Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2014, Article ID 635898, 7 pages
http://dx.doi.org/10.1155/2014/635898

Research Article

Fuzzy Optimization of Option Pricing Model and Its Application in Land Expropriation

Aimin Heng, Qian Chen, and Yingshuang Tan

College of Economics and Business Administration, Chongqing University, Chongqing 400044, China

Correspondence should be addressed to Aimin Heng; hengniu528@sisu.edu.cn

Received 11 April 2014; Accepted 8 June 2014; Published 17 July 2014

Academic Editor: Jian-Wen Peng

Option pricing is irreversible, fuzzy, and flexible. The fuzzy measure which is used for real option pricing is a useful supplement to the traditional real option pricing method. Based on the review of the concepts of the mean and variance of trapezoidal fuzzy number and the combination with the Carlsson-Fuller model, the trapezoidal fuzzy variable can be used to represent the current price of land expropriation and the sale price of land on the option day. Fuzzy Black-Scholes option pricing model can be constructed under fuzzy environment and problems also can be solved and discussed through numerical examples.

1. Introduction

Decision-makers with incomplete information may often produce errors in the process of observation and statistics due to the subjective cognition and interaction of the objective and subjective. At the same time, the preference of utility function to characterize also reflects heterogeneity and nonuniformity. The original study was not explicitly aware of the breakthrough of heterogeneity or nonuniformity; however, excessive seeking of the accuracy of input variables led to the ambiguity of output result. The traditional real option pricing method ignored this ambiguity and easily got wrong decisions. Therefore, the estimation of the pricing parameters made by decision-makers is fuzzy. The fuzzy measure which is used for real option pricing is a useful supplement to the traditional real option pricing method.

The B-S pricing formula usually uses the random probability to represent the uncertain factors, but in practical problems it often contains fuzziness and the fuzzy theory is a powerful tool to deal with them. With the precondition of considering the fuzziness of pricing parameters, many scholars have improved the classical theory of real option pricing. Carlsson et al. [1, 2] introduced the fuzzy concept to the traditional Black-Scholes model (B-S model), publishing a series of research results on fuzzy real option; for example, Yoshida [3] established the fuzzy real option model and concluded the enterprise value of fuzzy number. On the basis of traditional B-S model, it used trapezoidal fuzzy number to estimate the net present value of expected cash flows and introduced the concepts of mean value and variance of probability. It used dynamic decision tree to choose suitable investment opportunities for determining the execution time to delay the optimal option. But the study only got the maximized returns of the late strategic investment stage. According to Bellman dynamic programming principle, the maximized returns of the late strategic investment stage do not mean the maximized returns of the whole strategy document. Meanwhile, the intertemporal investment also did not consider the effect of the fuzzy volatility of interest rate on the whole income. Based on the model of equivalent martingale measure and the B-S model, considering that the stock price was a fuzzy random, Yoshida [3] proposed a symmetric triangular fuzzy number. He gave the model of European option pricing and hedge strategies by using the fuzzy overall assessment to define the fuzzy expectation with the assumption that the fuzzy degree was meant to be proportional to the stock price. Yoshida also took into account the investors' subjective judgment and constructed an optimized equation that uses the probability and fuzzy expectation to describe the randomness and fuzziness to solve the optimal stopping problems and study the discrete model of American put option under fuzzy optimization. This not only led to the best execution time of American option but also concluded the range or interval price of optimal expected option. Zmeškal [4] evaluated the enterprise value using the European option of fuzzy random variables. Based on the main introduction of fuzzy numbers, he introduced B-S formula and established the pricing model based on fuzzy random variables. Although the input variable of the model is fuzzy number, the output variable is interval value, and the model also takes into account the fuzziness of input parameters, the simple linear fuzzy number cannot accurately describe the uncertain form of income. Liu [5] took the fuzzy theory into pricing model of currency options, using the risk-free rate, volatility, and primary property prices as a fuzzy number, to discuss pricing currency that is under fuzzy environment. Under the assumption of the relaxed B-S, Wu [6] modified the classic B-S formula by means of the fuzzy theory and concluded the B-S formula of European option pricing formula under a fuzzy form, B-S formula of payment of dividends, and formula for the value of a put and call option. Having analyzed the existing business value and strategic synergy value of acquisition enterprises, Po and Hua [7] presented the Black-Sholes option pricing model of the existing business value as well as the fuzzy binomial option pricing model of strategic synergy value. By the help of B-S model, Zhu et al. [8] put forward an idea of transforming the expert evaluation interval of expected present value of cash flow into the normal fuzzy number and also verified the rationality that the latter can be employed to estimate the expected present value of cash flow. Based on the traditional option pricing method, Shuxia [9] made an analysis of the incomprehensiveness of the methods in existence. And in doing so, she found that the practical pricing problem was processed in an uncertain environment generated by randomness and fuzziness. And then, she not only compared and analyzed the research progress that emerging fuzzy option pricing theory had made but also made her own expectation considering the trend of the research. Below are other related researches: Kong and Kwok [10], Thiagarajah et al. [11], Chrysafis and Papadopoulos [12], Guerra et al. [13], Muzzioli and Reynaerts [14], Wang and Hwang [15], Liu [16], and Lee et al. [17].

