

Research Article

Nonlinear Constrained Adaptive Backstepping Tracking Control for a Hypersonic Vehicle with Uncertainty

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The control problem of a flexible hypersonic vehicle is presented, where input saturation and aerodynamic uncertainty are considered. A control-oriented model including aerodynamic uncertainty is derived for simple controller design due to the nonlinearity and complexity of hypersonic vehicle model. Then it is separated into velocity subsystem and altitude subsystem. On the basis of the integration of robust adaptive control and backstepping technique, respective controller is designed for each subsystem, where an auxiliary signal provided by an additional dynamic system is used to compensate for the control saturation effect. Then to deal with the "explosion of terms" problem inherent in backstepping control, a novel first-order filter is proposed. Simulation results are included to demonstrate the effectiveness of the adaptive backstepping control scheme.

1. Introduction

Air-breathing hypersonic vehicles (AHVs) are characterized by their unique design, incorporating a supersonic combustion ramjet engine located beneath the fuselage. This esoteric configuration results in strong coupling between the thrust and pitch dynamics of the vehicle, which in combination with flexible effects and static instability make the vehicle a challenging application for control [1]. In addition, there are sensitivity changes in the flight conditions, uncertain aerodynamic characteristics of the vehicle, and highly nonlinear nature of hypersonic vehicle dynamics. Thus the problem of control design is one of the key techniques for the application of flexible air-breathing hypersonic vehicle (FAHV), and the control system is required to have robustness to uncertainty. Since there is difficulty in accurately measuring and estimating vehicle's aerodynamic characteristics, only the longitudinal analytical model of FAHV proposed in [2] has been used for controller design. For the enormous complexity of the nonlinear dynamics of FAHV, linear control theory has been widely employed for flight control design based on a linearized model [3–7]. These controllers are designed based

on a linearized model which is obtained at specified trim condition or obtained by feedback linearization technique. With the development of nonlinear control theory, nonlinear control schemes are used to design the controller of FAHV [8–14].

Though the research mentioned above achieved satisfactory control performance, it has not considered input saturation. And it usually appears in many practical systems because the amplitudes of control inputs of almost all practical control systems are limited. The closed-loop system performance may be degraded severely or even lose stability if the input constraint is ignored. Some control methods are applied to handle input constraints [15-17]. For the flight control system, under the occurrence of input saturation, it is in open loop state. If the output of actuator does not return to linear work space, the hypersonic vehicle may lose stability or even disintegrates. Thus it is necessary to design the high reliability control system with input constraints. Many control approaches have been presented to design the controller for the hypersonic vehicle with the consideration of input constraints. Anti-windup control was developed to handle input constraint of hypersonic vehicles while

the uncertainty was not considered [18]. Model predictive control has been used popularly because of its inherent capability to implement input constraint directly at the level of control design [19]. However, it depends on the real-time receding horizon optimization, and the main barrier of it applied to hypersonic vehicle is online optimization and the determination of time-domain step size [20]. H_{∞} approach was proposed for a linearized FAHV model in the presence of uncertain parameters and input constraints, where the linearized model was obtained by the feedback linearization approach [21]. It should be pointed out that high-order derivatives of outputs need to be computed. By using the differential geometry principle and the total energy theory, advanced flight control laws were designed for hypersonic vehicle in the presence of actuator limitations [22]. Three adaptive fault control schemes were proposed for AHV in considering external disturbances, actuator faults, and input saturation [23]. The latter two control approaches did not need to know the upper bound of the external disturbances and the real minimum value of actuator efficiency factor in advance. An adaptive backstepping attitude controller was proposed for reentry RLV with input constraint and external disturbance in [24]. And in [25], an adaptive dynamic surface controller was proposed for a generic hypersonic flight vehicle with consideration of magnitude, rate, and bandwidth constraints on actuator signals. Then in [26], a novel integral term was introduced during dynamic surface control (DSC) scheme design procedure to improve the tracking performance of designed controller and avoid a large initial control signal. Moreover, a robust adaptive dynamic surface controller was investigated for a hypersonic vehicle in the presence of parametric model uncertainty and input saturation, where a compensation design was employed when the input saturations occurred [27]. An adaptive DSC scheme based on radial basis function neural network (RBFNN) was presented for a hypersonic vehicle under the magnitude, rate, and bandwidth constraints on actuator in [28]. Furthermore, nonlinear disturbance observer and RBFNN based sliding mode control were designed for a near space vehicle in [29], where RBFNN was constructed as a compensator to avoid the saturation nonlinearity of rudders.

The motivation of the research is to develop practical nonlinear robust control method for a FAHV model with aerodynamic uncertainty. The main contributions are summarized as follows.

First of all, a nonlinear control-oriented model derived from the curved-fitted model of FAHV without obtaining the linearized model at a trim condition or computing the high-order time derivatives of outputs. Based on the analysis of control-oriented model, it is reasonable to decompose it into two low-order subsystems: velocity subsystem and altitude subsystem. Then the available inputs are designed for subsystems with the reduction of computational burden.

In the second place, auxiliary signals are introduced to cope with input constraints which are provided by auxiliary systems, and the auxiliary signals are employed during the controller design and stability analysis procedure. Although input constraint is also handled by the additional system in [30], in this paper it does not need to construct dynamic robust term during the controller and additional system design procedure. So it simplifies the controller design and stability analysis.

Last but not least, the upper bound of uncertainties is not required to be known in advance. Adaptive law is designed to estimate the upper bound, and the robustness is ensured at the same time. The difference from our previous work [30] is that adaptive technique is employed to approximate compounded uncertainty. The estimation ability of the adaptive law can be clearly shown via theory analysis, and the estimation accuracy can be improved by choosing parameters. From theoretical and simulation aspects, the parameters of adaptive laws are determined more simply than that of RBFNN used in [30]. The "explosion of terms" problem is avoided by developing the novel first-order filter, and its advantage over the traditional first-order filter in DSC method is testified by simulation result.

2. Problem Formulation

2.1. Curved-Fitted Model of FAHV. The nonlinear equations of motion of FAHV used in this study are mentioned in [31]. The longitudinal dynamic equations of a FAHV, which describe velocity, altitude, flight path angle (FPA), angle of attack (AOA), pitch rate, and flexible modes are given as follows:

$$\dot{V} = \frac{(T\cos\alpha - D)}{m} - g\sin\gamma,\tag{1}$$

$$\dot{n} = V \sin \gamma, \tag{2}$$

$$\dot{\gamma} = \frac{(L+T\sin\alpha)}{(mV)} - g\frac{\cos\gamma}{V},\tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma},\tag{4}$$

$$\dot{q} = \frac{M_{yy}}{I_{yy}},\tag{5}$$

$$\ddot{\eta}_i = -2\xi_i \omega_i \dot{\eta}_i - \omega_i^2 \dot{\eta}_i + N_i, \quad i = 1, 2, 3.$$
(6)

This model is called a curve-fitted model (CFM) for FAHV and utilized for simulation only. In (1)–(6), the thrust *T*, drag *D*, lift *L*, pitching moment M_{yy} , and three generalized forces N_1 , N_2 , N_3 are complex algebraic functions of both system states and inputs that must be simplified to render the model analytically tractable. The flexible states η_1 , η_2 , η_3 are related to the deflections of the fore-body turn angle τ_1 and aft-body vertex angle τ_2 , denoted by $\Delta \tau_1$ and $\Delta \tau_2$, respectively. The approximations of the forces and moments are the same as those provided in [32], which can be expressed as

$$T \approx \overline{q} \left[\phi C_{T,\phi} \left(\alpha, \Delta \tau_1, M_{\infty} \right) + C_T \left(\alpha, \Delta \tau_1, M_{\infty}, A_d \right) \right],$$

$$D \approx \overline{q} S C_D \left(\alpha, \delta_e, \delta_c, \Delta \tau_1, \Delta \tau_2 \right),$$

$$L \approx \overline{q} S C_L \left(\alpha, \delta_e, \delta_c, \Delta \tau_1, \Delta \tau_2 \right),$$

