

Research Article

Projective Synchronization of Hyperchaotic Financial Systems

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Based on a special matrix structure, the projective synchronization control laws of the hyperchaotic financial systems are proposed in this paper. Put a hyperchaotic financial system as the drive system, via transformation of the system state variables, construct its response system, and then design the controller based on the special matrix structure. The given scheme is applied to achieve projective synchronization of the different hyperchaotic financial systems. Numerical experiments demonstrate the effectiveness of the method.

1. Introduction

Chaos in the economy was first discovered since 1985; it had made a tremendous impact on market economy. Chaos theory provides new approaches and ideas for financial crisis and other related issues. Synchronous development of the financial system is a real problem faced by many economists, such as how to achieve synchronous development in different countries and areas. In 1999, Mainieri and Rehacek [1] proposed a chaotic synchronous mapping; namely, drive-response system can be synchronous with a desired scaling factor. In recent decades, projective synchronization [2, 3] of the chaotic financial system is also of concern. Scholars realized projective synchronization of three-dimensional fractional-order chaotic financial system and the integer-order one, respectively. Hyperchaotic system has two or more positive Lyapunov exponents. The economic system is a high-dimensional nonlinear system, whose chaos is mostly super chaos [4]. Achieving synchronization of hyperchaotic financial system is closer to the actual condition, so some researchers have investigated the synchronization of hyperchaotic financial system [5, 6]. In this paper, based on a special matrix and conversion of state variables, we will realize projective synchronization of the identical and different hyperchaotic financial systems.

2. Projective Synchronization of Hyperchaotic Financial Systems

Consider the financial system [7] as follows:

$$\begin{aligned}\dot{x} &= z + (y - a)x, \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz,\end{aligned}\tag{1}$$

where x , y , and z represent the interest rate, investment demand, and price index, respectively. The parameter a is the saving, b is the per-investment cost, and c is the elasticity of demands of commercials. And they are positive constants.

Based on the chaotic finance of system (1), scholars found that the factors affecting the interest rate are related not only to investment demand and price index, but also to the average profit margin denoted by w . Therefore, the following improved chaotic finance system is constructed [8]:

$$\begin{aligned}\dot{x} &= z + (y - a)x + w, \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz, \\ \dot{w} &= -dxy - kw,\end{aligned}\tag{2}$$

where a, b, c, d , and k are the parameters of the system, and they are positive numbers. When $a = 0.9, b = 0.2, c = 1.5, d = 0.2$, and $k = 0.17$, system (2) presents hyperchaotic behavior [8]. When the initial value of system (2) is taken as $(1, 3, 2, 5)$ and $t \in (0, 1000)$, the phase portraits of system (2) are shown in Figures 1(a)–1(d).

Chaos of financial systems is mostly hyperchaos. In order to achieve financial synchronous development of developed and developing countries or different areas, we need to solve more problems of synchronization of hyperchaotic financial systems.

The projective synchronization discussed in this paper is defined as two relative chaotic dynamical systems can be synchronous with a desired scaling factor [2].

To discuss projective synchronization of the hyperchaotic financial system (2), we rewrite the system (2) as follows:

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{pmatrix} &= \begin{pmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \\ &+ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} xy \\ -x^2 \\ 0 \\ -dxy \end{pmatrix}. \end{aligned} \quad (3)$$

In order to facilitate design of controller, using the thought of state transition, we construct the following response system for the given system (3):

$$\begin{aligned} \begin{pmatrix} \dot{x}_s \\ \dot{y}_s \\ \dot{z}_s \\ \dot{w}_s \end{pmatrix} &= \begin{pmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{pmatrix} \begin{pmatrix} x_s \\ y_s \\ z_s \\ w_s \end{pmatrix} \\ &+ \alpha \left[\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} xy \\ -x^2 \\ 0 \\ -dxy \end{pmatrix} \right] + \mathbf{u}, \end{aligned} \quad (4)$$

where α is a scaling factor and \mathbf{u} is the external input control vector.

Denote $\mathbf{x} = (x, y, z, w)^T$, $\mathbf{y} = (x_s, y_s, z_s, w_s)^T$, and $\mathbf{e} = (e_1, e_2, e_3, e_4)^T$. If we define the error system as $\mathbf{e} = \mathbf{y} - \alpha\mathbf{x}$, we can get the error system

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{u}, \quad (5)$$

where

$$\mathbf{A} = \begin{pmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{pmatrix}. \quad (6)$$

