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Research Article

Robust H_∞ Filtering for Uncertain Neutral Stochastic Systems with Markovian Jumping Parameters and Time Delay

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This paper deals with the robust H_∞ filter design problem for a class of uncertain neutral stochastic systems with Markovian jumping parameters and time delay. Based on the Lyapunov-Krasovskii theory and generalized Finsler Lemma, a delay-dependent stability condition is proposed to ensure not only that the filter error system is robustly stochastically stable but also that a prescribed H_∞ performance level is satisfied for all admissible uncertainties. All obtained results are expressed in terms of linear matrix inequalities which can be easily solved by MATLAB LMI toolbox. Numerical examples are given to show that the results obtained are both less conservative and less complicated in computation.

1. Introduction

Time delay exists extensively in chemical process systems, communication systems, economic systems, microwave oscillator, and networked control systems. Meanwhile, because of the modeling inaccuracies and changes in the environment of the model parameter, uncertainties are unavoidable in the process of modeling. The appearance of time delay and uncertainties in many systems can often bring instability, oscillation, and poor performance; considerable attention has been focused on the stability analysis of uncertain time delays systems; see [1–6] and the references therein. On the other hand, a real system is often affected by external disturbances such as stochastic perturbations. The stochastic effects can also lead to oscillation, divergence, or other poor performances. Therefore, the stability study of stochastic systems with time delay has been paid great attention and a lot of significant results have been reported in the literature; see [7–12] and the references therein.

In the past few decades, filtering problem has been a hot issue in the fields of signal processing. Kalman filtering has been successfully applied in many fields such as manufacturing systems, economic systems, and maneuvered target tracking. However, the exact requirement of known dynamics system and precise noise statistics have restricted its practical application. In such a case, we can resort to H_∞ filtering [13–22] and L_2-L_∞ filtering [23–25]; see the references therein.

Markovian jump systems, originally raised by Krasovskii and Lidskii [26], are famous for the description of many dynamical practical systems whose structure and parameters are subject to random changes. Therefore, the stability analysis and filtering problem for Markovian jump systems have been studied [27–34]. For example, the stability analysis of impulsive stochastic neural networks with Markovian jump are studied in [27, 29, 31, 34]; H_∞ control and mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probabilities are, respectively, investigated in [28, 30, 32]. The design of reduced-order H_∞ filter for Markovian jumping systems with time delay is studied in [15]. The L_2-L_∞ filter design for stochastic time-varying delay systems with Markovian jumping parameters is considered in [33].

Many methods are proposed in the process of robust stochastic stability analysis and filtering design, which develops from the early solving Riccati equation to model transformation method and cross-terms bounding technique [5], free-weighting matrices method [11, 32], slack matrix variables [17–19, 24, 28], and delay-partitioning method [20]. However, model transformations may give rise to additional dynamics of the original systems [13], and cross-terms bounding techniques can bring conservatism. Moreover, as pointed out in [33], under certain circumstance, free matrix variables may not lessen the conservatism. In recent years, another popular method called Finsler Lemma is carried out so as to decrease the computational cost as well [5], and the Finsler Lemma in deterministic setting is extended to generalized Finsler Lemma in stochastic systems in [5, 33, 34].

On the other hand, many dynamical systems can be modeled by neutral systems which are organized by neutral functional differential equations. Other than retarded time delay systems containing delays only in states, a neutral time delay system contains delays in both its state and its derivatives of state. Recently, for neutral stochastic time delay systems, the stability analysis and filter design problem are mainly addressed in [4, 9, 19]. It is necessary to point out that the delay-dependent robust H_∞ filtering for uncertain neutral stochastic time delay system is studied in [19, 24]. Robust H_∞ filter design for neutral stochastic uncertain systems with time-varying delay is studied in [13]. To the best of the authors' knowledge, the H_∞ filtering problem has not been reported about uncertain neutral stochastic systems with Markovian jumping parameters and time delay, which motivates the present study.

Motivated by the works in [5, 13, 17, 18], the robust H_∞ filtering for uncertain neutral stochastic systems with Markovian jumping parameters and time delay is considered in this paper. By generalized Finsler Lemma, the robust stochastic stability condition is obtained. The presented condition is simple and efficient. Finally, the effectiveness of the approach is verified by illustrative examples including a comparison with some recent results.

Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. I is the identity matrix. $|\cdot|$ denotes Euclidean norm for vectors and $\|\cdot\|$ denotes the spectral norm of matrices. N denotes the set of all natural numbers; that is, $N = \{0,1,2,\ldots\}$. $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P})$ is a complete probability space with filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. M^T stands for the transpose of the matrix M. For symmetric matrices X and Y, the notation $X > Y$ (resp., $X \geq Y$) means that the $X-Y$ is positive definite (resp., positive semidefinite). $*$ denotes a block that is readily inferred by symmetry. $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$.

2. Problem Description

Consider the following uncertain neutral stochastic systems with Markovian jumping parameters and time delay:

$$
\begin{aligned}
d\left[x(t)-G(r_t)x(t-h)\right] &= \left[A(t,r_t)(t)x(t)\right.\\
&\quad \left.+A_1(t,r_t)(t)x(t-h)+B(r_t)v(t)\right]dt\\
&\quad +\left[D(t,r_t)(t)x(t)+D_1(t,r_t)(t)x(t-h)\right.\\
&\quad \left.+D_2(r_t)v(t)\right]dw(t),\\
dy(t) &= \left[C(r_t)x(t)+C_1(r_t)x(t-h)+C_2(r_t)v(t)\right]dt\\
&\quad +\left[E(r_t)(t)x(t)+E_1(r_t)x(t-h)+E_2(r_t)v(t)\right]dw(t),\\
z(t) &= L_1(r_t)x(t)+L_2(r_t)x(t-h)+L_3(r_t)v(t),\\
x(\theta) &= \psi(\tau),\\
&\qquad r_t = r_0\in S,\ \forall\tau\in[-h,0],
\end{aligned}
\tag{1}
$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $y(t)\in\mathbb{R}^m$ is the measured output, $v(t)\in\mathbb{R}^q$ is the disturbance input in $L_2[0,\infty)$, and $z(t)\in\mathbb{R}^p$ is the signal to be estimated. $A(t,r_t)(t)$, $A_1(t,r_t)(t)$, $B(r_t)$, $D(t,r_t)(t)$, $D_1(t,r_t)(t)$, $D_2(r_t)$, $C(r_t)$, $C_1(r_t)$, $C_2(r_t)$, $E(r_t)$, $E_1(r_t)$, $E_2(r_t)$, $L_1(r_t)$, $L_2(r_t)$, and $L_3(r_t)$ are the matrix functions of the random jumping process $r(t)$, where $r(t)$ is a finite-state Markovian jump process representing the system mode; that is, $r(t)$ takes discrete values in given finite set $\mathcal{S} = 1,2,\ldots,N$. Here $\psi(\cdot)$ is the initial condition and is assumed to be continuously differentiable on $[-h,0]$. Consider $h > 0$ indicates the time delay. $w(t)$ is a scalar Brownian motion (Wiener process) defined on the complete probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P})$ satisfying

$$
\begin{aligned}
\mathbf{E}\{dw(t)\} &= 0,\\
\mathbf{E}\left\{dw^2(t)\right\} &= dt.
\end{aligned}
\tag{2}
$$

The transition probability matrix of systems (1) is given by

$$
P_r\left(r_{t+\Delta}=j\mid r_t=i\right)=\begin{cases}\pi_{ij}\Delta+o(\Delta), & j\neq i\\ 1+\pi_{ii}\Delta+o(\Delta), & j=i,\end{cases}
\tag{3}
$$

where $\Delta > 0$, $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$, $\pi_{ij}\geq 0$, $\forall j\neq i$, is the transition rate from mode i at time t to mode j at time $t+\Delta$, and

$$
\pi_{ii} = -\sum_{j=1,j\neq i}^{j=N}\pi_{ij} < 0.
\tag{4}
$$

For the purpose of simplicity, in this paper, for each $r(t) = i\in\mathcal{S}$, $A(t,r_t)(t)$, $A_1(t,r_t)(t)$, and $B(r_t)$ are denoted by $A_i(t)$, $A_{1i}(t)$, B_i, and so on. In systems (1),

$$
\begin{aligned}
A_i(t) &= A_i+\Delta A_i(t),\\
A_{1i}(t) &= A_{1i}+\Delta A_{1i}(t),\\
D_i(t) &= D_i+\Delta D_i(t),\\
D_{1i}(t) &= D_{1i}+\Delta D_{1i}(t),
\end{aligned}
\tag{5}
$$

and A_i, A_{1i}, B_i, D_i, D_{1i}, D_{2i}, C_i, C_{1i}, C_{2i}, E_i, E_{1i}, E_{2i}, L_{1i}, L_{2i}, and L_{3i} are known real constant matrices with appropriate dimensions. $\Delta A_i(t)$, $\Delta A_{1i}(t)$, $\Delta D_i(t)$, and $\Delta D_{1i}(t)$ are unknown matrices representing norm-bounded parameter uncertainties, which are assumed to satisfy

$$
\begin{bmatrix}\Delta A_i(t) & \Delta A_{1i}(t)\\ \Delta D_i(t) & \Delta D_{1i}(t)\end{bmatrix}=\begin{bmatrix}M_{1i}\\ M_{2i}\end{bmatrix}F_i(t)\begin{bmatrix}N_{1i} & N_{2i}\end{bmatrix},
\tag{6}
$$

where M_{1i}, M_{2i}, N_{1i}, and N_{2i} are known real constant matrices and $F_i(\cdot):\mathbb{R}\to\mathbb{R}^{k\times l}$ is an unknown time-varying matrix function satisfying

$$
F_i^T(t)F_i(t)\leq I,\quad \forall t.
\tag{7}
$$

The parameter uncertainties $\Delta A_i(t)$, $\Delta A_{1i}(t)$, $\Delta D_i(t)$, and $\Delta D_{1i}(t)$ are said to be admissible if both (6) and (7) hold.