$$M_{yy} \approx z_T T + \overline{q} S \overline{c} C_M \left(\alpha, \delta_e, \delta_c, \Delta \tau_1, \Delta \tau_2 \right),$$

$$N_i \approx \overline{q} C_{N_i} \left(\alpha, \delta_e, \delta_c, \Delta \tau_1, \Delta \tau_2 \right), \quad i = 1, 2, 3;$$
(7)

the coefficients obtained from fitting the curves are given in the following:

$$\begin{split} C_{T,\phi} &= C_{T,\phi}^{\alpha} \alpha + C_{T,\phi}^{\alpha M_{co}^{-2}} \alpha M_{co}^{-2} + C_{T,\phi}^{\alpha \Delta \tau_{1}} \alpha \Delta \tau_{1} + C_{T,\phi}^{M_{co}^{-2}} M_{co}^{-2} \\ &+ C_{T,\phi}^{\Delta \tau_{1}^{-2}} \Delta \tau_{1}^{-2} + C_{T,\phi}^{\Delta \tau_{1}} \Delta \tau_{1} + C_{T,\phi}^{0}, \\ C_{T} &= C_{T}^{A_{d}} A_{d} + C_{T}^{\alpha} \alpha + C_{T}^{M_{co}^{-2}} M_{co}^{-2} + C_{T}^{\Delta \tau_{1}} \Delta \tau_{1} + C_{T}^{0}, \\ C_{D} &= C_{D}^{(\alpha + \Delta \tau_{1})^{2}} (\alpha + \Delta \tau_{1})^{2} + C_{D}^{(\alpha + \Delta \tau_{1})} (\alpha + \Delta \tau_{1}) \\ &+ C_{D}^{\delta_{e}^{2}} \delta_{e}^{2} + C_{D}^{\delta_{e}} \delta_{e} + C_{D}^{\delta_{c}^{2}} \delta_{c}^{2} + C_{D}^{\delta_{c}} \delta_{c} + C_{D}^{\alpha \delta_{e}} \alpha \delta_{e} \\ &+ C_{D}^{\alpha \delta_{c}} \alpha \delta_{c} + C_{D}^{\Delta \tau_{2}} \Delta \tau_{2} + C_{D}^{0}, \end{split}$$
(8)
$$C_{L} &= C_{L}^{\alpha} \alpha + C_{L}^{\delta_{e}} \delta_{e} + C_{M}^{\delta_{c}} \delta_{c} + C_{M}^{\Delta \tau_{1}} \Delta \tau_{1} + C_{M}^{\Delta \tau_{2}} \Delta \tau_{2} \\ &+ C_{L}^{0}, \\ C_{M} &= C_{M}^{\alpha} \alpha + C_{M}^{\delta_{e}} \delta_{e} + C_{M}^{\delta_{c}} \delta_{c} + C_{M}^{\Delta \tau_{1}} \Delta \tau_{1} + C_{M}^{\Delta \tau_{2}} \Delta \tau_{2} \\ &+ C_{M}^{0}, \\ C_{N_{i}} &= C_{N_{i}}^{\alpha} \alpha + C_{N_{i}}^{\delta_{e}} \delta_{e} + C_{N_{i}}^{\delta_{c}} \delta_{c} + C_{N_{i}}^{\Delta \tau_{1}} \Delta \tau_{1} + C_{N_{i}}^{\Delta \tau_{2}} \Delta \tau_{2} \\ &+ C_{N_{i}}^{0}. \end{split}$$

Here, the function arguments are removed in brief, M_{∞} is the free stream Mach number, and \overline{q} is the dynamic pressure; they are defined as $\overline{q} = 0.5\rho(h)V^2$ and $M_{\infty} = V/M_0$. Herein, $\rho(h)$ is the altitude dependent air-density, and M_0 is the speed of sound at a given altitude and temperature.

2.2. Control-Oriented Model and Control Objective. As discussed in [5], the simplified model is developed for controller design. It is called control-oriented model (COM) and derived from the CFM by neglecting the flexible dynamics since the measurements of the flexible states are not assumed to be available for feedback [13]. That is to say, the flexible dynamics are not considered directly at the control design level but are taken as perturbations on the COM, and their effects are evaluated in simulation. Besides, the input coupling terms are represented as perturbations. Namely, the COM is comprised of five rigid body dynamic equations (1)–(5).

It is noted that in this model a canard is used to cancel the lift-elevator coupling to cancel nonminimum phase. The canard deflection δ_c is a function of elevator deflection δ_e so that $\delta_c = k_{ec}\delta_e$, where $k_{ec} = -C_L^{\delta_e}/C_L^{\delta_c}$, is an interconnect gain. From (8), it is obvious that if k_{ec} is precisely specified, the canard would exactly cancel the lift due to the elevator deflection. However, in practice, an ideal interconnect gain is hard to achieve and thus it is impossible to exactly cancel the lift-elevator coupling [33]; it is assumed that $k_{ec} = k_{ec0} +$ Δk_{ec} is uncertain. Herein, the uncertainty of aerodynamic parameters except $C_L^{\delta_c}$, $C_L^{\delta_e}$ and k_{ec0} ($\Delta k_{ec} \neq 0$) is considered. For example, the uncertainty of aerodynamic parameter C_L^{α} is denoted as ΔC_L^{α} . Based on it, we can obtain the following expressions of aerodynamic uncertainty:

$$\Delta C_{L} = \Delta C_{L}^{\alpha} \alpha + \Delta C_{L}^{0} + \Delta C_{L}^{\Delta \tau_{1}} \Delta \tau_{1} + \Delta C_{L}^{\Delta \tau_{2}} \Delta \tau_{2}$$

$$+ C_{L}^{\delta_{c}} \Delta k_{ec} \delta_{e},$$

$$\Delta C_{D} = \Delta C_{D}^{(\alpha + \Delta \tau_{1})^{2}} (\alpha + \Delta \tau_{1})^{2} + \Delta C_{D}^{(\alpha + \Delta \tau_{1})} (\alpha + \Delta \tau_{1})$$

$$+ \Delta C_{D}^{\Delta \tau_{2}} \Delta \tau_{2} + \Delta C_{D}^{0} + \Delta C_{d},$$
(9)

where

$$\begin{split} \Delta C_{d} &= \left[\Delta C_{D}^{\delta_{e}^{2}} + \left(k_{ec0} + \Delta k_{ec}\right)^{2} \Delta C_{D}^{\delta_{e}^{2}} \right] \delta_{e}^{2} \\ &+ \left[\Delta C_{D}^{\delta_{e}} + \Delta C_{D}^{\delta_{c}} \left(k_{ec0} + \Delta k_{ec}\right) \right] \delta_{e} \\ &+ \left[\Delta C_{D}^{\alpha\delta_{e}} + \Delta C_{D}^{\alpha\delta_{c}} \left(k_{ec0} + \Delta k_{ec}\right) \right] \alpha \delta_{e}, \end{split}$$

$$\Delta C_{M} &= \Delta C_{M}^{\alpha} \alpha + \left[\Delta C_{M}^{\delta_{e}} + \Delta C_{M}^{\delta_{c}} \left(k_{ec0} + \Delta k_{ec}\right) \right] \delta_{e} \\ &+ \Delta C_{M}^{\Delta\tau_{1}} \Delta \tau_{1} + \Delta C_{M}^{\Delta\tau_{2}} \Delta \tau_{2} + \Delta C_{M}^{0}, \cr \Delta C_{T,\phi} &= \Delta C_{T,\phi}^{\alpha} \alpha + \Delta C_{T,\phi}^{\alpha\Delta\tau_{1}} \alpha \Delta \tau_{1} + \Delta C_{T,\phi}^{M-2} M_{\infty}^{-2} \\ &+ \Delta C_{T,\phi}^{0} + \Delta C_{T,\phi}^{\alpha\Delta\tau_{1}} \alpha \Delta \tau_{1} + \Delta C_{T,\phi}^{2} \Delta \tau_{1}^{2} \\ &+ \Delta C_{T,\phi}^{\Delta\tau_{1}} \Delta \tau_{1}, \cr \Delta C_{T} &= \Delta C_{T}^{A_{d}} A_{d} + \Delta C_{T}^{\alpha} \alpha + \Delta C_{T}^{M-2} M_{\infty}^{-2} + \Delta C_{T}^{\Delta\tau_{1}} \Delta \tau_{1} \quad (10) \\ &+ \Delta C_{T}^{0}, \cr \Delta C_{N_{i}} &= \Delta C_{N_{i}}^{\alpha} \alpha + \left(\Delta C_{N_{i}}^{\delta_{e}} + \Delta C_{N_{i}}^{\delta_{c}} k_{ec0} + \Delta C_{N_{i}}^{\delta_{c}} \Delta k_{ec} \right) \delta_{e} \\ &+ \Delta C_{N_{i}}^{0} + \Delta C_{N_{i}}^{\Delta\tau_{1}} \Delta \tau_{1} + \Delta C_{N_{i}}^{\Delta\tau_{2}} \Delta \tau_{2}, \cr i = 1, 2, 3. \end{split}$$