Theorem 1. *If we design the controller $\mathbf{u} = \mathbf{B}\mathbf{e}$ for the error system (5), then the system (5) is asymptotically stable at the origin, where*

$$\mathbf{B} = \begin{pmatrix} k_1 \cdot \left(\frac{b_{11}}{k_1}\right) & k_1 \cdot \left(-\frac{a_1}{k_2}\right) & k_1 \cdot \left(\frac{1}{k_3} - \frac{1}{k_1} - \frac{a_2}{k_3}\right) & k_1 \cdot \left(-\frac{1}{k_1} - \frac{a_3}{k_4}\right) \\ k_2 \cdot \left(\frac{a_1}{k_2}\right) & k_2 \cdot \left(\frac{b_{22}}{k_2}\right) & k_2 \cdot \left(-\frac{a_4}{k_3}\right) & k_2 \cdot \left(-\frac{a_5}{k_4}\right) \\ k_3 \cdot \left(\frac{a_2}{k_3}\right) & k_3 \cdot \left(\frac{a_4}{k_3}\right) & k_3 \cdot \left(\frac{b_{33}}{k_3}\right) & k_3 \cdot \left(-\frac{a_6}{k_4}\right) \\ k_4 \cdot \left(\frac{a_3}{k_4}\right) & k_4 \cdot \left(\frac{a_5}{k_4}\right) & k_4 \cdot \left(\frac{a_6}{k_4}\right) & k_4 \cdot \left(\frac{b_{44}}{k_4}\right) \end{pmatrix}, \quad (7)$$

$$b_{11} \leq a, \quad b_{22} \leq b, \quad b_{33} \leq c, \quad b_{44} \leq k;$$

$$a_i \in R \quad (i = 1, 2, 3, 4, 5, 6), \quad k_i \in R^+ \quad (i = 1, 2, 3, 4).$$

In order to prove Theorem 1, we introduce Lemma 2 firstly.

Lemma 2. *Suppose a dynamic system can be written as*

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{pmatrix} = \begin{pmatrix} k_1 a_{11} & k_1 a_{12} & \cdots & k_1 a_{1n} \\ k_2 a_{21} & k_2 a_{22} & \cdots & k_2 a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ k_n a_{n1} & k_n a_{n2} & \cdots & k_n a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}. \quad (8)$$

If the system (8) satisfies the following conditions [9]:

- (1) $\forall a_{ij} \in R$,
- (2) $a_{ij} = -a_{ji}$ ($i \neq j$),
- (3) $a_{ii} \leq 0$ (not all a_{ii} are equal to zero),
- (4) $\forall k_i > 0$,

then the states of system (8) will decrease to zero gradually.

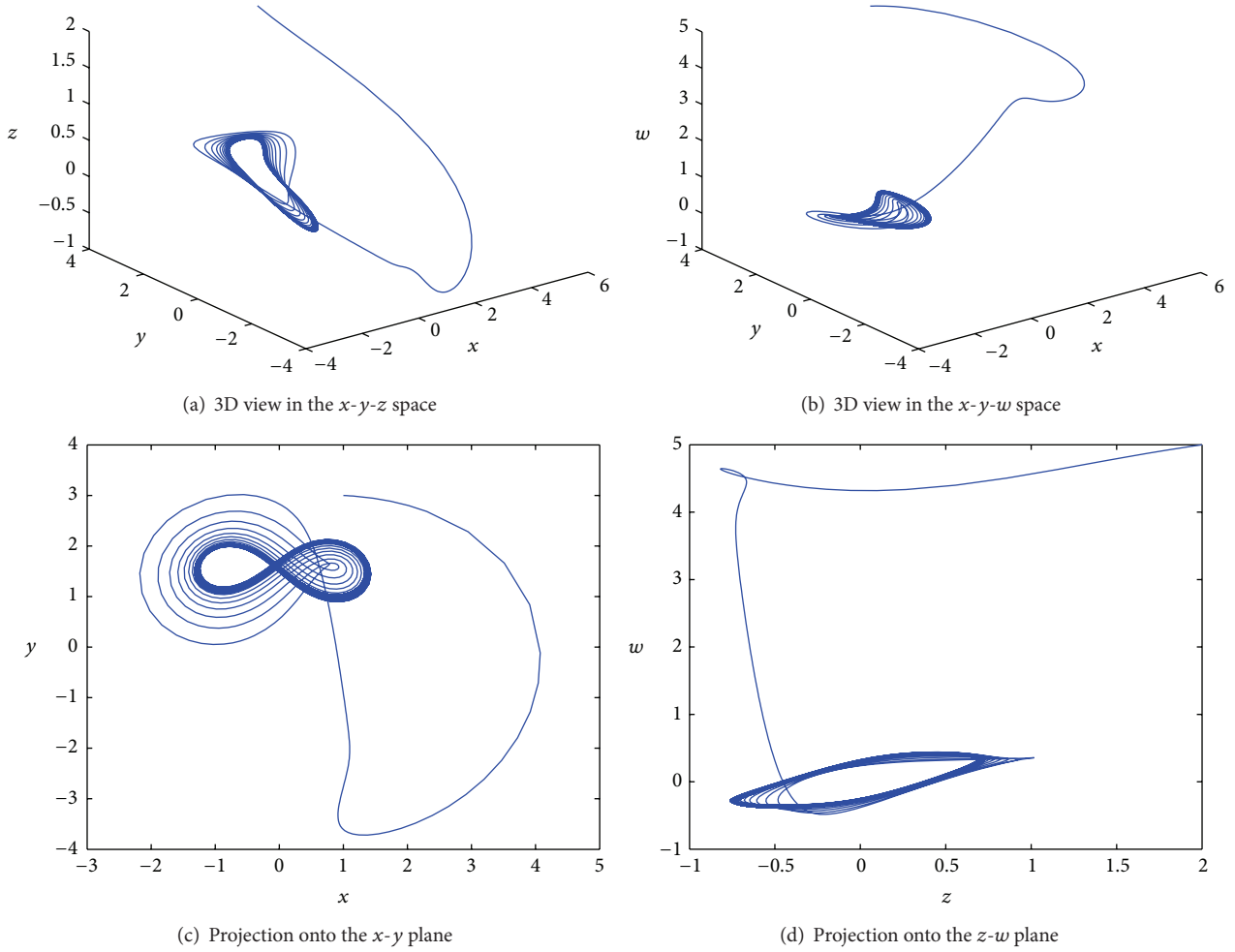


FIGURE 1: Phase portraits of hyperchaotic finance system (2).

