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Research Article

On Normalistic Vague Soft Groups and Normalistic Vague Soft Group Homomorphism

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We further develop the theory of vague soft groups by establishing the concept of normalistic vague soft groups and normalistic vague soft group homomorphism as a continuation to the notion of vague soft groups and vague soft homomorphism. The properties and structural characteristics of these concepts as well as the structures that are preserved under the normalistic vague soft group homomorphism are studied and discussed.

1. Introduction

Soft set theory introduced by Molodtsov in 1999 (see [1]) is a general mathematical tool that is commonly used to deal with imprecision, uncertainties, and vagueness that are pervasive in a lot of complicated problems affecting various areas in the real world. Since its inception, research on soft set theory as well as its generalizations and related theories such as soft algebra and soft topology has been developing at an exponential rate. Presently, research pertaining to other areas of generalizations of soft set theory such as fuzzy soft algebraic theory and vague soft algebraic theory is being carried out and is progressing at a rapid rate.

The study of soft algebra was initiated by Aktaş and Çağman in 2007 (see [2]) through the introduction of the notion of soft groups. A soft group is a parameterized family of subgroups which includes the algebraic structures of soft sets. Sezgin and Atagün on the other hand (see [3]) introduced the notion of normalistic soft groups and normalistic soft group homomorphism as an extension to the notion of soft groups introduced by [2]. All this led to the study of fuzzy soft algebra by Aygunoglu and Aygün (see [4]) who introduced the notion of fuzzy soft groups. In [4], the authors applied Rosenfeld's well-known concept of a fuzzy subgroup of a group (see [5]) to fuzzy soft set theory to introduce the concept of a fuzzy soft group of a group which extends the notion of soft groups to include the theory of fuzzy sets and fuzzy algebra.

The concept of vague soft sets which is a combination of the notion of soft sets and vague sets was introduced by Xu et al. (see [6]), as an extension to the notion of soft sets and fuzzy soft sets. Research in the area of vague soft algebra was initiated by Varol et al. (see [7]) who developed the theory of vague soft groups and defined the concepts of vague soft groups, normal vague soft groups, and vague soft homomorphism. Therefore it is now natural to further develop and investigate the concepts introduced in [7] and expand the theory by introducing related concepts as well as generalizations of the existing concepts.

In this paper, we contribute to the further development of the theory of vague soft groups. We define the notion of normalistic vague soft groups which is an extended albeit more comprehensive alternative to the concept of normal vague soft groups introduced in [7]. Varol et al. in [7] defined the concept of normal vague soft groups as an Abelian vague soft group; that is, $(\widehat{F}, A)$ is a vague soft group that satisfies the commutative law given by $t_{\widehat{F}_a}(x\cdot y) = t_{\widehat{F}_a}(y\cdot x)$ and $1 - f_{\widehat{F}_a}(x\cdot y) = 1 - f_{\widehat{F}_a}(y\cdot x)$. However, this definition is incomplete as there exist two other statements which describe the normality of a vague soft set that is also equivalent to the commutative law used in [7]. As such, in this paper we incorporate all these three equivalent statements into a single definition with the aim of establishing a more complete, comprehensive, and accurate representation of this concept compared to the notion of normal vague soft groups initiated in [7]. We choose to name this concept as normalistic vague soft groups in order to distinguish it from the existing concept of normal vague soft groups proposed in [7]. Subsequently, we use this concept of normalistic vague soft groups to define the notion of normalistic vague soft group homomorphism as a natural extension to the concept of vague soft homomorphism proposed in [7]. Furthermore, some of the properties and structural characteristics of the concept of normalistic vague soft groups are studied and illustrated with an example. Lastly, we prove that there exists a one-to-one correspondence between normalistic vague soft groups and some of the corresponding concepts in soft group theory and classical group theory.

2. Preliminaries

In this section, some important concepts and definitions on soft set theory, vague soft set theory, and hyperstructure theory will be presented.

Definition 1 (see [1]). A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the soft set (F, A) or as the ε-approximate elements of the soft set.

Definition 2 (see [8]). For a soft set (F, A), the set $\operatorname{Supp}(F, A) = \{x \in A : F(x) \neq \emptyset\}$ is called the *support* of the soft set (F, A). Thus a nonnull soft set is indeed a soft set with an empty support and a soft set (F, A) is said to be nonnull if $\operatorname{Supp}(F, A) \neq \emptyset$.

Definition 3 (see [2]). Let X be a group and let (F, A) be a soft set over X. Then (F, A) defined by $F : A \rightarrow P(X)$ is said to be a *soft group* over X if and only if $F(a) \leq X$, for each $a \in A$.

Definition 4 (see [2]). Let (F, A) and (H, K) be two soft groups over G. Then (H, K) is a *soft subgroup* of (F, A), written as $(H, K)\widetilde{<}(F, A)$, if

(i) $K \subset A$,

(ii) $H(x) < F(x)$ for all $x \in K$.

Definition 5 (see [3]). Let G be a group and let (F, A) be a nonnull soft set over G. Then (F, A) is called a *normalistic soft group* over G if $F(x)$ is a normal subgroup of G for all $x \in \operatorname{Supp}(F, A)$.

Definition 6 (see [9]). Let X be a space of points (objects) with a generic element of X denoted by x. A *vague set* V in X is characterized by a truth membership function $t_V : X \rightarrow [0,1]$ and a false membership function $f_V : X \rightarrow [0,1]$. The value $t_V(x)$ is a lower bound on the grade of membership of x derived from the evidence for x and $f_V(x)$ is a lower bound on the negation of x derived from the evidence against x. The values $t_V(x)$ and $f_V(x)$ both associate a real number in the interval $[0,1]$ with each point in X, where $t_V(x) + f_V(x) \leq 1$. This approach bounds the grade of membership of x to a subinterval $[t_V(x), 1 - f_V(x)]$ of $[0,1]$. Hence a vague set is a form of fuzzy set.