$$\begin{split} \Delta L &\approx \overline{q} S \Delta C_L, \\ \Delta D &\approx \overline{q} S \Delta C_D, \\ \Delta M_{yy} &\approx z_T \Delta T + \overline{q} S \overline{c} \Delta C_M, \\ \Delta T &= \overline{q} \left(\phi \Delta C_{T,\phi} + \Delta C_T \right), \\ \Delta N_i &\approx \overline{q} \Delta C_{N_i}, \end{split}$$

with

$$\Delta C_{d} = \left[C_{D}^{\delta_{e}^{2}} + \left(k_{ec0} + \Delta k_{ec}\right)^{2} C_{D}^{\delta_{c}^{2}}\right] \delta_{e}^{2} + \left[C_{D}^{\delta_{e}} + C_{D}^{\delta_{c}} \left(k_{ec0} + \Delta k_{ec}\right)\right] \delta_{e}$$
(11)
$$+ \left[C_{D}^{\alpha\delta_{e}} + C_{D}^{\alpha\delta_{c}} \left(k_{ec0} + \Delta k_{ec}\right)\right] \alpha \delta_{e}.$$

For the hypersonic vehicles control system, it is unavoidable that the actuator output is limited, especially the magnitude constraint of actuator input. The input constraints of hypersonic vehicle is required to maintain the conditions that sustain scramjet operation, the limit on fuel equivalence ratio is naturally induced. The thermal choking will occur if the constraint is violated, which could lead engine to be unstart which could jeopardize the mission, the vehicle, and its contents. The constraint on elevator deflection is mainly caused by the limits on control surface displacement. Because the actuator outputs are constrained, the input constraint of fuel equivalence ratio and elevator deflection are denoted as sat(ϕ) and sat(δ_e), respectively.

The expression of input saturation $sat(\phi)$ is as follows:

sat
$$(\phi) = \begin{cases} \phi_{\max}, & \phi \ge \phi_{\max} \\ \phi, & \phi_{\min} < \phi < \phi_{\max} \\ \phi_{\min}, & \phi \le \phi_{\min}, \end{cases}$$
 (12)

where ϕ is the desired control input to be designed in the following section and ϕ_{\min} and ϕ_{\max} are the minimum value and maximum value of fuel equivalence ratio, respectively.

The expression of input saturation sat(δ_e) is as follows:

$$\operatorname{sat}(\delta_{e}) = \begin{cases} \delta_{e\max}, & \delta_{e} \ge \delta_{e\max} \\ \delta_{e}, & \delta_{e\min} < \delta_{e} < \delta_{e\max} \\ \delta_{e\min}, & \delta_{e} \le \delta_{e\min}, \end{cases}$$
(13)

where δ_e is the desired control input to be designed in the following section and $\delta_{e_{\min}}$ and $\delta_{e_{\max}}$ are the minimum value and maximum value of elevator deflection, respectively.

There are four inputs in (1)–(5) and they are the diffuserarea-ratio A_d (it is fitted as $A_d = 1$ in this study), canard deflection, fuel equivalence ratio, and elevator deflection. The outputs to be controlled are selected as velocity and altitude. It is assumed that the states of the rigid body system are available and the controller design only utilizes the feedback from the rigid body states. Because the measurements of the flexible states are not assumed to be available for feedback [13], the flexible states are treated as disturbances. The control objective is to design fuel equivalence ratio and elevator deflection to make velocity and altitude track their command trajectories with aerodynamic uncertainty and input saturation.

3. Controller Design

It can be obtained from the aircraft model (1)–(5) and the aerodynamic formulations (7)-(8) that thrust affects the velocity and the fuel equivalence ratio affected by the thrust, so the velocity is mainly affected by fuel equivalence ratio. Moreover, it is reasonable to implement a separate control design, since elevator deflection has a dominant contribution towards the altitude change. The COM is decomposed into two subsystems firstly, and they are the velocity subsystem and the altitude subsystem. Then every subsystem is controlled separately by the available input. Dynamic inversion and robust adaptive control are synthesized to design control input (fuel equivalence ratio) for the first subsystem. Backstepping control and robust adaptive control are combined to design control input (elevator deflection) after the second subsystem is transformed into strict-feedback form. With the consideration of input saturation, the auxiliary signals which are provided by the auxiliary system are applied to cope with them. The auxiliary signals are employed during the controller design and stability analysis procedure. The detailed design procedures are given in the following two subsections.

3.1. Robust Adaptive Control for Velocity Subsystem. Dynamic inversion can achieve the decoupling between the input and output of a system. It can be used for different types of aircraft and adapted to a model change from simulation and flight tests, but it lacks robustness to uncertainty. The adaptive control can overcome the uncertainty, disturbance, and unmodeled dynamics. Therefore, in this subsection, the robust adaptive control is incorporated into the dynamic inversion to design the fuel equivalence ratio.

The dynamic of velocity can be written as

$$\dot{V} = f_V + g_V \text{sat}\left(\phi\right) + \Delta f_V, \tag{14}$$

where

$$f_{V} = \overline{q}C_{T,1}\cos\frac{\alpha}{m} - g\sin\gamma$$

$$-\frac{\overline{q}S\left(C_{D}^{(\alpha+\Delta\tau_{1})^{2}}\alpha^{2} + C_{D}^{(\alpha+\Delta\tau_{1})}\alpha + C_{D}^{0}\right)}{m},$$

$$g_{V} = \overline{q}C_{T,\phi,1}\cos\frac{\alpha}{m},$$

$$\Delta f_{V} = \left[\Delta T$$

$$+ \overline{q}\phi\left(C_{T,\phi}^{\alpha\Delta\tau_{1}}\alpha\Delta\tau_{1} + C_{T,\phi}^{\Delta\tau_{1}^{2}}\Delta\tau_{1}^{2} + C_{T,\phi}^{\Delta\tau_{1}}\Delta\tau_{1}\right) \qquad (15)$$

$$+ \overline{q}C_{T}^{\Delta\tau_{1}}\Delta\tau_{1}\right]\cos\frac{\alpha}{m}$$

$$-\frac{\left[D - \overline{q}S\left(C_{D}^{(\alpha+\Delta\tau_{1})^{2}}\alpha^{2} + C_{D}^{\alpha+\Delta\tau_{1}}\alpha + C_{D}^{0}\right) + \Delta D\right]}{m},$$

$$C_{T,1} = C_{T}^{A_{d}}A_{d} + C_{T}^{\alpha}\alpha + C_{T}^{M_{co}^{-2}}M_{co}^{-2} + C_{T,\phi}^{0}.$$

It is obvious that the desired control input may be larger than the actual control energy provided. Thus there is a difference between the desired control input and the actual control input, and it is described as

$$\Delta \phi = \operatorname{sat}(\phi) - \phi. \tag{16}$$

According to physical backgrounds of FAHV, it is reasonable to make the following assumption. Assumption 1. For the uncertain term Δf_V , there is a constant $\lambda_V > 0$ such that $|\Delta f_V| \le \lambda_V$.