In this paper, we make the following assumption on the matrix G_i in systems (1).

Assumption 1. The matrix G_i in systems (1) satisfies

$$\rho(G_i) < 1, \tag{8}$$

where the notation $\rho(G_i)$ denotes the spectral radius of G_i.

We now consider a full-order filter for systems (1) with the following form:

$$\begin{aligned}(\Sigma_f): d\hat{x}(t) &= A_{fi}\hat{x}(t)\,dt + B_{fi}\,dy(t),\\ \hat{z}(t) &= C_{fi}\hat{x}(t),\end{aligned} \tag{9}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the filter state, $\hat{z}(t) \in \mathbb{R}^p$ is the estimation of $z(t)$, and A_{fi}, B_{fi}, and C_{fi} $(i \in \mathcal{S})$ are the filter parameters with compatible dimensions to be determined.

Define

$$\begin{aligned}\xi(t) &= \left[x^T(t) \;\; \hat{x}^T(t)\right]^T,\\ e(t) &= z(t) - \hat{z}(t).\end{aligned} \tag{10}$$

Then the filtering error systems can be obtained as follows:

$$\begin{aligned}d\left[\xi(t) - \overline{G}_i K\xi(t-h)\right] &= \left[\overline{A}_i(t)\xi(t) + \overline{A}_{1i}(t)K\xi(t-h) + \overline{B}_i v(t)\right]dt + \left[\overline{D}_i(t)\xi(t) + \overline{D}_{1i}(t)K\xi(t-h) + \overline{D}_{2i}v(t)\right]d\omega(t),\\ e(t) &= \overline{L}_{1i}\xi(t) + L_{2i}K\xi(t-h) + L_{3i}v(t),\end{aligned} \tag{11}$$

where

$$\begin{aligned}\overline{A}_i(t) &= \overline{A}_i + \Delta\overline{A}_i(t),\\ \overline{A}_{1i}(t) &= \overline{A}_{1i} + \Delta\overline{A}_{1i}(t),\\ \overline{D}_i(t) &= \overline{D}_i + \Delta\overline{D}_i(t),\\ \overline{D}_{1i}(t) &= \overline{D}_{1i} + \Delta\overline{D}_{1i}(t)\end{aligned} \tag{12}$$

with

$$\begin{aligned}\overline{A}_i &= \begin{bmatrix} A_i & 0\\ B_{fi}C_i & A_{fi}\end{bmatrix},\\ \Delta\overline{A}_i(t) &= \begin{bmatrix}\Delta A_i(t) & 0\\ 0 & 0\end{bmatrix},\\ \overline{A}_{1i} &= \begin{bmatrix} A_{1i}\\ B_{fi}C_{1i}\end{bmatrix},\\ \Delta\overline{A}_{1i}(t) &= \begin{bmatrix}\Delta A_{1i}(t)\\ 0\end{bmatrix},\\ \overline{B}_i &= \begin{bmatrix} B_i\\ B_{fi}C_{2i}\end{bmatrix},\\ \overline{D}_i &= \begin{bmatrix} D_i & 0\\ B_{fi}E_i & 0\end{bmatrix},\\ \Delta\overline{D}_i(t) &= \begin{bmatrix}\Delta D_i(t) & 0\\ 0 & 0\end{bmatrix},\\ \overline{D}_{1i} &= \begin{bmatrix} D_{1i}\\ B_{fi}E_{1i}\end{bmatrix},\\ \Delta\overline{D}_{1i}(t) &= \begin{bmatrix}\Delta D_{1i}(t)\\ 0\end{bmatrix},\\ \overline{D}_{2i} &= \begin{bmatrix} D_{2i}\\ B_{fi}E_{2i}\end{bmatrix} K = \begin{bmatrix} I & 0\end{bmatrix},\\ \overline{L}_{1i} &= \begin{bmatrix} L_{1i} & -C_{fi}\end{bmatrix},\\ \overline{G}_i &= \begin{bmatrix} G_i\\ 0\end{bmatrix}.\end{aligned} \tag{13}$$

Then the problem of robust H_∞ filtering to be addressed in this paper is formulated as follows: given the uncertain stochastic delay systems (1) and a prescribed attenuation level $\gamma > 0$, design linear stochastic filter (Σ_f) as the form of (9) such that the filtering error systems (11) are robustly stochastically stable and under zero initial conditions, the following inequality holds:

$$\|e(t)\|_2 < \gamma\|v(t)\|_2 \tag{14}$$

for all nonzero $v(t) \in L_2[0,\infty)$ and all admissible uncertainties.

Before concluding this section, we introduce the following Lemmas, which will be used in the derivation of our main results in the next section.

Lemma 2. *For any vectors $x, y \in \mathbf{R}^n$ and any scalar $\epsilon > 0$, matrices D, F, E are real matrices of appropriate dimensions with $F^TF \le I$, then the following inequality hold:*

$$2x^TDFEy \le \epsilon^{-1}x^TDD^Tx + \epsilon y^TEE^Ty. \tag{15}$$

Proposition 3 ([5], generalized Finsler Lemma (GFL)). *Consider stochastic vector $\theta \in \mathbb{R}^n$, symmetric and positive matrix $\Theta \in \mathbb{R}^{n\times n}$, and matrix $\mathscr{B} \in \mathbb{R}^{m\times n}$ with rank $(\mathscr{B}) = r < n$. Let $\mathscr{B}^\perp$ represent the right orthogonal complement of $\mathscr{B}$, that is, $\mathscr{B}\mathscr{B}^\perp = 0$, then the following four statements are equivalent:*

($T1$) $\mathbf{E}\{\theta^T\Theta\theta\} < 0$, $\forall\theta \ne 0$, $t > t_0$, $\mathbf{E}\{\mathscr{B}\theta\} = 0$;

($T2$) $\mathscr{B}^{\perp T}\Theta\mathscr{B}^\perp < 0$;

($T3$) $\exists\epsilon \in \mathbb{R} : \Theta - \epsilon\mathscr{B}^T\mathscr{B} < 0$;

($T4$) $\exists\Lambda \in \mathbb{R}^{n\times m} : \Theta + \Lambda\mathscr{B} + \mathscr{B}^T\Lambda^T < 0$.

Remark 4. Based on generalized Finsler Lemma, the stability of neutral stochastic systems with time delay has been studied in [5, 34]. In addition, it should be noted that the stochastic systems in [5, 34] are not Markovian jump systems. And the filtering problem for stochastic time delay systems with Markovian jumping parameters is considered in [20]. However, it should be pointed out that the systems in [20] do not include any analysis of neutral phenomena. So H_∞ filtering for neutral stochastic systems with Markovian jumping parameters and time delay is considered in this paper.

3. Main Results

Theorem 5. *Consider the uncertain neutral stochastic Markovian jump systems (1). For given scalars $\gamma > 0$, $h > 0$, systems (1) are robustly stochastically stable for all admissible uncertainties satisfying (6) and (7), if there exist symmetric positive matrices $P_i > 0$, $R > 0$, $Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} > 0$, and scalar $\epsilon_i > 0$ satisfying*

$$\Omega_i = \begin{bmatrix} \Omega_{11i} & \Omega_{12i} & -K^T R G_i & P_i \overline{B}_i & h\overline{A}_i^T K^T R & \overline{D}_i^T P_i & \overline{L}_{1i}^T & P_i \overline{M}_{1i} \\ * & \Omega_{22i} & \Omega_{23i} & -\overline{G}_i^T P_i \overline{B}_i & h\overline{A}_{1i}^T K^T R & \overline{D}_{1i}^T P_i & L_{2i}^T & -\overline{G}_i^T P_i \overline{M}_{1i} \\ * & * & \Omega_{33i} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & h\overline{B}_i^T K^T R & \overline{D}_{2i}^T P_i & L_{3i}^T & 0 \\ * & * & * & * & -R & 0 & 0 & hRK\overline{M}_{1i} \\ * & * & * & * & * & -P_i & 0 & P_i \overline{M}_{2i} \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\epsilon_i \end{bmatrix} < 0, \quad (16)$$