In a word, the documents above apply fuzzy theory to options pricing theory, but the results of most of the fuzzy option pricing models are still within an interval or just some fuzzy numbers. This requires to further narrow down the range of possible prices or to conduct necessary researches on the expected value, optimistic value, and the pessimistic value of fuzzy option and this is what we should do to make it more practical. All the above are the matters we should take into account. The profit and cost of this project will be measured by the improved trapezoidal fuzzy number in the following dissertation and influence exerted by the fuzzy fluctuation of risky-free interest on the option value interval will be also taken into consideration. We will discuss several major modified models of practical option pricing in depth on the ground that related concepts about variance and average value of trapezoidal fuzzy number have been reviewed. As changes of the external environment make the investment on agriculture land irreversible, fuzzy and flexible, possible profit of agriculture land bought in by potential opportunities and the added value obtained by taking advantage of all kinds of chances in a smart way cannot be ignored when compensating for land expropriation. And this means that attention should be paid not only to the certain value of agriculture land but also to the uncertain value of agriculture land, namely, the fuzzy value. On this basis and combined with Carlsson-Fuller model, we will make use of trapezoidal fuzzy variable to show the present price and the price of land on option expiration date during land expropriation and then we can build a B-S option pricing model by the help of statistics to figure it out.

Definition 1 (see [2, 3]). A fuzzy set $A \in F$ is called a trapezoidal fuzzy number with core $[a, b]$, left width α, and right width β if its membership function has the following form:

$$A(t) = \begin{cases} 1 - \dfrac{a-t}{\alpha} & \text{if } a - \alpha \le t < a \\ 1 & \text{if } a \le t \le b \\ 1 - \dfrac{t-b}{\beta} & \text{if } b < t \le b + \beta \\ 0 & \text{Otherwise.} \end{cases} \quad (1)$$

And we may use the notation $A = (a, b, \alpha, \beta)$, $\forall \gamma \in [0, 1]$; then it can easily be shown that

$$[A]^{\gamma} = [a - (1-\gamma)\alpha, b - (1-\gamma)\beta]. \quad (2)$$

The support of A is $(a - \alpha, b + \beta)$.

A trapezoidal fuzzy number with core (a, b) may be seen as a context-dependent description (α and β define the context) of the property: "the value of a real variable is approximately in (a, b)."

Let $[A]^{\gamma} = (a_1(\gamma), b_1(\gamma), \alpha_1(\gamma), \beta_1(\gamma))$, $[B]^{\gamma} = (a_2(\gamma), b_2(\gamma), \alpha_2(\gamma), \beta_2(\gamma))$ be fuzzy numbers and let $\lambda \in \mathbb{R}$ be a real number; using the extension principle we can verify the following rules for addition and scalar multiplication of fuzzy numbers:

$$\begin{aligned} [A+B]^{\gamma} &= [a_1(\gamma) + a_2(\gamma), b_1(\gamma) + b_2(\gamma), \\ &\qquad \alpha_1(\gamma) + \alpha_2(\gamma), \beta_1(\gamma) + \beta_2(\gamma)], \\ [A-B]^{\gamma} &= [a_1(\gamma) - b_2(\gamma), b_1(\gamma) - a_2(\gamma), \\ &\qquad \alpha_1(\gamma) + \alpha_2(\gamma), \beta_1(\gamma) + \beta_2(\gamma)], \\ [\lambda A]^{\gamma} &= \lambda [A]^{\gamma}. \end{aligned} \quad (3)$$

From Carlsson and Fuller [2] and Yoshida [3], it is easy to see the (crisp) possibility mean (or expected) value of A and the (possibility) variance of A as (it is easy to see that if $[A]^{\gamma} = (a_1(\gamma), b_1(\gamma), \alpha_1(\gamma), \beta_1(\gamma))$ is a trapezoidal fuzzy number)

$$\begin{aligned} E(A) &= \int_0^1 \gamma (a - (1-\gamma)\alpha + b + (1-\gamma)\beta)\, d\gamma \\ &= \frac{a+b}{2} + \frac{\beta - \alpha}{6}. \end{aligned} \quad (4)$$