The tracking error of velocity is defined as

$$z_V = V - V_d, \tag{17}$$

where V_d is the reference command of velocity. The time derivative of (17) is

$$\dot{z}_V = f_V + g_V \phi + g_V \Delta \phi - \dot{V}_d + \Delta f_V.$$
(18)

Considering the saturation of fuel equivalence ratio (12), inspired by the work in [34], the following auxiliary system is constructed:

 $\dot{\sigma}_{\phi}$

$$= \begin{cases} -k_{\sigma_{\phi}}\sigma_{\phi} - \frac{\left(\left|z_{V}g_{V}\Delta\phi\right| + 0.5\Delta\phi^{2}\right)}{\sigma_{\phi}} - \Delta\phi, & \left|\sigma_{\phi}\right| \ge \psi_{\phi} \\ 0, & \left|\sigma_{\phi}\right| < \psi_{\phi}, \end{cases}$$
(19)

where $\psi_{\phi} > 0$.

The fuel equivalence ratio is designed as

$$\phi = g_V^{-1} \left(-k_V \left(z_V - \sigma_\phi \right) - f_V + \dot{V}_d - \frac{\hat{\lambda}_V c_V z_V}{|z_V| + \varepsilon_V} \right).$$
(20)

The adaptive law for $\hat{\lambda}_V$ is

$$\dot{\hat{\lambda}}_V = \frac{a_V c_V z_V^2}{|z_V| + \varepsilon_V},\tag{21}$$

where $k_V, a_V, \varepsilon_V > 0, c_V > 1$.

Taking the estimation error and tracking error into account, the Lyapunov function is constructed as

$$Y_V = \frac{1}{2}z_V^2 + \frac{1}{2a_V}\tilde{\lambda}_V^2 + \frac{1}{2}\sigma_{\phi}^2,$$
 (22)

where $\tilde{\lambda}_V = \hat{\lambda}_V - \lambda_V$ is the estimation error of λ_V .

Using the derivative of Y_V respective to time,

$$\dot{Y}_V = z_V \dot{z}_V + \frac{1}{a_V} \tilde{\lambda}_V \dot{\hat{\lambda}}_V + \sigma_\phi \dot{\sigma}_\phi.$$
(23)

From (18)–(21), we have

$$\dot{Y}_{V} = -k_{V}z_{V}^{2} + k_{V}\sigma_{\phi}z_{V} + z_{V}g_{V}\Delta\phi - \frac{\lambda_{V}c_{V}z_{V}^{2}}{|z_{V}| + \varepsilon_{V}} + z_{V}\Delta f_{V} - k_{\sigma_{\phi}}\sigma_{\phi}^{2} - \left(|z_{V}g_{V}\Delta\phi| + 0.5\Delta\phi^{2}\right)$$
(24)
$$- \sigma_{\phi}\Delta\phi.$$

Since

$$z_{V}g_{V}\Delta\phi - |z_{V}g_{V}\Delta\phi| \leq 0,$$

$$z_{V}\Delta f \leq |z_{V}|\lambda_{V},$$

$$k_{V}\sigma_{\phi}z_{V} - \sigma_{\phi}\Delta\phi \leq \frac{k_{V}}{2}z_{V}^{2} + \frac{k_{V}+1}{2}\sigma_{\phi}^{2} + \frac{1}{2}\Delta\phi^{2},$$
(25)

then

$$\dot{X}_{V} \leq -k_{V}z_{V}^{2} - \left(k_{\sigma_{\phi}} - \frac{k_{V}+1}{2}\right)\sigma_{\phi}^{2} - \frac{\lambda_{V}c_{V}z_{V}^{2}}{\left|z_{V}\right| + \varepsilon_{V}} + \left|z_{V}\right|\lambda_{V},$$
(26)

as long as $|z_V| \ge \varepsilon_V/(c_V - 1)$, $k_{\sigma_{\phi}} - (k_V + 1)/2 > 0$,

$$\dot{Y}_{V} \leq -k_{V}z_{V}^{2} - \left(k_{\sigma_{\phi}} - \frac{k_{V}+1}{2}\right)\sigma_{\phi}^{2} \leq 0.$$
 (27)

3.2. Robust Adaptive Backstepping Control for Altitude Subsystem. The altitude subsystem includes dynamic equations of altitude, FPA, AOA, and pitch rate. The altitude is controlled through the tracking of FPA reference command γ_d , which is derived from altitude reference command h_d .

The tracking error of altitude is defined as

$$\mathbf{z}_h = h - h_d,\tag{28}$$

where h_d is altitude reference command.

The dynamic of (28) is $\dot{z}_h = V \sin \gamma - \dot{h}_d$. Because FPA is very small during the cruise phase, $\sin \gamma \approx \gamma$, then

$$\dot{z}_h = V\gamma - \dot{h}_d. \tag{29}$$

The FPA reference command is defined as γ_d and the tracking error of FPA is $z_{\gamma} = \gamma - \gamma_d$. Then (29) becomes

$$\dot{z}_h = z_h V \gamma_d + z_h V z_v - \dot{h}_d. \tag{30}$$

And the FPA reference command γ_d is chosen as

$$\gamma_d = \frac{\left(-k_h z_h + \dot{h}_d\right)}{V},\tag{31}$$

where $k_h > 0$ is the parameter to be designed.

The Lyapunov function is constructed as

$$Y_{h} = \frac{1}{2}z_{h}^{2}.$$
 (32)

Based on (30) and (31), the time derivative of (32) satisfies

$$\dot{Y}_h = -k_h z_h^2 + z_h V z_\gamma. \tag{33}$$

In the next step, the time derivative of γ_d needs to be computed. But from (31), the term \dot{V} is needed, and there is aerodynamic uncertainty in the formulations \dot{V} , so it is difficult to compute $\dot{\gamma}_d$. Thus, the derivative of γ_d is estimated by the following first-order filter:

$$\dot{\bar{\gamma}}_{d} = -\frac{e_{f\gamma}}{\tau_{\gamma}} - l_{\gamma} \tanh\left(\xi_{\gamma} l_{\gamma} e_{f\gamma}\right), \qquad (34)$$

where $e_{f\gamma} = \overline{\gamma}_d - \gamma_d$ are the filter estimation error, τ_{γ} is the filter time constant, and $\xi_{\gamma}, l_{\gamma} > 0$ are constants.

Remark 2. $\dot{\gamma}_d$ is approximated by the first-order filter (34), and $\dot{\gamma}_d$ is used for the controller design in the next step. Obviously, with l_{γ} assumed to be zero, the first-order filter is reduced to a classical integral filter as used in DSC method $(\dot{\gamma}_d = -e_{f\gamma}/\tau_{\gamma})$. With this filter, the measurement noise can be eliminated in the virtual control effort. Compared with the classical first-order filter in the DSC method, the fast transient response of filter can be obtained. Moreover, compared with the filter that includes saturation function component, the hyperbolic tangent function component is superior to the saturation function component; thus the performances of the filter proposed in this paper can be improved.

The dynamics of FPA, AOA, and pitch rate are rewritten as the following strict-feedback formulation:

$$\dot{\gamma} = f_{\gamma} + g_{\gamma} \alpha + \Delta f_{\gamma}, \tag{35}$$

$$\dot{\alpha} = f_{\alpha} + g_{\alpha}q + \Delta f_{\alpha}, \qquad (36)$$

$$\dot{q} = f_q + g_q \operatorname{sat}(\delta_e) + \Delta f_q, \tag{37}$$

where

$$\begin{split} f_{\gamma} &= \overline{q}\phi \\ &\cdot \sin \frac{\alpha \left[\left(C_{T,\phi}^{M_{co}^{-2}} M_{co}^{-2} + C_{T,\phi}^{\alpha} \alpha + C_{T,\phi}^{\alpha M_{co}^{-2}} \alpha M_{co}^{-2} + C_{T,\phi}^{0} \right) \right]}{(mV)} \\ &- g \cos \frac{\gamma}{(mV)} + \overline{q} \\ &\cdot \sin \frac{\alpha \left[\left(C_{T}^{A_{d}} A_{d} + C_{T}^{\alpha} \alpha + C_{T}^{0} + C_{T}^{M_{co}^{-2}} M_{co}^{-2} \right) + \overline{q}SC_{L}^{0} \right]}{(mV)}, \\ f_{\alpha} &= -f_{\gamma} - \frac{\overline{q}SC_{L}^{\alpha} \alpha}{(mV)}, \\ f_{q} &= \frac{\left[z_{T}T_{1} + \overline{q}S\overline{c} \left(C_{M}^{\alpha} \alpha + C_{M}^{0} \right) \right]}{I_{yy}}, \\ T_{1} &= \overline{q} \left(\phi C_{T,\phi,1} + C_{T,1} \right), \\ g_{\gamma} &= \frac{\overline{q}SC_{L}^{\alpha}}{(mV)}, \\ g_{\alpha} &= 1, \\ g_{q} &= \frac{\overline{q}S\overline{c} \left(C_{M}^{\delta_{c}} + C_{M}^{\delta_{c}} k_{ec0} \right)}{I_{yy}}, \\ \Delta f_{\gamma} &= \left[\Delta T + \overline{q}\phi \left(C_{T,\phi}^{\alpha\Delta\tau_{1}} \alpha \Delta\tau_{1} + C_{T,\phi}^{\Delta\tau_{1}^{2}} \Delta\tau_{1}^{2} + C_{T,\phi}^{\Delta\tau_{1}} \Delta\tau_{1} \right) \\ &+ \overline{q}C_{T}^{\Delta\tau_{1}} \Delta\tau_{1} \right] \sin \frac{\alpha}{(mV)} + \frac{\Delta L}{(mV)} \\ &+ \frac{\overline{q}S \left(C_{L}^{\Delta\tau_{1}} \Delta\tau_{1} + C_{L}^{\Delta\tau_{2}} \Delta\tau_{2} \right)}{(mV)}, \end{split}$$

$$\Delta f_{\alpha} = -\Delta f_{\gamma},$$

$$\Delta f_{q} = \frac{\left[z_{T}\Delta T + \overline{q}S\overline{c}\left(\Delta C_{M} + C_{M}^{\Delta\tau_{1}}\Delta\tau_{1} + C_{M}^{\Delta\tau_{2}}\Delta\tau_{2}\right)\right]}{I_{yy}}.$$
(38)

The difference between the desired control input and the actual control input is described as

$$\Delta \delta_e = \operatorname{sat}\left(\delta_e\right) - \delta_e. \tag{39}$$

It is noted that the structure of (35)-(37) possesses a strict-feedback form, where the uncertain terms do not satisfy the matched condition, and it makes backstepping control philosophy applicable. Here, robust adaptive control is incorporated into backstepping control to design control input (elevator deflection) and the states AOA and pitch rate are taken as the virtual control inputs. The order of the altitude subsystem is four; it will induce repeated differentiations of virtual control inputs and may cause "explosion of terms" problem. What is more, the time derivatives of virtual control inputs are needed in the next step, but there are nonlinearity and uncertainty in (35)–(37); it is difficult to obtain the time derivatives. And it may cost large computational load even if the time derivatives can be computed. To cope with this situation, the time derivatives of virtual control inputs are estimated by the novel first-order filter.

According to physical backgrounds, it is reasonable to make the following assumption.

Assumption 3. For the uncertain terms Δf_{γ} , Δf_{α} , Δf_{q} , there exist constants λ_{γ} , λ_{α} , $\lambda_{q} > 0$ such that $|\Delta f_{\gamma}| \leq \lambda_{\gamma}$, $|\Delta f_{\alpha}| \leq \lambda_{\alpha}$, $|\Delta f_{q}| \leq \lambda_{q}$.

From (35), the dynamic of tracking error of FPA is written as

$$\dot{z}_{\gamma} = f_{\gamma} + g_{\gamma}\alpha - \dot{\gamma}_d + \Delta f_{\gamma}. \tag{40}$$

The virtual control input α_d is designed as

$$\alpha_d = g_{\gamma}^{-1} \left(-k_{\gamma} z_{\gamma} - f_{\gamma} + \dot{\gamma}_d - \frac{\hat{\lambda}_{\gamma} c_{\gamma} z_{\gamma}}{\left| z_{\gamma} \right| + \varepsilon_{\gamma}} - z_h V \right), \quad (41)$$

with the adaptive law of \hat{r}_{v}

$$\dot{\widehat{\lambda}}_{\gamma} = \frac{a_{\gamma}c_{\gamma}z_{\gamma}^{2}}{\left|z_{\gamma}\right| + \varepsilon_{\gamma}} - a_{\gamma}\widehat{\lambda}_{\gamma}, \qquad (42)$$

where $a_{\nu}, c_{\nu}, \varepsilon_{\nu} > 0$ are parameters to be designed.

Considering the estimation error and tracking error, the Lyapunov function is constructed as

$$Y_{\gamma} = \frac{1}{2}z_{\gamma}^{2} + \frac{1}{2a_{\gamma}}\tilde{\lambda}_{\gamma}^{2} + \frac{1}{2}e_{f\gamma}^{2}, \qquad (43)$$

where $\tilde{\lambda}_{\gamma}$ is estimation error, $\tilde{\lambda}_{\gamma} = \hat{\lambda}_{\gamma} - \lambda_{\gamma}$.

The time derivative of (43) is

$$\dot{Y}_{\gamma} = z_{\gamma} \left(f_{\gamma} + g_{\gamma} \alpha - \dot{\gamma}_d + \Delta f_{\gamma} \right) + \frac{1}{a_{\gamma}} \tilde{\lambda}_{\gamma} \dot{\hat{\lambda}}_{\gamma} + e_{f\gamma} \dot{e}_{f\gamma}.$$
(44)

From (34), filter error, and [35], if $l_{\gamma} > |\dot{\gamma}_d|_{max}$, the term $e_{f\gamma}\dot{e}_{f\gamma}$ yields

$$e_{f\gamma}\dot{e}_{f\gamma} \leq -\frac{e_{f\gamma}^{2}}{\tau_{\gamma}} - l_{\gamma} \tanh\left(\xi_{\gamma}l_{\gamma}\left(e_{f\gamma}\right)\right)e_{f\gamma} + \dot{\gamma}_{d}\left|e_{f\gamma}\right|$$

$$\leq -\frac{e_{f\gamma}^{2}}{\tau_{\gamma}} + \frac{k}{\xi_{\gamma}},$$
(45)

where k is a constant that satisfies $k = e^{-k+1}$; that is, k = 0.2758.

Based on (41), (42), (45), and $-\tilde{\lambda}_{\alpha}\hat{\lambda}_{\alpha} \leq -\tilde{\lambda}_{\alpha}^2/2 + \lambda_{\alpha}^2/2$, (44) yields

$$\begin{split} \dot{Y}_{\gamma} &= -k_{\gamma}z_{\gamma}^{2} + z_{\gamma}\Delta f_{\gamma} + z_{\gamma}g_{\gamma}z_{\alpha} + \frac{\tilde{\lambda}_{\gamma}c_{\gamma}z_{\gamma}^{2}}{\left|z_{\gamma}\right| + \varepsilon_{\gamma}} - \frac{\hat{\lambda}_{\gamma}c_{\gamma}z_{\gamma}^{2}}{\left|z_{\gamma}\right| + \varepsilon_{\gamma}} \\ &- \tilde{\lambda}_{\gamma}\hat{\lambda}_{\gamma} - z_{\gamma}Vz_{h} - \frac{e_{f\gamma}^{2}}{\tau_{\gamma}} + \frac{k}{\lambda_{\gamma}} \\ &\leq -k_{\gamma}z_{\gamma}^{2} + z_{\gamma}\lambda_{\gamma} - \frac{\lambda_{\gamma}c_{\gamma}z_{\gamma}^{2}}{\left|z_{\gamma}\right| + \varepsilon_{\gamma}} - \frac{\tilde{\lambda}_{\gamma}^{2}}{2} + \frac{\lambda_{\gamma}^{2}}{2} + z_{\gamma}g_{\gamma}z_{\alpha}} \\ &- z_{\gamma}Vz_{h} - \frac{e_{f\gamma}^{2}}{\tau_{\gamma}} + \frac{k}{\xi_{\gamma}}. \end{split}$$

$$(46)$$

The derivative of α_d is estimated by the following firstorder filter:

$$\dot{\overline{\alpha}}_{d} = -\frac{e_{f\alpha}}{\tau_{\alpha}} - l_{\alpha} \tanh\left(\xi_{\alpha}l_{\alpha}e_{f\alpha}\right),\tag{47}$$

where $e_{f\alpha} = \overline{\alpha}_d - \alpha_d$ are the filter estimation errors, τ_{α} is the filter time constant, and $\xi_{\alpha}, l_{\alpha} > 0$ are constants.