where

$$\begin{aligned} \Omega_{11i} &= P_i \overline{A}_i + \overline{A}_i^T P_i + \sum_{j=1}^{N} \pi_{ij} P_j + K^T Q_1 K + \epsilon_i \widetilde{N}_{1i}^T \widetilde{N}_{1i} \\ &\quad - K^T R K, \\ \Omega_{12i} &= P_i \overline{A}_{1i} - \overline{A}_i^T P_i \overline{G}_i - \sum_{j=1}^{N} \pi_{ij} P_j \overline{G}_i + K^T Q_2 \\ &\quad + \epsilon_i \widetilde{N}_{1i}^T N_{2i} + K^T R (G_i + I), \\ \Omega_{22i} &= -\overline{G}_i^T P_i \overline{A}_{1i} - \overline{A}_{1i}^T P_i \overline{G}_i + \overline{G}_i^T \sum_{j=1}^{N} \pi_{ij} P_j \overline{G}_i + Q_3 - Q_1 \\ &\quad + \epsilon_i N_{2i}^T N_{2i} - (G_i + I)^T R (G_i + I), \\ \Omega_{23i} &= -Q_2 + (G_i + I)^T R G_i, \\ \Omega_{33i} &= -Q_3 - G_i^T R G_i, \\ \widetilde{N}_{1i} &= N_{1i} K = [N_{1i} \quad 0], \\ \overline{M}_{1i} &= \begin{bmatrix} M_{1i} \\ 0 \end{bmatrix}, \\ \overline{M}_{2i} &= \begin{bmatrix} M_{2i} \\ 0 \end{bmatrix}. \end{aligned} \quad (17)$$

Proof. For the purpose of convenience, the following notations are adopted:

$$\begin{aligned} \overline{z}(t) &= \xi(t) - \overline{G}_i K \xi(t-h), \\ f_v(t, \xi(t), i) &= \overline{A}_i(t) \xi(t) + \overline{A}_{1i}(t) K \xi(t-h) \\ &\quad + \overline{B}_i v(t), \\ g_v(t, \xi(t), i) &= \overline{D}_i(t) \xi(t) + \overline{D}_{1i}(t) K \xi(t-h) \\ &\quad + \overline{D}_{2i} v(t), \end{aligned} \quad (18)$$

and then the filtering error systems (11) become

$$d\overline{z}(t) = f_v(t, \xi(t), i)\, dt + g_v(t, \xi(t), i)\, dw(t). \quad (19)$$

Choose the Lyapunov-Krasovskii functional candidate as follows:

$$\begin{aligned} V(t, \xi(t), i) &= \overline{z}^T(t) P_i \overline{z}(t) + \int_{t-h}^{t} \eta^T(s) K^T Q K \eta(s)\, ds \\ &\quad + h \int_{-h}^{0} \int_{t+\beta}^{t} f_v^T(s, \xi(s), i) K^T R K f_v(s, \xi(s), i)\, ds\, d\beta, \end{aligned} \quad (20)$$

where $\eta(t) = \left[\xi^T(t) \quad \xi^T(t-h)\right]^T$. According to Itô's differential formula, the stochastic differential along systems (11) is

$$\begin{aligned} dV(t, \xi(t), i) &= \mathscr{L} V(t, \xi(t), i)\, dt \\ &\quad + 2\overline{z}^T(t) P_i g_v(t, \xi(t), i)\, dw(t), \end{aligned} \quad (21)$$

where

$$\begin{aligned}\mathscr{L}V(t,\xi(t),i) &= 2\bar{z}^T(t)P_i f_v(t,\xi(t),i) \\ &\quad + g_v^T(t,\xi(t),i)P_i g_v(t,\xi(t),i) \\ &\quad + \bar{z}^T(t)\sum_{j=1}^{N}\pi_{ij}P_j\bar{z}(t) + \eta^T(t)K^TQK\eta(t) \\ &\quad - \eta^T(t-h)K^TQK\eta(t-h) \\ &\quad + h^2 f_v^T(t,\xi(t),i)K^TRKf_v(t,\xi(t),i) \\ &\quad - \int_{t-h}^{t} f_v^T(s,\xi(s),i)K^ThRKf_v(s,\xi(s),i)\,ds.\end{aligned} \tag{22}$$

Firstly, we show that the filtering error systems (11) with $v(t)=0$ are robustly stochastically stable.

Taking mathematical expectation on both sides of system (19) and by virtue of $\mathbf{E}\{dw(t)=0\}$, we obtain

$$\mathbf{E}\{d\bar{z}(t)\} = \mathbf{E}\{f_v(t,\xi(t),i)\}\,dt. \tag{23}$$

Integrating both sides of (23) from $t-h$ to t, we have

$$\begin{aligned}\int_{t-h}^{t}\mathbf{E}\Big\{&-I\xi(t) + \left(\overline{G}_iK + I\right)\xi(t-h) - \overline{G}_iK\xi(t-2h) \\ &+ hf_v(s,\xi(s),i)\Big\}\,ds = 0.\end{aligned} \tag{24}$$

From the definition of $f_v(t,\xi(t),i)$ with $v(t)=0$ and (24), it is easy to obtain

$$\int_{t-h}^{t}\mathbf{E}\{\mathscr{B}_i\varsigma_i\}\,ds = 0, \tag{25}$$

where

$$\begin{aligned}\mathscr{B}_i &= \begin{bmatrix} -I & \overline{A}_i(t) & \overline{A}_{1i}(t) & 0 & 0 \\ 0 & -K & K\overline{G}_i + I & -K\overline{G}_i & hI \end{bmatrix}, \\ \varsigma_i &= \left[f_v^T(t,\xi(t),i) \;\; \xi^T(t) \;\; \xi^T(t-h)K^T \;\; \xi^T(t-2h)K^T \;\; f_v^T(s,\xi(s),i)K^T\right]^T.\end{aligned} \tag{26}$$

The right orthogonal complements of $\mathscr{B}_i$ is

$$\mathscr{B}_i^{\perp} = \begin{bmatrix} \overline{A}_i^T(t) & I & 0 & 0 & \dfrac{K^T}{h} \\ \overline{A}_{1i}^T(t) & 0 & I & 0 & -\dfrac{\left(K\overline{G}_i + I\right)^T}{h} \\ 0 & 0 & 0 & I & \dfrac{\left(K\overline{G}_i\right)^T}{h} \end{bmatrix}^T. \tag{27}$$

Taking mathematical expectation on both sides of (21) and then substituting (22) into (21), we have

$$\begin{aligned}\frac{[d\mathbf{E}V(t,\xi(t),i)]}{dt} &= \mathbf{E}\{\mathscr{L}V(t,\xi(t),i)\} \\ &\leq \mathbf{E}\left\{\frac{1}{h}\int_{t-h}^{t}\varsigma_i^T\Gamma_i\varsigma_i ds\right\} \\ &= \frac{1}{h}\int_{t-h}^{t}\mathbf{E}\left\{\varsigma_i^T\Gamma_i\varsigma_i\right\}ds,\end{aligned} \tag{28}$$

where

$$\begin{aligned}\Gamma_i &= \begin{bmatrix} h^2K^TRK & P_i & -P_i\overline{G}_i & 0 & 0 \\ * & \Gamma_{22i} & \Gamma_{23i} & 0 & 0 \\ * & * & \Gamma_{33i} & -Q_2 & 0 \\ * & * & * & -Q_3 & 0 \\ * & * & * & * & -h^2R \end{bmatrix}, \\ \Gamma_{22i} &= \sum_{j=1}^{N}\pi_{ij}P_j + \overline{D}_i^T(t)P_i\overline{D}_i(t) + K^TQ_1K, \\ \Gamma_{23i} &= -\sum_{j=1}^{N}\pi_{ij}P_j\overline{G}_i + \overline{D}_i^T(t)P_i\overline{D}_{1i}(t) + K^TQ_2, \\ \Gamma_{33i} &= \overline{G}_i^T\sum_{j=1}^{N}\pi_{ij}P_j\overline{G}_i + \overline{D}_{1i}^T(t)P_i\overline{D}_{1i}(t) + Q_3 - Q_1.\end{aligned} \tag{29}$$

