Research Article

Energy and Momentum of Bianchi Type VI_h Universes

S. K. Tripathy,[1,2] B. Mishra,[1,3] G. K. Pandey,[1,4] A. K. Singh,[1,5] T. Kumar,[1,5] and S. S. Xulu[6]

[1]*National Institute of Technology, Patna, Bihar 800005, India*
[2]*Department of Physics, Indira Gandhi Institute of Technology, Sarang, Dhenkanal, Odisha 759146, India*
[3]*Department of Mathematics, Birla Institute of Technology and Science Pilani, Hyderabad Campus, Hyderabad 500078, India*
[4]*Patna Science College, Patna University, Patna, Bihar 800005, India*
[5]*Physics Department, Patna University, Patna, Bihar 800005, India*
[6]*Department of Computer Science, University of Zululand, Kwa-Dlangezwa 3886, South Africa*

Correspondence should be addressed to S. K. Tripathy; tripathy_sunil@rediffmail.com

We obtain the energy and momentum of the Bianchi type VI_h universes using different prescriptions for the energy-momentum complexes in the framework of general relativity. The energy and momentum of the Bianchi VI_h universes are found to be zero for the parameter $h = -1$ of the metric. The vanishing of these results supports the conjecture of Tryon that the universe must have a zero net value for all conserved quantities. This also supports the work of Nathan Rosen with the Robertson-Walker metric. Moreover, it raises an interesting question: "Why is the $h = -1$ case so special?"

1. Introduction

The local distribution of energy and momentum has remained a challenging domain of research in the context of Einstein's general relativity. Einstein proposed the first energy-momentum complex [1] that follows the covariant conservation laws by including the energy and momenta of gravitational fields along with those of matter and nongravitational fields. The energy-momentum due to the gravitational field turns out to be a nontensorial object. The choice of the gravitational field pseudotensor (nontensor) is not unique and therefore, following Einstein, many authors prescribed different definitions of energy-momentum complexes based on the canonical approach (e.g., Tolman [2], Papapetrou [3], Landau and Lifshitz (LL) [4], Bergmann and Thompson (BT) [5], and Weinberg [6]). The Tolman definition is essentially the same as that of Einstein; however, these two definitions differ in form and sometimes it is easier to use Tolman's definition. This was explained by Virbhadra [7]. The main concern in the use of these definitions is that they are coordinate-dependent. However, with these definitions, meaningful and reasonable results can be obtained if "Cartesian coordinates" (also called quasi-Cartesian or quasi-Minkowskian for asymptotically Minkowskian space-times) are used. Some coordinate-independent definitions have also been proposed by Møller [8], Komar [9] and Penrose [10]. The coordinate-independent prescriptions, including the quasi-local mass of Penrose [10], were found to have some serious shortcomings as these are limited to a certain class of symmetries only (see in [7] and also references therein).

The issue of energy localization and the coordinate dependence of these definitions gained momentum with renewed interests after the works of Virbhadra and his collaborators (notably, Nathan Rosen, the most famous collaborator of Albert Einstein, of the Einstein-Rosen bridge, the EPR paradox, and the Einstein-Rosen gravitational waves fame) who found a striking similarity in the results for different energy-momentum prescriptions. They considered numerous space-times [11–25] and obtained seminal results that rejuvenated this field of research.

Virbhadra [7] further investigated whether or not these energy-momentum complexes lead to the same results for the most general nonstatic spherically symmetric metric and found that they disagree. Virbhadra and his collaborators [13–25] observed that if the calculations of energy-momentum are done in Kerr-Schild Cartesian coordinates, then the energy-momentum complexes of Papapetrou [3], Landau and Lifshitz [4], and Weinberg [6] produce the same result as in the Einstein definition. However, if the computations are made in Schwarzschild Cartesian coordinates, these energy-momentum complexes disagree [7]. Xulu [26] confirmed this by obtaining the energy and momentum for the most general nonstatic spherically symmetric system using the Møller definition and found a different result in general form than those obtained by Einstein's definition. Xulu and others [26–54] obtained many important results in this field. However, till now, there is no completely acceptable definition for energy and momentum distributions in general relativity even though prescriptions in teleparallel gravity claim to provide a satisfactory solution to this problem [55, 56]. Gad calculated the energy and momentum densities of stiff fluid case using the prescriptions of Einstein, Begmann-Thompson, and Landau-Liftshitz in both the general relativity and the teleparallel gravity and found that different prescriptions do not provide the same results in both theories. Also, they have shown that both the general relativity and the teleparallel gravity are equivalent [56]. Similar results have also been obtained by Aygün and coworkers in [57, 58] where the authors have concluded that energy-momentum definitions are identical not only in general relavity but also in teleparallel gravity.