Define the error signal of AOA as

$$z_{\alpha} = \alpha - \overline{\alpha}_d. \tag{48}$$

From (36), the time derivative of z_{α} is

$$\dot{z}_{\alpha} = f_{\alpha} + g_{\alpha}q + \Delta f_{\alpha} - \frac{\dot{\overline{\alpha}}}{\alpha}, \qquad (49)$$

The virtual control input q_d is designed as

$$q_d = g_{\alpha}^{-1} \left(-k_{\alpha} z_{\alpha} - f_{\alpha} + \dot{\overline{\alpha}}_d - \frac{\widehat{\lambda}_{\alpha} c_{\alpha} z_{\alpha}}{|z_{\alpha}| + \varepsilon_{\alpha}} - g_{\gamma} z_{\gamma} \right), \quad (50)$$

with the adaptive law of $\hat{\lambda}_{\alpha}$,

$$\dot{\widehat{\lambda}}_{\alpha} = \frac{a_{\alpha}c_{\alpha}z_{\alpha}^{2}}{|z_{\alpha}| + \varepsilon_{\alpha}} - a_{\alpha}\widehat{\lambda}_{\alpha}, \qquad (51)$$

where $a_{\alpha}, c_{\alpha}, \varepsilon_{\alpha} > 0$ are parameters to be designed.

Considering the estimation error and tracking error, the Lyapunov function is constructed as

$$Y_{\alpha} = \frac{1}{2}z_{\alpha}^2 + \frac{1}{2a_{\alpha}}\widetilde{\lambda}_{\alpha}^2 + \frac{1}{2}e_{f\alpha}^2, \qquad (52)$$

where $\hat{\lambda}_{\alpha}$ is the estimation of λ_{α} and $\tilde{\lambda}_{\alpha} = \hat{\lambda}_{\alpha} - \lambda_{\alpha}$ is the estimation error.

From (50) and (51), the time derivative of (52) is

$$\begin{split} \dot{Y}_{\alpha} &= z_{\alpha} \left(f_{\alpha} + g_{\alpha}q + \Delta f_{\alpha} - \dot{\overline{\alpha}}_{d} \right) + \frac{1}{a_{\alpha}} \tilde{\lambda}_{\alpha} \dot{\overline{\lambda}}_{\alpha} + e_{f\alpha} \dot{e}_{f\alpha} \\ &= -k_{\alpha} z_{\alpha}^{2} + z_{\alpha} \Delta f_{\alpha} - z_{\alpha} g_{\gamma} z_{\gamma} + z_{\alpha} g_{\alpha} z_{q} + \frac{\tilde{\lambda}_{\alpha} c_{\alpha} z_{\alpha}^{2}}{|z_{\alpha}| + \varepsilon_{\alpha}} \\ &- \frac{\tilde{\lambda}_{\alpha} c_{\alpha} z_{\alpha}^{2}}{|z_{\alpha}| + \varepsilon_{\alpha}} + e_{f\alpha} \dot{e}_{f\alpha} \\ &\leq -k_{\alpha} z_{\alpha}^{2} + |z_{\alpha}| \lambda_{\alpha} - \frac{\lambda_{\alpha} c_{\alpha} z_{\alpha}^{2}}{|z_{\alpha}| + \varepsilon_{\alpha}} - \tilde{\lambda}_{\alpha} \hat{\lambda}_{\alpha} - z_{\alpha} g_{\gamma} z_{\gamma} \\ &+ z_{\alpha} g_{\alpha} z_{q} + e_{f\alpha} \dot{e}_{f\alpha}; \end{split}$$
(53)

from (47), filter error, and [35], if $l_{\alpha} > |\dot{\alpha}_d|_{\max}$, the term $e_{f\alpha} \dot{e}_{f\alpha}$ yields

$$e_{f\alpha}\dot{e}_{f\alpha} \le -\frac{e_{f\alpha}^2}{\tau_{\alpha}} + \frac{k}{\xi_{\alpha}}.$$
(54)

Based on (54) and $-\tilde{\lambda}_{\alpha}\hat{\lambda}_{\alpha} \leq -\tilde{\lambda}_{\alpha}^2/2 + \lambda_{\alpha}^2/2$, (53) yields

$$\dot{Y}_{\alpha} \leq -k_{\alpha}z_{\alpha}^{2} + \left|z_{\alpha}\right|\lambda_{\alpha} - \frac{\lambda_{\alpha}c_{\alpha}z_{\alpha}^{2}}{\left|z_{\alpha}\right| + \varepsilon_{\alpha}} - \frac{\tilde{\lambda}_{\alpha}^{2}}{2} - z_{\alpha}g_{\gamma}z_{\gamma} + z_{\alpha}g_{\alpha}z_{q} - \frac{e_{f\alpha}^{2}}{\tau_{\alpha}} + \frac{k}{\xi_{\alpha}} + \frac{\lambda_{\alpha}^{2}}{2}.$$
(55)

The derivative of q_d is estimated by the following first-order filter:

$$\dot{\bar{q}}_d = -\frac{e_{fq}}{\tau_q} - l_q \tanh\left(\xi_q l_q e_{fq}\right),\tag{56}$$

where $e_{fq} = \overline{q}_d - q_d$ are the filter estimation errors, τ_q is the filter time constant, and $\xi_q, l_q > 0$ are constants.

The error signal of pitch rate is

$$z_q = q - \overline{q}_d. \tag{57}$$

On the basis of (37) and (39), the time derivative of (57) is

$$\dot{z}_q = f_q + g_q \delta_e + g_q \Delta \delta_e + \Delta f_q - \dot{\overline{q}}_d.$$
(58)

Inspired by the work in [34], an auxiliary system is used to handle saturation of elevator deflection (13):

$$\sigma_{\delta_{e}} = \begin{cases} -k_{\delta_{e}}\sigma_{\delta_{e}} - \frac{\left(\left|e_{q}g_{q}\Delta\delta_{e}\right| + 0.5\Delta\delta_{e}^{-2}\right)}{\sigma_{\delta_{e}}} - \Delta\delta_{e}, & \left|\sigma_{\delta_{e}}\right| \ge \psi_{\delta_{e}} \\ 0, & \left|\sigma_{\delta_{e}}\right| < \psi_{\delta_{e}}, \end{cases}$$
(59)

where $\psi_{\delta_e} > 0$.

The elevator deflection is designed as

$$\delta_{e} = g_{q}^{-1} \left(-k_{q} \left(z_{q} - \sigma_{\delta_{e}} \right) + \dot{\overline{q}}_{d} - f_{q} - \frac{\widehat{\lambda}_{q} c_{q} z_{q}}{\left| z_{q} \right| + \varepsilon_{q}} - g_{\alpha} z_{\alpha} \right).$$

$$(60)$$

The adaptive law for $\hat{\lambda}_a$ is

$$\dot{\widehat{\lambda}}_q = \frac{a_q c_q z_q^2}{\left|z_q\right| + \varepsilon_q} - a_q \widehat{\lambda}_q,\tag{61}$$

where k_q , k_α , a_q , $\varepsilon_q > 0$, $c_q > 1$.

Remark 4. In the traditional sliding mode control, the *sign* function may cause chattering problem, and it may induce that the virtual control input in the backstepping control cannot be tracked accurately by the next subsystem. What is worse, for the hypersonic vehicle system, the chattering may lead to the disintegration of the vehicle rudder. To eliminate the chattering problem, the continuous robust term with norm-type switched function $\hat{\lambda}_l c_l z_l / (|z_l| + \varepsilon_l)$ $(l = V, \gamma, \alpha, q)$ is used during control input design.