Definition 7 (see [6]). A pair $(\widehat{F}, A)$ is called a *vague soft set* over U where $\widehat{F}$ is a mapping given by $\widehat{F} : A \rightarrow V(U)$ and $V(U)$ is the power set of vague sets on U. In other words, a vague soft set over U is a parameterized family of vague sets of the universe U. Every set $\widehat{F}(e)$ for all $e \in A$, from this family, may be considered as the set of e-approximate elements of the vague soft set $(\widehat{F}, A)$. Hence the vague soft set $(\widehat{F}, A)$ can be viewed as consisting of a collection of approximations of the following form:

$$\begin{aligned}\left(\widehat{F}, A\right) &= \left\{\widehat{F}\left(e_i\right) : i = 1,2,3,\ldots\right\} \\ &= \left\{\frac{\left[t_{\widehat{F}(e_i)}\left(x_i\right), 1 - f_{\widehat{F}(e_i)}\left(x_i\right)\right]}{x_i} : i = 1,2,3,\ldots\right\}\end{aligned} \tag{1}$$

for all $e \in A$ and for all $x \in U$.

Definition 8 (see [10]). Let $(\widehat{F}, A)$ be a vague soft set over X. The *support* of $(\widehat{F}, A)$ denoted by $\operatorname{Supp}(\widehat{F}, A)$ is defined as

$$\begin{aligned}&\operatorname{Supp}\left(\widehat{F}, A\right) \\ &\quad = \left\{a \in A : t_{\widehat{F}_a}(x) \neq 0, 1 - f_{\widehat{F}_a}(x) \neq 0 \text{ i.e. } \widehat{F}(a) \neq \widehat{0}\right\}\end{aligned} \tag{2}$$

for all $x \in X$.

It is to be noted that a *null vague soft set* is a vague soft set where both the truth and false membership functions are equal to zero. Therefore, a vague soft set $(\widehat{F}, A)$ is said to be nonnull if $\operatorname{Supp}(\widehat{F}, A) \neq \emptyset$.

Definition 9 (see [10]). The *extended intersection* of two vague soft sets $(\widehat{F}, A)$ and $(\widehat{G}, B)$ over a universe X, which is denoted as $(\widehat{F}, A)\widehat{\cap}(\widehat{G}, B) = (\widehat{H}, C)$, is a vague soft set $(\widehat{H}, C)$, where $C = A \cup B$ and, for every $c \in C$ and $x \in X$,

$$\begin{aligned}t_{\widehat{H}_c}(x) &= \begin{cases} t_{\widehat{F}_c}(x) & c \in A - B, \\ t_{\widehat{G}_c}(x) & c \in B - A, \\ \min\left(t_{\widehat{F}_c}(x), t_{\widehat{G}_c}(x)\right) & c \in B \cap A, \end{cases} \\ 1 - f_{\widehat{H}_c}(x) & \\ &= \begin{cases} 1 - f_{\widehat{F}_c}(x) & c \in A - B, \\ 1 - f_{\widehat{G}_c}(x) & c \in B - A, \\ \min\left(1 - f_{\widehat{F}_c}(x), 1 - f_{\widehat{G}_c}(x)\right) & c \in B \cap A, \end{cases}\end{aligned} \tag{3}$$

where $t_{\widehat{H}_c}$ and $f_{\widehat{H}_c}$ are the upper and lower bounds of $\widehat{H}_c$, respectively. This relationship can be written as $\widehat{H}_c(x) = \widehat{F}_c(x)\widetilde{\cap}\widehat{G}_c(x)$ for every $x \in X$.

Definition 10 (see [10]). The *restricted intersection* of two vague soft sets $(\widehat{F}, A)$ and $(\widehat{G}, B)$ over a universe X, which is denoted as $(\widehat{F}, A)\Cap(\widehat{G}, B) = (\widehat{H}, C)$, is a vague soft set $(\widehat{H}, C)$, where $C = A \cap B$ and, for every $c \in C$ and $x \in X$,

$$t_{\widehat{H}_c}(x) = \min\left\{t_{\widehat{F}_c}(x), t_{\widehat{G}_c}(x)\right\},$$
$$1 - f_{\widehat{H}_c}(x) = \min\left\{1 - f_{\widehat{F}_c}(x), 1 - f_{\widehat{G}_c}(x)\right\}, \quad (4)$$

where $t_{\widehat{H}_c}$ and $f_{\widehat{H}_c}$ are the upper and lower bounds of $\widehat{H}_c$, respectively. This relationship can be written as $\widehat{H}_c(x) = \widehat{F}_c(x) \cap \widehat{G}_c(x)$.

Definition 11 (see [7]). Let $(\widehat{F}, A)$ be a vague soft set over U. Then for every $\alpha, \beta \in [0, 1]$, where $\alpha \le \beta$, the (α, β)-*cut* or the *vague soft cut* of $(\widehat{F}, A)$ is a subset of U which is defined as follows:

$$\left(\widehat{F}, A\right)_{(\alpha,\beta)} = \left\{x \in U : t_{\widehat{F}_a}(x) \ge \alpha, 1 - f_{\widehat{F}_a}(x) \ge \beta, \text{ i.e. } \widehat{F}_a(x) \ge [\alpha, \beta]\right\} \quad (5)$$

for every $a \in A$.

Definition 12 (see [10]). Let $(\widehat{F}, A)$ be a vague soft set over U. Then for every $\alpha \in [0, 1]$, the α-*cut* of $(\widehat{F}, A)$, denoted as $(\widehat{F}, A)_{(\alpha,\alpha)}$, is a subset of U which is defined as follows:

$$\left(\widehat{F}, A\right)_{(\alpha,\alpha)} = \left\{x \in U : t_{\widehat{F}_a}(x) \ge \alpha, 1 - f_{\widehat{F}_a}(x) \ge \alpha, \text{ i.e. } \widehat{F}_a(x) \ge [\alpha, \alpha]\right\} \quad (6)$$

for every $a \in A$.

Definition 13 (see [10]). Let $(\widehat{F}, A)$ be a vague soft set over X and let G be a nonnull subset of X. Then $(\widehat{F}, A)_G$ is called a *vague soft characteristic set* of G in $[0, 1]$ and the lower bound and the upper bound of $(\widehat{F}_a)_G$ are defined as follows:

$$t_{(\widehat{F}_a)_G}(x) = 1 - f_{(\widehat{F}_a)_G}(x) \begin{cases} s & \text{if } x \in G, \\ w & \text{otherwise,} \end{cases} \quad (7)$$

where $(\widehat{F}_a)_G$ is a subset of $(\widehat{F}, A)_G$, $x \in X$, $s, w \in [0, 1]$, and $s > w$.

3. Vague Soft Groups

In this section, the concept of vague soft groups and some important results pertaining to this concept introduced in [7] are presented. These definitions and results will be extended to the concept of normalistic vague soft groups in the next section.

