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## Research Article

# Characterizing Vibrating Cantilevers for Liquid Viscosity and Density Sensing

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Miniaturized liquid sensors are essential devices in online process or condition monitoring. In case of viscosity and density sensing, microacoustic sensors such as quartz crystal resonators or SAW devices have proved particularly useful. However, these devices basically measure a thin-film viscosity, which is often not comparable to the macroscopic parameters probed by conventional viscometers. Miniaturized cantilever-based devices are interesting alternatives for such applications, but here the interaction between the liquid and the oscillating beam is more involved. In our contribution, we describe a measurement setup, which allows the investigation of this interaction for different beam cross-sections. We present an analytical model based on an approximation of the immersed cantilever as an oscillating sphere comprising the effective mass and the intrinsic damping of the cantilever and additional mass and damping due to the liquid loading. The model parameters are obtained from measurements with well-known sample liquids by a curve fitting procedure. Finally, we present the measurement of viscosity and density of an unknown sample liquid, demonstrating the feasibility of the model.

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## 1. INTRODUCTION

For many applications like online process or condition monitoring, the liquid parameters viscosity and mass density are of high relevance. The use of conventional laboratory equipment is often not applicable due to its cost, space requirements, and other preconditions, for example, vibration-free mounting. Furthermore, sample taking for such devices often involves manual labor, tending to be time consuming and error-prone.

Microacoustic sensors like quartz thickness shear mode (TSM) resonators [1, 2] and surface acoustic wave (SAW) devices, for example, [3] have proved particularly useful alternatives to traditional viscometers [4]. However, these devices measure viscosity at comparatively high-shear rates and small vibration amplitudes. For non-Newtonian liquids, the results are, therefore, not directly comparable to those obtained from conventional viscometers. For complex liquids such as emulsions, it has also been shown that microacoustic devices may not be sufficient to detect rheological effects which are present only on the macroscopic scale [5].

Micromachined vibrating structures usually feature lower resonance frequencies and higher vibration amplitudes, making them more suitable for non-Newtonian and complex liquids [6]. Microcantilevers commonly used in atomic force microscopy [7–9] have been successfully used as liquid property sensors. They allow for simultaneous measurement of the liquid's viscosity and mass density, requiring sample volumes of less than 1 nL [10]. However, a highly sensitive optical readout is required to determine the beam's vibration amplitudes. When immersed in liquid, the cantilevers face strong deterioration of the quality factor due to high-dissipative effects [10]. Consequently, the vibration amplitudes drops even more, limiting the sensor's measurement range to low-viscous liquids. In other works, micromachined cantilevers and doubly clamped beams driven by Lorentz forces [6, 11–13] or by the piezoelectric effect [14, 15] have been utilized as liquid property sensors, and the feasibility of these sensors has been demonstrated for viscosities in the range up to several Pa·s.

In this contribution, we characterize resonating cantilevers for the measurement of mass density and viscosity

of liquids. They measure viscosity in a rheological domain which is more comparable to that probed by conventional laboratory instruments. The cantilevers feature piezoelectric excitation as well as piezoelectric readout. The vibrating part is about 55 mm long, but since only the cantilever tip is immersed in the liquid, the induced damping of the cantilever vibration is kept low. The sensors, therefore, exhibit high-quality factors, ranging from 20 to 60 even for highly viscous liquids. Consequently, the detection of the resonances could be accomplished by a simple readout electronics, and the measurement range is greatly extended. On the other hand, the sensitivity of the sensor is decreased. Furthermore, the sensor principle allows attaching different tips of well-defined geometries to the cantilevers.

When the cantilever tip is immersed in a liquid, the resonance frequency and the damping of the cantilever are influenced by the viscosity and density of the liquid. However, the cantilevers do not show a simple relationship between the result of such a measurement and the liquid parameters. Recently, several models have been devised to give a proper description of the interaction of a vibrating cantilever and the surrounding liquid, for example, [16–18] but most of these models assume fully immersed cantilevers, which are long and thin, that is, the width  $w$  is much smaller than the length  $L$ . For the designs considered in this work, only the cantilever tip is immersed in the liquid. For modeling the sensor-fluid interaction, the cantilever length  $L$  must therefore be replaced by the dipping depth  $d$ , which is in the same range as  $w$ . A solution for cantilevers featuring  $w \approx L$ , and accordingly  $w \approx d$  is given in [19]. In [14] and other works, the influence of the liquid loading on the cantilever’s frequency characteristics has been successfully modeled by approximating the forces acting on the cantilever tip by those acting on a sphere oscillating in a fluid. The results indicate that the cantilever tip is subject to an additional mass loading and an additional damping caused by the surrounding liquid.

In our work, we present a vibrating cantilever sensor and a setup for the measurement of viscosity and density. The interaction of the sensor and the liquid in which the tip is immersed is modeled by an oscillating sphere. Possible simplifications of the model are discussed, and a general model is devised. This model allows for simple calibration of the sensor in a set of liquids with well-known properties. Therefore, the knowledge of mechanical and electrical properties of the cantilever is not required, the model parameters are instead obtained from the calibration procedure.