In order to prove the robust stochastic stability of the filtering error systems (11) with $v(t)=0$, it suffices to show

$$\mathbf{E}\left\{\varsigma_i^T\Gamma_i\varsigma_i\right\} < 0. \tag{30}$$

By virtue of Proposition 3, (30) is equivalent to

$$\mathbf{E}\left\{\widehat{\Gamma}_i\right\} = \mathbf{E}\left\{\mathscr{B}_i^{\perp T}\Gamma_i\mathscr{B}_i^{\perp}\right\} < 0, \tag{31}$$

where

$$\hat{\Gamma}_i = \begin{bmatrix} \hat{\Gamma}_{11i} & \hat{\Gamma}_{12i} & -K^T RK\overline{G}_i \\ * & \hat{\Gamma}_{22i} & \hat{\Gamma}_{23i} \\ * & * & \hat{\Gamma}_{33i} \end{bmatrix},$$

$$\begin{aligned}
\hat{\Gamma}_{11i} &= h^2\overline{A}_i^T(t)K^T RK\overline{A}_i(t) + P_i\overline{A}_i(t) + \overline{A}_i^T(t)P_i \\
&\quad + \sum_{j=1}^N \pi_{ij}P_j + \overline{D}_i^T(t)P_i\overline{D}_i(t) + k^T Q_1 K \\
&\quad - K^T RK, \\
\hat{\Gamma}_{12i} &= h^2\overline{A}_i^T(t)K^T RK\overline{A}_{1i}(t) + P_i\overline{A}_{1i}(t) - \overline{A}_i^T(t)P_i\overline{G}_i \\
&\quad - \sum_{j=1}^N \pi_{ij}P_j\overline{G}_i + \overline{D}_i^T(t)P_i\overline{D}_{1i}(t) + K^T Q_2 \\
&\quad + K^T R\left(K\overline{G}_i + I\right), \\
\hat{\Gamma}_{22i} &= h^2\overline{A}_{1i}^T(t)K^T RK\overline{A}_{1i}(t) - \overline{G}_i^T P_i\overline{A}_{1i}(t) \\
&\quad - \overline{A}_{1i}^T(t)P_i\overline{G}_i + \overline{G}_i^T\sum_{j=1}^N \pi_{ij}P_j\overline{G}_i \\
&\quad + \overline{D}_{1i}^T(t)P_i\overline{D}_{1i}(t) + Q_3 - Q_1 \\
&\quad - \left(K\overline{G}_i + I\right)^T R\left(K\overline{G}_i + I\right), \\
\hat{\Gamma}_{23i} &= -Q_2 + \left(K\overline{G}_i + I\right)^T RK\overline{G}_i, \\
\hat{\Gamma}_{33i} &= -Q_3 - \left(K\overline{G}_i\right)^T RK\overline{G}_i.
\end{aligned} \tag{32}$$

It is obvious that $\hat{\Gamma}_i < 0$ are implied by (16) according to Schur complements. Therefore, if (16) is feasible, then filtering error systems (11) with $v(t) = 0$ are robustly stochastically stable.

Next, we will establish the H_∞ performance for the filtering error systems (11) under the zero initial condition.

From the definition of $f_v(t,\xi(t),i)$ and together with (24), it implies

$$\int_{t-h}^{t} \mathbf{E}\left\{\mathscr{B}_{vi}\varsigma_{vi}\right\} ds = 0, \tag{33}$$

where

$$\begin{aligned}
\mathscr{B}_{vi} &= \begin{bmatrix} -I & \overline{A}_i(t) & \overline{A}_{1i}(t) & 0 & 0 & \overline{B}_i \\ 0 & -K & K\overline{G}_i + I & -K\overline{G}_i & hI & 0 \end{bmatrix}, \\
\varsigma_{vi} &= \left[f_v^T(t,\xi(t),i) \quad \xi^T(t) \quad \xi^T(t-h)K^T \quad \xi^T(t-2h)K^T \quad f_v^T(s,\xi(s),i)K^T \quad v^T(t)\right]^T.
\end{aligned} \tag{34}$$

The right orthogonal complements of $\mathscr{B}_{vi}$ are

$$\mathscr{B}_{vi}^{\perp} = \begin{bmatrix} \overline{A}_i^T(t) & I & 0 & 0 & \dfrac{K^T}{h} & 0 \\ \overline{A}_{1i}^T(t) & 0 & I & 0 & -\dfrac{\left(K\overline{G}_i + I\right)^T}{h} & 0 \\ 0 & 0 & 0 & I & \dfrac{\left(K\overline{G}_i\right)^T}{h} & 0 \\ \overline{B}_i^T & 0 & 0 & 0 & 0 & I \end{bmatrix}^T. \tag{35}$$

Noticing (21)-(22) together with $\mathbf{E}\{dw(t)\} = 0$, we can obtain

$$\begin{aligned}
\frac{\left[d\mathbf{E}V(t,\xi(t),i)\right]}{dt} &= \mathbf{E}\left\{\mathscr{L}V(t,\xi(t),i)\right\} \\
&\le \mathbf{E}\left\{\frac{1}{h}\int_{t-h}^{t} \varsigma_{vi}^T\Gamma_{vi}\varsigma_{vi}ds\right\} \\
&= \frac{1}{h}\int_{t-h}^{t} \mathbf{E}\left\{\varsigma_{vi}^T\Gamma_{vi}\varsigma_{vi}\right\} ds,
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
\Gamma_{vi} &= \begin{bmatrix} \Gamma_i & \Gamma_v \\ * & \Gamma_{66i} \end{bmatrix}, \\
\Gamma_v &= \begin{bmatrix} 0 & \Gamma_{26i}^T & \Gamma_{36i}^T & 0 & 0 \end{bmatrix}^T, \\
\Gamma_{26i} &= \overline{D}_i^T(t)P_i\overline{D}_{2i}, \\
\Gamma_{36i} &= \overline{D}_{1i}^T(t)P_i\overline{D}_{2i}, \\
\Gamma_{66i} &= \overline{D}_{2i}^T P_i\overline{D}_{2i},
\end{aligned} \tag{37}$$

and set

$$J = \mathbf{E}\left\{\int_0^\infty \left[e^T(t)e(t) - \gamma^2 v^T(t)v(t)\right]dt\right\}. \tag{38}$$

Adding the right side of (38) to both sides of (36) and integrating both sides of (36) from 0 to ∞ and then taking the zero initial condition into account, we can acquire

$$J \le \frac{1}{h}\int_0^\infty \int_{t-h}^{t} \mathbf{E}\left\{\varsigma_{vi}^T\mathscr{H}_{vi}\varsigma_{vi}\right\} ds\,dt, \tag{39}$$

where

$$\mathscr{H}_{vi} = \Gamma_{vi} + \Pi^T\Pi + \operatorname{diag}\left\{0,0,0,0,0,-\gamma^2 I\right\}, \quad (40)$$

and $\Pi = \begin{bmatrix} 0 & \overline{L}_{1i} & L_{2i} & K & 0 & 0 & L_{3i} \end{bmatrix}$.

According to Proposition 3, $\mathbf{E}\{\varsigma_{vi}^T \mathscr{H}_{vi}\varsigma_{vi}\} < 0$ is equivalent to

$$\Theta_i = \mathscr{B}_{vi}^{\perp T}\mathscr{H}_{vi}\mathscr{B}_{vi}^{\perp} = \begin{bmatrix} \Theta_{11i} & \Theta_{12i} & -K^TRK\overline{G}_i & \Theta_{14i} \\ * & \Theta_{22i} & \Theta_{23i} & \Theta_{24i} \\ * & * & \Theta_{33i} & 0 \\ * & * & * & \Theta_{44i} \end{bmatrix} < 0, \quad (41)$$

where

$$\begin{aligned}
\Theta_{11i} &= \widehat{\Gamma}_{11i} + \overline{L}_{1i}^T\overline{L}_{1i},\\
\Theta_{12i} &= \widehat{\Gamma}_{12i} + \overline{L}_{1i}^T L_{2i},\\
\Theta_{22i} &= \widehat{\Gamma}_{22i} + L_{2i}^T L_{2i},\\
\Theta_{23i} &= \widehat{\Gamma}_{23i},\\
\Theta_{33i} &= \widehat{\Gamma}_{33i},\\
\Theta_{14i} &= h^2\overline{A}_i^T(t)K^TRK\overline{B}_i + P_i\overline{B}_i + \overline{D}_i^T(t)P_i\overline{D}_{2i} + \overline{L}_{1i}^T L_{3i},\\
\Theta_{24i} &= h^2\overline{A}_{1i}^T(t)K^TRK\overline{B}_i - \overline{G}_i^TP_i\overline{B}_i + \overline{D}_{1i}^T(t)P_i\overline{D}_{2i} + L_{2i}^TL_{3i},\\
\Theta_{44i} &= h^2\overline{B}_i^TK^TRK\overline{B}_i + \overline{D}_{2i}^TP_i\overline{D}_{2i} + L_{3i}^TL_{3i} - \gamma^2 I.
\end{aligned} \quad (42)$$