Definition 14 (see [7]). Let X be a group and let $(\widehat{F}, A)$ be a vague soft set over X. Then $(\widehat{F}, A)$ is called a *vague soft group* over X if and only if, for every $a \in A$ and $x, y \in X$, the following conditions are satisfied:

(i) $t_{\widehat{F}_a}(xy) \ge \min\{t_{\widehat{F}_a}(x), t_{\widehat{F}_a}(y)\}$ and $1 - f_{\widehat{F}_a}(xy) \ge \min\{1 - f_{\widehat{F}_a}(x), 1 - f_{\widehat{F}_a}(y)\}$; that is, $\widehat{F}_a(xy) \ge \min(\widehat{F}_a(x), \widehat{F}_a(y))$,

(ii) $t_{\widehat{F}_a}(x^{-1}) \ge t_{\widehat{F}_a}(x)$ and $1 - f_{\widehat{F}_a}(x^{-1}) \ge 1 - f_{\widehat{F}_a}(x)$; that is, $\widehat{F}_a(x^{-1}) \ge \widehat{F}_a(x)$.

In other words, for every $a \in A$, $\widehat{F}_a$ is a *vague subgroup* in Rosenfeld's sense.

Proposition 15 (see [7]). *Let $(\widehat{F}, A)$ be a vague soft group over X and let e be the identity element of X. Then for every $a \in A$ and $x \in X$,*

(i) *$t_{\widehat{F}_a}(x^{-1}) = t_{\widehat{F}_a}(x)$ and $1 - f_{\widehat{F}_a}(x^{-1}) = 1 - f_{\widehat{F}_a}(x)$; that is, $\widehat{F}_a(x^{-1}) = \widehat{F}_a(x)$,*

(ii) *$t_{\widehat{F}_a}(e) \ge t_{\widehat{F}_a}(x)$ and $1 - f_{\widehat{F}_a}(e) \ge 1 - f_{\widehat{F}_a}(x)$; that is, $\widehat{F}_a(e) \ge \widehat{F}_a(x)$.*

Proposition 16 (see [7]). *Let $(\widehat{F}, A)$ be a vague soft set. Then $(\widehat{F}, A)$ is vague soft group if and only if, for every $a \in A$ and $x, y \in X$,*

$$t_{\widehat{F}_a}\left(xy^{-1}\right) \ge \min\left\{t_{\widehat{F}_a}(x), t_{\widehat{F}_a}(y)\right\},$$
$$1 - f_{\widehat{F}_a}\left(xy^{-1}\right) \ge \min\left\{1 - f_{\widehat{F}_a}(x), 1 - f_{\widehat{F}_a}(y)\right\}, \quad (8)$$
$$\text{that is, } \widehat{F}_a\left(x \cdot y^{-1}\right) \ge \min\left(\widehat{F}_a(x), \widehat{F}_a(y)\right).$$

4. Normalistic Vague Soft Groups

In this section, we propose the concept of normalistic vague soft groups and study some of the fundamental properties and structural characteristics of this concept.

Theorem 17 (see [7]). *Let $(\widehat{F}, A)$ be a vague soft set over X. Then the following statements are equivalent for each $a \in A$ and for every $x, y \in X$:*

(i) *$t_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) \ge t_{\widehat{F}_a}(x)$ and $1 - f_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) \ge 1 - f_{\widehat{F}_a}(x)$; that is, $\widehat{F}_a(y \cdot x \cdot y^{-1}) \ge \widehat{F}_a(x)$ or*

(ii) *$t_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) = t_{\widehat{F}_a}(x)$ and $1 - f_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) = 1 - f_{\widehat{F}_a}(x)$; that is, $\widehat{F}_a(y \cdot x \cdot y^{-1}) = \widehat{F}_a(x)$ or*

(iii) *$t_{\widehat{F}_a}(x \cdot y) = t_{\widehat{F}_a}(y \cdot x)$ and $1 - f_{\widehat{F}_a}(x \cdot y) = 1 - f_{\widehat{F}_a}(y \cdot x)$; that is, $\widehat{F}_a(x \cdot y) = \widehat{F}_a(y \cdot x)$.*

If condition (iii) is satisfied, then $(\widehat{F}, A)$ is said to be an Abelian vague soft set over X.

Definition 18. Let X be a group and let $(\widehat{F}, A)$ be a nonnull vague soft group over X. Then $(\widehat{F}, A)$ is called a *normalistic vague soft group* over X if, for every $a \in \text{Supp}(\widehat{F}, A)$ and for every $x, y \in X$, either one of the following conditions is satisfied:

(i) $t_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) \geq t_{\widehat{F}_a}(x)$ and $1 - f_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) \geq 1 - f_{\widehat{F}_a}(x)$; that is, $\widehat{F}_a(y \cdot x \cdot y^{-1}) \geq \widehat{F}_a(x)$ or

(ii) $t_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) = t_{\widehat{F}_a}(x)$ and $1 - f_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) = 1 - f_{\widehat{F}_a}(x)$; that is, $\widehat{F}_a(y \cdot x \cdot y^{-1}) = \widehat{F}_a(x)$ or

(iii) $t_{\widehat{F}_a}(x \cdot y) = t_{\widehat{F}_a}(y \cdot x)$ and $1 - f_{\widehat{F}_a}(x \cdot y) = 1 - f_{\widehat{F}_a}(y \cdot x)$; that is, $\widehat{F}_a(x \cdot y) = \widehat{F}_a(y \cdot x)$.

In other words, for every $a \in \operatorname{Supp}(\widehat{F}, A)$ and $x \in X$, $\widehat{F}_a$ is a *normal vague subgroup* of X in Rosenfeld's sense. If $\widehat{F}_a$ satisfies condition (iii), then $(\widehat{F}, A)$ is called an *Abelian vague soft group*.

From Definition 18, it is obvious that all normalistic vague soft groups are vague soft groups but the converse is not necessarily true.