We introduce the (possibility) variance of $[A]^{\gamma} = (a_1(\gamma), b_1(\gamma))$ as

$$\delta^2(A) = \int_0^1 \gamma\left[\frac{a_1(\gamma)+b_1(\gamma)}{2} - a_1(\gamma)\right]^2 + \left[\frac{a_1(\gamma)+b_1(\gamma)}{2} - b_1(\gamma)\right]^2 d\gamma = \frac{1}{2}\int_0^1 \gamma\left[b_1(\gamma) - a_1(\gamma)\right] d\gamma. \quad (5)$$

The variance of A is defined as the expected value of the squared deviations between the arithmetic mean and the endpoints of its level sets; that is, the possibility variance of A is defined as the expected value of the left-hand endpoint between the arithmetic mean and the endpoints of its level sets.

It is easy to see that if $[A]^{\gamma} = (a_1(\gamma), b_1(\gamma), \alpha_1(\gamma), \beta_1(\gamma))$ is a trapezoidal fuzzy number then

$$\delta^2(A) = \frac{(b-a)^2}{4} + \frac{(b-a)(\alpha+\beta)}{6} + \frac{(\alpha+\beta)^2}{24}. \quad (6)$$

2. The Model and Its Application in Land Expropriation

(1) B-S Model under the Fuzzy Environment and Its Application in Land Expropriation. The B-S option pricing model is composed of Fischer Black and Myron Scholes in 1973. Black and Scholes made a major breakthrough by deriving a differential equation that must be satisfied by the price of any derivative security dependent on a nondividend paying stock. According to the B-S, we can get the expected value in the random case:

$$E[\max(V-I, 0)]. \quad (7)$$

The theoretical price of call option value is expected to be risk-free interest rate discount result:

$$C = e^{-r\Delta T} E[\max(V-I, 0)]. \quad (8)$$

So we can get the formula

$$C = Ve^{-r\Delta T} N(d_1) - Ie^{-r\Delta T} N(d_2), \quad (9)$$

where

$$d_1 = \frac{\text{In}(V/I) + \left(r + (1/2)\sigma^2\right)\Delta T}{\sigma\Delta T}, \qquad d_2 = \frac{\text{In}(V/I) + \left(r - (1/2)\sigma^2\right)\Delta T}{\sigma\Delta T}, \quad (10)$$

in which σ is the volatility, r is risk-free interest rate, and $N(x)$ is the probability of less than d under the condition of the standard normal distribution. I is the exercise price, V is the current stock price, and T is the options expiration time.

The V and I usually are assumed to be a fixed number at the pricing formula, but because the value of the underlying asset and the present value of cash flow and prices have greater uncertainty and are usually not a certain number, V and I cannot be characterized by a single number, so this assumption is not realistic. We can be expressed by using a possibility distribution of the form, such as the use of trapezoidal fuzzy number. Then according to Carlsson and Fuller [2], Yoshida [3], and Zmeškal [4] of the related literature, we can get the real option pricing formula under the fuzzy environment

$$\begin{aligned}\text{FROV} = \big(&V_1 e^{-\mu\Delta T} N(d_1) - I_2 e^{-r\Delta T} N(d_2),\\ &V_2 e^{-\mu\Delta T} N(d_1) - I_1 e^{-r\Delta T} N(d_2),\\ &\alpha_1 e^{-\mu\Delta T} N(d_1) + \beta_2 e^{-r\Delta T} N(d_2),\\ &\beta_1 e^{-\mu\Delta T} N(d_1) + \alpha_2 e^{-r\Delta T} N(d_2)\big)\end{aligned} \quad (11)$$

$$d_1 = \frac{\ln(E(V)/E(I)) + \left(r - \mu + \left(\mu^2/2\right)\right)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}. \quad (12)$$

That is, $\mu \in (0, r)$ is drift, and $r > 0$ is risk-free interest rate. The core of the trapezoidal fuzzy numbers V lies in the interval $[V_1, V_2]$. $(V_2 + \beta_1)$ and $(V_1 - \alpha_1)$, respectively, mean the upper and lower bounds of the uncertain factor of the expected project profit cash flow. Similarly, The core of the trapezoidal fuzzy number I is the most likely value of the cost of the present value of expected cash flows and it lies in the interval $[I_1, I_2]$. $(I_2 + \beta_2)$ and $(I_1 - \alpha_2)$, respectively, mean the upper and lower bounds of the expected investment cost.