Bianchi type models are spatially homogeneous and anisotropic universe models. These models are nine in number, but their classification permits splitting them in two classes. There are six models in class A (I, II, VI_1, VII, VIII, and IX) and five in class B (III, IV, V, VI_h, and VII_h). Spatially homogeneous cosmological models play an important role in understanding the structure and properties of the space of all cosmological solutions of Einstein field equations. These spatially homogeneous and anisotropic models are the exact solutions of Einstein field equations and are more general than the Friedman models in the sense that they can provide interesting results pertaining to the anisotropy of the universe. Here, it is worth to mention that the issue of global anisotropy has gained a lot of research interest in recent times. The standard cosmological model (ΛCDM) based upon the spatial isotropy and flatness of the universe is consistent with the data from precise measurements of the CMB temperature anisotropy [59] from the Wilkinson Microwave Anisotropy Probe (WMAP). However, the ΛCDM model suffers from some anomalous features at large scale and signals a deviation from the usual geometry of the universe. Recently released Planck data [60–63] show a slight red shift of the primordial power spectrum from the exact scale invariance. It is clear from the Planck data that the ΛCDM model does not fit well to the temperature power spectrum at low multipoles. Also, precise measurements from WMAP predict asymmetric expansion with one direction expanding differently from the other two transverse directions at equatorial plane [64] which

signals a nontrivial topology of the large scale geometry of the universe (see [65, 66] and references therein).

In recent times this pressing issue of the energy and momentum localization has been studied widely by many authors using different space-times and definitions of energy-momentum complexes. The importance of the study of energy and momentum distribution lies in the fact that it helps us getting an idea of the effective gravitational mass of metrics of certain symmetries and can put deep insight into the gravitational lensing phenomena [67–73]. In fact, the energy-momentum distribution in space-time can be interpreted as effective gravitational mass if the space-time has certain symmetry. However, negative and positive energy distributions in space-times always indicate divergent and convergent gravitational lensing, respectively.

Using Einstein definition, Banerjee and Sen [74] studied the energy distribution with Bianchi type I (BI) space-time. Xulu [33] calculated the total energy in BI universes using the prescriptions of LL, Pappapetrou, and Weinberg. Radinschi [39] calculated the energy of a model of the universe based on the Bianchi type VI_0 metric using the energy-momentum complexes of LL and of Papapetrou. She found that the energy due to the matter plus field is equal to zero. Aydoğdu and Salti [75], using the Møller's tetrad, investigated the energy of the BI universe. In another work, Aydogdu and Salti [76] calculated the energy of the LRS Bianchi type II metric to get consistent results.

In this paper, we obtain the energy and momentum for a more general homogeneous and anisotropic Bianchi type VI_h metric and its transformation by using different prescriptions for the energy-momentum complex in general relativity. The Bianchi type VI_h model has already been shown to provide interesting results in cosmology in connection with the late time accelerated expansion of the universe when the contribution to the matter field comes from one-dimensional cosmic strings and bulk viscosity [77]. In the present work, we have used the convention that Latin indices take values from 0 to 3 and Greek indices run from 1 to 3. We use geometrized units where $G = 1$ and $c = 1$. The composition of the paper is as follows. In Section 2, we have written the energy-momentum tensor for the Bianchi type VI_h space-time. In Section 3, the Einstein energy-momentum complex is discussed, following which we investigate the energy-momentum complex of Landau and Lifshitz, Papapetrou, and Bergmann Thomson for the assumed metric in the subsequent subsections. In the last section, we summarize our results.

2. Bianchi Type VI_h Space-Time

We have considered the Bianchi type VI_h space-time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{2hx} dz^2. \qquad (1)$$

The metric potentials A, B, and C are functions of cosmic time t only. Further, x^i, $i = 1, 2, 3, 0$, respectively, denote the coordinates x, y, z, and t. The exponent h is time-independent and can assume integral values in the range

$h = -1, 0, 1$. It should be mentioned here that these are not vacuum solutions as $T_k^i \neq 0$ for all i and k.

For metric (1), the determinant of the metric tensor g and the contravariant components of the tensor are, respectively, given as

$$g = |g_{ab}| = -A^2 B^2 C^2 e^{2(h+1)x},$$

$$g^{00} = -1,$$

$$g^{11} = \frac{1}{A^2},$$

$$g^{22} = \frac{1}{B^2 e^{2x}}, \tag{2}$$

$$g^{33} = \frac{1}{C^2 e^{2hx}}.$$

In general relativity, the energy-momentum tensor is given by $8\pi T_i^j = R_i^j - (1/2)Rg_i^j$, where R_i^j is the Ricci tensor and R is the Ricci scalar. The nonvanishing components of the energy-momentum tensor for the Bianchi type VI_h space time are given below (this is not a new result; however, we put it here because we need it for analysing and discussing results):

$$8\pi T_1^1 = \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{h}{A^2},$$

$$8\pi T_2^2 = \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2},$$

$$8\pi T_3^3 = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2}, \tag{3}$$

$$8\pi T_0^0 = -\frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} + \frac{1+h+h^2}{A^2},$$

$$8\pi T_0^1 = (1+h)\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - h\frac{\dot{C}}{C},$$

where the overhead dots hereafter denote ordinary time derivatives.

