\mu\nu} = \operatorname{diag}\left\{-\frac{1}{G^{2}(r)}, G^{2}(r), \frac{1}{F^{2}(r)}, \frac{1}{F^{2}(r)\sin^{2}\theta}\right\}.$$
(3.4)

Having spherical symmetry for the general form of the tetrad and using the coordinate transformation, the tetrad components can be written in a matrix form as

$$h_{a}^{\mu} = \begin{pmatrix} \frac{1}{G(r)} & 0 & 0 & 0\\ 0 & G(r)\sin\theta\cos\phi & \frac{\cos\theta\cos\phi}{F(r)} & -\frac{\sin\phi}{F(r)\sin\theta}\\ 0 & G(r)\sin\theta\sin\phi & \frac{\cos\theta\sin\phi}{F(r)} & \frac{\cos\phi}{F(r)\sin\theta}\\ 0 & G(r)\cos\theta & -\frac{\sin\theta}{F(r)} & 0 \end{pmatrix}.$$
 (3.5)

3.1. Asymptotically Flat Black Holes

For the asymptotically flat black holes, the line element (3.3) becomes [26]

$$ds^{2} = -\frac{(r-r_{+})(r-r_{-})}{(r-r_{0})^{2-2n}(r+r_{0})^{2n}}dt^{2} + \frac{(r-r_{0})^{2-2n}(r+r_{0})^{2n}}{(r-r_{+})(r-r_{-})}dr^{2} + \frac{(r+r_{+})^{2n}}{(r-r_{0})^{2n}}d\Omega^{2},$$
(3.6)

where r_{\pm} , r_0 are constants, $0 \le n \le 1$, and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi$.

Case a = b = 1

Taking a = b = 1 in the action (3.2) the metric becomes [26]

$$ds^{2} = -\left(1 - \frac{2m}{r}\right)dt^{2} + \left(1 - \frac{2m}{r}\right)^{-1}dr^{2} + r(r - 2r_{0})d\Omega^{2}.$$
(3.7)

Taking $G^2(r) = (1 - (2m/r))$ and $F^2(r) = r(r - 2r_0)$ in (3.5) and then using it in (2.5) one can obtain the Freud superpotential as follows:

$$\mathfrak{S}_0^{01} = \frac{2\sin\theta}{\kappa} \left[2m + r_0 - \frac{2r_0}{r} - r + (r - 2m)^{1/2} (r - 2r_0)^{1/2} \right]. \tag{3.8}$$

Using this result in the energy integral (2.8) the energy density is found as

$$E(r) = 2m + \frac{e^{-\varphi_0}Q^2}{4m} \left(1 - \frac{2m}{r}\right) - r + (r - 2m)^{1/2} \left(r - \frac{e^{-\varphi_0}Q^2}{2m}\right)^{1/2},$$
(3.9)

where φ_0 is a scalar field when $r \to \infty$. Finally the total energy when $r \to \infty$ for the asymptotically flat black holes (a = b = 1) is

$$E = m. \tag{3.10}$$

Here $r_0 = Q^2 e^{-\varphi_0}/(2m)$, Q_e is electrical charge, Q_m is magnetic charge, and the Q is total charge $(Q^2 = Q_e^2 + Q_m^2)$.

Case $a = 1, b \ll 1$

In this case the line element is [26]

$$ds^{2} = -\frac{(r-r_{+})(r-r_{-})}{(r^{2}-r_{0}^{2})}dt^{2} + \frac{(r^{2}-r_{0}^{2})}{(r-r_{+})(r-r_{-})}dr^{2} + (r^{2}-r_{0}^{2})d\Omega^{2}.$$
 (3.11)

If one can use $G^2(r) = (r - r_+)(r - r_-)/(r^2 - r_0^2)$ and $F^2(r) = (r^2 - r_0^2)$ in (3.5), is (2.5) obtained as follows:

$$\mathfrak{S}_{0}^{01} = \frac{2\sin\theta}{\kappa} \bigg[(r-r_{+})^{(1/2)} (r-r_{-})^{1/2} - \frac{r(r-r_{+})(r-r_{-})}{r^{2}-r_{0}^{2}} \bigg].$$
(3.12)

The energy density obtained by (2.8) is

$$E(r) = \left[r^2 - 2mr + \left(Q_e^2 + Q_m^2\right)e^{-\phi_0} - r_0^2\right]^{1/2} - r\left[1 - \frac{2mr - \left(Q_e^2 + Q_m^2\right)e^{-\phi_0}}{r^2 - r_0^2}\right],$$
(3.13)

where $r_0 = (Q_e^2 - Q_m^2)e^{-\phi_0}/2m$ and $r_{\pm} = m \pm [m^2 + r_0^2 - (Q_e^2 + Q_m^2)e^{-\phi_0}]^{1/2}$. One can easily see that, for $Q_m = 0$, $Q_e = Q$ and under the coordinate transformation $r \pm r_0 \rightarrow r$, the case transforms in to a Garfinkle-Horowitz-Strominger (GHS) dilaton black hole [27]. Therefore the total energy for GHS dilaton black hole is [28]

$$E = m - \frac{Q^2 e^{-\varphi_0}}{r}.$$
 (3.14)