The model is applied to the problem of land expropriation in the case. Due to the current value of I and the option expiration day present value of city land sale value cash flow V is fuzzy. It can be represented by trapezoidal fuzzy numbers, the current value: $I = [I_1, I_2, \alpha_2, \beta_2]$. The core of the trapezoidal fuzzy numbers I lies in the interval $[I_1, I_2]$ most likely. $(I_2 + \beta_2)$, $(I_1 - \alpha_2)$, respectively, mean the upper and lower bounds of the uncertain factor of the expected project profit cash flow. The option expiration city land transfer price is expressed as trapezoidal fuzzy number $V = [V_1, V_2, \alpha_1, \beta_1]$. The core of the trapezoidal fuzzy numbers V lies in the interval $[V_1, V_2]$ most likely. $(V_2 + \beta_1)$, $(V_1 - \alpha_1)$, respectively, mean the upper and lower bounds of expected transfer value of the expected project profit cash flow. We can put the expropriation of land has the right at a European call option, then, the real option pricing formula under the fuzzy environment by (11).

(2) Fuzzy Optimization Problem of Sequence of Real Option Pricing Model and Its Application in Land Expropriation. Geske [18] presents a theory and a method for pricing options on options or compound options. The method can be generalized to value many corporate liabilities and call corrects some important biases of the B-S model. The compound call option formula which is itself an option on the assets of

the firm. This perspective incorporates leverage effects into option pricing and consequently the variance of the rate of return on the stock is not constant as B-S assumed but is a function of the level of the stock price. The B-S formula is shown to be a special case of the compound option formula. Geske [18] thought that many investment opportunities in essence had sequence properties and the next opportunity might be performed only after the execution of the first opportunity. Based on the assumption of a continuous time, the Geske model thought of an enterprise where products could face the market only after the two processes, which was the initial development and research and completed development. In fact, if land projects choose the acquisition of investment stage, supposed to go through two stages of initial acquisition and completed acquisition, then there are features of its own sequence of growth option, and the option is European call option. Land acquisition and investment behavior in succession are connected with the mutual influence and restrict each other. The Geske model through a simple standard option pricing model based on the assets of standard normal distribution, if change in variable normal distribution hypothesis, the option pricing formula can be expressed as

$$C = Ve^{-rT}M\left(k,h;\rho\right) - Ie^{-rT}M\left(k-\sigma\sqrt{t},k-\sigma\sqrt{T};\rho\right) - I^{*}e^{-rT}N_1\left(k-\sigma\sqrt{t}\right), \tag{13}$$

in which

$$k = \frac{\text{In}\left(F/F_c\right)+(1/2)\sigma^2 t}{\sigma\sqrt{t}}, \qquad h = \frac{\text{In}\left(F/K\right)+(1/2)\sigma^2 T}{\sigma\sqrt{T}}, \qquad \rho = \sqrt{\frac{t}{T}}. \tag{14}$$

V is the options expiration date of current value of city land sale value of cash flow. F_c is d the first buyer's option, is the critical value of land project delivery, and can be calculated using the Black-Scholes model. The first variable is less than k in $M(k,h;\rho)$. The second variable is less than h and two correlation coefficients for the ρ standard normal distribution of the cumulative probability function. σ is the volatility. I is the initial acquisition cost. I^{*} is a complete cost of the land requisition required. r is risk-free interest rate. T is a sequence of time to expiration of the option, and t is the first time to expiration of the option. The Geske model and the trapezoidal fuzzy number can be obtained by combining the fuzzy Geske model environment:

$$\begin{aligned}
\text{FROV} = {} & \left[V_1,V_2,\alpha_1,\beta_1\right]N_2\left(h+\sigma\sqrt{t_1},k+\sigma\sqrt{t_2};\sqrt{t_1/t_2}\right) \\
& - \left[I_1,I_2,\alpha_2,\beta_2\right]e^{-rt_2}N_2\left(h,k;\sqrt{t_1/t_2}\right) \\
& - \left[I_1',I_2',\alpha_2',\beta_2'\right]e^{-rt_1}N_1\left(h\right) \\
= {} & \Big\{V_1N_2\left(h+\sigma\sqrt{t_1},k+\sigma\sqrt{t_2};\sqrt{t_1/t_2}\right) \\
& - I_2e^{-rt_2}N_2\left(h,k;\sqrt{t_1/t_2}\right) - I_2'e^{-rt_1}N_1\left(h\right), \\
& V_2N_2\left(h+\sigma\sqrt{t_1},k+\sigma\sqrt{t_2};\sqrt{t_1/t_2}\right) \\
& - I_1e^{-rt_2}N_2\left(h,k;\sqrt{t_1/t_2}\right) - I_1'e^{-rt_1}N_1\left(h\right), \\
& \alpha_1N_2\left(h+\sigma\sqrt{t_1},k+\sigma\sqrt{t_2};\sqrt{t_1/t_2}\right) \\
& + \beta_2e^{-rt_2}N_2\left(h,k;\sqrt{t_1/t_2}\right) + \beta_2'e^{-rt_1}N_1\left(h\right), \\
& \beta_1N_2\left(h+\sigma\sqrt{t_1},k+\sigma\sqrt{t_2};\sqrt{t_1/t_2}\right) \\
& + \alpha_2e^{-rt_2}N_2\left(h,k;\sqrt{t_1/t_2}\right) + \alpha_2'e^{-rt_1}N_1\left(h\right)\Big\}
\end{aligned} \tag{15}$$