Proposition 19. *Let $(\widehat{F}, A)$ be a nonnull vague soft set over X and $B \subset A$. If $(\widehat{F}, A)$ is a normalistic vague soft group over X, then $(\widehat{F}, B)$ is a normalistic vague soft group over X if it is nonnull.*

Proof. Let $(\widehat{F}, A)$ be a normalistic vague soft group over X. Then $\widehat{F}_a$ is a normal vague subgroup of X for every $a \in \operatorname{Supp}(\widehat{F}, A)$. Since $B \subset A$, $(\widehat{F}, B)$ is a vague soft subgroup of $(\widehat{F}, A)$. Therefore $\widehat{F}_b$ is a normal vague subgroup of $(\widehat{F}, A)$ for every $b \in \operatorname{Supp}(\widehat{F}, B)$. This implies that $\widehat{F}_b$ is a normal vague subgroup of X for every $b \in \operatorname{Supp}(\widehat{F}, B)$, because $(\widehat{F}, B)$ is a vague soft subgroup of $(\widehat{F}, A)$. Then $(\widehat{F}, B)$ is a normalistic vague soft group over X. □

Proposition 20. *Let $(\widehat{F}, A)$ be a normalistic vague soft group over X and let $(\widehat{F}, A)^+$ be a nonnull vague soft set over X which is as defined below:*

$$\begin{aligned}(\widehat{F}, A)^{+} = \Big\{(\widehat{F}_a)^{+} = \big\{x \in X : t_{(\widehat{F}_a)^+}(x) = t_{\widehat{F}_a}(x) + 1 \\ - t_{\widehat{F}_a}(e), 1 - f_{(\widehat{F}_a)^+}(x) = 1 - f_{\widehat{F}_a}(x) + f_{\widehat{F}_a}(e)\big\}\Big\},\end{aligned} \tag{9}$$

for every $a \in \operatorname{Supp}(\widehat{F}, A)$ and $x \in X$ while e is the identity element of group X. Then $(\widehat{F}, A)^+$ is a normalistic vague soft group over X.

Proof. $\widehat{F}_a$ is a normal vague subgroup of X for each $a \in \operatorname{Supp}(\widehat{F}, A)$. Now let $x, y \in (\widehat{F}, A)^+$ and $a \in \operatorname{Supp}(\widehat{F}, A)^+$. Then $t_{(\widehat{F}_a)^+}(x) = t_{\widehat{F}_a}(x) + 1 - t_{\widehat{F}_a}(e)$ and $1 - f_{(\widehat{F}_a)^+}(x) = 1 - f_{\widehat{F}_a}(x) + f_{\widehat{F}_a}(e)$ and also $t_{(\widehat{F}_a)^+}(y) = t_{\widehat{F}_a}(y) + 1 - t_{\widehat{F}_a}(e)$ and $1 - f_{(\widehat{F}_a)^+}(y) = 1 - f_{\widehat{F}_a}(y) + f_{\widehat{F}_a}(e)$. This means that $(\widehat{F}_a)^+(x) = \widehat{F}_a(x) + 1 - \widehat{F}_a(e)$ and $(\widehat{F}_a)^+(y) = \widehat{F}_a(y) + 1 - \widehat{F}_a(e)$. Thus the following is obtained:

$$\begin{aligned} t_{(\widehat{F}_a)^+}(xy) &= t_{\widehat{F}_a}(xy) + 1 - t_{\widehat{F}_a}(e) \\ &\geq \min\{t_{\widehat{F}_a}(x), t_{\widehat{F}_a}(y)\} + 1 - t_{\widehat{F}_a}(e) \\ &= \min\{t_{\widehat{F}_a}(x) + 1 - t_{\widehat{F}_a}(e), t_{\widehat{F}_a}(y) + 1 - t_{\widehat{F}_a}(e)\} \\ &= \min\{t_{(\widehat{F}_a)^+}(x), t_{(\widehat{F}_a)^+}(y)\}\end{aligned} \tag{10}$$

and $t_{(\widehat{F}_a)^+}(x^{-1}) = t_{\widehat{F}_a}(x^{-1}) + 1 - t_{\widehat{F}_a}(e) \geq t_{\widehat{F}_a}(x) + 1 - t_{\widehat{F}_a}(e) = t_{(\widehat{F}_a)^+}(x)$.

Similarly, it can be proven that $1 - f_{(\widehat{F}_a)^+}(xy) \geq \min\{1 - f_{(\widehat{F}_a)^+}(x), 1 - f_{(\widehat{F}_a)^+}(y)\}$ and $1 - f_{(\widehat{F}_a)^+}(x^{-1}) \geq 1 - f_{(\widehat{F}_a)^+}(x)$. Therefore $(\widehat{F}_a)^+$ is vague subgroup of X. Next we prove normality:

$$\begin{aligned} t_{(\widehat{F}_a)^+}(y \cdot x \cdot y^{-1}) &= t_{\widehat{F}_a}(y \cdot x \cdot y^{-1}) + 1 - t_{\widehat{F}_a}(e) \\ &\geq t_{\widehat{F}_a}(x) + 1 - t_{\widehat{F}_a}(e) = t_{(\widehat{F}_a)^+}(x).\end{aligned} \tag{11}$$

Similarly, we obtain $1 - f_{(\widehat{F}_a)^+}(y \cdot x \cdot y^{-1}) \geq 1 - f_{(\widehat{F}_a)^+}(x)$. Hence it has been proven that $(\widehat{F}_a)^+$ is a normal vague subgroup of X. As such, $(\widehat{F}, A)^+$ is a normalistic vague soft group over X. □

Theorem 21. *Let G be a nonnull subset of X, let $(\widehat{F}, A)$ be a normalistic vague soft group over X, let $(\widehat{F}, A)_G$ be a vague soft characteristic set over X, and let $(\widehat{F}, A)^+$ be a vague soft set over X as defined in Proposition 20. If $(\widehat{F}, A)^+$ is a normalistic vague soft set over X, then G is a normal subgroup of X.*