According to Schur complement and (12), we can obtain

$$\Theta_i = \begin{bmatrix} \Pi_{11i} & \Pi_{12i} & -K^TRG_i & P_i\overline{B}_i & h\overline{A}_i^TK^TR & \overline{D}_i^TP_i & \overline{L}_{1i}^T \\ * & \Pi_{22i} & \Pi_{23i} & -G_i^TP_i\overline{B}_i & h\overline{A}_{1i}^TK^TR & \overline{D}_{1i}^TP_i & L_{2i}^T \\ * & * & \Pi_{33i} & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & h\overline{B}_i^TK^TR & \overline{D}_{2i}^TP_i & L_{3i}^T \\ * & * & * & * & -R & 0 & 0 \\ * & * & * & * & * & -P_i & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} + \begin{bmatrix} \Theta_{11i} & \Theta_{12i} & 0 & 0 & h\Delta\overline{A}_i^T(t)K^TR & \Delta\overline{D}_i^T(t)P_i & 0 \\ \Theta_{21i} & \Theta_{22i} & 0 & 0 & h\Delta\overline{A}_{1i}^T(t)K^TR & \Delta\overline{D}_{1i}^T(t)P_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ hRK\Delta\overline{A}_i(t) & hRK\Delta\overline{A}_{1i}(t) & 0 & 0 & 0 & 0 & 0 \\ P_i\Delta\overline{D}_i^T(t) & P_i\Delta\overline{D}_{1i}(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (43)$$

where

$$\begin{aligned}
\Pi_{11i} &= P_i\overline{A}_i + \overline{A}_i^TP_i + \sum_{j=1}^{N}\pi_{ij}P_j + k^TQ_1K - K^TRK,\\
\Pi_{12i} &= P_i\overline{A}_{1i} - \overline{A}_i^TP_i\overline{G}_i - \sum_{j=1}^{N}\pi_{ij}P_j\overline{G}_i + K^TQ_2 + K^TR\left(K\overline{G}_i + I\right),\\
\Pi_{22i} &= -\overline{G}_i^TP_i\overline{A}_{1i} - \overline{A}_{1i}^TP_i\overline{G}_i + \overline{G}_i^T\sum_{j=1}^{N}\pi_{ij}P_j\overline{G}_i + Q_3 - Q_1 - \left(K\overline{G}_i + I\right)^TR\left(K\overline{G}_i + I\right),\\
\Pi_{23i} &= -Q_2 + \left(K\overline{G}_i + I\right)^TRK\overline{G}_i,\\
\Pi_{33i} &= -Q_3 - \left(K\overline{G}_i\right)^TRK\overline{G}_i,\\
\Theta_{11i} &= P_i\Delta\overline{A}_i(t) + \Delta\overline{A}_i^T(t)P_i,\\
\Theta_{12i} &= P_i\Delta\overline{A}_{1i}(t) - \Delta\overline{A}_i^T(t)P_i\overline{G}_i,\\
\Theta_{21i} &= \Delta\overline{A}_{1i}^T(t)P_i - \overline{G}_i^TP_i\Delta\overline{A}_i(t),\\
\Theta_{22i} &= -\overline{G}_i^TP_i\Delta\overline{A}_{1i}(t) - \Delta\overline{A}_{1i}^T(t)P_i\overline{G}_i.
\end{aligned} \quad (44)$$

Then by Lemma 2, it can be seen that

$$\begin{bmatrix} \Theta_{11i} & \Theta_{12i} & 0 & 0 & h\Delta\overline{A}_i^T(t)K^TR & \Delta\overline{D}_i^T(t)P_i & 0 \\ \Theta_{21i} & \Theta_{22i} & 0 & 0 & h\Delta\overline{A}_{1i}^T(t)K^TR & \Delta\overline{D}_{1i}^T(t)P_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ hRK\Delta\overline{A}_i(t) & hRK\Delta\overline{A}_{1i}(t) & 0 & 0 & 0 & 0 & 0 \\ P_i\Delta\overline{D}_i^T(t) & P_i\Delta\overline{D}_{1i}(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \Upsilon F_i(t)\Omega + \Omega^T F_i(t)^T\Upsilon^T \le \frac{1}{\epsilon_i}\Upsilon\Upsilon^T + \epsilon_i\Omega^T\Omega, \quad (45)$$

where $\Upsilon = \left[\overline{M}_{1i}^T P_i \;\; -\overline{M}_{1i}^T P_i\overline{G}_i \;\; 0 \;\; 0 \;\; h\overline{M}_{1i}K^TR \;\; \overline{M}_{2i}^T P_i \;\; 0\right]^T$, $\Omega = \left[\widetilde{N}_{1i} \;\; N_{2i} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right]$.

By (41), (46), and Schur complements, $\Theta_i < 0$ holds if and only if $\Omega_i < 0$. This completes the proof. □

Remark 6. Theorem 5 is established based on GFL. For the sake of reducing the computational complexity, similar to [6, 8, 10], the first two equivalent conditions of Proposition 3 are adopted in this paper.

Now we are in a position to present the H_∞ filter design for uncertain neutral stochastic system with Markovian jumping parameters and time delay based on Theorem 5.

Theorem 7. *Consider systems (1), for given scalars $h > 0$, $\gamma > 0$; then there exists a linear stochastic full-order filter with the form (9), such that filter error systems (11) are robustly stochastically stable and satisfy prescribed H_∞ disturbance attenuation level γ for all admissible uncertainties (6) and (7) if there exist symmetric positive matrices $X_i > 0$, $F_i > 0$, $Q = \left[\begin{smallmatrix} Q_1 & Q_2 \\ * & Q_3 \end{smallmatrix}\right] > 0$, and $R > 0$ and matrices A_{Fi}, B_{Fi}, C_{Fi}, scalars $\epsilon_i > 0$, such that the following LMI holds:*

$$\begin{bmatrix} \Upsilon_{11i} & \Upsilon_{12i} & \Upsilon_{13i} & -RG_i & \Upsilon_{15i} & hA_i^TR & \Upsilon_{17i} & \Upsilon_{18i} & L_{1i}^T - C_{Fi}^T & \Upsilon_{110i} \\ * & \Upsilon_{22i} & \Upsilon_{23i} & 0 & \Upsilon_{25i} & 0 & 0 & 0 & -C_{Fi}^T & -F_iM_{1i} \\ * & * & \Upsilon_{33i} & \Upsilon_{34i} & \Upsilon_{35i} & hA_{1i}^TR & \Upsilon_{37i} & \Upsilon_{38i} & L_{2i}^T & -G_i^TX_iM_{1i} \\ * & * & * & \Upsilon_{44i} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & hB_i^TR & \Upsilon_{57i} & \Upsilon_{58i} & L_{3i}^T & 0 \\ * & * & * & * & * & -R & 0 & 0 & 0 & hRM_{1i} \\ * & * & * & * & * & * & -X_i + F_i & 0 & 0 & \Upsilon_{710i} \\ * & * & * & * & * & * & * & -F_i & 0 & -F_iM_{2i} \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon_i I \end{bmatrix} < 0, \quad (46)$$

where

$$\Upsilon_{11i} = (X_i - F_i)A_i + A_i^T(X_i - F_i) + \sum_{j=1}^N \pi_{ij}(X_j - F_j) + Q_1 - R + \epsilon_i N_{1i}^T N_{1i},$$

$$\Upsilon_{12i} = -A_i^T F_i + C_i^T B_{Fi}^T + A_{Fi}^T,$$

$$\Upsilon_{22i} = A_{Fi} + A_{Fi}^T + \sum_{j=1}^N \pi_{ij}F_j,$$

$$\Upsilon_{13i} = (X_i - F_i)A_{1i} - A_i^T X_i G_i + C_i^T B_{Fi}^T G_i + A_{Fi}^T G_i + \sum_{j=1}^N \pi_{ij}(F_j - X_j)G_i + Q_2 + R(G_i + I) + \epsilon_i N_{1i}^T N_{2i},$$

$$\Upsilon_{23i} = -F_iA_{1i} + B_{Fi}C_{1i} + A_{Fi}^T G_i + \sum_{j=1}^N \pi_{ij}F_jG_i,$$

$$\Upsilon_{33i} = -G_i^T X_i A_{1i} - A_{1i}^T X_i^T G_i + G_i^T B_{Fi}C_{1i} + C_{1i}^T B_{Fi}^T G_i + G_i^T\sum_{j=1}^N \pi_{ij}X_jG_i + Q_3 - Q_1 - (G_i + I)^T R(G_i + I) + \epsilon_i N_{2i}^T N_{2i},$$

$$
\begin{aligned}
\Upsilon_{34i} &= -Q_2 + (G_i + I)^T RG_i, \\
\Upsilon_{44i} &= -Q_3 - G_i^T RG_i, \\
\Upsilon_{15i} &= (X_i - F_i) B_i, \\
\Upsilon_{25i} &= -F_i B_i + B_{Fi} C_{2i}, \\
\Upsilon_{35i} &= -G_i^T X_i B_i + G_i^T B_{Fi} C_{2i}, \\
\Upsilon_{17i} &= D_i^T (X_i - F_i), \\
\Upsilon_{37i} &= D_{1i}^T (X_i - F_i), \\
\Upsilon_{57i} &= D_{2i}^T (X_i - F_i), \\
\Upsilon_{18i} &= -D_i^T F_i + E_i^T B_{Fi}^T, \\
\Upsilon_{38i} &= -D_{1i}^T F_i + E_{1i}^T B_{Fi}^T, \\
\Upsilon_{58i} &= -D_{2i}^T F_i + E_{2i}^T B_{Fi}^T, \\
\Upsilon_{110i} &= (X_i - F_i) M_{1i}, \\
\Upsilon_{710i} &= (X_i - F_i) M_{2i}.
\end{aligned} \tag{47}
$$

Meanwhile, the filter parameters are given by

$$
\begin{aligned}
A_{fi} &= F_i^{-1} A_{Fi}, \\
B_{fi} &= F_i^{-1} B_{Fi}, \\
C_{fi} &= C_{Fi}.
\end{aligned} \tag{48}
$$

Proof. We note that, from (48), it is easy to see $\left[\begin{smallmatrix} X_i & -F_i \\ -F_i & F_i \end{smallmatrix}\right] > 0$, and $X_i > F_i > 0$. Define

$$
P_i = \begin{bmatrix} X_i & -F_i \\ -F_i & F_i \end{bmatrix}; \tag{49}
$$

then applying Schur complement, $X_i - F_i F_i^{-1} F_i = X_i - F_i > 0$ guarantees $P_i > 0$. Let

$$
\mathcal{J} = \operatorname{diag}\{T, I, I, I, I, T, I, I, I\}, \tag{50}
$$

where $T = \left[\begin{smallmatrix} I & 0 \\ I & I \end{smallmatrix}\right]$. Substituting P_i and (12) into (16), then pre- and postmultiplying (16) by $\mathcal{J}^T$ and $\mathcal{J}$, respectively, and using (51), the desired result (48) follows immediately. This completes the proof. □

Remark 8. Theorem 7 considers the H_∞ filtering problem for uncertain neutral stochastic time delay systems with Markovian jumping parameters. It should be noted that the proposed conditions are formulated in terms of LMIs. Therefore, by MATLAB LMI toolbox, for given different h or γ, the lower bound of performance index γ and the upper bound of h can be efficiently obtained by solving a generalized eigenvalue problem.