in which

$$\begin{aligned}
h &= \left[V_1,V_2,\alpha_1,\beta_1\right]\frac{\text{In}\left(E\left(V_0\right)/E\left(V_c\right)\right)+\left(r-(1/2)\sigma^2\right)t_1}{\sigma\sqrt{t_1}}, \\
k &= \left[V_1,V_2,\alpha_1,\beta_1\right] \\
&\quad\times\frac{\text{In}\left(E\left(V_0\right)/E\left(I_1,I_2,\alpha_2,\beta_2\right)\right)+\left(r-(1/2)\sigma^2\right)t_2}{\sigma\sqrt{t_2}}, \\
a_2 &= a_1-\sigma\sqrt{t_1}, \qquad b_2 = b_1-\sigma\sqrt{t_2}.
\end{aligned} \tag{16}$$

V_c is the sequence of options that should value the implementation of land acquisition, and V_{c_1} can be calculated by the following equation:

$$\left[V_1,V_2,\alpha_1,\beta_1\right]N_1\left(K+\sigma\sqrt{t_2-t_1}\right) - \left[I_1,I_2,\alpha_2,\beta_2\right]e^{-r(t_2-t_1)}N\left(k\right) - \left[I_1',I_2',\alpha_2',\beta_2'\right] = 0, \tag{17}$$

in which $[V_1,V_2,\alpha_1,\beta_1]$ is described by trapezoidal fuzzy number option expiration day present value of city land sale value of cash flow. $E(V_c)$ is the first buyer's option, is the critical value of land project delivery, and can be calculated using the Black-Scholes model. In $N_2(h+\sigma\sqrt{t_1},k+\sigma\sqrt{t_2};\sqrt{t_1/t_2})$ the first variable is less than a_1, the second variable is less than b_1, and two variable correlation coefficients are probability function of standard normal cumulative distribution of $\sqrt{t_2/t_1}$. N_1 is one-dimensional normal distribution function; $[I_1,I_2,\alpha_2,\beta_2]$ is the initial acquisition costs of trapezoidal fuzzy numbers. $[I_1',I_2',\alpha_2',\beta_2']$ is a trapezoidal fuzzy number representation of the complete cost land acquisition phase. r is risk-free interest rate. T is a sequence of time to expiration of the option.

(3) Fuzzy Optimization Problem of Elettro and Rossella Model and Its Application in Land Expropriation. E. Agliardi and R. Agliardi [19] established a generalization of the Geske formula for compound options which is derived in the case of time-dependent volatility and interest rate. A comparison with the Geman, El Karoui, and Rochet formula is also provided. Such a generalization seems to be more appropriate for the evaluation of compound real (growth) options under the fuzzy environment. The fuzzy real option pricing model of fuzzy optimization is as follows:

$$\begin{aligned}
\text{FROV} = {} & [V_1, V_2, \alpha_1, \beta_1] N_2(q_2(t), p_2(t); s(t)) \\
& - [I_1, I_2, \alpha_2, \beta_2] e^{-\int_t^T r(\tau)d\tau} N_2(q_1(t), p_1(t); s(t)) \\
& - [I_1', I_2', \alpha_2', \beta_2'] e^{-\int_t^T r(\tau)d\tau} N_1(q_1(t)) \\
= {} & \Big\{ V_1 N_2(q_2(t), p_2(t); s(t)) \\
& \quad - I_2 e^{-\int_t^T r(\tau)d\tau} N_2(q_1(t), p_1(t); s(t)) \\
& \quad - I_2' e^{-\int_t^T r(\tau)d\tau} N_1(q_1(t)), \\
& \quad V_2 N_2(q_2(t), p_2(t); s(t)) \\
& \quad - I_1 e^{-\int_t^T r(\tau)d\tau} N_2(q_1(t), p_1(t); s(t)) \\
& \quad - I_1' e^{-\int_t^T r(\tau)d\tau} N_1(q_1(t)), \\
& \quad \alpha_1 N_2(q_2(t), p_2(t); s(t)) \\
& \quad + \beta_2 e^{-\int_t^T r(\tau)d\tau} N_2(q_1(t), p_1(t); s(t)) \\
& \quad + \beta' e^{-\int_t^T r(\tau)d\tau} N_1(q_1(t)), \\
& \quad \beta_1 N_2(q_2(t), p_2(t); s(t)) \\
& \quad + \alpha_2 e^{-\int_t^T r(\tau)d\tau} N_2(q_1(t), p_1(t); s(t)) \\
& \quad + \alpha' e^{-\int_t^T r(\tau)d\tau} N_1(q_1(t))_2 \Big\},
\end{aligned} \tag{18}$$