Proof. Suppose that G is a nonnull subset of X and $(\widehat{F}, A)^+$ is a normalistic vague soft group over X. Then $(\widehat{F}_a)^+$ is a normal vague subgroup of X for every $a \in \operatorname{Supp}(\widehat{F}, A)^+$. Now let $x, y \in G$ and $a \in \operatorname{Supp}(\widehat{F}, A)^+$. Thus we obtain $t_{(\widehat{F}_a)^+_G}(x) = t_{(\widehat{F}_a)^+_G}(y) = s$ and $1 - f_{(\widehat{F}_a)^+_G}(x) = 1 - f_{(\widehat{F}_a)^+_G}(y) = s$. Since $(\widehat{F}_a)^+$ is a normal vague subgroup of X, we obtain the following:

$$\begin{aligned} t_{(\widehat{F}_a)^+_G}(xy^{-1}) &= t_{(\widehat{F}_a)_G}(xy^{-1}) + 1 - t_{(\widehat{F}_a)_G}(e) \\ &\geq \min\{t_{(\widehat{F}_a)_G}(x), t_{(\widehat{F}_a)_G}(y^{-1})\} + 1 - t_{(\widehat{F}_a)_G}(e) \\ &\geq \min\{t_{(\widehat{F}_a)_G}(x), t_{(\widehat{F}_a)_G}(y)\} + 1 - t_{(\widehat{F}_a)_G}(e) \\ &= \min\{t_{(\widehat{F}_a)_G}(x) + 1 - t_{(\widehat{F}_a)_G}(e), t_{(\widehat{F}_a)_G}(y) + 1 \\ &\quad - t_{(\widehat{F}_a)_G}(e)\} = \min\{t_{(\widehat{F}_a)^+_G}(x), t_{(\widehat{F}_a)^+_G}(y)\} \\ &= \min\{s, s\} = s.\end{aligned} \tag{12}$$

Similarly, it can be proven that $1 - f_{(\widehat{F}_a)^+_G}(xy^{-1}) \geq s$ too. This means that $(\widehat{F}_a)^+_G(xy^{-1}) \geq s$ and therefore $xy^{-1} \in G$. Hence G

Table 1: Cayley table.

$\cdot$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

is a subgroup of X. To prove normality, let $x \in G$ and $y \in X$. Then we obtain

$$\begin{aligned} t_{(\widehat{F}_a)^+_G}\left(y \cdot x \cdot y^{-1}\right) &= t_{(\widehat{F}_a)_G}\left(y \cdot x \cdot y^{-1}\right) + 1 - t_{(\widehat{F}_a)_G}(e) \\ &= t_{(\widehat{F}_a)_G}(x) + 1 - t_{(\widehat{F}_a)_G}(e) \\ &= t_{(\widehat{F}_a)^+_G}(x) = s. \end{aligned} \tag{13}$$

In the same manner, it can be proven that $1 - f_{(\widehat{F}_a)^+_G}(y \cdot x \cdot y^{-1}) = s$. Thus $(\widehat{F}_a)^+_G(y \cdot x \cdot y^{-1}) = s$ and this implies that $y \cdot x \cdot y^{-1} \in G$. As such, it is proven that G is a normal subgroup of X. □

Definition 22. Let $(\widehat{F}, A)$ be a normalistic vague soft group over X. Then

(i) $(\widehat{F}, A)$ is called a *trivial normalistic vague soft group* over X if $\widehat{F}(a) = \{e_X\}$ for all $a \in \operatorname{Supp}(\widehat{F}, A)$ and the truth and false membership function of $(\widehat{F}, A)$ are defined as follows:

$$\begin{aligned} t_{\widehat{F}_a}(x) &= \begin{cases} 1 & \text{if } a \in \operatorname{Supp}\left(\widehat{F}, A\right), \\ 0 & \text{otherwise,} \end{cases} \\ f_{\widehat{F}_a}(x) &= \begin{cases} 0 & \text{if } a \in \operatorname{Supp}\left(\widehat{F}, A\right), \\ 1 & \text{otherwise} \end{cases} \end{aligned} \tag{14}$$

for every $a \in A$ and $x \in X$.

(ii) $(\widehat{F}, A)$ is called an *absolute normalistic vague soft group* over X if $\widehat{F}(a) = X$ for all $a \in \operatorname{Supp}(\widehat{F}, A)$ and the truth and false membership function of $(\widehat{F}, A)$ are defined as follows:

$$\begin{aligned} t_{\widehat{F}_a}(x) &= \begin{cases} 1 & \text{if } a \in \operatorname{Supp}\left(\widehat{F}, A\right), \\ 0 & \text{otherwise,} \end{cases} \\ f_{\widehat{F}_a}(x) &= \begin{cases} 0 & \text{if } a \in \operatorname{Supp}\left(\widehat{F}, A\right), \\ 1 & \text{otherwise.} \end{cases} \end{aligned} \tag{15}$$

Example 23. Let $X = \{\{1, -1, i, -i\}, \times\}$ be a group with the operations of group X as given in the Cayley table in Table 1.

(i) Then let $A = \{1, -1\}$ and let $(\widehat{F}, A)$ be a nonnull vague soft group over X, where $\widehat{F} : A \to V(X)$ is a set-valued function that is as defined below:

$$\widehat{F}(a) = \left\{x \in X : x = a^2\right\} \tag{16}$$

for all $a \in A$. Then $\widehat{F}(1) = \widehat{F}(-1) = \{1\} = \{e_X\}$ and $\widehat{F}(i) = \widehat{F}(-i) = -1$.

The truth membership function and false membership function of $(\widehat{F}, A)$ are as given below:

$$\begin{aligned} t_{\widehat{F}_a}(x) &= \begin{cases} 1 & \text{if } x = a^2 = 1, \\ 0 & \text{otherwise,} \end{cases} \\ f_{\widehat{F}_a}(x) &= \begin{cases} 0 & \text{if } x = a^2 = 1, \\ 1 & \text{otherwise} \end{cases} \end{aligned} \tag{17}$$

for all $a \in A$ and $x \in X$. Since $\widehat{F}(a) = \{e_X\} = 1$ for all $a \in \operatorname{Supp}(\widehat{F}, A)$, $\widehat{F}(a)$ is a trivial normal vague subgroup of X. As such, $(\widehat{F}, A)$ is a trivial normalistic vague soft group over X.

(ii) Now let $B = \{i, -i\}$ and let $(\widehat{G}, B)$ be a nonnull vague soft group over X, where $\widehat{G} : B \to V(X)$ is a set-valued function that is as defined below:

$$\widehat{G}(b) = \{x \in X : x = a^n \text{ for any } n \in \mathbb{N}\} \tag{18}$$

for all $b \in B$. Thus $\widehat{G}(i) = \widehat{G}(-i) = \{1, -1, i, -i\} = X$, $\widehat{G}(1) = \{1\}$, and

$$\widehat{G}(-1) = \begin{cases} 1 & \text{if } n \text{ is an even integer,} \\ -1 & \text{if } n \text{ is an odd integer.} \end{cases} \tag{19}$$

The truth membership function and false membership function of $(\widehat{G}, B)$ are as given below:

$$\begin{aligned} t_{\widehat{G}_b}(x) &= \begin{cases} 1 & \text{if } x = a^n = X, \\ 0 & \text{otherwise,} \end{cases} \\ f_{\widehat{G}_b}(x) &= \begin{cases} 0 & \text{if } x = a^n = X, \\ 1 & \text{otherwise} \end{cases} \end{aligned} \tag{20}$$

for every $b \in B$ and $x \in X$. Hence $\widehat{G}(b) = X$ for all $b \in \operatorname{Supp}(\widehat{G}, B)$ and therefore $\widehat{G}(b)$ is an absolute normal vague subgroup of X. As such, $(\widehat{G}, B)$ is an absolute normalistic vague soft group over X.