Now we would like to proceed to present the H_∞ filtering for uncertain neutral stochastic time delay systems without Markovian jumping parameters. Considering the system (Σ) without the Markovian jumping parameters, the following systems can be obtained:

$$
\begin{aligned}
(\Sigma_D) : d[x(t) - Gx(t-h)] &= [A(t)x(t) + A_1(t)x(t-h) + Bv(t)]\,dt \\
&\quad + [D(t)x(t) + D_1(t)x(t-h) + D_2 v(t)]\,dw(t),
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
dy(t) &= [Cx(t) + C_1 x(t-h) + C_2 v(t)]\,dt \\
&\quad + [Ex(t) + E_1 x(t-h) + E_2 v(t)]\,dw(t),
\end{aligned} \tag{52}
$$

$$
z(t) = L_1 x(t) + L_2 x(t-h) + L_3 v(t), \tag{53}
$$

$$
\begin{aligned}
x(\theta) &= \psi(\tau), \\
r_t &= r_0 \in S, \quad \forall \tau \in [-h, 0],
\end{aligned} \tag{54}
$$

where

$$
\begin{aligned}
A(t) &= A + \Delta A(t), \\
A_1(t) &= A_1 + \Delta A_1(t), \\
D(t) &= D + \Delta D(t), \\
D_1(t) &= D_1 + \Delta D_1(t),
\end{aligned} \tag{55}
$$

and $\Delta A(t)$, $\Delta A_1(t)$, $\Delta D(t)$, and $\Delta D_1(t)$ are unknown matrices satisfying

$$
\begin{bmatrix} \Delta A(t) & \Delta A_1(t) \\ \Delta D(t) & \Delta D_1(t) \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(t) \begin{bmatrix} N_1 & N_2 \end{bmatrix}. \tag{56}
$$

We can obtain the following Corollary 9 for system (Σ_D).

Corollary 9. *Consider the system (Σ_D), for given scalars $h > 0$, $\gamma > 0$; then there exists a linear stochastic full-order filter with the form (9), if there exist symmetric positive matrices $F > 0$, $X > 0$, $R > 0$, $Q = \left[\begin{smallmatrix} Q_1 & Q_2 \\ * & Q_3 \end{smallmatrix}\right] > 0$, matrices A_F, B_F, C_F, and scalar $\epsilon > 0$, such that the following LMI holds:*

$$\Xi=\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & -RG & \Xi_{15} & hA^TR & \Xi_{17} & \Xi_{18} & L_1^T-C_F^T & (X-F)M_1\\ * & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 & 0 & 0 & -C_F^T & -FM_1\\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & hA_1^TR & \Xi_{37} & \Xi_{38} & L_2^T & -G^TXM_1\\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -\gamma^2I & hB^TR & \Xi_{57} & \Xi_{58} & L_3^T & 0\\ * & * & * & * & * & -R & 0 & 0 & 0 & hRM_1\\ * & * & * & * & * & * & -X+F & 0 & 0 & (X-F)M_2\\ * & * & * & * & * & * & * & -F & 0 & -FM_2\\ * & * & * & * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & * & * & * & -\epsilon I\end{bmatrix}<0, \quad (57)$$

where

$$\begin{aligned}
\Xi_{11} &= (X-F)A + A^T(X-F) + Q_1 - R + \epsilon N_1^T N_1,\\
\Xi_{12} &= -A^TF + C^TB_F^T + A_F^T,\\
\Xi_{13} &= (X-F)A_1 - A^TXG + C^TB_F^TG + A_F^TG\\
&\quad + \epsilon N_1^T N_2 + Q_2 + R(G+I),\\
\Xi_{15} &= (X-F)B,\\
\Xi_{17} &= D^T(X-F),\\
\Xi_{18} &= -D^TF + E^TB_F^T,\\
\Xi_{22} &= A_F + A_F^T,\\
\Xi_{23} &= -FA_1 + B_FC_1 + A_F^TG,\\
\Xi_{25} &= -FB + B_FC_2,\\
\Xi_{33} &= -G^TXA_1 - A_1^TXG + G^TB_FC_1 + C_1^TB_F^TG + Q_3\\
&\quad - Q_1 - (G+I)^T R(G+I) + \epsilon N_2^T N_2,\\
\Xi_{34} &= -Q_2 + (G+I)^T RG,\\
\Xi_{35} &= -G^TXB + G^TB_FC_2,\\
\Xi_{37} &= D_1^T(X-F),\\
\Xi_{38} &= -D_1^TF + E_1^TB_F^T,\\
\Xi_{44} &= -Q_3 - G^TRG,\\
\Xi_{57} &= D_2^T(X-F),\\
\Xi_{58} &= -D_2^TF + E_2^TB_F^T;
\end{aligned} \quad (58)$$

then the robust H_∞ filtering problem is solvable. Furthermore, the parameters of the desired robust H_∞ filter can be given as

$$\begin{aligned}
A_f &= F^{-1}A_F,\\
B_f &= F^{-1}B_F,\\
C_f &= C_F.
\end{aligned} \quad (59)$$

Remark 10. The proof of Corollary 9 follows the same lines as that in the proof of Theorem 7, so the detailed procedure is omitted here. When $D_2 = 0$, $E = 0$, $E_1 = 0$, and $E_2 = 0$, systems (52)–(55) in this paper reduces to systems (1) in [18]. It is noticed that the filtering problem studied in [18] is a special case of this paper. For comparisons of our results with that in [18], see Example 1 in detail.

4. Numerical Examples

In this section, numerical examples and simulations are given to illustrate the validity and benefits of the proposed approach.

Example 1. Consider systems (52)–(55) without Markovian jumping parameters as follows:

$$A=\begin{bmatrix}-1 & 0\\ 0 & -0.9\end{bmatrix},$$

$$A_1=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},$$

$$B=\begin{bmatrix}0.2 & 0\\ -0.1 & 0.1\end{bmatrix},$$

$$D=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},$$

$$D_1=\begin{bmatrix}0.1 & 0\\ 0.3 & -0.2\end{bmatrix},$$

$$D_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},$$

Table 1: Upper bounds of h for different γ (Example 1).

γ	0.3	0.8	1.2	1.8	2.4	3.0
h by Corollary 9	1.079	1.786	1.954	2.089	2.167	2.219

$$C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0 & 0.15 \\ -0.1 & 0.1 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix},$$
$$E = \begin{bmatrix} 0.5 & 0.5 \\ 1.0 & 0 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 1.0 & 0.2 \\ 0.3 & -1 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 0.2 & 0.5 \\ 0.3 & 0.4 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.1 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 0.1 & -0.15 \\ 0 & 0.15 \end{bmatrix},$$
$$L_3 = \begin{bmatrix} 0.2 & 0 \\ -0.15 & 0.1 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.2 \end{bmatrix},$$
$$G = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix},$$
$$N_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}. \quad (60)$$

For different given noise attenuation levels γ, the upper bounds of delay for systems (52)–(55) in Corollary 9 of this paper are presented in Table 1. For different given time delays, the lower bounds of noise attenuation level γ for systems (52)–(55) in Corollary 9 of this paper are provided in Table 2.

In particular, when $D_2 = 0$, $E = 0$, $E_1 = 0$, and $E_2 = 0$, systems (52)–(55) in this paper reduce to systems (1) in [18]. For different given noise attenuation levels γ, the comparison of the upper bounds of delay in [18] with our results is presented in Table 3, and, for different given time delays, the lower bounds of noise attenuation level γ for systems (1) in [18] and the same systems in this paper are provided in Table 4.

Table 2: Lower bounds of γ for different h (Example 1).

h	0.5	1.0	1.5	2.0	2.2	2.4
γ by Corollary 9	0.275	0.291	0.473	1.364	2.745	10.940

Table 3: Upper bounds of h for different γ (Example 1).