Prove Theorem 1 as follows.

Proof. Let controller be $\mathbf{u} = B\mathbf{e}$, where $B = (b_{ij})_{4 \times 4}$ is a 4×4 order constant matrix to be designed; $\mathbf{e} = (e_1, e_2, e_3, e_4)^T$. We rewrite the error system (5) as

$$\dot{\mathbf{e}} = (c + B)\mathbf{e}, \quad (9)$$

where

$$C = (c_{ij})_{4 \times 4}$$

$$= \begin{pmatrix} k_1 \cdot \left(-\frac{a}{k_1}\right) & 0 & k_1 \cdot \left(\frac{1}{k_1}\right) & k_1 \cdot \left(\frac{1}{k_1}\right) \\ 0 & k_2 \cdot \left(-\frac{b}{k_2}\right) & 0 & 0 \\ k_3 \cdot \left(-\frac{1}{k_3}\right) & 0 & k_3 \cdot \left(-\frac{c}{k_3}\right) & 0 \\ 0 & 0 & 0 & k_4 \cdot \left(-\frac{k}{k_4}\right) \end{pmatrix}. \quad (10)$$

In order to make the system (9) satisfy Lemma 2, design matrix B to satisfy conditions (11) and (12) as follows:

$$-\frac{a}{k_1} + \frac{b_{11}}{k_1} \leq 0,$$

$$-\frac{b}{k_2} + \frac{b_{22}}{k_2} \leq 0,$$

$$-\frac{c}{k_3} + \frac{b_{33}}{k_3} \leq 0,$$

$$-\frac{k}{k_4} + \frac{b_{44}}{k_4} \leq 0,$$

(11)

$$\frac{b_{12}}{k_1} = -\frac{b_{21}}{k_2},$$

$$\frac{1}{k_1} + \frac{b_{13}}{k_1} = -\left(-\frac{1}{k_3} + \frac{b_{31}}{k_3}\right),$$

$$\frac{1}{k_1} + \frac{b_{14}}{k_1} = -\frac{b_{14}}{k_4},$$

$$\frac{b_{23}}{k_2} = -\frac{b_{32}}{k_3},$$

$$\begin{aligned}\frac{b_{24}}{k_2} &= -\frac{b_{42}}{k_4}, \\ \frac{b_{34}}{k_3} &= -\frac{b_{43}}{k_4}.\end{aligned}\quad (12)$$

It can be seen from (11) that $b_{11} \leq a$, $b_{22} \leq b$, $b_{33} \leq c$, and $b_{44} \leq k$.

Let the coefficient matrix of (12) be H , and let its augmented matrix be \bar{H} . After calculation, we can get $r(H) = r(\bar{H})$, and the numbers of unknowns are greater than the

numbers of (12), so (12) has infinitely many solutions. Note that all solutions of (12) are as follows:

$$\begin{aligned}\eta &= (b_{12}, b_{21}, b_{13}, b_{31}, b_{14}, b_{41}, b_{23}, b_{32}, b_{24}, b_{42}, b_{34}, b_{43}) \\ &= \left(-\frac{a_1 k_1}{k_2}, a_1, \frac{k_1}{k_3} - 1 - \frac{a_2 k_1}{k_3}, a_2, -1 - \frac{a_3 k_1}{k_4}, a_3, \right. \\ &\quad \left. -\frac{a_4 k_2}{k_3}, a_4, -\frac{a_5 k_2}{k_4}, a_5, -\frac{a_6 k_3}{k_4}, a_6 \right),\end{aligned}\quad (13)$$

where $a_i \in R$ ($i = 1, 2, 3, 4, 5, 6$); $k_i \in R^+$ ($i = 1, 2, 3, 4$).

Therefore integrate (11) and (12) to obtain the matrix B :

$$B = \begin{pmatrix} k_1 \cdot \left(\frac{b_{11}}{k_1} \right) & k_1 \cdot \left(-\frac{a_1}{k_2} \right) & k_1 \cdot \left(\frac{1}{k_3} - \frac{1}{k_1} - \frac{a_2}{k_3} \right) & k_1 \cdot \left(-\frac{1}{k_1} - \frac{a_3}{k_4} \right) \\ k_2 \cdot \left(\frac{a_1}{k_2} \right) & k_2 \cdot \left(\frac{b_{22}}{k_2} \right) & k_2 \cdot \left(-\frac{a_4}{k_3} \right) & k_2 \cdot \left(-\frac{a_5}{k_4} \right) \\ k_3 \cdot \left(\frac{a_2}{k_3} \right) & k_3 \cdot \left(\frac{a_4}{k_3} \right) & k_3 \cdot \left(\frac{b_{33}}{k_3} \right) & k_3 \cdot \left(-\frac{a_6}{k_4} \right) \\ k_4 \cdot \left(\frac{a_3}{k_4} \right) & k_4 \cdot \left(\frac{a_5}{k_4} \right) & k_4 \cdot \left(\frac{a_6}{k_4} \right) & k_4 \cdot \left(\frac{b_{44}}{k_4} \right) \end{pmatrix}.\quad (14)$$