Considering the estimation error and tracking error, the Lyapunov function is constructed as

$$Y_{q} = \frac{1}{2}z_{q}^{2} + \frac{1}{2a_{q}}\tilde{\lambda}_{q}^{2} + \frac{1}{2}e_{fq}^{2} + \frac{1}{2}\sigma_{\delta_{e}}^{2}, \qquad (62)$$

where $\hat{\lambda}_q$ is the estimation of λ_q and $\tilde{\lambda}_q = \hat{\lambda}_q - \lambda_q$ is the estimation error.

Using the derivative of Y_q respective to time,

$$\dot{Y}_{q} = z_{q} \left(f_{q} + g_{q} \delta_{e} - \dot{\overline{q}}_{d} + \Delta f_{q} \right) + \frac{1}{a_{q}} \tilde{\lambda}_{q} \dot{\overline{\lambda}}_{q} + e_{fq} \dot{e}_{fq}$$

$$+ \sigma_{\delta_{e}} \dot{\sigma}_{\delta_{e}}.$$
(63)

If $l_q > |\dot{q}_d|_{\text{max}}$, the term satisfies

$$e_{fq}\dot{e}_{fq} \le -\frac{e_{fq}^2}{\tau_q} + \frac{k}{\xi_q}.$$
(64)

From (59)-(61), the inequality (63) satisfies

$$\begin{split} \dot{Y}_{q} &\leq -k_{q}z_{q}^{2} - k_{\alpha}^{2}z_{\alpha}z_{q} + k_{\alpha}z_{q}^{2} + k_{q}\sigma_{\delta_{e}}z_{q} + z_{q}g_{q}\Delta\delta_{e} \\ &- \frac{\widehat{\lambda}_{q}c_{q}z_{q}^{2}}{\left|z_{q}\right| + \varepsilon_{q}} + z_{q}\Delta f_{q} + \frac{\widetilde{\lambda}_{q}c_{q}z_{q}^{2}}{\left|z_{q}\right| + \varepsilon_{q}} - \widetilde{\lambda}_{q}\widehat{\lambda}_{q} \\ &- z_{q}g_{\alpha}z_{\alpha} - k_{\sigma_{\delta_{e}}}\sigma_{\delta_{e}}^{2} - \left(\left|z_{q}g_{q}\Delta\delta_{e}\right| + \frac{\Delta\delta_{e}^{2}}{2}\right) \\ &- \sigma_{\delta_{e}}\Delta\delta_{e} - \frac{e_{fq}^{2}}{\tau_{q}} + \frac{k}{\xi_{q}}. \end{split}$$
(65)

Since

$$\begin{aligned} z_{q}g_{q}\Delta\delta_{e} - \left|z_{q}g_{q}\Delta\delta_{e}\right| &\leq 0, \\ z_{q}\Delta f \leq \left|z_{q}\right|\lambda_{q}, \\ k_{q}\sigma_{\delta_{e}}z_{q} - \sigma_{\delta_{e}}g_{q}\Delta\delta_{e} &\leq \frac{1}{2}k_{q}z_{q}^{2} + \frac{k_{q}+1}{2}\sigma_{\delta_{e}}^{2} + \frac{1}{2}\Delta\delta_{e}^{2}, \\ -\tilde{\lambda}_{q}\hat{\lambda}_{q} &\leq -\frac{\tilde{\lambda}_{q}^{2}}{2} + \frac{\lambda_{q}^{2}}{2}, \end{aligned}$$
(66)

the following inequality holds:

$$\dot{Y}_{q} \leq -\left(\frac{k_{q}}{2} - k_{\alpha}\right)z_{q}^{2} - z_{q}g_{\alpha}z_{\alpha}$$

$$-\left(k_{\sigma_{\delta e}} - \frac{k_{q} + 1}{2}\right)\sigma_{\delta_{e}}^{2} + \left|z_{q}\right|\lambda_{q} - \frac{\lambda_{q}c_{q}z_{q}^{2}}{\left|z_{q}\right| + \varepsilon_{q}} \qquad (67)$$

$$-\frac{\tilde{\lambda}_{q}^{2}}{2} - \frac{e_{fq}^{2}}{\tau_{q}} + \frac{k}{\xi_{q}} + \frac{\lambda_{q}^{2}}{2}.$$

Remark 5. This paper contains the following different aspects compared with existing results [26, 27, 30] that investigate the controller design of FAHV. (a) The robustness of the scheme developed herein is shown through aerodynamic uncertainty, whereas the robustness of the designed scheme in [26] is evaluated through different fuel levels. Although in [30] input constraint is handled by the additional system, we do not need to construct dynamic robust term when designing controller and additional system in this paper. It simplifies the controller design and stability analysis. Moreover, the novel first-order filter that is different from the filter used in [30] is developed to handle "explosion of terms" problem. (b) The difference of this paper compared with [27] is as follows. On the one hand, the two subsystems do not needed to be transformed into linear parameterized form. On the other hand, input constraint is handled by constructing additional system in this paper, whereas the application of compensation technique in [27] coped with input constraint.

The Lyapunov function for the altitude subsystem is constructed as

$$Y = Y_h + Y_v + Y_a + Y_q.$$
 (68)

Using the derivative of it respective to time, then

$$\dot{Y} = \dot{Y}_h + \dot{Y}_\gamma + \dot{Y}_\alpha + \dot{Y}_q. \tag{69}$$

From (27), (33), (46), (55), and (67), the following inequality holds:

$$\begin{split} \dot{Y} &\leq -k_h z_h^2 - k_\gamma z_\gamma^2 - k_\alpha z_\alpha^2 - \left(\frac{k_q}{2} - k_\alpha\right) z_q^2 \\ &- \left(k_{\sigma_{\delta e}} - \frac{k_q + 1}{2}\right) \sigma_{\delta_e}^2 - \frac{e_{f\gamma}^2}{\tau_\gamma} - \frac{e_{f\alpha}^2}{\tau_\alpha} - \frac{e_{fq}^2}{\tau_q} + \frac{k}{\xi_\gamma} \\ &+ \frac{k}{\xi_\alpha} + \frac{k}{\xi_q} + z_h V z_\gamma - z_\gamma V z_h + z_\gamma g_\gamma z_\alpha - z_\alpha g_\gamma z_\gamma \end{split}$$

$$+ z_{\alpha}g_{\alpha}z_{q} - z_{q}g_{\alpha}z_{\alpha} + \left|z_{\gamma}\right|\lambda_{\gamma} - \frac{\lambda_{\gamma}c_{\gamma}z_{\gamma}^{2}}{\left|z_{\gamma}\right| + \varepsilon_{\gamma}}$$

$$+ \left|z_{\alpha}\right|\lambda_{\alpha} - \frac{\lambda_{\alpha}c_{\alpha}z_{\alpha}^{2}}{\left|z_{\alpha}\right| + \varepsilon_{\alpha}} + \left|z_{q}\right|\lambda_{q} - \frac{\lambda_{q}c_{q}z_{q}^{2}}{\left|z_{q}\right| + \varepsilon_{q}} - \frac{\widetilde{\lambda}_{\gamma}^{2}}{2}$$

$$- \frac{\widetilde{\lambda}_{\alpha}^{2}}{2} - \frac{\widetilde{\lambda}_{q}^{2}}{2} + \frac{\lambda_{\gamma}^{2}}{2} + \frac{\lambda_{\alpha}^{2}}{2} + \frac{\lambda_{q}^{2}}{2}.$$
(70)