3. Energy-Momentum Complexes

3.1. Einstein Energy-Momentum Complex. The Einstein energy-momentum complex is [1]

$$\Theta_i^k = \frac{1}{16\pi} H_{i,l}^{kl}, \tag{4}$$

where

$$H_i^{kl} = -H_i^{lk} = \frac{g_{in}}{\sqrt{-g}} \left[-g\left(g^{kn}g^{lm} - g^{ln}g^{km}\right)\right]_{,m}. \tag{5}$$

Θ_0^0 and Θ_α^0 stand for the energy and momentum density components, respectively. The energy and momentum components are obtained through a volume integration

$$P_i = \iiint \Theta_i^0 dx^1 dx^2 dx^3. \tag{6}$$

By applying Gauss' theorem, the above equation can also be reduced to

$$P_i = \frac{1}{16\pi} \iint H_i^{0\alpha} n_\alpha dS, \tag{7}$$

where n_α is the outward unit normal vector over the infinitesimal surface element dS. P_0 and P_α stand for the energy and momentum components, respectively. The required nonvanishing components of H_i^{kl} for line element (1) are given by

$$H_0^{01} = -\frac{2BC}{A}(1+h)e^{x(1+h)},$$

$$H_1^{01} = -2ABC\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)e^{x(1+h)},$$

$$H_2^{02} = -2ABC\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right)e^{x(1+h)}, \tag{8}$$

$$H_3^{03} = -2ABC\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)e^{x(1+h)}.$$

Using (8), we obtain the components of energy and momentum densities as

$$\Theta_0^0 = -\frac{BC}{8\pi A}(1+h)^2 e^{x(1+h)},$$

$$\Theta_1^0 = -\frac{ABC}{8\pi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)(1+h)e^{x(1+h)}, \tag{9}$$

$$\Theta_2^0 = \Theta_3^0 = 0.$$

If the dependence of A, B, and C on the time coordinate t were known, one could evaluate the surface integral. It is clear from the above results that, for the specific choices of h, that is, $h = 0$ and 1, the energy of the VI_h universe in the Einstein prescription is not zero. However, it is interesting to note that the energy and momentum densities vanish for $h = -1$.

3.2. Landau and Lifshitz Energy-Momentum Complex. The symmetric Landau and Lifshitz energy-momentum complex is [4]

$$L^{ik} = \frac{1}{16\pi} \lambda_{,lm}^{iklm}, \tag{10}$$

where

$$\lambda^{iklm} = -g\left(g^{ik}g^{lm} - g^{il}g^{km}\right). \tag{11}$$

Here L^{00} and $L^{\alpha 0}$ are the energy and momentum density components.

The energy and momentum can be defined through the volume integral

$$P^i = \iiint L^{i0} dx^1 dx^2 dx^3. \tag{12}$$

Using Gauss' theorem, the energy and momentum components become

$$P^i = \frac{1}{16\pi} \iint \lambda_{,m}^{i0\alpha m} n_\alpha dS. \tag{13}$$

The required nonvanishing components of λ^{iklm} for the present model are obtained as

$$\lambda^{0011} = -B^2C^2e^{2x(1+h)},$$

$$\lambda^{1010} = B^2C^2e^{2x(1+h)},$$

$$\lambda^{0022} = -A^2C^2e^{2hx},$$

$$\lambda^{2020} = A^2C^2e^{2hx}, \tag{14}$$

$$\lambda^{0033} = -A^2B^2e^{2x},$$

$$\lambda^{3030} = A^2B^2e^{2x}.$$

Using these components in (13), we obtain

$$P^0 = -\frac{xr}{2}(1+h)B^2C^2e^{2x(1+h)},$$

$$P^1 = \frac{xr}{2}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)B^2C^2e^{2x(1+h)},$$

$$P^2 = \frac{yr}{2}\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)A^2C^2e^{2hx}, \tag{15}$$

$$P^3 = \frac{zr}{2}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)A^2B^2e^{2x},$$

where $r = \sqrt{x^2+y^2+z^2}$. For the Landau and Lifshitz prescription, the energy of the universe is nonzero for $h = 0$ and 1 and it vanishes for $h = -1$.