So system (9) becomes

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} k_1 \cdot \left(-\frac{a}{k_1} + \frac{b_{11}}{k_1} \right) & k_1 \cdot \left(-\frac{a_1}{k_2} \right) & k_1 \cdot \left(\frac{1}{k_3} - \frac{a_2}{k_3} \right) & k_1 \cdot \left(-\frac{a_3}{k_4} \right) \\ k_2 \cdot \left(\frac{a_1}{k_2} \right) & k_2 \cdot \left(-\frac{b}{k_2} + \frac{b_{22}}{k_2} \right) & k_2 \cdot \left(-\frac{a_4}{k_3} \right) & k_2 \cdot \left(-\frac{a_5}{k_4} \right) \\ k_3 \cdot \left(-\frac{1}{k_3} + \frac{a_2}{k_3} \right) & k_3 \cdot \left(\frac{a_4}{k_3} \right) & k_3 \cdot \left(-\frac{c}{k_3} + \frac{b_{33}}{k_3} \right) & k_3 \cdot \left(-\frac{a_6}{k_4} \right) \\ k_4 \cdot \left(\frac{a_3}{k_4} \right) & k_4 \cdot \left(\frac{a_5}{k_4} \right) & k_4 \cdot \left(\frac{a_6}{k_4} \right) & k_4 \cdot \left(-\frac{k}{k_4} + \frac{b_{44}}{k_4} \right) \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}.\quad (15)$$

According to Lemma 2, the system (15) is asymptotically stable at the origin; namely, the system (5) is asymptotically stable at the origin. \square

Remark 3. The error system (5) is asymptotically stable at the origin; namely, the system (3) and system (4) achieve the projective synchronization.

We take $b_{11} = 0.5$, $b_{22} = 0.1$, $b_{33} = 1$, and $b_{44} = 0.1$; $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 4$, $a_5 = 5$, and $a_6 = 6$; $k_1 = 1$, $k_2 = 2$, $k_3 = 3$, and $k_4 = 4$. Then if we take $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$, $k = 0.17$, and $\alpha = 1/2$; the initial values of drive system (3) and response system (4) are taken as (1, 2, 0.5, 0.5) and (1, 3, 2, 5), respectively. So the initial value of error system (5) is (0, 1, 1.5, 4.5), and the state variables of system (5) varying with time t are shown in Figures 2(a)–2(e).

Remark 4. To make the structure of the controller as simple as possible, we take $b_{ii} = 0$, $i = 1, 2, 3, 4$, $a_i = 0$, $i = 1, 2, \dots, 6$, and $k_i = 1$, $i = 1, 2, 3, 4$. So we can get control matrix

$$B = \begin{pmatrix} 0 & 0 & k_1 \cdot \left(\frac{1}{k_3} - \frac{1}{k_1} \right) & k_1 \cdot \left(-\frac{1}{k_1} \right) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},\quad (16)$$

$$k_i \in R^+, \quad i = 1, 2, 3, 4$$

which is a special case.

Remark 5. When $\alpha = 1$, the system (3) and system (4) are of complete synchronization [10]; when $\alpha = -1$, the system (3) and system (4) are antisynchronization [11].