in which

$$\begin{aligned}
q_1(t) &= \frac{\text{In}(E(V)/E(V_c)) + \int_t^{T_1} r(\tau) - (1/2)\sigma^2(\tau)\, d\tau}{\sqrt{\int_t^{T_1} \sigma^2(\tau)\, d\tau}}, \\
q_2(t) &= \frac{\text{In}(E(V)/E(V_c)) + \int_t^{T_1} r(\tau) + (1/2)\sigma^2(\tau)\, d\tau}{\sqrt{\int_t^{T_1} \sigma^2(\tau)\, d\tau}}, \\
p_1(t) &= \frac{\text{In}(E(V)/E(I)) + \int_t^{T_2} r(\tau) - (1/2)\sigma^2(\tau)\, d\tau}{\sqrt{\int_t^{T_2} \sigma^2(\tau)\, d\tau}}, \\
p_2(t) &= \frac{\text{In}(E(V)/E(I)) + \int_t^{T_2} r(\tau) + (1/2)\sigma^2(\tau)\, d\tau}{\sqrt{\int_t^{T_2} \sigma^2(\tau)\, d\tau}}, \\
\rho(t) &= \frac{\sqrt{\int_t^{T_1} \sigma^2(\tau)\, d\tau}}{\sqrt{\int_t^{T_2} \sigma^2(\tau)\, d\tau}};
\end{aligned} \tag{19}$$

V_c is the sequence of options and should value the implementation of land acquisition: V_c = that value of V such that.

The model is applied to the problem of land expropriation in the case analogously. $[V_1, V_2, \alpha_1, \beta_1]$ is described by trapezoidal fuzzy number option expiration day present value of city land sale value of cash flow. $E(V_c)$ is the first buyer's option, is the critical value of land project delivery, and can be calculated using the Black-Scholes model. $[I_1, I_2, \alpha_2, \beta_2]$ is the initial acquisition costs of trapezoidal fuzzy numbers. $[I_1', I_2', \alpha_2', \beta_2']$ is a trapezoidal fuzzy number representation of the complete cost land acquisition phase. Elettro and Rossella model is generalization of the Geske formula for compound options, which is derived in the case of time-dependent volatility and interest rate. Such generalization seems to be more appropriate for the evaluation of compound real (growth) options.

Sequence option and compound option have been widely used in the value evaluation model of project under the fuzzy environment, such as real options, the technological innovation of enterprises in manufacturing industry of optimal investment decision, because most investment opportunities has order properties, namely has the property of options, so selecting the innovation of enterprise technology investment stage has a sequence of growth options and investment behavior and there is an interaction and mutual restriction bettween them. In fact, land acquisition projects investments usually contain multiple processes; each process contains adoption. Therefore, land acquisition has the right to choose to stop at each stage or postpone the land acquisition projects in order to maximize the benefit. Of course, this model only discussed the fuzzy real option pricing model sequence of the same volatility, the volatility of different fuzzy sequence of real option pricing model, the expected rate of fuzzy real option pricing model of sequence of time-varying function returns and volatility, and multistage compound growth option of fuzzy real option pricing model; we also do the corresponding discussion. At the same time, in the above we mainly studied the European option pricing problem based on fuzzy theory and the American option pricing problem and the two-fork tree option pricing problem based on fuzzy theory, we do no research on it.

3. An Approach to Real Option Valuation

For example, after the fuzzy assessment and treatment, under fuzzy environment we use integrated land as specific calculation of the option value. As the only unobservable variable in the model of option pricing, the volatility of city land value can use historical volatility to be estimated. In this study, it is supposed that the farmland circulation can be deferred for one year. If time deposits at the bank bring in 3 percent a year and the risk-free interest rate takes this as the standard 3 percent. Because this paper is based on the annual data, here the value leakage rate (dividend rate) is the risk free rate; that is, the risk-free interest rate is 3%. If correlative parameters determined above are substituted into formula (6), it can be concluded.