## 2. SENSOR FABRICATION

The cantilever sensors used in this work are based on commercially available lead zirconate titanate (PZT) bimorph bending actuators (Argillon GmbH, Redwitz, Germany). They feature a length of 49.95 mm, a width of 7.2 mm, and a total thickness of 0.8 mm [20]. The cantilevers consist of two piezoelectric PZT layers on both sides of a carbon fiber substrate (Figure 1(a)). The PZT layers are polarized in thickness direction. Electrodes (1, 2, 3) allow for excitation

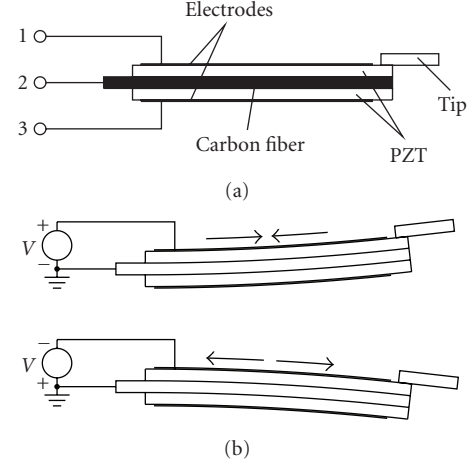


FIGURE 1: (a) PZT bending actuator with attached tip. The bending actuator consists of a carbon fiber substrate and two piezoelectric layers. The electrodes (1, 2, 3) allow for excitation and readout of the sensor. (b) Applying a voltage to the upper layer leads to contraction or elongation of the upper layer as indicated by the arrows.

TABLE 1: Tip geometries (rectangular cross-section) and tip materials.

Tip	Width $w$	Thickness	Material
Tip A	7 mm	0.5 mm	silicon
Tip B	5 mm	0.5 mm	silicon
Tip C	4 mm	0.3 mm	brass
Tip D	2 mm	0.5 mm	silicon

of the actuator. Applying a voltage between the top electrode (electrode 1) and the center electrode (2 in Figure 1(a)), which is used as ground electrode (Figure 1(b)) causes a contraction or elongation in the upper PZT layer but not in the substrate, and, therefore, deforms the cantilever. In our setup (Figure 2(a)) a sinusoidal voltage is applied, which leads to bending vibrations of the beam. A maximum voltage of 200 V can be applied to the cantilever. The actual beam deflection is determined by measuring the voltage at the sensing electrode.

The bending actuator is clamped at one end, whereas different tips of well-defined cross-sections are attached to the free end. These tips are immersed in the sample liquid. The tip geometries and materials are given in Table 1. The clamping fixture is mounted on a rigid frame allowing for vertical ( $x$ -direction) positioning of the sensor and preventing vibrations of the entire setup. A lock-in amplifier (LIA) measures the sensor voltage  $V_s$ , resulting in the cantilever’s frequency response. As voltage source  $V_d$ , the internal oscillator of the lock-in amplifier is used.

The cantilever exhibits several resonant vibration modes. As an example, Figure 3(a) shows the deflection ( $z$ -direction) for a cantilever with tip A (Table 1) vibrating in air. The corresponding sensing electrode voltage is shown in the diagram below (Figure 3(b)). The resonance frequencies are 100 Hz and 851 Hz for the first and second modes, respectively. Further resonances were found at 2456 Hz

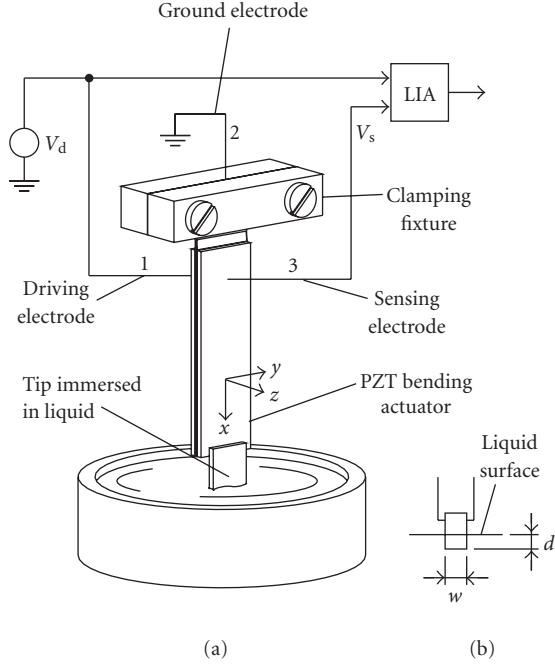


FIGURE 2: Measurement setup. The PZT bending actuator is rigidly clamped at one end. To the free end of the cantilever, a tip of well-defined geometry (width  $w$ ) is attached and immersed in the sample liquid (dipping depth  $d$ ). The interaction of the cantilever tip and the liquid changes the frequency characteristics, which is measured by the lock-in amplifier (LIA).

(3rd mode) and 4870 Hz (4th mode). The mode shapes for a uniform cantilevered beam without a load at the tip are given in Figure 4 [21].

### 3. THEORETICAL MODEL

The vibration behavior of the piezoelectric bending actuator is described by the Euler-Bernoulli beam equation [21]:

$$EI \frac{\partial^4 \psi(x, t)}{\partial x^4} + \mu \frac{\partial^2 \psi(x, t)}{\partial t^2} = \frac{\partial^2 M_p(x, t)}{\partial x^2}, \quad (1)$$

where  $EI$  and  $\mu$  are the effective bending stiffness and the effective mass per unit length of the composite beam,  $\psi(x, t)$  is the beam deflection in  $z$ -direction (Figure 2), and  $M_p$  is the actuating moment due to the piezoelectric effect. Since  $M_p$  is considered to be a constant moment along the entire beam,  $\partial^2 M_p / \partial x^2 = 0$ . Consequently, the boundary conditions for the clamped-free beam are

$$\begin{aligned} \psi(0, t) &= 0, \\ \frac{\partial \psi(0, t)}{\partial x} &= 0, \\ \frac{\partial^2 \psi(L, t)}{\partial x^2} &= \frac{1}{EI} M_p, \\ \frac{\partial^3 \psi(L, t)}{\partial x^3} &= -\frac{1}{EI} F, \end{aligned} \quad (2)$$