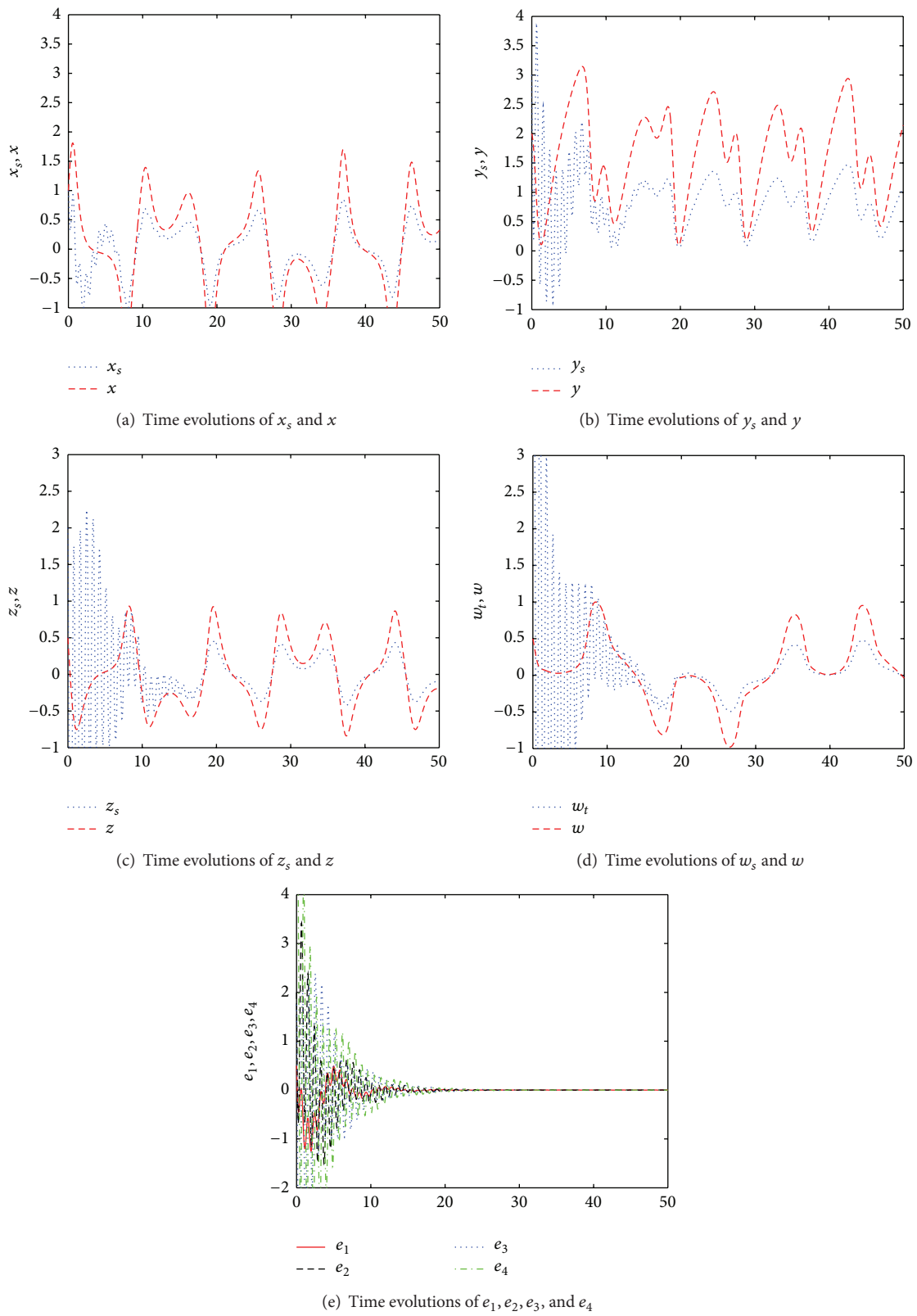


FIGURE 2: Time evolutions of the state variables of system (5).