3.3. Papapetrou Energy-Momentum Complex.
The symmetric energy-momentum complex of Papapetrou is given by [3]

$$\Sigma^{ik} = \frac{1}{16\pi}\mathcal{N}^{iklm}_{,lm}, \tag{16}$$

where

$$\mathcal{N}^{iklm} = \sqrt{-g}\left(g^{ik}\eta^{lm}-g^{il}\eta^{km}+g^{lm}\eta^{ik}-g^{lk}\eta^{im}\right), \tag{17}$$

with

$$\eta^{ik} = \mathrm{diag}\left(-1,1,1,1\right). \tag{18}$$

Here Σ^{00} and $\Sigma^{\alpha 0}$ are, respectively, the energy and energy current (momentum) density components. The energy and momentum can be defined as

$$P^i = \iiint \Sigma^{i0}dx^1dx^2dx^3. \tag{19}$$

For time-independent metrics, one can have

$$P^i = \frac{1}{16\pi}\iint \mathcal{N}^{i0\alpha\beta}_{,\beta}n_\alpha dS. \tag{20}$$

The nonvanishing components of $\mathcal{N}^{iklm}$ required to obtain the energy and momentum density components for the space-time described by line element (1) are

$$\mathcal{N}^{1001} = \mathcal{N}^{2002} = \mathcal{N}^{3003} = ABCe^{x(1+h)},$$

$$\mathcal{N}^{0011} = -ABCe^{x(1+h)}\left(1+\frac{1}{A^2}\right),$$

$$\mathcal{N}^{1010} = ABCe^{x(1+h)}\left(\frac{1}{A^2}\right),$$

$$\mathcal{N}^{0022} = -ABCe^{x(1+h)}\left(1+\frac{1}{B^2e^{2x}}\right), \tag{21}$$

$$\mathcal{N}^{2020} = ABCe^{x(1+h)}\left(\frac{1}{B^2e^{2x}}\right),$$

$$\mathcal{N}^{0033} = -ABCe^{x(1+h)}\left(1+\frac{1}{C^2e^{2hx}}\right),$$

$$\mathcal{N}^{3030} = ABCe^{x(1+h)}\left(\frac{1}{C^2e^{2hx}}\right).$$

The Papapetrou energy and energy current density components are obtained by using the above components in (21) as

$$\Sigma^{00} = -\frac{(1+h)^2}{16\pi}ABC\left(1+\frac{1}{A^2}\right)e^{x(1+h)},$$

$$\Sigma^{10} = \frac{1+h}{16\pi}\left[\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{1}{A^2}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right)\right]$$
$$\cdot ABCe^{x(1+h)}, \tag{22}$$

$$\Sigma^{20} = 0,$$

$$\Sigma^{30} = 0.$$

Like the previous cases, it can be concluded from the above components that the energy of the universe in the Papapetrou prescription is nonzero for $h = 0$ and 1 and it becomes zero for $h = -1$. Therefore, the energy and momentum components both are zero for $h = -1$.

3.4. Bergmann-Thompson Energy-Momentum Complex.
The Bergmann-Thompson energy-momentum complex is [5]

$$\mathbf{B}^{ik} = \frac{1}{16\pi}\left[g^{il}\mathscr{B}^{km}_l\right]_{,m}, \tag{23}$$

where

$$\mathscr{B}^{km}_l = \frac{g_{ln}}{\sqrt{-g}}\left[-g\left(g^{kn}g^{mp}-g^{mn}g^{kp}\right)\right]_{,p}. \tag{24}$$

Here $\mathbf{B}^{00}$ and $\mathbf{B}^{\alpha 0}$ are the energy and momentum densities, respectively. The energy and momentum can be defined as

$$P^i = \iiint \mathbf{B}^{i0}dx^1dx^2dx^3. \tag{25}$$

Using Gauss's theorem, the energy and momentum components can be expressed as

$$P^i = \frac{1}{16\pi}\iint \mathscr{B}^{i0\alpha}n_\alpha dS. \tag{26}$$

In order to calculate the energy and momentum distribution for Bianchi type VI_h space-time using the Bergmann and Thompson energy-momentum complex, we require the following nonvanishing components of $\mathscr{B}_l^{km}$:

$$\mathscr{B}_1^{01} = 2ABCe^{x(1+h)}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right),$$

$$\mathscr{B}_2^{02} = 2ABCe^{x(1+h)}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right), \qquad (27)$$

$$\mathscr{B}_3^{03} = 2ABCe^{x(1+h)}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right).$$

The energy and momentum density components can be obtained by using (27) in (23) as

$$\mathbf{B}^{00} = \mathbf{B}^{20} = \mathbf{B}^{30} = 0,$$

$$\mathbf{B}^{10} = \left(\frac{1+h}{8\pi}\right)\frac{BCe^{x(1+h)}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \qquad (28)$$

It is clear that the energy of the Bianchi VI_h universe is zero in Bergmann-Thompson prescription $\forall h = 0, \pm 1$. However, the energy and all momentum components vanish for $h = -1$.