γ	0.3	0.8	1.2	1.8	2.4	3.0
h by Theorem 3 [18]	1.157	1.808	1.926	1.991	2.025	2.046
h by Corollary 9	1.409	2.138	2.263	2.335	2.371	2.394

Table 4: Lower bounds of γ for different h (Example 1).

h	0.5	1	1.5	2	2.1	2.2
γ by Theorem 3 [18]	0.264	0.270	0.630	1.919	8.235	infeasible
γ by Corollary 9	0.264	0.266	0.340	0.712	0.766	0.925

Besides, Theorem 2 in [13] fails to give a feasible solution. The number of decision variables of Theorem 3 in [18] is $(13n^2 + 5n + 2)/2$, which is the same as that in Corollary 9 of this paper. From Tables 3 and 4, we can see that our proposed method is less conservative than that in [18].

Now in the case when $\gamma = 2.4$, $h_{\max} = 2.167$, we resort to the MATLAB LMI control toolbox to solve the LMI (59), and the feasible solution can be obtained as follows:

$$\begin{aligned} d\hat{x}(t) &= \begin{bmatrix} -2.6806 & -0.1892 \\ -8.4386 & -2.0417 \end{bmatrix} \hat{x}(t)\, d(t) \\ &\quad + \begin{bmatrix} 0.0232 & 0.0476 \\ -0.1573 & 0.3199 \end{bmatrix} dy(t), \\ \hat{z}(t) &= \begin{bmatrix} -0.3127 & 0.0474 \\ 0.1536 & -0.0621 \end{bmatrix} \hat{x}(t). \end{aligned} \quad (61)$$

The initial conditions are also taken as $x(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} -2 & 2 \end{bmatrix}^T$. The simulation results of the state response of the system are plotted in Figures 1–3. The filter state $x_1(t)$ and its estimation $\hat{x}_1(t)$ and state $x_2(t)$ and its estimation $\hat{x}_2(t)$ are given in Figures 1 and 2, respectively. Figure 3 depicts the estimation error $e(t) = z(t) - \hat{z}(t)$. The simulation results demonstrate that the designed H_∞ filters are feasible, effective and the stochastic stability of the error systems is ensured.

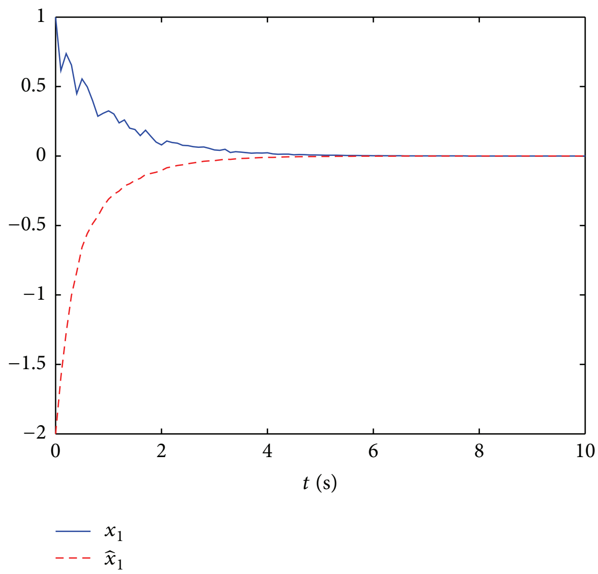

FIGURE 1: State $x_1(t)$ and its estimation $\hat{x}_1(t)$ in Example 1.

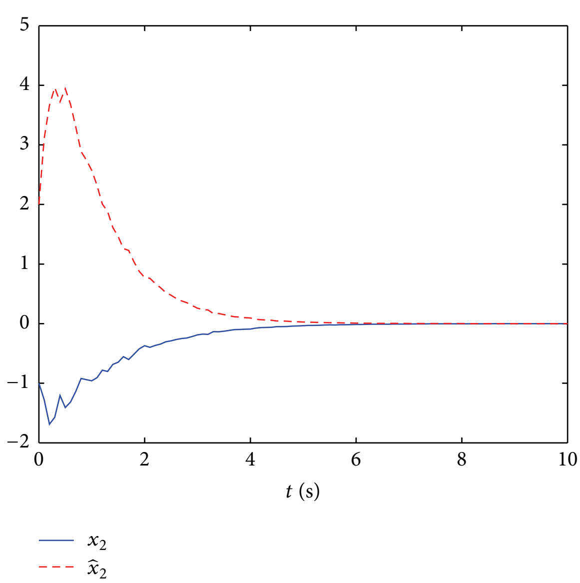

FIGURE 2: State $x_2(t)$ and its estimation $\hat{x}_2(t)$ in Example 1.

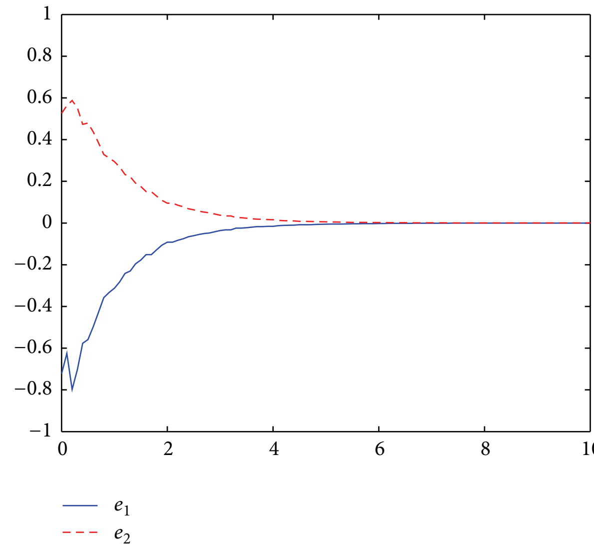

FIGURE 3: The error responses $e_1(t)$ and $e_2(t)$ in Example 1.

Example 2. Consider systems (1) with Markovian jumping parameters as follows.

Mode 1. Consider

$$A_1 = \begin{bmatrix} -2.0 & 0 \\ 0 & -1.9 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.15 & 0.4 \\ 0.2 & 0.3 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 0.3 & 0.9 \\ 0.2 & 0.1 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 1.0 & 0.2 \\ 0.3 & -1 \end{bmatrix},$$

$$E_{11} = \begin{bmatrix} 0.2 & 0.5 \\ 0.3 & 0.4 \end{bmatrix},$$

$$E_{21} = \begin{bmatrix} 0.5 & 0.5 \\ 1.0 & 0 \end{bmatrix},$$

$$L_{11} = \begin{bmatrix} 0.8 & 0.6 \\ 0.4 & 0.3 \end{bmatrix},$$

$$L_{21} = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.8 \end{bmatrix},$$

$$L_{31} = \begin{bmatrix} 0.6 & 0.5 \\ 0.3 & 0.4 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$M_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$N_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$N_{21} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}. \tag{62}$$

Mode 2. Consider

$$A_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} -1.0 & 0 \\ -1.0 & -0.5 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.15 & 0.4 \\ 0.2 & 0.31 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$D_{12} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$G_2 = \begin{bmatrix} -0.6 & 0 \\ 0 & -0.6 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$C_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$C_{22} = \begin{bmatrix} 0.3 & 0.9 \\ 0.2 & 0.15 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 0.5 & 0.6 \\ 1.2 & 0 \end{bmatrix},$$
$$E_{12} = \begin{bmatrix} 1.0 & 0.3 \\ 0.4 & -1.0 \end{bmatrix},$$
$$E_{22} = \begin{bmatrix} 0.25 & 0.5 \\ 0.3 & 0.4 \end{bmatrix},$$
$$L_{12} = \begin{bmatrix} 0.8 & 0.5 \\ 0.6 & 0.8 \end{bmatrix},$$

Table 5: Upper bounds of h for different γ (Example 2).

γ	0.5	2	3	4	5
h by Theorem 7	0.104	0.232	0.311	0.343	0.361

Table 6: Lower bounds of γ for different h (Example 2).

h	0.1	0.2	0.3	0.4
γ by Theorem 7	1.493	1.806	2.774	15.047

$$L_{22} = \begin{bmatrix} 0.4 & 0.8 \\ 0.3 & 0.5 \end{bmatrix},$$
$$L_{32} = \begin{bmatrix} 0.5 & 1.0 \\ 0.6 & 0.4 \end{bmatrix},$$
$$M_{12} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$M_{22} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$N_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$N_{22} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$\pi_{11} = -1,$$
$$\pi_{22} = -2,$$
$$\pi_{12} = 1,$$
$$\pi_{21} = 2. \tag{63}$$

In this section, the purpose of this example is to design a full-order filter in the form of (9) such that the filtering error system is robustly stochastically stable for all admissible uncertainties and satisfies the required H_∞ performance level.

By Theorem 7, for different given noise attenuation levels γ, the upper bounds of delay for systems (1) are presented in Table 5. For different given time delays, the lower bounds of noise attenuation level γ for systems (1) are provided in Table 6.