Theorem 24. *Let $(\widehat{F}, A)$ and $(\widehat{G}, B)$ be normalistic vague soft groups over X. Then consider the following.*

(i) *If $(\widehat{F}, A)$ and $(\widehat{G}, B)$ are trivial normalistic vague soft groups over X, then $(\widehat{F}, A)\widetilde{\cap}(\widehat{G}, B)$ is a trivial normalistic vague soft group over X.*

(ii) *If $(\widehat{F}, A)$ and $(\widehat{G}, B)$ are absolute normalistic vague soft groups over X, then $(\widehat{F}, A)\widetilde{\cap}(\widehat{G}, B)$ is an absolute normalistic vague soft group over X.*

(iii) *If $(\widehat{F}, A)$ is a trivial normalistic vague soft group over X and $(\widehat{G}, B)$ is an absolute normalistic vague soft group over X, then $(\widehat{F}, A)\widetilde{\Cap}(\widehat{G}, B)$ is a trivial normalistic vague soft group over X.*

Proof. The proofs are straightforward and are therefore omitted. □

Theorem 25. *Let $\varphi : X \to Y$ be a group epimorphism and let $(\widehat{F}, A)$ be a normalistic vague soft group over X. Then $(\varphi(\widehat{F}), \operatorname{Supp}(\widehat{F}, A))$ is a normalistic vague soft group over Y.*

Proof. Since $(\widehat{F}, A)$ is a normalistic vague soft group over X, it must be nonnull and therefore it follows that $(\varphi(\widehat{F}), \operatorname{Supp}(\widehat{F}, A))$ must be nonnull too. Now let $a \in A$ and assume that, for every $y_1, y_2 \in Y$, there exists $x_1, x_2 \in X$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Thus we obtain the following:

$$\begin{aligned}
&\varphi\left(\widehat{F}_a\right)\left(\varphi\left(x_1\right)\cdot\varphi\left(x_2\right)\right)\\
&\quad=\left(\varphi\left(\widehat{F}\right)\right)_a\left(y_1\cdot y_2\right)\\
&\quad=\left(\left(\varphi\left(\widehat{F}\right)\right)_a\left(y_1\right)\right)\cdot\left(\left(\varphi\left(\widehat{F}\right)\right)_a\left(y_2\right)\right)\\
&\quad=\left(\left(\varphi\left(\widehat{F}\right)\right)_a\left(y_2\right)\right)\cdot\left(\left(\varphi\left(\widehat{F}\right)\right)_a\left(y_1\right)\right)\\
&\quad=\left(\varphi\left(\widehat{F}\right)\right)_a\left(y_2\cdot y_1\right)=\varphi\left(\widehat{F}_a\right)\left(\varphi\left(x_2\right)\cdot\varphi\left(x_1\right)\right).
\end{aligned} \tag{21}$$

As such, $(\varphi(\widehat{F}), \operatorname{Supp}(\widehat{F}, A))$ is a normalistic vague soft group over Y. □

Theorem 26. *Let $\varphi : X \to Y$ be a group isomorphism and let $(\widehat{G}, B)$ be a normalistic vague soft group over Y. Then $(\varphi^{-1}(\widehat{G}), \operatorname{Supp}(\widehat{G}, B))$ is a normalistic vague soft group over X.*

Proof. The proof is similar to that of Theorem 25 and is therefore omitted. □

Theorem 27. *Let $(\widehat{F}, A)$ and $(\widehat{G}, B)$ be normalistic vague soft groups over X and Y, respectively, and let $\varphi : X \to Y$ be a group epimorphism. Then consider the following.*

(i) *If $\widehat{F}(a) = Ker(\varphi)$ for all $a \in \operatorname{Supp}(\widehat{F}, A)$, then $(\varphi(\widehat{F}), \operatorname{Supp}(\widehat{F}, A))$ is a trivial normalistic vague soft group over Y.*

(ii) *If $(\widehat{F}, A)$ is an absolute normalistic vague soft group over X, then $(\varphi(\widehat{F}), \operatorname{Supp}(\widehat{F}, A))$ is an absolute normalistic vague soft group over Y.*

(iii) *If φ is injective and $\widehat{G}(b) = \varphi(X)$ for every $b \in \operatorname{Supp}(\widehat{G}, B)$, then $(\varphi^{-1}(\widehat{G}), \operatorname{Supp}(\widehat{G}, B))$ is an absolute normalistic vague soft group over X.*

(iv) *If φ is injective and $(\widehat{G}, B)$ is a trivial normalistic vague soft group over Y, then $(\varphi^{-1}(\widehat{G}), \operatorname{Supp}(\widehat{G}, B))$ is a trivial normalistic vague soft group over X.*

Proof. (i) Let φ be a group epimorphism from X to Y. Then the kernel of φ is as given below:

$$\operatorname{Ker}(\varphi) = \{x \in X : \varphi(x) = e_Y\}, \tag{22}$$

where e_Y is the identity element of Y. Let $\widehat{F}(a) = \operatorname{Ker}(\varphi)$ for every $a \in \operatorname{Supp}(\widehat{F}, A)$. Then for every $a \in \operatorname{Supp}(\widehat{F}, A)$ and $x \in X$, it follows that $\varphi(\widehat{F})(a) = \varphi(\widehat{F}(a)) = \varphi(x) = \{e_Y\}$. Then the truth and false membership function of $(\varphi(\widehat{F}), \operatorname{Supp}(\widehat{F}, A))$ are as given below:

$$\begin{aligned}
t_{\varphi(\widehat{F}_\alpha)}(x) &= \begin{cases} 1 & \text{if } a \in \operatorname{Supp}\left(\widehat{F}, A\right), \\ 0 & \text{otherwise}, \end{cases}\\
f_{\varphi(\widehat{F}_\alpha)}(x) &= \begin{cases} 0 & \text{if } a \in \operatorname{Supp}\left(\widehat{F}, A\right), \\ 1 & \text{otherwise} \end{cases}
\end{aligned} \tag{23}$$

for every $a \in A$ and $x \in X$. As such, $(\varphi(\widehat{F}), \operatorname{Supp}(\widehat{F}, A))$ is a trivial normalistic vague soft group over Y.