4. Summary

In the present study, we have obtained energy and momentum distributions for the spatially homogeneous and anisotropic Bianchi type VI_h metric using Einstein, Landau and Lifshitz, Papapetrou, and Bergmann-Thompson complexes in the framework of general relativity. Bianchi type VI_h space-time has an edge over the usual Friedman-Robertson-Walker (FRW) in the sense that it can handle the anisotropic spatial expansion. In the present study, we found that the energy and momentum vanish for the Bianchi VI_{-1} universe. The only exception to this is the case of Landau and Lifshitz where although energy density components vanish, the momentum density components do not vanish in general for $h = -1$. Virbhadra (refer to [7] and references in this paper) however pointed out that the Landau and Lifshitz complex does not work as good as the Einstein complex and the latter is the best for energy-momentum calculations. Other energy-momentum complexes agree with that of Einstein's for the $h = -1$ case. Equation (3) shows that the energy and momentum densities due to matter and nongravitational fields are nonzero even for $h = -1$. However, as energy-momentum complexes include effects of the gravitational field as well, the total result comes to be zero.

The observation of the cosmic microwave background radiation by Penzias and Wilson [78, 79] in the year 1965 strongly supports that some version of the big bang theory is correct and it also suggested a remarkable conjecture regarding the total energy and momentum of the universe. Tryon [80] assumed that our universe appeared from nowhere about 10^{10} years ago. He pondered that the conventional laws of physics need not have been violated at the time of creation of the universe. Therefore, he proposed that our universe must have a zero net value for all conserved quantities. Further, he presented arguments suggesting that the net energy and momentum of our universe may be indeed zero. Tryon gave a particular big bang model and according to that our universe is a fluctuation of the vacuum and he predicted a homogeneous, isotropic, and closed universe consisting of equal amount of matter and antimatter. Tryon, in the same paper, also mentioned an excellent topological argument by Bergmann that any closed universe has zero energy. Later, Rosen [81], in the year 1994, considered a closed homogeneous isotropic universe described by the Friedman-Robertson-Walker (FRW) metric and found that the total energy of the universe is zero (sadly, he passed away in the following year (1995)). Excited by these results, Xulu [33] studied energy and momentum in Bianchi type I universes and his results supported the conjecture of Tryon.

In view of the satisfying results mentioned in the above paragraph, the outcome of our research that the energy and momentum of the Bianchi VI_{-1} universe are zero is indeed a very important result. However, one would ask "Why is this true only for the $h = -1$ case?" History of science has records that coincidence of results usually points out something very important. Thus, it remains to be investigated: "Why is the $h = -1$ case so special?" It is likely that the outcome of these investigations would have important implications for general relativity and relativistic astrophysics.

Disclosure

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

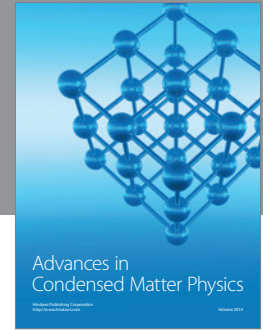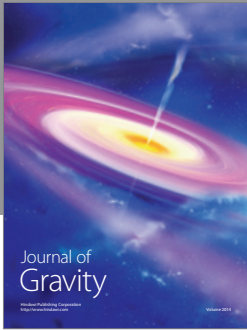# References

[1] A. Einstein, "Zur allgemeinen Relativitatstheorie," *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin*, vol. 778, 1915.

[2] R. C. Tolman, *Relativity, Thermodynamics and Cosmology*, Oxford University Press, Oxford, UK, 1934.

[3] A. Papapetrou, "Einstein's theory of gravitation and flat space," *Proceedings of the Royal Irish Academy Section A*, vol. 52, p. 11, 1948.

[4] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Addison-Wesley, Reading, Mass, USA, 1951.

[5] P. G. Bergmann and R. Thompson, "Spin and angular momentum in general relativity," *Physical Review*, vol. 89, no. 2, pp. 400–407, 1953.

[6] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of General Theory of Relativity*, Wiley, New York, NY, USA, 1972.

[7] K. S. Virbhadra, "Naked singularities and Seifert's conjecture," *Physical Review D*, vol. 60, no. 10, Article ID 104041, 6 pages, 1999.

[8] C. Møller, "On the localization of the energy of a physical system in the general theory of relativity," *Annals of Physics*, vol. 4, no. 4, pp. 347–371, 1958.

[9] A. Komar, "Covariant conservation laws in general relativity," *Physical Review*, vol. 113, article 934, 1959.

[10] R. Penrose, "Quasi-local mass and angular momentum in general relativity," *Proceedings of the Royal Society of London A*, vol. 381, pp. 53–63, 1982.

[11] N. Rosen and K. S. Virbhadra, "Energy and momentum of cylindrical gravitational waves," *General Relativity and Gravitation*, vol. 25, no. 4, pp. 429–433, 1993.