The total energy obtained by taking $Q_e = Q_m = 0$ (Schwarzschild solution) for electrically or magnetically charged black holes is Schwarzschild solution

$$E = m. \tag{3.15}$$

Case $a = b \gg 1$

In this case the line element is written by [26]

$$ds^{2} \approx -\left(1 - \frac{2m}{r - 2r_{0}}\right)dt^{2} + \left(1 - \frac{2m}{r - 2r_{0}}\right)^{-1}dr^{2} + (r - 2r_{0})^{2}d\Omega^{2}.$$
 (3.16)

Using $G^2(r) = (1 - (2m/(r - 2r_0)))$ and $F(r) = (r - 2r_0)$ in the (3.5), (2.5) is obtained as

$$\mathfrak{S}_0^{01} = \frac{2\sin\theta}{\kappa} \left\{ 2m + (r - 2r_0) \left[\left(1 - \frac{2m}{r - 2r_0} \right)^{1/2} - 1 \right] \right\}.$$
(3.17)

The energy density is obtained as

$$E(r) = 2m + \left(r - \frac{a^2 e^{-a\phi} Q^2}{2m_0}\right) \left[\left(1 - \frac{4mm_0}{2rm_0 - a^2 e^{-a\phi} Q^2}\right)^{1/2} - 1 \right],$$
(3.18)

and the total energy for $a = b \gg 1$ is found as

$$E = m, \tag{3.19}$$

where $r_0 \approx a^2 e^{-a\phi} Q^2 / 4m_0$ and $m_0 \approx m + r_0$.

3.2. Asymptotically Nonflat Black Holes

When we consider asymptotically non-flat black holes, the line element (3.3) becomes [26]

$$ds^{2} = -\left(\frac{r}{2r_{0}}\right)^{2n} \left[1 - \frac{2m}{(1-n)r}\right] dt^{2} + \left(\frac{2r_{0}}{r}\right)^{2n} \left[1 - \frac{2m}{(1-n)r}\right]^{-1} dr^{2} + r^{2} \left(\frac{2r_{0}}{r}\right)^{2n} d\Omega^{2},$$
(3.20)

where $r_{+} = 2m/(1+n)$, $r_{-} = 0$, and $n = 1/(1+a^{2})$.

Case a = b = 1

Now, the metric has the form [26]

$$ds^{2} = -\left(\frac{r-4m}{2r_{0}}\right)dt^{2} + \left(\frac{r-4m}{2r_{0}}\right)^{-1}dr^{2} + 2rr_{0}d\Omega^{2}.$$
(3.21)

Considering $G^2(r) = ((r - 4m)/2r_0)$ and $F^2(r) = 2rr_0$ in (3.5) and then using it in (2.5) one can obtain the necessary component of the Freud superpotentials as follows:

$$\mathfrak{F}_0^{01} = \frac{2\sin\theta}{\kappa} \Big[2m - \frac{r}{2} + r^{1/2}(r - 4m)^{1/2} \Big]. \tag{3.22}$$

Substituting this result into the (2.8) the energy distribution is found as

$$E(r) = 2m - \frac{r}{2} + r^{1/2}(r - 4m)^{1/2}.$$
(3.23)

When we take the limit $r \to \infty$, we see that the energy distribution diverges.

Case $a = b \ll 1$

At this point the metric becomes [26]

$$ds^{2} = -\left(\frac{r}{2r_{0}}\right)^{2} \left[1 - \frac{2m}{a^{2}r}\right] dt^{2} + \left(\frac{2r_{0}}{r}\right)^{2} \left[1 - \frac{2m}{a^{2}r}\right]^{-1} dr^{2} + 4r_{0}^{2}d\Omega^{2}.$$
 (3.24)

If we use $G^2(r) = (r/2r_0)^2[1 - (2m/a^2r)]$ and $F(r) = 2r_0$ in (3.5) and (2.5), the calculated component of the Freud superpotentials is

$$\mathfrak{F}_{0}^{01} = \frac{2r\sin\theta}{\kappa} \left(1 - \frac{2m}{a^{2}r}\right)^{1/2}.$$
(3.25)

From (2.8) the energy is found

$$E(r) = r \left(1 - \frac{2m}{a^2 r} \right)^{1/2}.$$
 (3.26)

At large distances, the total energy diverges.