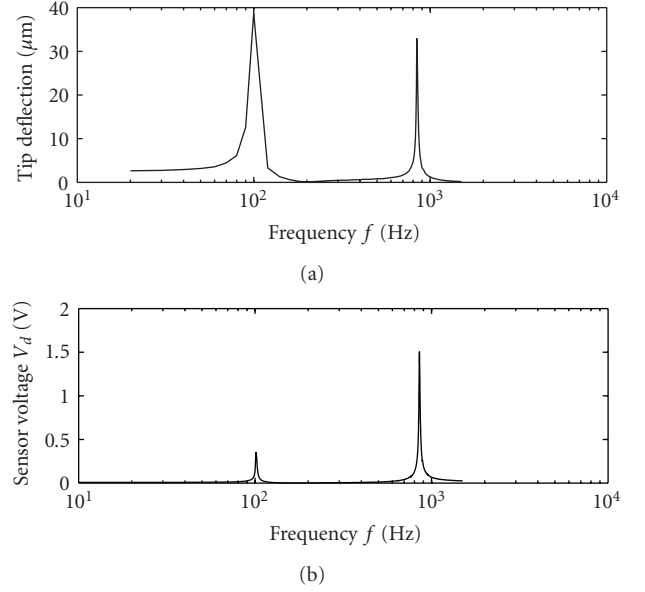


FIGURE 3: Frequency characteristics of a PZT cantilever with tip A (Table 1),  $V_d = 400 \text{ mV}_{\text{rms}}$  driving voltage, vibrating in air. (a) The tip deflection (peak-peak) was measured using a Polytec OFV-5000/OFV-505 laser vibrometer. (b) The diagram below shows the corresponding sensing electrode rms voltage.

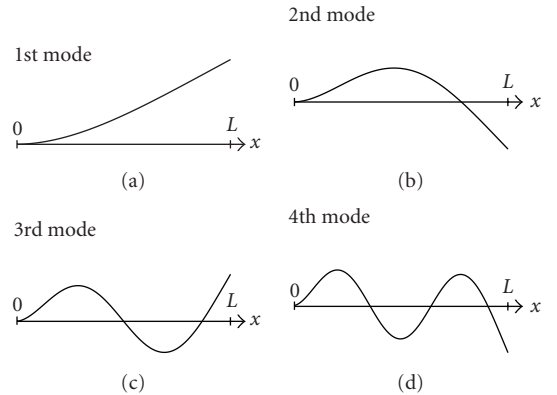


FIGURE 4: Mode shapes of a uniform cantilevered beam. The mode shapes have been obtained from solutions of the Euler-Bernoulli beam equation and are also roughly valid for the piezoelectric bimorph.

where  $L$  is the length of the beam and  $F$  is the force acting on the tip due to the interaction with the surrounding liquid. The actuating moment  $M_p$  is given by [22]

$$M_p = \int_{A_p} Y_p d_{31} E_z z dA = w_b Y_p d_{31} V_d z_m, \quad (3)$$

where  $A_p$ ,  $Y_p$ , and  $d_{31}$  are the cross-sectional area, Young's modulus, and the piezoelectric modulus of the actuating layer and  $E_z$  is the electric field in this layer in  $z$ -direction.  $w_b$  is the width of the bending actuator,  $z_m$  is the mean distance of the actuating layer from the beam center, and  $V_d$  is the excitation voltage (Figure 2(a)).

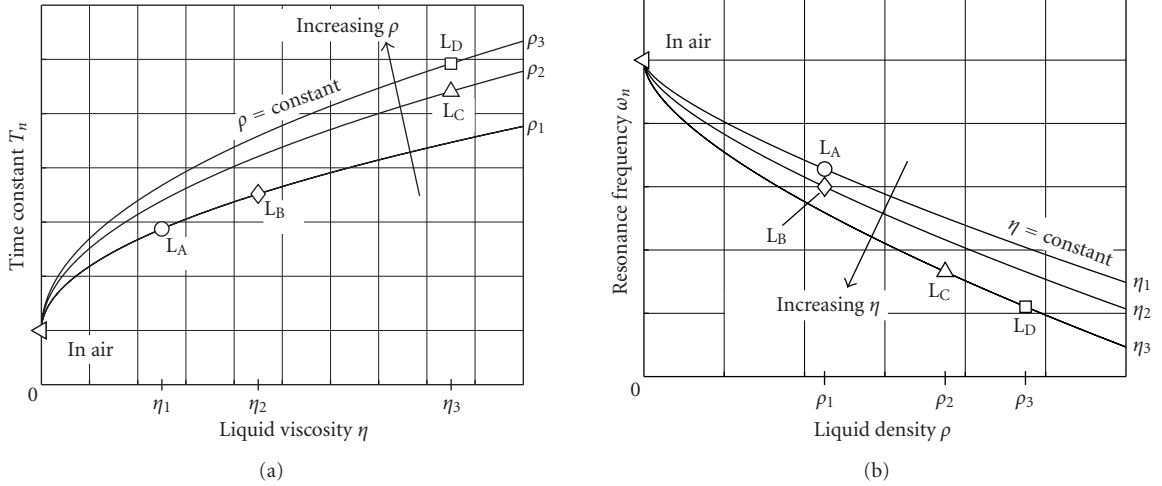


FIGURE 5: Time constant  $T_n$  and resonance frequency  $\omega_n$  of a resonant cantilever dipping in 4 different sample liquids  $L_A, L_B, L_C,$  and  $L_D$  as given by the generalized models (15) and (16).