[12] K. S. Virbhadra, "Gravitational and electromagnetic fields of a charged tachyon," *Pramana*, vol. 45, no. 2, pp. 175–179, 1995.

[13] K. S. Virbhadra, "Energy associated with a Kerr-Newman black hole," *Physical Review D: Particles and Fields*, vol. 41, no. 4, pp. 1086–1090, 1990.

[14] K. S. Virbhadra, "Angular momentum distribution in Kerr-Newman spacetime," *Physical Review D*, vol. 42, no. 4, pp. 1066–1067, 1990.

[15] K. S. Virbhadra, "Energy distribution in Kerr-Newman spacetime in Einstein's as well as Møller's prescriptions," *Physical Review D*, vol. 42, no. 8, p. 2919, 1990.

[16] A. Chamorro and K. S. Virbhadra, "A radiating dyon solution," *Pramana*, vol. 45, no. 2, pp. 181–187, 1995.

[17] A. Chamorro and K. S. Virbhadra, "Energy associated with charged dilaton black holes," *International Journal of Modern Physics D*, vol. 5, no. 3, p. 251, 1996.

[18] K. S. Virbhadra and J. C. Parikh, "Gravitational energy of a stringy charged black hole," *Physics Letters B*, vol. 317, no. 3, pp. 312–314, 1993.

[19] K. S. Virbhadra and J. C. Parikh, "A conformal scalar dyon black hole solution," *Physics Letters B*, vol. 331, no. 3-4, pp. 302–304, 1994.

[20] K. S. Virbhadra, "Janis-Newman-Winicour and Wyman solutions are the same," *International Journal of Modern Physics A*, vol. 12, p. 4831, 1997.

[21] K. S. Virbhadra, S. Jhingan, and P. S. Joshi, "Nature of singularity in Einstein-massless scalar theory," *International Journal of Modern Physics D*, vol. 6, pp. 357–362, 1997.

[22] K. S. Virbhadra, "Exact solutions of Einstein and Einstein-scalar equations in 2 + 1 dimensions," *Pramana*, vol. 44, no. 4, pp. 317–322, 1995.

[23] K. S. Virbhadra, "Energy and momentum in Vaidya spacetime," *Pramana*, vol. 38, no. 1, pp. 31–35, 1992.

[24] K. S. Virbhadra, "A comment on the energy-momentum pseudotensor of Landau and Lifshitz," *Physics Letters A*, vol. 157, no. 4-5, pp. 195–197, 1991.

[25] J. M. Aguirregabiria, A. Chamorro, and K. S. Virbhadra, "Energy and angular momentum of charged rotating black holes," *General Relativity & Gravitation*, vol. 28, no. 11, pp. 1393–1400, 1996.

[26] S. S. Xulu, "Møller energy of the nonstatic spherically symmetric space-time," *Astrophysics and Space Science*, vol. 283, no. 1, pp. 23–32, 2003.

[27] E. C. Vagenas, "Energy distribution in the dyadosphere of a Reissner-Nordström black hole in Møller's prescription," *Modern Physics Letters A*, vol. 21, no. 25, pp. 1947–1956, 2006.

[28] E. C. Vagenas, "Energy distribution in a BTZ black hole spacetime," *International Journal of Modern Physics D*, vol. 14, pp. 573–585, 2005.

[29] T. Multamaki, A. Putaja, I. Vilja, and E. C. Vagenas, "Energy-momentum complexes in $f(R)$ theories of gravity," *Classical and Quantum Gravity*, vol. 25, no. 7, Article ID 075017, 2008.

[30] S. S. Xulu, "Tolman energy of a stringy charged black hole," *International Journal of Theoretical Physics*, vol. 37, no. 6, pp. 1773–1777, 1998.

[31] S. S. Xulu, "Energy distribution of charged dilaton black holes," *International Journal of Modern Physics D*, vol. 7, pp. 773–777, 1998.

[32] S. S. Xulu, "Energy associated with schwarzschild black hole in a magnetic universe," *International Journal of Modern Physics A*, vol. 15, no. 19, p. 2979, 2000.

[33] S. S. Xulu, "Total energy of the Bianchi type I universes," *International Journal of Theoretical Physics*, vol. 39, no. 4, pp. 1153–1161, 2000.

[34] S. S. Xulu, "Energy distribution in Melvin's magnetic universe," *International Journal of Modern Physics A*, vol. 15, no. 30, p. 4849, 2000.

[35] S. S. Xulu, "Møller energy for the Kerr-Newman metric," *Modern Physics Letters A: Particles and Fields, Gravitation, Cosmology, Nuclear Physics*, vol. 15, no. 24, pp. 1511–1517, 2000.