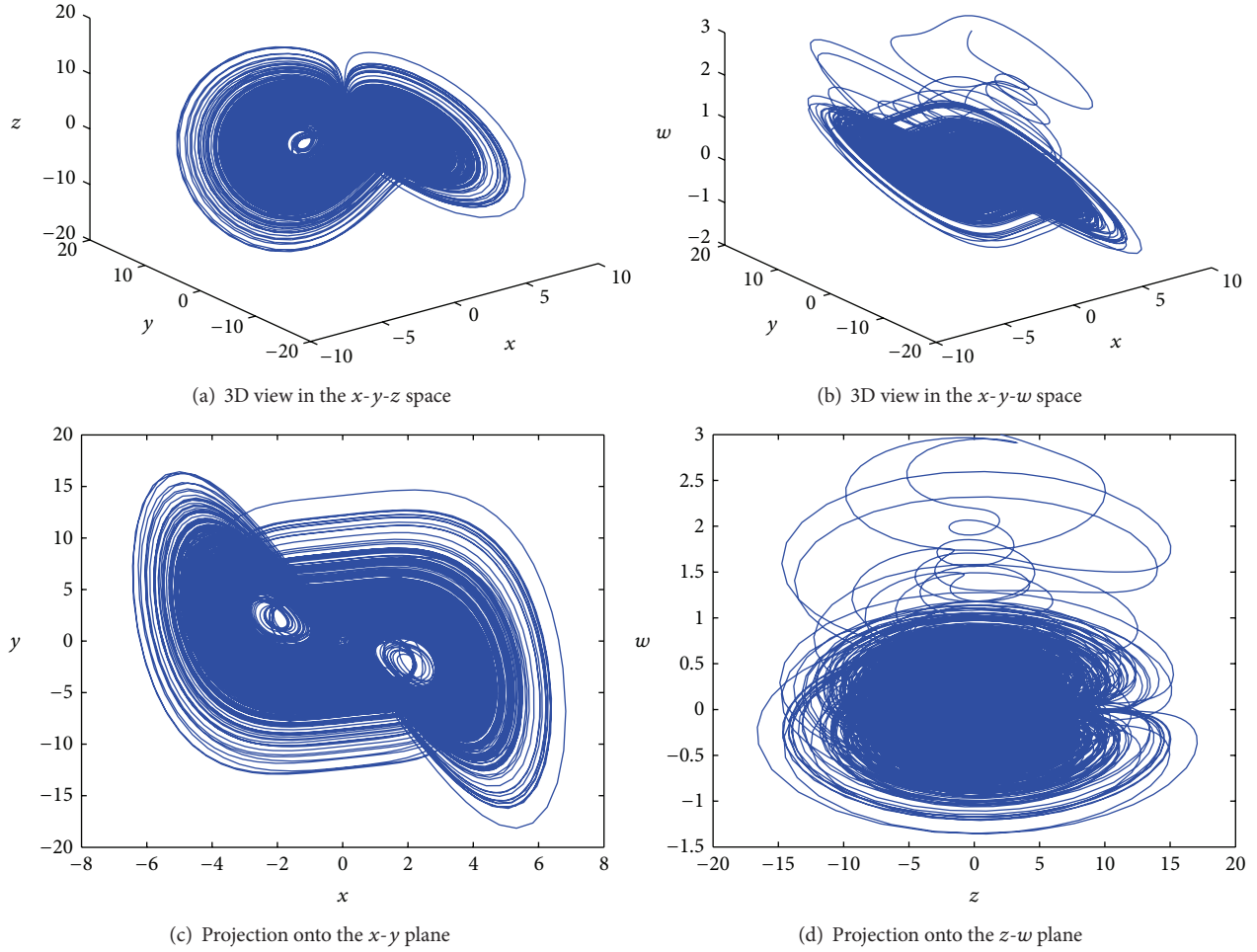


FIGURE 3: Phase portraits of hyperchaotic finance system (17).

Remark 6. When $k_1 = k_2 = k_3 = k_4 = 1$, the coefficient matrix of system (15) is the antisymmetric matrix in [12]. That is to say, they have the same control scheme.

3. Projective Synchronization of Two Different Hyperchaotic Financial Systems

In the real world, we will not only meet synchronization of two identical hyperchaotic financial systems, but also encounter more often the one of two different hyperchaotic financial systems. Here we consider the projective synchronization of the system (2) and the following system (17).

Ding et al. [13] introduced a state feedback controller w_t and constructed a new hyperchaotic financial system:

$$\begin{aligned} \dot{x}_t &= -a'(x_t + y_t) + w_t, \\ \dot{y}_t &= -y_t - a'x_t z_t, \\ \dot{z}_t &= b' + a'x_t y_t, \\ \dot{w}_t &= -c'x_t z_t - d'w_t, \end{aligned} \quad (17)$$

where a' and b' are parameters of the system (17), c' is a constant ($c' = 0.2$), and d' is a control parameter. And when $a' = 3$, $b' = 15$, $c' = 0.2$, and $d' = 0.12$, system (17) presents

hyperchaotic behavior [13]; the initial value of system (17) is taken as $(1.5, 3.2, 2, 3)$; take $t \in (0, 1000)$; the phase portraits of system (17) are shown in Figures 3(a)–3(d).

Let the system (3) be the drive system, and let the system (17) be response system. So the system (17) becomes

$$\begin{aligned} \begin{pmatrix} \dot{x}_t \\ \dot{y}_t \\ \dot{z}_t \\ \dot{w}_t \end{pmatrix} &= \begin{pmatrix} -a' & -a' & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d' \end{pmatrix} \begin{pmatrix} x_t \\ y_t \\ z_t \\ w_t \end{pmatrix} \\ &+ \begin{pmatrix} 0 \\ -a'x_t z_t \\ a'x_t y_t \\ -c'x_t z_t \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ b' \\ 0 \end{pmatrix} + \mathbf{u}. \end{aligned} \quad (18)$$

We denote

$$A = \begin{pmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{pmatrix}, \quad \mathbf{h}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