The measured voltage  $V_s$  is calculated by applying

$$\begin{aligned} D_z &= d_{31}T_{xx} + \varepsilon E_z, \\ T_{xx} &= \frac{1}{K_{11}}(S_{xx} - d_{31}E_z) \end{aligned} \quad (4)$$

to the sensing PZT layer, where  $D_z, T_{xx},$  and  $S_{xx}$  are electric displacement, stress, and strain.  $\varepsilon$  and  $K_{11}$  are the permittivity and the compliance of the PZT layer. Since  $V_s$  is measured by means of a voltage amplifier in our setup, the current  $I_s$  from the sensing electrode vanishes, and, therefore, the charge:

$$Q = \int_{A_e} D_z dA = 0, \quad (5)$$

where  $A_e$  is the surface area of the sensing electrode. From (4), (5), and  $S_{xx} = -z\partial^2\psi(x, t)/\partial x^2,$

$$V_s(t) = K_v \frac{\partial\psi(L, t)}{\partial x} \quad (6)$$

is obtained, where  $K_v$  is a constant depending on material and geometry parameters [22]. The voltage at the sensing electrode is given by the slope of the beam deflection  $\psi(x, t)$  at the free end,  $x = L,$  of the cantilever. This fact is greatly confirmed by the measurements given by Figure 3. Despite the lower tip deflection amplitude of the second mode the resulting output voltage is higher than for the first mode of vibration.

The interaction with the liquid surrounding, the cantilever tip can be modeled by approximating the vibrating cantilever as an oscillating sphere immersed in a liquid [14], exhibiting an effective sphere radius  $R$  and an effective sphere mass. The force  $F$  acting on such a sphere is given by [23]

$$F = 6\pi\eta R \left(1 + \frac{R}{\delta}\right) \frac{du}{dt} + 3\pi R^2 \sqrt{\frac{2\eta\rho}{\omega}} \left(1 - \frac{2R}{9\delta}\right) \frac{d^2u}{dt^2}, \quad (7)$$

where  $u$  is the sphere displacement,  $R$  is the sphere radius,  $\omega$  is the angular oscillation frequency, and  $\delta$  is the depth of penetration of the acoustic wave, which is given by

$$\delta = \sqrt{\frac{2\eta}{\omega\rho}}. \quad (8)$$

The tips are attached to the free ends of the cantilevers, therefore, the sphere displacement is  $u = \psi(L, t).$

In principle, solving the equations given above yields the frequency characteristics of the vibrating cantilever with a tip immersed in liquid for all modes of vibration. As the cantilever represents a composite structure involving layers featuring different material properties, effective parameters have to be used in the beam equation, for example, for Young's modulus. As also the material parameters of the layers are not, or only partly, available in the required accuracy and since the solution of the beam equation would require the solution of a higher order system in order to determine the coefficients by a suitable expansion (e.g., eigenmode expansion), we have approximated the resonance behavior of the sensor in the vicinity of the first mode resonance frequency as a second-order system, given by

$$(M_e + M_i) \frac{d^2u}{dt^2} + (b_e + b_i) \frac{du}{dt} + Ku = F_0 e^{j\omega t}, \quad (9)$$

where  $u$  is the deflection of the cantilever tip in  $z$ -direction (Figure 2),  $M_e$  and  $b_e$  are the effective mass and the intrinsic damping of the cantilever and the tip,  $K$  is the spring constant, and  $F_0$  and  $\omega$  are the driving force amplitude and angular frequency.  $M_i$  and  $b_i$  are the induced mass and damping due to the liquid loading, given by (7)

$$M_i = 3\pi R^2 \sqrt{\frac{2\eta\rho}{\omega}} \left(1 - \frac{2R}{9\delta}\right), \quad (10)$$

$$b_i = 6\pi\eta R \left(1 + \frac{R}{\delta}\right). \quad (11)$$

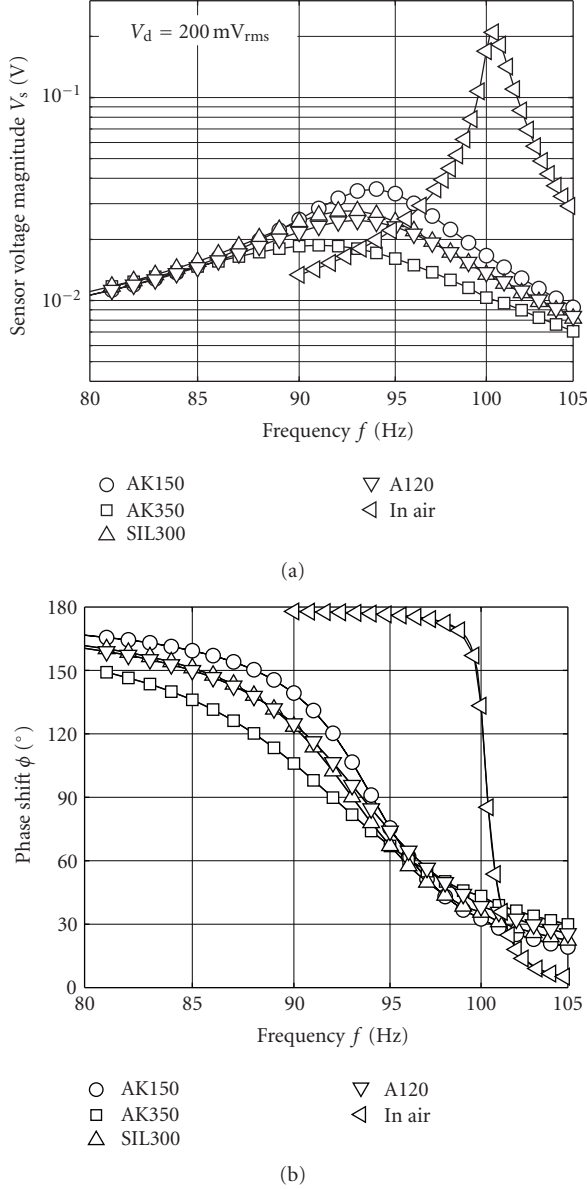


FIGURE 6: Magnitude and phase response of a cantilever with tip A vibrating in air and immersed in different liquids (Table 2). The dipping depth  $d$  is 2 mm, the driving voltage  $V_d = 200 \text{ mV}_{\text{rms}}$ .