The current value of city land is $I = (4000, 6000, 4500, 4500)$, and on the maturity date of option, the following is the city land sales:

$$V = (5500, 7500, 5000, 5000), \qquad r = 3\%, \qquad \mu = 0.02, \qquad T = 3. \tag{20}$$

Then

$$\begin{aligned} E(I) &= \int_0^1 \gamma\left(a - (1-\gamma)\alpha + b + (1-\gamma)\beta\right) \\ &= \frac{V_1 + V_2}{2} + \frac{\beta_1 - \alpha_1}{6} = 5000, \\ E(V) &= \frac{I_2 + I_1}{2} - \frac{\beta_2 - \alpha_2}{6} = 6500, \\ \delta(V) &= \sqrt{\frac{(V_2 - V_1)^2}{4} + \frac{(V_2 - V_1)(\alpha_1 + \beta_1)}{6} + \frac{(\alpha_1 + \beta_1)^2}{24}} \\ &= 2315.48, \\ \sigma &= \frac{\delta(V)}{E(V)} = 44.85\%, \\ N(d_1) &= N\left(\frac{\ln(E(V)/E(I)) + \left(r - \mu + \mu^2/2\right)T}{\sigma\sqrt{T}}\right) \\ &= 0.6443, \\ N(d_2) &= N\left(d_1 - \sigma\sqrt{T}\right) = 0.3156. \end{aligned} \tag{21}$$

Therefore, when circulation of farmland is comprehensive land, the selection value is

$$\begin{aligned} \text{FROV} = \big(& V_1 e^{-\mu\Delta T} N(d_1) - I_2 e^{-r\Delta T} N(d_2), \\ & V_2 e^{-\mu\Delta T} N(d_1) - I_1 e^{-r\Delta T} N(d_2), \\ & \alpha_1 e^{-\mu\Delta T} N(d_1) + \beta_2 e^{-r\Delta T} N(d_2), \\ & \beta_1 e^{-\mu\Delta T} N(d_1) + \alpha_2 e^{-r\Delta T} N(d_2)\big) \\ = (& 1854.72, 3562.55, 4146.03, 4146.03). \end{aligned} \tag{22}$$

And the expected value is 2708.64; the most possible interval is within (1854.72, 3562.55); the number of the left point is 2291.31; the number of the right point is 7708.58. With the same method, we can work out the selective value of the land at the time of the agriculture lands being transformed into other forms, which is based on the other two models and, of course, some related numbers under fuzzy circumstance. The example above only discusses how to apply practical option to land expropriation if the volatilities are the same. Likewise, the similar way can be used here to calculate the results and discuss the problem of applying the real option with a different volatility and the sequential real option into the process of land expropriation according to the fact.

4. Conclusion

As an instrument to hedge against risk, option is extremely popular among financial marketers and thus pricing the option scientifically and accurately is in its imperial need when it comes to get rid of the risks of the financial market. For the time being, although there is increasing amount of application of fuzzy theory in real option pricing, we are in lack of widely agreed views on how to exercise the fuzzy optimization model in real option pricing theory for the reason that the influence of fuzziness on the market cannot be quantified. So, fuzzy option pricing is still a project deserving researching on. Considering that the decision-makers' estimation about the pricing parameter is fuzzy, we give priority to the totality of the heterogeneity utility function when it comes to deal with the heterogeneity and make breakthrough in uniform reason of option pricing. Utility function shows its preference; namely, its heterogeneity and changes of expected utility can be achieved by reconstructing measurement. Also, because the decision-makers information is always inconclusive, the statistics which people observe and count are inevitably inaccurate. The development of the fuzzy mathematics in modern society makes the fuzzy measuring a successful instrument of practicers' cognition capacity. The model of applying fuzzy measuring into option pricing derives from the attempt to supplement the classical pricing methods. Yet, unnecessarily, the original researches laid too much emphasis on pursuing the accuracy of the input numbers limiting it within the frame of uniform reason, instead of recognizing clearly the heterogeneity or the breakthrough in the no uniform reason. On the contrary, it leads to fuzziness of output result.

After analyzing the feature of the real option of the land expropriation, we introduce the fuzzy theory into the real option pricing model when evaluating the value of the project. By combining the fuzzy expected value and the variance with Carlsson-Fuller model, we use the trapezoidal fuzzy variable to indicate the present price of the city land and the price when option is expired, constructing a Black-Scholes option pricing model in fuzzy circumstance to work it out through number of examples. This method overcomes the impossibility of predicting model parameter caused by lack of enough statistics and fuzziness of land expropriation by employing a series of operations such as the fuzzy optimization of the Black-Scholes model with the same fluctuation ratio; E. Agliardi and R. Agliardi [19] established a generalization of the Geske formula for compound options which is derived in the case of time-dependent volatility and interest rate. By the help of these mentioned fuzzy-optimized models, we discuss the following problems with numbers.

How to apply real option with the same fluctuation and also the sequential real option pricing model to land expropriation and a generalization seems to be more appropriate for the evaluation of compound real options to land expropriation under the fuzzy environment.