[36] S. S. Xulu, "Bergmann-Thomson energy-momentum complex for solutions more general than the Kerr-Schild class," *International Journal of Theoretical Physics*, vol. 46, no. 11, pp. 2915–2922, 2007.

[37] S. S. Xulu, "Energy in the dyadosphere of a Reissner-Nordström black hole," *Chinese Journal of Physics*, vol. 44, no. 5, pp. 348–357, 2006.

[38] S. S. Xulu, "Bergmann-Thomson energy of a charged rotating black hole," *Foundations of Physics Letters*, vol. 19, no. 6, pp. 603–609, 2006.

[39] I. Radinschi, "Energy associated with the blanchi type VI0 universe," *Chinese Journal of Physics*, vol. 39, no. 5, p. 393, 2001.

[40] I. Radinschi, "The energy distribution of the Bianchi type VI0 universe," *Chinese Journal of Physics*, vol. 39, no. 3, pp. 231–235, 2001.

[41] M. Sharif, "Energy and momentum in spacetime homogeneous Godel type metrics," *International Journal of Modern Physics D*, vol. 13, pp. 1019–1028, 2004.

[42] V. C. de Andrade, L. C. Guillen, and J. G. Pereira, "Gravitational energy-momentum density in teleparallel gravity," *Physical Review Letters*, vol. 84, no. 20, pp. 4533–4536, 2000.

[43] S. L. Loi and T. Vargas, "The energy of Bianchi type I and II universes in teleparallel gravity," *Chinese Journal of Physics*, vol. 43, p. 901, 2005.

[44] O. Aydogdu, "Gravitational energy-momentum density in Bianchi type II space-times," *International Journal of Modern Physics D*, vol. 15, no. 4, pp. 459–468, 2006.

[45] O. Aydogdu and M. Salti, "The momentum 4-vector in bulk viscous Bianchi type-V space-time," *Czechoslovak Journal of Physics*, vol. 56, no. 8, pp. 789–798, 2006.

[46] S. Aygün and I. Tarhan, "Energy–momentum localization for Bianchi type-IV Universe in general relativity and teleparallel gravity," *Pramana*, vol. 78, no. 4, pp. 531–548, 2012.

[47] C. Zachos, "Comment on 'noncommutativity as a possible origin of the ultrahigh-energy cosmic ray and the Tev photon paradoxes'," *Modern Physics Letters A*, vol. 19, no. 19, pp. 1483–1487, 2004.

[48] R. M. Gad, "Energy and momentum associated with solutions exhibiting directional type singularities," *General Relativity and Gravitation*, vol. 38, no. 3, pp. 417–424, 2006.

[49] A. M. Abbassi and S. Mirshekari, "Energy-momentum density of gravitational waves," *International Journal of Modern Physics A*, vol. 23, no. 27-28, pp. 4569–4577, 2008.

[50] S. Mirshekari and A. M. Abbassi, "Energy-momentum prescriptions in general spherically symmetric spacetimes," *Modern Physics Letters A: Particles and Fields, Gravitation, Cosmology, Nuclear Physics*, vol. 24, no. 10, article 747, 2009.

[51] S. Mirshekari and A. M. Abbassi, "On energy distribution of two space-times with planar and cylindrical symmetries," *International Journal of Modern Physics A: Particles and Fields; Gravitation; Cosmology; Nuclear Physics*, vol. 24, no. 4, pp. 789–797, 2009.

[52] P. Halpern and M. Pecorino, "The localized energy distribution of dark energy star solutions," *ISRN Astronomy and Astrophysics*, vol. 2013, Article ID 939876, 4 pages, 2013.

[53] I.-C. Yang and I. Radinschi, "The energy of the gamma metric in the Møller prescription," *Modern Physics Letters A*, vol. 17, no. 18, pp. 1159–1163, 2002.

[54] I.-C. Yang and I. Radinschi, "On the difference of energy between the Einstein and Møller prescription," *Chinese Journal of Physics*, vol. 42, p. 40, 2004.

[55] M. Sharif and A. Jawad, "Energy contents of some well-known solutions in teleparallel gravity," *Astrophysics and Space Science*, vol. 331, no. 1, pp. 257–263, 2011.

[56] R. M. Gad, "Energy and momentum densities of cosmological models, with equation of state $\rho = \mu$, in general relativity and teleparallel gravity," *International Journal of Theoretical Physics*, vol. 46, no. 12, pp. 3263–3274, 2007.

[57] S. Aygün, H. Baysal, and İ. Tarhan, "Energy momentum of Marder universe in teleparallel gravity," *International Journal of Theoretical Physics*, vol. 46, no. 10, pp. 2607–2616, 2007.

[58] S. Aygün, İ. Tarhan, and H. Baysal, "Scalar field theory and energy-momentum problem of Yilmaz-Rosen metric in general relativity and teleparallel gravity," *Astrophysics and Space Science*, vol. 314, no. 4, pp. 323–330, 2008.