For sample liquids exhibiting high viscosities, and low-vibration frequencies,  $R$  is negligible compared to  $\delta$ , leading to a simplification of (10) and (11) [14]. For the considered cantilever tips and dipping depths (Table 1), the effective sphere radius  $R$  is in the range of a few millimeters. The expected penetration depths  $\delta$  for the sample liquids used in this work were calculated for a vibration frequency of 100 Hz and are given in Table 2. The results show that  $\delta$  is in the same range as  $R$ , and the prerequisites for said simplification are not fulfilled.

Consequently, the consideration of the characteristic penetration depth  $\delta$  leads to a frequency dependence of the liquid mass loading  $M_i$ , (cf. (10)), and the liquid damping  $b_i$

(cf. (11)). In the following, both  $M_i$  and  $b_i$  are considered constant in the vicinity of the resonance frequency, which is justified by the high-quality factors and the accompanied narrow bandwidths of the resonances. Therefore, solving the differential equation (9) using the Laplace transform yields

$$U(s) = \frac{1}{1 + (2D_n/\omega_n)s + (1/\omega_n^2)s^2} F(s), \quad (12)$$

where  $U(s)$  and  $F(s)$  are the Laplace transforms of the tip deflection  $u$  and the driving force:

$$\begin{aligned} \frac{1}{\omega_n^2} &= \frac{M_e + M_i}{K}, \\ \frac{2D_n}{\omega_n} &= \frac{b_e + b_i}{K}, \end{aligned} \quad (13)$$

where  $\omega_n$  is the resonance frequency and  $D_n$  the damping factor of the respective vibration mode. From (7) and (13), one obtains

$$\begin{aligned} \omega_n^2 &= \omega_{n,\text{air}}^2 \frac{1}{1 + (2\pi R^3/3M_e)\rho + (6\pi R^2/\sqrt{2}M_e)(1/\sqrt{\omega_n})\sqrt{\eta\rho}}, \\ \frac{2D_n}{\omega_n} &= \frac{2D_{n,\text{air}}}{\omega_{n,\text{air}}} \left( 1 + \frac{6\pi R}{b_e}\eta + \frac{6\pi R^2}{\sqrt{2}b_e}\sqrt{\omega_n}\sqrt{\eta\rho} \right), \end{aligned} \quad (14)$$

where  $\omega_{n,\text{air}}$  and  $D_{n,\text{air}}$  are the resonance frequency and damping factor of the cantilever without liquid loading. The oscillating sphere model shows that the cantilever's resonance frequency is affected by a term related to the liquid density, and a second term containing the viscosity-density product, whereas the time constant  $T_n = 2D_n/\omega_n$  depends on the viscosity and the viscosity-density product.

For the interpretation of our measurement results involving rectangular cross-sections, we devised a more generalized model. Based on (14) and by introducing four independent coefficients  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$ , we have

$$\omega_n^2 = \omega_{n,\text{air}}^2 \frac{1}{1 + c_1\rho + c_2(1/\sqrt{\omega_n})\sqrt{\eta\rho}}, \quad (15)$$

$$T_n = T_{n,\text{air}}(1 + c_3\eta + c_4\sqrt{\omega_n}\sqrt{\eta\rho}), \quad (16)$$

where  $T_{n,\text{air}}$  is the in-air time constant  $2D_{n,\text{air}}/\omega_{n,\text{air}}$ . The values of the parameters  $c_i$  are determined by the tip size and geometry, the effective cantilever mass and damping, the dipping depth  $d$  (Figure 2(b)), and the respective mode of vibration  $n$ .

Figure 5 elucidates the model given by (15) and (16). We consider four liquids  $L_A$ ,  $L_B$ ,  $L_C$ , and  $L_D$  of which  $L_A$  and  $L_B$  are of the same density  $\rho_1$ , whereas  $L_C$  and  $L_D$  are of the same viscosity  $\eta_3$ . The expected values of  $\omega_n$  and  $T_n$  are given by markers in the figure. The curves in the figure represent the results expected for liquids of the same viscosity and density, respectively.

For the determination of viscosity and density of an unknown liquid, (15) and (16) have to be solved for  $\eta$  and  $\rho$ . The system of equations can be written as

$$\begin{aligned} A &= B\rho + C\sqrt{\eta\rho}, \\ D &= E\sqrt{\eta\rho} + F\eta, \end{aligned} \quad (17)$$