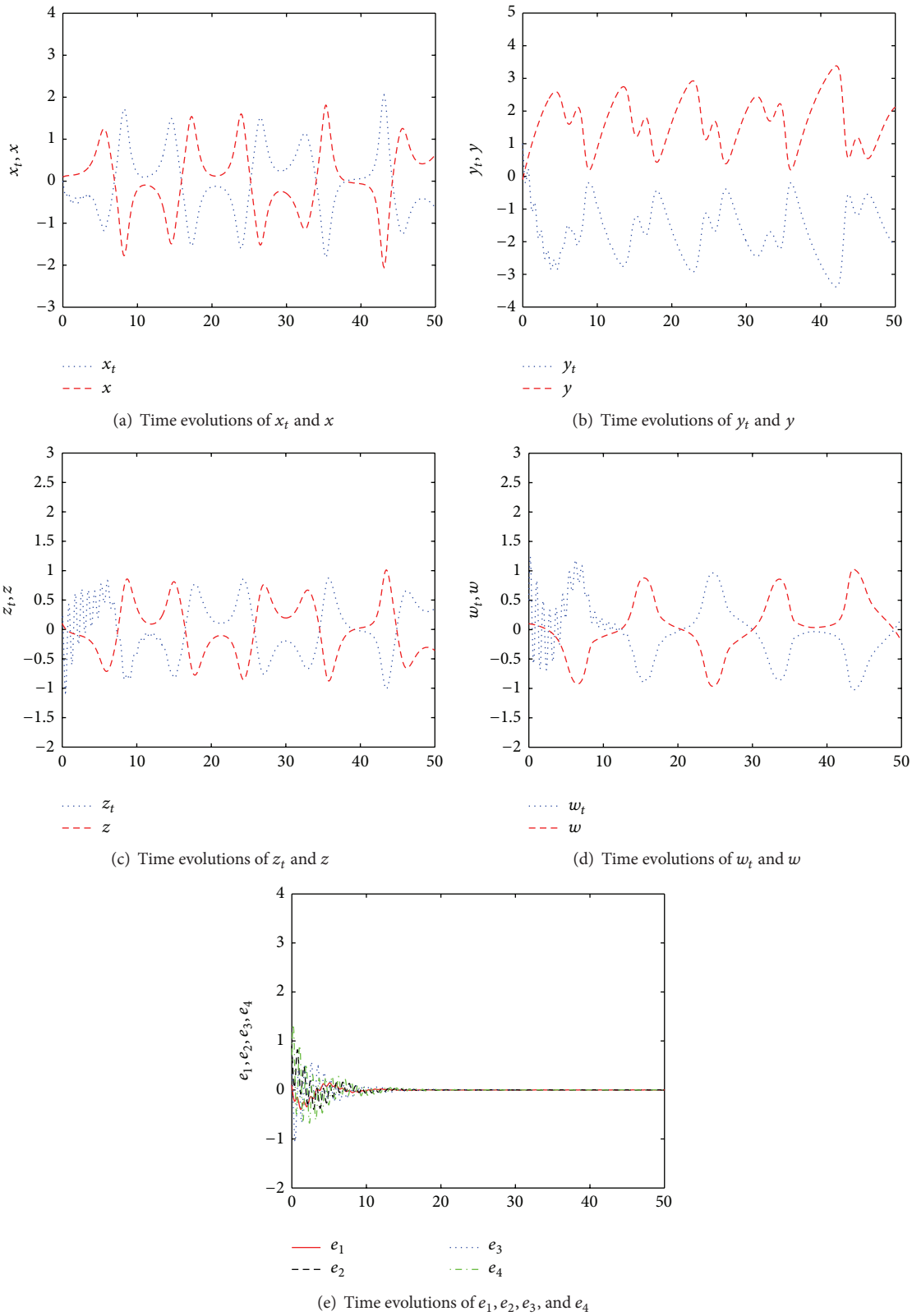


FIGURE 4: Time evolutions of the state variables of system (3) and system (20).

$$f(\mathbf{x}) = \begin{pmatrix} xy \\ -x^2 \\ 0 \\ -dxy \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}, \quad + \alpha \left[\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} xy \\ -x^2 \\ 0 \\ -dxy \end{pmatrix} \right] + E\mathbf{e}. \quad (20)$$

$$D = \begin{pmatrix} -a' & -a' & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d' \end{pmatrix}, \quad \mathbf{h}_2 = \begin{pmatrix} 0 \\ 0 \\ b' \\ 0 \end{pmatrix},$$

$$g(\mathbf{y}) = \begin{pmatrix} 0 \\ -a'x_tz_t \\ a'x_t y_t \\ -c'x_tz_t \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} x_t \\ y_t \\ z_t \\ w_t \end{pmatrix}. \quad (19)$$

Let $\mathbf{u} = (A - D)\mathbf{y} + \alpha(\mathbf{h}_1 + f(\mathbf{x})) - \mathbf{h}_2 - g(\mathbf{y}) + E\mathbf{e}$, where $E = (e_{ij})_{4 \times 4}$ is a 4×4 order constant matrix to be designed, and $\mathbf{e} = (e_1, e_2, e_3, e_4)^T$. So the response system (18) becomes

$$\begin{pmatrix} \dot{x}_t \\ \dot{y}_t \\ \dot{z}_t \\ \dot{w}_t \end{pmatrix} = \begin{pmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{pmatrix} \begin{pmatrix} x_t \\ y_t \\ z_t \\ w_t \end{pmatrix}$$

We also define the error system as $\mathbf{e} = \mathbf{y} - \alpha\mathbf{x}$. Let the system (20) minus the α times of the system (3); we get the error system

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -k \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} + E \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}. \quad (21)$$

Theorem 7. For the error system (21), let the controller be

$$\mathbf{u} = (A - D)\mathbf{y} + \alpha(\mathbf{h}_1 + f(\mathbf{x})) - \mathbf{h}_2 - g(\mathbf{y}) + E\mathbf{e}, \quad (22)$$

where

$$E = \begin{pmatrix} k_1 \cdot \left(\frac{e_{11}}{k_1}\right) & k_1 \cdot \left(-\frac{a_1}{k_2}\right) & k_1 \cdot \left(\frac{1}{k_3} - \frac{1}{k_1} - \frac{a_2}{k_3}\right) & k_1 \cdot \left(-\frac{1}{k_1} - \frac{a_3}{k_4}\right) \\ k_2 \cdot \left(\frac{a_1}{k_2}\right) & k_2 \cdot \left(\frac{e_{22}}{k_2}\right) & k_2 \cdot \left(-\frac{a_4}{k_3}\right) & k_2 \cdot \left(-\frac{a_5}{k_4}\right) \\ k_3 \cdot \left(\frac{a_2}{k_3}\right) & k_3 \cdot \left(\frac{a_4}{k_3}\right) & k_3 \cdot \left(\frac{e_{33}}{k_3}\right) & k_3 \cdot \left(-\frac{a_6}{k_4}\right) \\ k_4 \cdot \left(\frac{a_3}{k_4}\right) & k_4 \cdot \left(\frac{a_5}{k_4}\right) & k_4 \cdot \left(\frac{a_6}{k_4}\right) & k_4 \cdot \left(\frac{e_{44}}{k_4}\right) \end{pmatrix}, \quad (23)$$

$$e_{11} \leq a, \quad e_{22} \leq b, \quad e_{33} \leq c, \quad e_{44} \leq k,$$

$$a_i \in \mathbb{R}, \quad i = 1, 2, \dots, 6; \quad k_i \in \mathbb{R}^+, \quad i = 1, 2, 3, 4.$$

Then the system (21) is asymptotically stable at the origin.