[59] G. Hinshaw, J. L. Weiland, R. S. Hill et al., "Five-year Wilkinson microwave anisotropy probe observations: data processing, sky maps, and basic results," *The Astrophysical Journal Supplement Series*, vol. 180, no. 2, p. 225, 2009.

[60] P. A. R. Ade, N. Aghanim, and M. I. R. Alves, "*Planck* 2013 results. I. Overview of products and scientific results," *Astronomy & Astrophysics*, vol. 571, article A1, 2014.

[61] P. A. R. Ade, N. Aghanim, C. Armitage-Caplan et al., "Planck 2013 results. XVI. Cosmological parameters," *Astronomy & Astrophysics*, vol. 571, article A16, 66 pages, 2014.

[62] P. A. R. Ade, N. Aghanim, C. Armitage-Caplan et al., "*Planck* 2013 results. XXII. Constraints on inflation," *Astronomy & Astrophysics*, vol. 571, article A22, 2014.

[63] P. A. R. Ade, N. Aghanim, C. Armitage-Caplan et al., "Planck 2013 results. XXIV. Constraints on primordial non-gaussianity," *Astronomy & Astrophysics*, vol. 571, article A24, 2014.

[64] R. V. Buiny, A. Berera, and T. W. Kephart, "Asymmetric inflation: exact solutions," *Physical Review D*, vol. 73, Article ID 063529, 2006.

[65] M. Watanabe, S. Kanno, and J. Soda, "Inflationary universe with anisotropic hair," *Physical Review Letters*, vol. 102, no. 19, Article ID 191302, 4 pages, 2009.

[66] S. K. Tripathy, "Late time acceleration and role of skewness in anisotropic models," *Astrophysics and Space Science*, vol. 350, no. 1, pp. 367–374, 2014.

[67] C.-M. Claudel, K. S. Virbhadra, and G. F. Ellis, "The geometry of photon surfaces," *Journal of Mathematical Physics*, vol. 42, no. 2, pp. 818–838, 2001.

[68] K. S. Virbhadra, D. Narasimha, and S. M. Chitre, "Role of the scalar field in gravitational lensing," *Astronomy and Astrophysics*, vol. 337, no. 1, 1998.

[69] K. S. Virbhadra and G. F. R. Ellis, "Schwarzschild black hole lensing," *Physical Review D*, vol. 62, no. 8, Article ID 084003, 8 pages, 2000.

[70] K. S. Virbhadra and G. F. Ellis, "Gravitational lensing by naked singularities," *Physical Review D*, vol. 65, no. 10, Article ID 103004, 10 pages, 2002.

[71] K. S. Virbhadra and C. R. Keeton, "Relativistic images of Schwarzschild black hole lensing," *Physical Review D*, vol. 79, Article ID 083004, 2009.

[72] K. S. Virbhadra and C. R. Keeton, "Time delay and magnification centroid due to gravitational lensing by black holes and naked singularities," *Physical Review D*, vol. 77, Article ID 124014, 2008.

[73] K. S. Virbhadra, "Relativistic images of Schwarzschild black hole lensing," *Physical Review D*, vol. 79, no. 8, Article ID 083004, 25 pages, 2009.

[74] N. Banerjee and S. Sen, "Einstein pseudotensor and total energy of the universe," *Pramana*, vol. 49, no. 6, pp. 609–615, 1997.

[75] O. Aydoğdu and M. Salti, "Energy of the universe in bianchi-type i models in møller's tetrad theory of gravity," *Astrophysics and Space Science*, vol. 299, no. 3, pp. 227–232, 2005.

[76] O. Aydogdu and M. Salti, "Energy density associated with the Bianchi type-II space-time," *Progress of Theoretical Physics*, vol. 115, no. 1, pp. 63–71, 2006.

[77] S. K. Tripathy and D. Behera, "Bianchi type-VI_h string cloud cosmological models with bulk viscosity," *Astrophysics and Space Science*, vol. 330, no. 1, pp. 191–201, 2010.

[78] A. A. Penzias and R. W. Wilson, "A measurement of excess antenna temperature at 4080 Mc/s," *The Astrophysical Journal Letters*, vol. 142, pp. 419–421, 1965.

[79] A. A. Penzias and R. W. Wilson, "Measurement of the flux density of CAS a at 4080 Mc/s," *The Astrophysical Journal Letters*, vol. 142, p. 1149, 1965.

[80] E. P. Tryon, "Is the universe a vacuum fluctuation?" *Nature*, vol. 246, no. 5433, pp. 396–397, 1973.

[81] N. Rosen, "The energy of the universe," *General Relativity and Gravitation*, vol. 26, no. 3, pp. 319–321, 1994.