(ii) The proof is similar to the proof of part (i) and is therefore omitted.

(iii) Let $\varphi : X \to Y$ be an injective group epimorphism and, for every $b \in \operatorname{Supp}(\widehat{G}, B)$, $\widehat{G}(b) = \varphi(X)$. Thus $\widehat{G}(b) = \varphi(X) = Y$ for every $b \in \operatorname{Supp}(\widehat{G}, B)$ because φ is injective. Therefore by Definition 22(ii), $(\widehat{G}, B)$ is an absolute normalistic vague soft group over Y. As such, we obtain

$$\begin{aligned}
\varphi^{-1}\left(\widehat{G}\right)(b) &= \varphi^{-1}\left(\widehat{G}(b)\right) = \varphi^{-1}\left(\varphi(X)\right) = \varphi^{-1}(Y)\\
&= X
\end{aligned} \tag{24}$$

for every $b \in \operatorname{Supp}(\widehat{G}, B)$ because φ is injective. Thus for every $b \in \operatorname{Supp}(\widehat{G}, B)$, $\varphi^{-1}(\widehat{G}(b)) = X$. This means that $(\varphi^{-1}(\widehat{G}), \operatorname{Supp}(\widehat{G}, B))$ is an absolute normalistic vague soft group over X with truth and false membership function as given below:

$$\begin{aligned}
t_{\varphi^{-1}(\widehat{G}_b)}(y) &= \begin{cases} 1 & \text{if } b \in \operatorname{Supp}\left(\widehat{G}, B\right), \\ 0 & \text{otherwise}, \end{cases}\\
f_{\varphi^{-1}(\widehat{G}_b)}(y) &= \begin{cases} 0 & \text{if } b \in \operatorname{Supp}\left(\widehat{G}, B\right), \\ 1 & \text{otherwise} \end{cases}
\end{aligned} \tag{25}$$

for every $y \in Y$. Hence $(\varphi^{-1}(\widehat{G}), \operatorname{Supp}(\widehat{G}, B))$ is an absolute normalistic vague soft group over X.

(iv) The proof is similar to the proof of part (iii) and is therefore omitted. □

5. The Homomorphism of Normalistic Vague Soft Groups

In [7], the notion of vague soft functions was introduced whereas the concepts of the image and preimage of a vague soft set under a vague soft function were introduced in [10]. Here we extend these concepts to include normalistic vague soft groups and subsequently introduce the notion of normalistic vague soft group homomorphism. Lastly, we prove that this homomorphism preserves normalistic vague soft groups.

Definition 28 (see [7]). Let $\varphi : X \to Y$ and $\psi : A \to B$ be two functions, where A and B are the set of parameters for the classical sets X and Y, respectively. Let $(\widehat{F}, A)$ and $(\widehat{G}, B)$ be vague soft groups over X and Y, respectively. Then the ordered pair (φ, ψ) is called a *vague soft function* from $(\widehat{F}, A)$ to $(\widehat{G}, B)$, denoted as $(\varphi, \psi) : (\widehat{F}, A) \to (\widehat{G}, B)$.

Definition 29 (see [10]). Let $(\widehat{F}, A)$ and $(\widehat{G}, B)$ be two vague soft groups over X and Y, respectively. Let $(\varphi, \psi) : (\widehat{F}, A) \to (\widehat{G}, B)$ be a vague soft function.

(i) The *image* of $(\widehat{F}, A)$ under the vague soft function (φ, ψ), denoted as $(\varphi, \psi)(\widehat{F}, A)$, is a vague soft set over Y, which is defined as

$$(\varphi, \psi)\left(\widehat{F}, A\right) = \left(\varphi\left(\widehat{F}\right), \psi(A)\right), \tag{26}$$

where $\varphi(\widehat{F}_a)(\varphi(x)) = (\varphi(\widehat{F}))_{\psi(a)}(y)$ for every $a \in A$, $x \in X$, and $y \in Y$.

(ii) The *preimage* of $(\widehat{G}, B)$ under the vague soft function (φ, ψ), denoted as $(\varphi, \psi)^{-1}(\widehat{G}, B)$, is a vague soft set over X, which is defined as

$$(\varphi, \psi)^{-1}\left(\widehat{G}, B\right) = \left(\varphi^{-1}\left(\widehat{G}\right), \psi^{-1}(B)\right), \tag{27}$$

where $\varphi^{-1}(\widehat{G}_b)(\varphi^{-1}(y)) = (\varphi^{-1}(\widehat{G}))_{\psi^{-1}(b)}(x)$ for every $b \in B$, $x \in X$, and $y \in Y$.

If φ and ψ are injective (surjective), then the vague soft function (φ, ψ) is said to be injective (surjective).

Next we introduce the notion of normalistic vague soft group homomorphism as an extension to the notion of vague soft homomorphism introduced in [7].

Definition 30. Let $(\widehat{F}, A)$ and $(\widehat{G}, B)$ be normalistic vague soft groups over X and Y, respectively. Then the vague soft function $(\varphi, \psi) : (\widehat{F}, A) \to (\widehat{G}, B)$ is called as follows.

(a) It is called *normalistic vague soft group homomorphism* if the following conditions are satisfied:

(i) φ is a homomorphism from X to Y,

(ii) ψ is a function from A into B,

(iii) $\varphi(\widehat{F}(x)) = \widehat{G}(\psi(x))$ for every $x \in A$.

Then $(\widehat{F}, A)$ is said to be *normalistic vague soft homomorphic* to $(\widehat{G}, B)$ and this is denoted as $(\widehat{F}, A) \sim_N (\widehat{G}, B)$.

(b) It is called *normalistic vague soft group isomorphism* if the following conditions are satisfied:

(i) φ is an isomorphism from X to Y,

(ii) ψ is a bijective mapping from A to B,

(iii) $\varphi(\widehat{F}(x)) = \widehat{G}(\psi(x))$ for every $x \in A$.

Then $(\widehat{F}, A)$ is said to be *normalistic vague soft isomorphic* to $(\widehat{G}, B)$ and this is denoted as $(\widehat{F}, A) \cong_N (\widehat{G}, B)$.