We can conduct similar calculation and discussion according to the situation of real life. Of course, as to other untouched researches such as how to optimize the fuzzy sequential real option pricing model with different fluctuation ratio, how to optimize the fuzzy sequential real option pricing model regarding expected profit percentage and fluctuation ratio as time-varying function, and how to optimize the fuzzy real option pricing model with sequentially increased option over two phases including the American option pricing problem and binary tree option pricing problem both based on fuzzy theory, we can make full use of the similar research approach.

Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by Chongqing Social and Science Commission (2013) (no. 2013YBFX105), by the State Scholarship Fund, by the Ministry of Education of China (2011) (no. 11XJC820001), by the National Postdoctoral Science Foundation (2013) (no. 126555), by Chongqing Science and Technology Commission (2013) (no. cstc2013jccxA00048), by Science and Technology Commission of Chongqing Shapingba District (2013) (no. 201316), and by Science and Technology Project of Yubei District, Chongqing, China (no. 2012 - social-20).

References

[1] C. Carlsson, R. Fullér, M. Heikkilä, and P. Majlender, "A fuzzy approach to R&D project portfolio selection," *International Journal of Approximate Reasoning*, vol. 44, no. 2, pp. 93–105, 2007.

[2] C. Carlsson and R. Fuller, "A fuzzy approach to real option valuation," *Fuzzy Sets and Systems*, vol. 139, no. 2, pp. 297–312, 2003.

[3] Y. Yoshida, "The valuation of European options in uncertain environment," *European Journal of Operational Research*, vol. 145, no. 1, pp. 221–229, 2003.

[4] Z. Zmeškal, "Application of the fuzzy-stochastic methodology to appraising the firm value as a European call option," *European Journal of Operational Research*, vol. 135, no. 2, pp. 303–310, 2001.

[5] F. Liu, "Pricing currency options based on fuzzy techniques," *European Journal of Operational Research*, vol. 193, no. 2, pp. 530–540, 2009.

[6] H. Wu, "Using fuzzy sets theory and Black-Scholes formula to generate pricing boundaries of European options," *Applied Mathematics and Computation*, vol. 185, no. 1, pp. 136–146, 2007.

[7] H. H. Po and Q. W. Hua, "Method and application of enterprise value evaluation in M&A based on real options," *Systems Engineering-Theory Methodology Applications*, no. 2, pp. 11–16, 2005.

[8] D. M. Zhu, T. C. Zhang, D. L. Chen, and H.X. Gao, "A new fuzzy pricing approach to real options," *Journal of Northeastern University (Natural Science)*, vol. 29, no. 11, pp. 1544–1547, 2008.

[9] L. Shuxia, "Recent developments in option pricing theory under fuzzy environment," *Journal of Xidian University*, no. 5, pp. 51–55, 2008.

[10] J. J. Kong and Y. K. Kwok, "Real options in strategic investment games between two asymmetric firms," *European Journal of Operational Research*, vol. 181, no. 2, pp. 967–985, 2007.

[11] K. Thiagarajah, S. S. Appadoo, and A. Thavaneswaran, "Option valuation model with adaptive fuzzy numbers," *Computers & Mathematics with Applications*, vol. 53, no. 5, pp. 831–841, 2007.

[12] K. A. Chrysafis and B. K. Papadopoulos, "On theoretical pricing of options with fuzzy estimators," *Journal of Computational and Applied Mathematics*, vol. 223, no. 2, pp. 552–566, 2009.

[13] M. L. Guerra, L. Sorini, and L. Stefanini, "Parametrized fuzzy numbers for option pricing," in *Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE '07)*, pp. 1–6, IEEE, London, UK, July 2007.

[14] S. Muzzioli and H. Reynaerts, "American option pricing with imprecise risk-neutral probabilities," *International Journal of Approximate Reasoning*, vol. 49, no. 1, pp. 140–147, 2008.

[15] J. Wang and W.-L. Hwang, "A fuzzy set approach for R&D portfolio selection using a real options valuation model," *Omega*, vol. 35, no. 3, pp. 247–257, 2007.

[16] B. Liu, *Uncertainty Theory*, vol. 154 of *Studies in Fuzziness and Soft Computing*, Springer, Berlin, Germany, 2nd edition, 2007.

[17] C. Lee, G. Tzeng, and S. Wang, "A new application of fuzzy set theory to the Black-Scholes option pricing model," *Expert Systems with Applications*, vol. 29, no. 2, pp. 330–342, 2005.

[18] R. Geske, "The valuation of compound options," *Journal of Financial Economics*, vol. 7, no. 1, pp. 63–81, 1979.

[19] E. Agliardi and R. Agliardi, "A generalization of the Geske formula for compound options," *Elsevier Mathematical Social Sciences*, vol. 45, no. 1, pp. 75–82, 2003.