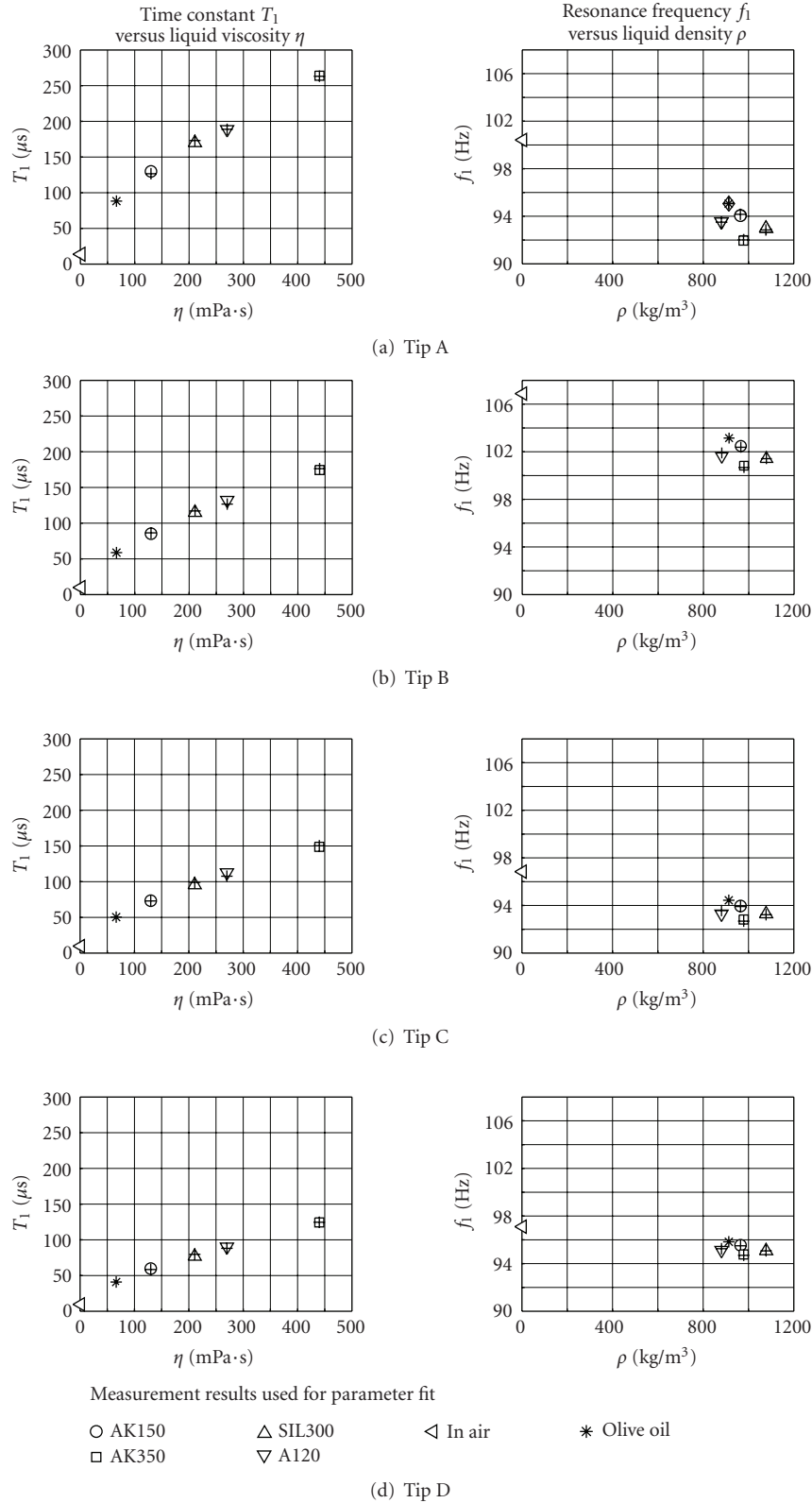


FIGURE 7: Resonance frequency  $\omega_1$  and time constant  $T_1$  obtained for the four different tips and a variety of sample liquids. The crosses (+) in the diagrams represent the calculated values obtained from the parameter fit and show good agreement between the experimental results and the model.

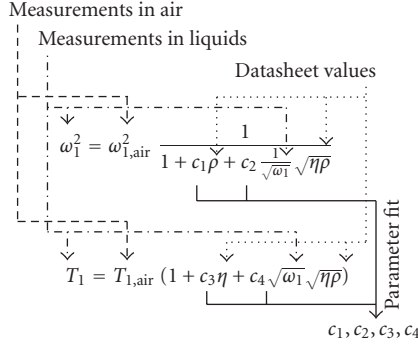


FIGURE 8: Obtaining the model parameters  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$  from measurements (Figure 7) in air and in reference liquids with known viscosities and densities (Table 2).

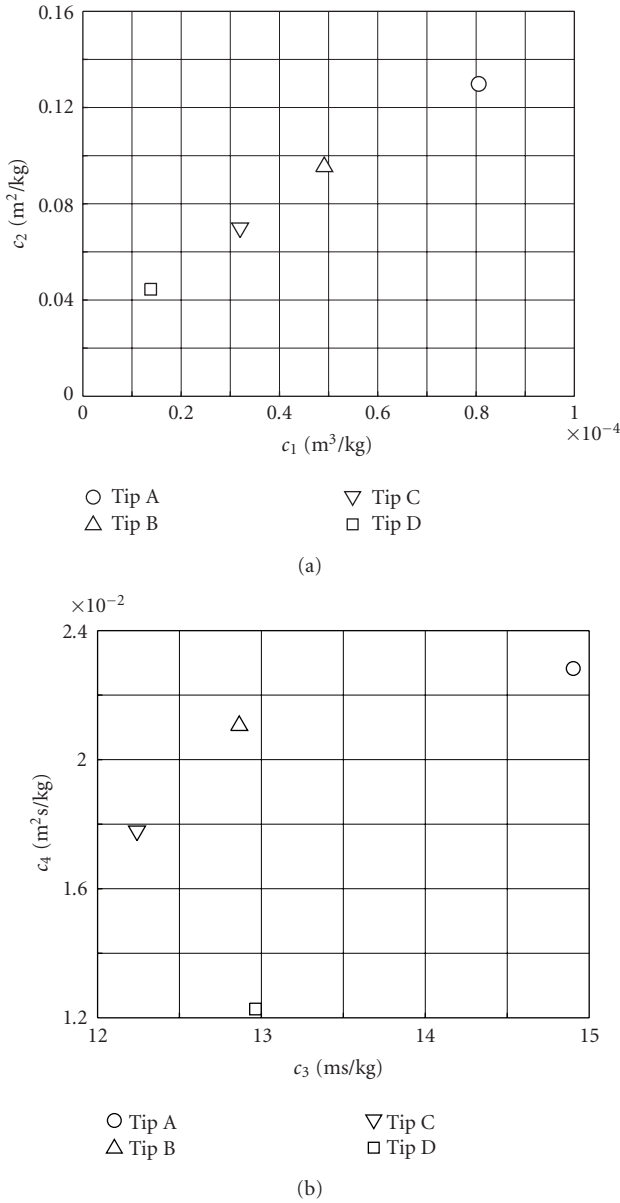


FIGURE 9: Fitted parameter values  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$  obtained by fitting the model equations to the measurement results.