Theorem 31. *Let $(\widehat{F}, A)$ and $(\widehat{G}, B)$ be normalistic vague soft groups over X and Y, respectively, and let $(\varphi, \psi) : (\widehat{F}, A) \to (\widehat{G}, B)$ be a normalistic vague soft group homomorphism. Then consider the following:*

(i) *$(\varphi, \psi)(\widehat{F}, A)$ is a normalistic vague soft group over Y.*

(ii) *$(\varphi, \psi)^{-1}(\widehat{G}, B)$ is a normalistic vague soft group over X.*

Proof. The proofs follow from Definitions 18 and 30 and are therefore omitted. □

Theorem 32. *Let X, Y, and Z be groups, let $(\widehat{F}, A)$, $(\widehat{G}, B)$, and $(\widehat{J}, C)$ be normalistic vague soft groups over X, Y, and Z, respectively, and let $(\varphi, \psi) : (\widehat{F}, A) \to (\widehat{G}, B)$ and $(\varphi^*, \psi^*) : (\widehat{G}, B) \to (\widehat{J}, C)$ be normalistic vague soft group homomorphisms. Then $(\varphi^* \circ \varphi, \psi^* \circ \psi)$ is a normalistic vague soft group homomorphism from $(\widehat{F}, A)$ to $(\widehat{J}, C)$.*

Proof. Suppose that (φ, ψ) represents a normalistic vague soft homomorphism from $(\widehat{F}, A)$ to $(\widehat{G}, B)$. Then (φ, ψ) is a vague soft function such that $\varphi : X \to Y$ is a group homomorphism from X into Y and $\psi : A \to B$ is a function from A to B. Also (φ, ψ) satisfies the following condition:

$$\varphi\left(\widehat{F}(x)\right) = \widehat{G}(\psi(x)) \quad \forall x \in \operatorname{Supp}\left(\widehat{F}, A\right). \tag{28}$$

Now suppose that (φ^*, ψ^*) represents a normalistic vague soft homomorphism from $(\widehat{G}, B)$ to $(\widehat{J}, C)$. Then (φ^*, ψ^*) is a vague soft function such that $\varphi^* : Y \to Z$ represents a group homomorphism from Y to Z, $\psi^* : B \to C$ is a function from A into B, and the following condition is satisfied:

$$\varphi^*\left(\widehat{G}(x)\right) = \widehat{J}(\psi^*(x)) \quad \forall x \in \operatorname{Supp}\left(\widehat{G}, B\right). \tag{29}$$

Therefore we have $\varphi^* \circ \varphi : X \to Z$ which is a group homomorphism from X into Z and the composition $\psi^* \circ \psi : A \to C$ represents a function from A into C. Therefore it follows that $(\varphi^* \circ \varphi, \psi^* \circ \psi)$ is a vague soft function from $(\widehat{F}, A)$ to $(\widehat{J}, C)$. Furthermore for all $x \in \operatorname{Supp}(\widehat{F}, A)$ we have

$$\begin{aligned}(\varphi^* \circ \varphi)\left(\widehat{F}(x)\right) &= \varphi^*\left(\varphi\left(\widehat{F}(x)\right)\right) = \varphi^*\left(\widehat{G}(\psi(x))\right) \\ &= \widehat{J}(\psi^*(\psi(x))) = \widehat{J}((\psi^* \circ \psi)(x)).\end{aligned} \tag{30}$$

Hence $(\varphi^* \circ \varphi, \psi^* \circ \psi)$ is a normalistic vague soft group homomorphism from $(\widehat{F}, A)$ to $(\widehat{J}, C)$. □

Theorem 33. *Let X and Y be groups and let $(\widehat{F}, A)$ and $(\widehat{G}, B)$ be vague soft sets over X and Y, respectively. If $(\widehat{F}, A)$ is a normalistic vague soft group over X and $(\widehat{F}, A) \cong_N (\widehat{G}, B)$, then $(\widehat{G}, B)$ is a normalistic vague soft group over Y.*

Proof. Since $(\widehat{F}, A) \cong_N (\widehat{G}, B)$, there exists a vague soft group isomorphism from $(\widehat{F}, A)$ to $(\widehat{G}, B)$. This means that there exists an isomorphism φ from X to Y and a bijective mapping ψ from A to B which satisfies $\varphi(\widehat{F}(x)) = \widehat{G}(\psi(x))$ for every $x \in A$. Now suppose that $(\widehat{F}, A)$ is a normalistic vague soft group over X. Then $\widehat{F}(x)$ is a normal vague subgroup of X for all $x \in \operatorname{Supp}(\widehat{F}, A)$. Thus $\varphi(\widehat{F}(x))$ is a normal vague subgroup of Y for all $x \in \operatorname{Supp}(\widehat{F}, A)$. Since ψ is a bijective mapping, for all $y \in \operatorname{Supp}(\widehat{G}, B) \subseteq B$, there exists an $x \in A$ such that $y = \psi(x)$. Hence we have $\varphi(\widehat{F}(x)) = \widehat{G}(y) = \widehat{G}(\psi(x))$ for all

$x \in A$. As such, $\widehat{G}(y)$ is a normal vague subgroup of Y for all $y \in \mathrm{Supp}(\widehat{G}, B)$. □

Corollary 34. *Let* $(\widehat{F}, A)$ *and* $(\widehat{G}, B)$ *be normalistic vague soft groups over* X *and* Y*, respectively, and* $(\widehat{F}, A) \cong_N (\widehat{G}, B)$*. If* $\widehat{F}(x)$ *is a normal vague subgroup of* X*, then* $\widehat{G}(\psi(x))$ *is a normal vague subgroup of* Y *and* $\widehat{F}(x) \cong_N \widehat{G}(\psi(x))$*.*

6. Conclusion

In this paper, we continue to further develop the initial theory of vague soft groups. We successfully introduced the novel concept of normalistic vague soft group homomorphisms as an extension to the notion of vague soft homomorphism. We also further developed and studied the concept of normalistic vague soft groups through the introduction of the notion of trivial normalistic vague soft groups and absolute normalistic vague soft groups as well as studying the behavior of normalistic vague soft groups under the normalistic vague soft group homomorphism. Lastly, it is proven that the homomorphic image and preimage of a normalistic vague soft group are preserved under the normalistic vague soft group homomorphism.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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