Now in the case when $\gamma = 2$, $h_{\max} = 0.232$, we resort to the MATLAB LMI control toolbox to solve the LMI (48), and we obtain the solution as follows:

$$Q_1 = \begin{bmatrix} 9.6525 & -1.9471 \\ -1.9471 & 1.6603 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} -10.1587 & -0.2297 \\ 2.2216 & -1.6233 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 10.8069 & -0.0873 \\ -0.0873 & 2.3599 \end{bmatrix},$$

$$R = \begin{bmatrix} 39.1583 & -0.3727 \\ -0.3727 & 8.8082 \end{bmatrix},$$

$$X_1 = \begin{bmatrix} 2.8376 & 0.1553 \\ 0.1553 & 2.4043 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 4.3724 & 0.4730 \\ 0.4730 & 5.8205 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.2942 & 0.2896 \\ 0.2896 & 1.1395 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 1.2783 & 0.9020 \\ 0.9020 & 2.8727 \end{bmatrix},$$

$$\epsilon_1 = 1.1107,$$

$$\epsilon_2 = 8.1439,$$

$$A_{F1} = \begin{bmatrix} -0.9169 & -0.6283 \\ -1.6111 & -2.5633 \end{bmatrix},$$

$$B_{F1} = \begin{bmatrix} 0.1262 & 0.0730 \\ 0.2392 & 0.3601 \end{bmatrix},$$

$$C_{F1} = \begin{bmatrix} 0.5690 & 0.9257 \\ 0.4004 & 1.0310 \end{bmatrix},$$

$$A_{F2} = \begin{bmatrix} -1.4295 & -1.3599 \\ -2.9406 & -2.5256 \end{bmatrix},$$

$$B_{F2} = \begin{bmatrix} 1.8563 & -0.7369 \\ 3.0463 & -1.0766 \end{bmatrix},$$

$$C_{F2} = \begin{bmatrix} 1.1602 & 1.1086 \\ 0.7364 & 0.9807 \end{bmatrix}. \tag{64}$$

Therefore, the full-order stochastic filter parameters are given as follows:

$$A_{f1} = \begin{bmatrix} -2.3001 & 0.1251 \\ -0.8294 & -2.2813 \end{bmatrix},$$

$$B_{f1} = \begin{bmatrix} 0.2963 & -0.0837 \\ 0.1346 & 0.3373 \end{bmatrix},$$

$$C_{f1} = \begin{bmatrix} 0.5690 & 0.9257 \\ 0.4004 & 1.0310 \end{bmatrix},$$

$$A_{f2} = \begin{bmatrix} -0.5087 & -0.5697 \\ -0.8639 & -0.7003 \end{bmatrix},$$

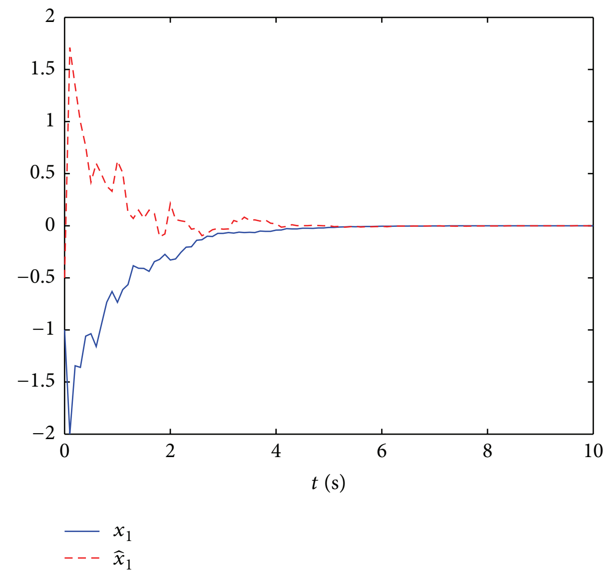

Figure 4: State $x_1(t)$ and its estimation $\hat{x}_1(t)$ in Example 2.

$$B_{f2} = \begin{bmatrix} 0.9043 & -0.4008 \\ 0.7765 & -0.2489 \end{bmatrix},$$

$$C_{f2} = \begin{bmatrix} 1.1602 & 1.1086 \\ 0.7364 & 0.9807 \end{bmatrix}. \tag{65}$$

The initial conditions are also taken as $x(0) = [-1 \;\; 1.5]^T$ and $\hat{x}(0) = [-0.5 \;\; 1]^T$. The simulation results of the state response of the system are plotted in Figures 4–7. The filter state $x_1(t)$ and its estimation $\hat{x}_1(t)$ and state $x_2(t)$ and its estimation $\hat{x}_2(t)$ are, respectively, given in Figures 4 and 5. Assuming the simulation step size $\Delta = 0.1$, one of the possible realizations of the Markovian jumping mode is plotted in Figure 6. Figure 7 depicts the estimation error $e(t) = z(t) - \hat{z}(t)$. It is clearly observed from the simulation results that the designed H_∞ filter satisfies the specified requirements and the expected objectives are well achieved.

Remark 3. In order to reduce conservatism, many methods such as cross-terms bounding techniques, free-weighting matrices method, and cross-terms bounding techniques are often adopted in the stability analysis of stochastic systems. In this paper, the generalized Finsler Lemma is utilized in uncertain neutral stochastic systems, which can bring the low conservatism and less computational cost.

5. Conclusions

In this paper, the robust filtering problem for a class of uncertain neutral stochastic systems with Markovian jumping parameters and time delay has been considered. Based on the Lyapunov-Krasovskii functional theory and generalized Finsler Lemma, a delay-dependent sufficient condition is

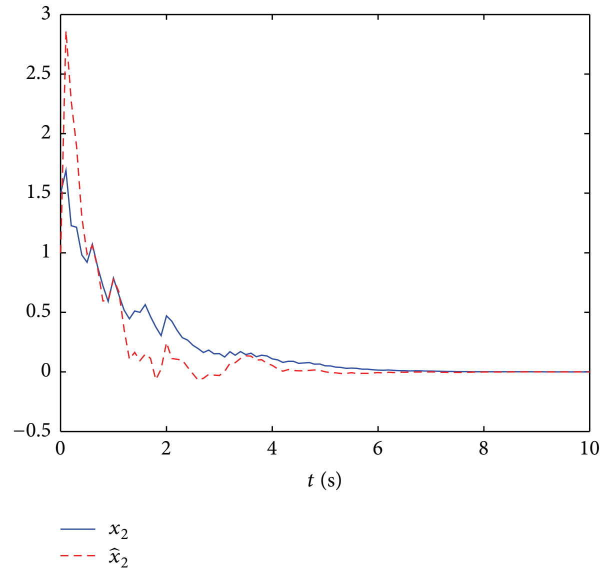

FIGURE 5: State $x_2(t)$ and its estimation $\hat{x}_2(t)$ in Example 2.

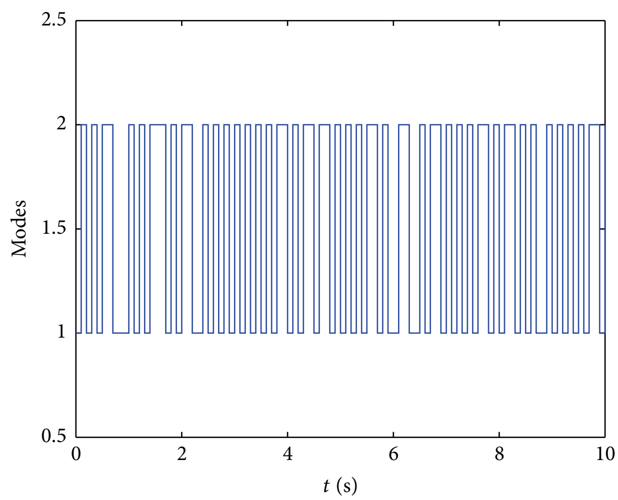

FIGURE 6: Markovian jumping mode in Example 2.

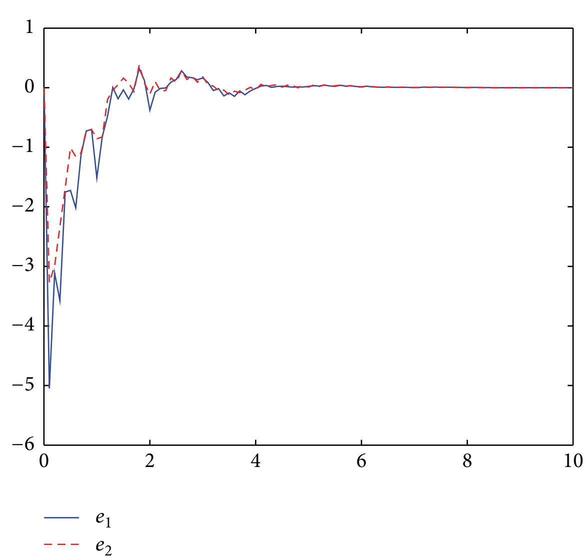

FIGURE 7: The error responses $e_1(t)$ and $e_2(t)$ in Example 2.

proposed for the existence of H_∞ filters which reduces the conservatism. The obtained result ensures the robust stability and a prescribed H_∞ performance level of the filtering error system for all admissible uncertainties. Two numerical examples and simulations have been presented to demonstrate the usefulness and effectiveness of the proposed filter design method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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