where  $A = \omega_{n,\text{air}}^2/\omega_n^2 - 1$ ,  $B = c_1$ ,  $C = c_2/\sqrt{\omega_n}$ ,  $D = T_n/T_{n,\text{air}} - 1$ ,  $E = c_4\sqrt{\omega_n}$ , and  $F = c_3$ . Solving the system of equations yields

$$\begin{aligned} \rho_{1,2} &= -\frac{1}{2} \frac{ACE - 2ABF - C^2D}{B(BF - CE)} \\ &\pm \sqrt{\frac{1}{4} \frac{(ACE - 2ABF - C^2D)^2}{B^2(BF - CE)^2} - \frac{A^2F}{B(BF - CE)}}, \\ \eta_{1,2} &= -\frac{1}{2} \frac{CDE - 2BDF - AE^2}{F(BF - CE)} \\ &\pm \sqrt{\frac{1}{4} \frac{(CDE - 2BDF - AE^2)^2}{F^2(BF - CE)^2} - \frac{BD^2}{F(BF - CE)}}. \end{aligned} \quad (18)$$

For the cantilevers used in this work, it turns out that  $BF - CE = c_1c_3 - c_2c_4 < 0$ . Furthermore, all the coefficients of (17) are positive, that is,  $A, B, C, D, E, F > 0$ . Therefore, in (18), only that  $\rho_{1,2}$  and  $\eta_{1,2}$ , respectively, are positive which exhibit a plus before the root sign, that is,

$$\begin{aligned} \rho &= -\frac{1}{2} \frac{(c_2c_4 - 2c_1c_3) \left( \frac{\omega_{n,\text{air}}^2}{\omega_n^2} - 1 \right) - \frac{c_2^2}{\omega_n} \left( \frac{T_n}{T_{n,\text{air}}} - 1 \right)}{c_1(c_1c_3 - c_2c_4)} \\ &+ \left\{ \frac{1}{4} \frac{\left[ (c_2c_4 - 2c_1c_3) \left( \frac{\omega_{n,\text{air}}^2}{\omega_n^2} - 1 \right) - \frac{c_2^2}{\omega_n} \left( \frac{T_n}{T_{n,\text{air}}} - 1 \right) \right]^2}{c_1^2(c_1c_3 - c_2c_4)^2} \right. \\ &\quad \left. - \frac{c_3 \left( \frac{\omega_{n,\text{air}}^2}{\omega_n^2} - 1 \right)^2}{c_1(c_1c_3 - c_2c_4)} \right\}^{1/2}, \\ \eta &= -\frac{1}{2} \frac{(c_2c_4 - 2c_1c_3) \left( \frac{T_n}{T_{n,\text{air}}} - 1 \right) - c_4^2\omega_n \left( \frac{\omega_{n,\text{air}}^2}{\omega_n^2} - 1 \right)}{c_3(c_1c_3 - c_2c_4)} \\ &+ \left\{ \frac{1}{4} \frac{\left[ (c_2c_4 - 2c_1c_3) \left( \frac{T_n}{T_{n,\text{air}}} - 1 \right) - c_4^2\omega_n \left( \frac{\omega_{n,\text{air}}^2}{\omega_n^2} - 1 \right) \right]^2}{c_3^2(c_1c_3 - c_2c_4)^2} \right. \\ &\quad \left. - \frac{c_1 \left( \frac{T_n}{T_{n,\text{air}}} - 1 \right)^2}{c_3(c_1c_3 - c_2c_4)} \right\}^{1/2}. \end{aligned} \quad (19)$$

#### 4. MEASUREMENTS

With the setup depicted in Figure 2(a), the frequency response of the cantilevers is examined. The lock-in amplifier (Stanford Research SR830) is used to drive the cantilever and to measure the sensor voltage  $V_s$  and the phase shift  $\phi(j\omega)$  between the driving voltage, that is, the driving force and the sensor voltage, that is, the actual cantilever deflection.

TABLE 2: Reference values (dynamic viscosity  $\eta$  and mass density  $\rho$ ) of the sample liquids used in the measurements and the expected depth of penetration  $\delta$  at an angular frequency of  $\omega = 2\pi \cdot 100$  Hz, see (8). The dynamic viscosity was measured using a Brookfield LVDV+II-CP cone/plate rheometer at an ambient temperature of 23°C.

Sample liquid	Viscosity $\eta$	Density $\rho$	$\delta$ (100 Hz)
AK150 silicone oil	130 mPa·s	965 kg/m <sup>3</sup>	0.65 mm
AK350 silicone oil	440 mPa·s	979 kg/m <sup>3</sup>	1.2 mm
SIL300 silicone oil	211 mPa·s	1078 kg/m <sup>3</sup>	0.79 mm
Alcatel A120 oil	270 mPa·s	881 kg/m <sup>3</sup>	0.99 mm

The measurements are carried out with a sinusoidal driving voltage of 200 mV<sub>rms</sub> and within a frequency range from 70 to 110 Hz. At the resonance frequency of the first mode, a maximum tip deflection of 19  $\mu$ m (peak-peak) was measured in air by means of a Polytec OFV-5000/OFV-505 laser vibrometer.

A variety of oils are used as test liquids: AK150, AK350 (Wacker Chemie), and SIL300 are silicone oils, and Alcatel 120 (A120) oil. They exhibit liquid densities in the range from 881 to 1078 kg/m<sup>3</sup> and viscosities from 145 to 440 mPa·s. These nominal liquid parameters are obtained from data sheets and measurements by means of a Brookfield LVDV+II-CP cone/plate rheometer (Table 2). The parameters of the liquids vary with temperature. As especially the viscosity is highly temperature dependent, temperature control of the liquid container has been established by means of a peltier heater/cooler system. All results presented here have been obtained at 23°C.