Proof. If $\mathbf{u} = (A - D)\mathbf{y} + \alpha(\mathbf{h}_1 + f(\mathbf{x})) - \mathbf{h}_2 - g(\mathbf{y}) + E\mathbf{e}$, according to the definition of the error system, the error system of the driven system (3) and response system (18) is

system (21); namely,

$$\dot{\mathbf{e}} = (C + E)\mathbf{e}. \quad (24)$$

System (24) has the same structure with the system (9), so we can get $E = B$; namely,

$$E = \begin{pmatrix} k_1 \cdot \left(\frac{e_{11}}{k_1}\right) & k_1 \cdot \left(-\frac{a_1}{k_2}\right) & k_1 \cdot \left(\frac{1}{k_3} - \frac{1}{k_1} - \frac{a_2}{k_3}\right) & k_1 \cdot \left(-\frac{1}{k_1} - \frac{a_3}{k_4}\right) \\ k_2 \cdot \left(\frac{a_1}{k_2}\right) & k_2 \cdot \left(\frac{e_{22}}{k_2}\right) & k_2 \cdot \left(-\frac{a_4}{k_3}\right) & k_2 \cdot \left(-\frac{a_5}{k_4}\right) \\ k_3 \cdot \left(\frac{a_2}{k_3}\right) & k_3 \cdot \left(\frac{a_4}{k_3}\right) & k_3 \cdot \left(\frac{e_{33}}{k_3}\right) & k_3 \cdot \left(-\frac{a_6}{k_4}\right) \\ k_4 \cdot \left(\frac{a_3}{k_4}\right) & k_4 \cdot \left(\frac{a_5}{k_4}\right) & k_4 \cdot \left(\frac{a_6}{k_4}\right) & k_4 \cdot \left(\frac{e_{44}}{k_4}\right) \end{pmatrix}. \quad (25)$$

Therefore, the system (24) becomes

$$\dot{\mathbf{e}} = (C + E) \mathbf{e}, \tag{26}$$

where

$$C + E = \begin{pmatrix} k_1 \cdot \left(-\frac{a}{k_1} + \frac{e_{11}}{k_1}\right) & k_1 \cdot \left(-\frac{a_1}{k_2}\right) & k_1 \cdot \left(\frac{1}{k_3} - \frac{a_2}{k_3}\right) & k_1 \cdot \left(-\frac{a_3}{k_4}\right) \\ k_2 \cdot \left(\frac{a_1}{k_2}\right) & k_2 \cdot \left(-\frac{b}{k_2} + \frac{e_{22}}{k_2}\right) & k_2 \cdot \left(-\frac{a_4}{k_3}\right) & k_2 \cdot \left(-\frac{a_5}{k_4}\right) \\ k_3 \cdot \left(-\frac{1}{k_3} + \frac{a_2}{k_3}\right) & k_3 \cdot \left(\frac{a_4}{k_3}\right) & k_3 \cdot \left(-\frac{c}{k_3} + \frac{e_{33}}{k_3}\right) & k_3 \cdot \left(-\frac{a_6}{k_4}\right) \\ k_4 \cdot \left(\frac{a_3}{k_4}\right) & k_4 \cdot \left(\frac{a_5}{k_4}\right) & k_4 \cdot \left(\frac{a_6}{k_4}\right) & k_4 \cdot \left(-\frac{k}{k_4} + \frac{e_{44}}{k_4}\right) \end{pmatrix}. \tag{27}$$

According to Lemma 2, the system (26) tends to zero gradually; namely, the system (21) is driven to the origin gradually. \square

Remark 8. The system (21) tended to zero gradually; that is, the drive system (3) and response system (18) achieve the projective synchronization.

Generally, we take $e_{11} = 0.5$, $e_{22} = 0.1$, $e_{33} = 1$, and $e_{44} = 0.1$; $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 4$, $a_5 = 5$, and $a_6 = 6$; $k_1 = 1$, $k_2 = 2$, $k_3 = 3$, and $k_4 = 4$. And we take $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$, $k = 0.17$, and $\alpha = -1$; the initial values of drive system (3) and response system (17) are taken as $(0.1, -0.1, 0.1, 0.1)$ and $(0, 1, 0.2, 0.5)$, respectively. So the initial value of error system (21) is $(-0.1, 1.1, 0.1, 0.4)$, and the state variables of system (21) varying with time t are shown in Figures 4(a)–4(e).

4. Conclusions

Based on a special matrix structure and the conversion of the state variable, we design the controllers to achieve the projective synchronization of hyperchaotic financial systems in this paper. And the control law is also suitable for synchronous control of other chaotic systems. Theoretical analysis and simulation results show the effectiveness of the method.

Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

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