Case $a = b \gg 1$

Here the metric becomes [26]

$$ds^{2} = -\left(\frac{r}{2r_{0}}\right)^{2/a^{2}} \left[1 - \frac{2a^{2}m}{(a^{2} - 1)r}\right] dt^{2} + \left(\frac{2r_{0}}{r}\right)^{2/a^{2}} \left[1 - \frac{2a^{2}m}{(a^{2} - 1)r}\right]^{-1} dr^{2} + r^{2} \left(\frac{2r_{0}}{r}\right)^{2/a^{2}} d\Omega^{2}.$$
(3.27)

If we have $G(r) = (r/2r_0)^{1/a^2} [1 - (2a^2m/(a^2 - 1)r)]^{1/2}$ and $F(r) = r(2r_0/r)^{1/a^2}$ in (3.5), then (2.5) is obtained as follows:

$$\mathfrak{S}_{0}^{01} = \frac{2\sin\theta}{\kappa} r^{(a^{2}+1)/a^{2}} \left(1 - \frac{2a^{m}}{(a^{2}-1)r}\right)^{1/2} \left[1 - \left(1 - \frac{2a^{2}m}{(a^{2}-1)r}\right)^{1/2} r^{1/a^{2}}\right].$$
(3.28)

Using Freud superpotential in the (2.8) the obtained energy distribution is

$$E(r) = r^{(a^2+1)/a^2} \left(1 - \frac{2a^m}{(a^2-1)r}\right)^{1/2} \left[1 - \left(1 - \frac{2a^2m}{(a^2-1)r}\right)^{1/2} r^{1/a^2}\right].$$
 (3.29)

When $a \to \infty$ and $r \to \infty$, the total energy becomes

$$E = m. \tag{3.30}$$

Case $|a| \neq |b|$

For this final case the metric becomes [26]

$$ds^{2} = -\frac{(r-r_{+})(r-r_{-})}{2rr_{0}}dt^{2} + \frac{2rr_{0}}{(r-r_{+})(r-r_{-})}dr^{2} + 2rr_{0}d\Omega^{2}.$$
 (3.31)

If we consider $G^2(r) = (r - r_+)(r - r_-)/2rr_0$ and $F^2(r) = 2rr_0$ the component of the Freud superpotentials is calculated as

$$\mathfrak{S}_{0}^{01} = \frac{\sin\theta}{\kappa} \bigg\{ \bigg[r_{+} + r_{-} + 2(r - r_{+})^{1/2} (r - r_{-})^{2} \bigg] - \frac{r_{+}r_{-}}{r} - r \bigg\}.$$
(3.32)

Now the energy distribution is

$$E(r) = 2m - \frac{q_e^2 q_m^2}{2rr_0^2} - \frac{r}{2} + \left(r^2 - 4mr + \frac{q_e^2 q_m^2}{r_0^2}\right)^{1/2}.$$
(3.33)



Figure 1: The plot for the Möller energy versus radius, in the case of the asymptotically flat black holes.

It can be easily checked that, at large distances, the total energy diverges. Here, q_e and q_m are electromagnetic charges which are related to components of the electromagnetic field strength given by $F_{tr} = (1/2q_e)dt \wedge dr$ and $F_{\theta\varphi} = q_m \sin\theta d\theta \wedge d\varphi$.

4. Summary and Discussion

According to Lessner [29] perspective, the Møller energy-momentum complex can be evaluated in any coordinate system. Hence, this framework is the most powerful one in calculating the energy and momentum distributions associated with spacetime.

In the present work, in order to compute the energy distribution associated with some specific black holes in the Einstein-Maxwell-Dilaton Axion theory, we focus on the Møller energy-momentum distribution in the teleparallel gravity. For the asymptotically flat charged black holes, for all the cases (a = b = 1, $a = b \gg 1$, and a = 1, $b \ll 1$), it is found that the energy distribution depends on the mass m and the charge Q. The corresponding teleparallel Møller total energy is obtained proportional to gravitational mass (Schwarzschild mass) when

$$\lim_{r \to \infty} E(r) = m. \tag{4.1}$$

It is given the plot for the Möller energy versus radius, in the case of the asymptotically flat black holes in Figure 1. According to graph it is seen that the energy of the case a = b = 1 approaches to the gravitational mass faster than the others.

Considering asymptotically non-flat charged black holes, for both the cases a = b = 1 and $a = b \ll 1$ the energies diverge. Only for the case $a = b \gg 1$ the total energy is proportional to *m*. The energy distribution for the case $|a| \neq |b|$ diverges as well. For small values of *a* and *b* it is seen that the energy distributions tend toward infinity. If the values of *a* and *b* are large enough the corresponding teleparallel energy will be equal to the gravitational mass.

These results agree well with the previous results [30–35] obtained by using the general relativity version of the Møller energy-momentum complex. The energy is confined to the region of nonvanishing energy-momentum tensor of matter and all nongravitational fields [36]. The results are quite important in the theory of teleparallel gravity, since this theory provides more satisfactory solution of the energy-momentum problem than general relativity [10].

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