The dipping depth (Figure 2(b)) was adjusted by lowering the sensor until the tip touches the liquid surface, and then adding the desired  $d = 2$  mm, using a micrometer screw. For liquids exhibiting low-surface tensions, like those considered in this work, a concave meniscus is formed at the liquid-tip interface. We expect the resulting liquid surface shape to increase the effective dipping depth, and, therefore, to influence the sensor's behavior. Consequently, the sensor principle would be limited to liquids with similar surface tensions, which can be assumed for the considered oils.

Figure 6 summarizes the measurement results at mode 1 for tip A in air and immersed in the sample liquids. As expected from (15) and (16), the liquid loading decreases the resonance frequency  $\omega_1$  and increases the damping factor  $D_1$ .

From the cantilever's frequency response, we extract the first mode resonance frequency  $\omega_1 = 2\pi f_1$  and the damping factor  $D_1$  by fitting a second-order transfer function:

$$\phi(j\omega) = \arg \left\{ \frac{1}{1 + j\omega(2D_1/\omega_1) - (\omega^2/\omega_1^2)} \right\} \quad (20)$$

to the phase shift measurements (Figure 6) with respect to the parameters  $\omega_1$  and  $D_1$ .

Figure 7 shows the results for the cantilevers with tips A, B, C, and D (Table 1) and the sample liquids (Table 2). The left column of Figure 7 shows the measured time constant  $T_1 = 2D_1/\omega_1$  versus the dynamic viscosity  $\eta$  of the sample liquids. The results for each cantilever appear to be nearly

TABLE 3: Determination of viscosity and density of an olive oil sample by means of vibrating cantilevers. The density  $\rho$  and the viscosity  $\eta$  have been calculated from the measurement results  $\omega_1 = 2\pi f_1$  and  $T_1$  using (19). The density and viscosity obtained by means of weighting and a cone-plate rotational viscometer, respectively, are  $\eta_{\text{ref}} = 66.4$  mPa·s and  $\rho_{\text{ref}} = 913$  kgm<sup>-3</sup>.

Cantilever	$f_1$	$T_1$	Viscosity $\eta$	Density $\rho$
Tip A	95.07 Hz	88.24 $\mu$ s	66.4 mPa·s	919.7 kgm <sup>-3</sup>
Tip B	103.15 Hz	58.80 $\mu$ s	62.9 mPa·s	921.4 kgm <sup>-3</sup>
Tip C	94.44 Hz	50.43 $\mu$ s	64.6 mPa·s	920.1 kgm <sup>-3</sup>
Tip D	95.83 Hz	40.88 $\mu$ s	67.6 mPa·s	908.6 kgm <sup>-3</sup>

lying on a single trend curve, indicating that  $T_1$  is dominantly influenced by the liquid's dynamic viscosity. According to model equation (16), the value of  $c_4$  thus must be small compared to  $c_3$ .

Figure 7, right column, depicts the resonance frequency  $f_1$  versus the liquid density. The results are widely spread in the  $\rho$ - $f_1$  plane. Obviously, the resonance frequency of the cantilever tip immersed in the liquid is strongly influenced by both the liquid's density and viscosity. In our model, this relationship is described by (15).

Using the measurement results (Figure 7), the model parameters  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$  can be determined. Figure 8 illustrates the model equations (15) and (16) and the parameter fit procedure. The values of  $\omega_{1,\text{air}}$  and  $T_{1,\text{air}}$  are obtained from a single measurement of the vibrating cantilever in air. Furthermore, each measurement in a sample liquid yields a pair of  $(\omega_1, T_1)$ . The values of viscosity and density,  $\eta$  and  $\rho$ , of each sample liquid are also known (Table 2). Finally, a fit algorithm applied to the model equations results in the model parameters  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$  for a particular cantilever. The results of this parameter fit are depicted in Figure 9. The diagram shows the model parameters  $c_1$ ,  $c_2$ ,  $c_3$ ,  $c_4$  for the cantilevers with different tips. It is important to note that the parameters do not only depend on the width of the attached tip, but also on the mass and the intrinsic damping of the entire cantilever. Therefore, the parameter values do not necessarily decrease with the tip width, as shown by the location of the tip D marker in the  $c_3$ - $c_4$  plane (Figure 9). The PZT bending actuator, with tip D has being attached, was not sealed by a protective coating, and thus features a lower intrinsic damping. However, the parameter fitting process described above yields parameter values considering such different behaviors of the cantilevers.

For a validation of the model, the parameters determined above were used to calculate the values of  $\omega_1$  and  $T_1$  for each tip-liquid combination. These calculated values are indicated by crosses (+) in Figure 7 and are in good agreement with the experimental data.

At last, we use the model parameters obtained above to determine the viscosity and density of an olive oil sample. The measurement results, that is, the resonance frequencies  $\omega_1$  and the time constants  $T_1$  of the cantilevers immersed in the sample liquid are given in Table 3. From these results, the density and the viscosity of the liquid are calculated using (19), respectively.



## 5. CONCLUSION

The change of the dynamic behavior of a vibrating cantilever allows to investigate the physical properties of liquids. Various types of small tips of different geometries are attached to the cantilever and immersed into the sample solutions. The liquid surrounding the cantilever tip changes both the resonance frequency and the damping of the entire cantilever structure. To be able to conclude, from the measured frequency response to the liquid's parameters, an analytical model is needed. The developed model is based on the forces acting on an oscillating sphere in liquid, but generalized model parameters are used to consider the actual geometries of the applied cantilever tips. These parameters, furthermore, include the electrical and mechanical characteristics of the beam, which, therefore, must not be known. The model proved to be well suited for the characterization of various cantilevers and tip geometries by measuring in liquids with known density and viscosity. To extract the model parameters from the measured data, a curve fitting procedure was performed. The obtained parameters are specific for each cantilever tip and allow the subsequent simultaneous determination of density and viscosity of unknown liquids. Compared to other works, the sensor features high-quality factors of the considered resonance mode even for highly viscous liquids, greatly extending the measurement range.

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