As long as $|z_{\gamma}| \ge \varepsilon_{\gamma}/(c_{\gamma}-1), |z_{\alpha}| \ge \varepsilon_{\alpha}/(c_{\alpha}-1), |z_{q}| \ge \varepsilon_{q}/(c_{q}-1), c_{\gamma} > 1, c_{\alpha} > 1, c_{q} > 1$, the above inequality yields

$$\dot{Y} \leq -k_{h}z_{h}^{2} - k_{\gamma}z_{\gamma}^{2} - k_{\alpha}z_{\alpha}^{2} - \left(\frac{k_{q}}{2} - k_{\alpha}\right)z_{q}^{2} - \left(k_{\sigma_{\delta e}} - \frac{k_{q} + 1}{2}\right)\sigma_{\delta_{e}}^{2} - \frac{\tilde{\lambda}_{\gamma}^{2}}{2} - \frac{\tilde{\lambda}_{\alpha}^{2}}{2} - \frac{\tilde{\lambda}_{q}^{2}}{2} - \frac{e_{f\gamma}^{2}}{\tau_{\gamma}} - \frac{e_{f\alpha}^{2}}{\tau_{\alpha}} - \frac{e_{fq}^{2}}{\tau_{q}} + \frac{k}{\xi_{\gamma}} + \frac{k}{\xi_{\alpha}} + \frac{k}{\xi_{q}} + \frac{\lambda_{\gamma}^{2}}{2} + \frac{\lambda_{\alpha}^{2}}{2} + \frac{\lambda_{q}^{2}}{2}$$
(71)

 $\leq -2\varepsilon Y + c.$

Here $\varepsilon = \min\{k_h, k_\gamma, k_\alpha, k_q/2 - k_\alpha, 1/2, 1/\tau_\gamma, 1/\tau_\alpha, 1/\tau_q, k_{\sigma_{\delta e}} - (k_q + 1)/2\}$ and $c = k/\xi_\gamma + k/\xi_\alpha + k/\xi_q + \lambda_\gamma^2/2 + \lambda_\alpha^2/2 + \lambda_q^2/2$.

The convergence domain of z_h can be expressed as the following compact set:

$$R_h = \left\{ z_h \mid \left| z_h \right| \le \sqrt{c/\varepsilon} \right\}. \tag{72}$$

So far, the stability of the subsystem is proven.

4. Simulations

Simulations are carried out to illustrate the effectiveness of the robust adaptive backstepping control scheme proposed in the previous section. The equations of motion (1)–(6) are used for simulations. The vehicle model parameters and the initial flight condition of the vehicle dynamics are referred to in [36]. The fuel level is assumed to be 50%. Input constraints that are used to test the capability of the developed controller to handle the input constraints are set as $\phi_{\min} = 0.1$, $\phi_{\max} = 1.2$, $\delta_{e\min} = -0.2618$, and $\delta_{e\max} = -\delta_{e\min}$. Parameters adopted for the control inputs, adaptive laws, and auxiliary systems are given as follows: $k_V = k_\gamma = k_\alpha = 2$, $k_h = 0.0001$, $k_q = 5$, k_ϕ , $k_{\delta_e} = 1.2$, $\varepsilon_V = 0.5$, c_V , c_γ , c_α , $c_q = 1.01$, ψ_ϕ , $\psi_{\delta_e} = 0.001$, ξ_γ , ξ_α , $\xi_q = 0.5$, l_γ , l_α , $l_q = 0.1$. $\varepsilon_\gamma = 0.0001$, $\varepsilon_\alpha = \varepsilon_q = 0.001$, $a_V = 0.05$, a_γ , a_α , $a_q = 0.1$, $\tau_\gamma = \tau_\alpha = \tau_q = 0.05$.

As shown in adaptive laws (21), $\hat{\lambda}_V$ is monotone increasing. Its overincrease may cause the control input to largely increase. So the adaptive law is revised to suppress the overincreasing of $\hat{\lambda}_V$:

$$\dot{\widehat{\lambda}}_{V} = \begin{cases} \frac{a_{V}c_{V}|z_{V}|^{2}}{|z_{V}| + \varepsilon_{V}}, & |z_{V}| > \frac{\varepsilon_{V}}{c_{V} - 1}\\ 0, & |z_{V}| \le \frac{\varepsilon_{V}}{c_{V} - 1}. \end{cases}$$
(73)

It is noted that after the error signal z_V entering the stability region, the estimation change rate $\hat{\lambda}_V$ remains at zero, which means that $\hat{\lambda}_V$ will not change; then the control input will not be too large.

To test the performance of the designed strategy, the maneuver simulation is carried out, where the maneuver uses separate reference commands of altitude and velocity. The reference command of velocity is chosen as 2500 ft/s. The reference command of altitude is represented as $h_{\text{ref}} = h_0 + \Delta h$. Here $\Delta h = 5000k_1$ and k_1 is generated as follows:

$$k_{1} = \begin{cases} 0, & t \in [0 \ 100) \\ \frac{1}{40000 (t - 100)^{2}}, & t \in [100 \ 150) \\ \frac{1}{400 (t - 150)} + \frac{1}{16}, & t \in [150 \ 500) \\ 1 - \frac{1}{40000 (t - 550)^{2}}, & t \in [500 \ 550) \\ 1, & t \in [550 \ 650) & (74) \\ 1 - \frac{1}{40000 (t - 650)^{2}}, & t \in [650 \ 700) \\ \frac{1}{400 (1050 - t)} + \frac{1}{16}, & t \in [700 \ 1050) \\ \frac{1}{40000 (t - 1100)^{2}}, & t \in [1050 \ 1100) \\ 0, & t \in [1100 \ 1200) . \end{cases}$$

As demonstrated in (74), the maneuver begins at 100 s. The vehicle climbs about 5000 ft between 100 s to 550 s and descents about 5000 ft between 650 s and 1100 s. Finally, the altitude remains constant $h_0 = 85000$ ft after 1100 s. In addition, reference commands are generated from a second-order prefilter with a natural frequency $\omega_f = 0.03$ rad/s and a damping ratio $\xi_f = 0.95$. Two uncertain cases are included: case 1: 10% of uncertainty of the aerodynamic parameters is taken into consideration; case 2: 20% of uncertainty of the aerodynamic parameters is taken into account.

The simulation results are shown in Figures 1–5. Besides, some local time responses are given in the corresponding figures to clearly demonstrate the dynamic process. Altitude and velocity achieve their stable tracking of their respective reference commands as given in Figure 1. It is noted that the FPA reference commands are approximately the same in case 1 and case 2 from Figure 1(c). As shown in Figure 2, the control inputs remain within their constraints. Other states are shown in Figures 3 and 4. In two cases, the pitch rate converges to 0 rad/s, and three flexible states are bounded between the values -0.1 and 0.8 during the whole maneuver. Compared simulation result between the proposed first-order filter of this paper and the traditional first-order filter in DSC method is given in Figure 5. It is clear that the estimation errors of the proposed filters in this paper



FIGURE 1: Time response to altitude, velocity, and FPA: (a) altitude, (b) velocity, and (c) FPA.



FIGURE 2: Time response to control inputs: (a) fuel equivalence ratio and (b) elevator deflection.

are all smaller than that of the traditional first-order filters in DSC method.

On the basis of simulation result, the following conclusion can be concluded. (1) The steady state values of AOA, elevator deflection, and three flexible states in case 1 are larger than those of case 2. The flexible states are bounded, and the pitch rate converges to zero. (2) The fuel equivalence ratio and elevator deflection are kept in their constraints. The designed controller achieves the satisfactory tracking performance despite the presence of aerodynamic uncertainty and input constraints.

5. Conclusions

The robust adaptive backstepping control is developed for a nonlinear longitudinal model of FAHV, where input constraints and aerodynamic uncertainties are taken into consideration. The nonlinear COM is decomposed into two subsystems to reduce the complexity of the controller design, and then the control inputs are designed for those two subsystems. The influence of the input constraints is analyzed via auxiliary system, whose state is utilized at the level of controller design and stability analysis. Moreover, to



FIGURE 3: Time response of AOA and pitch rate: (a) AOA and (b) pitch rate.

eliminate the "explosion of terms" problem, the novel firstorder filters are constructed. Simulation results assure that the proposed control strategy can guarantee the stable tracking of the respective reference trajectories of the velocity and the altitude despite input constraints and uncertainty.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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FIGURE 4: Time response of flexible states: (a) flexible state η_1 , (b) flexible state η_2 , and (c) flexible state η_3 .



(c) Filter estimation error e_{fq}

FIGURE 5: Time responses of traditional first-order filter and novel first-order filter proposed in this paper: (a) filter estimation error $e_{f\gamma}$, (b) filter estimation error $e_{f\alpha}$, and (c) filter estimation error e_{fq} .

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