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Research Article

Multiple Positive Periodic Solutions for Two Kinds of Higher-Dimension Impulsive Differential Equations with Multiple Delays and Two Parameters

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By applying the fixed point theorem, we derive some new criteria for the existence of multiple positive periodic solutions for two kinds of n-dimension periodic impulsive functional differential equations with multiple delays and two parameters: $x_i'(t) = a_i(t)x_i(t) - \lambda b_i(t) f_i(t, x(t), x(t-\tau_1(t)), \ldots, x(t-\tau_n(t))))$, a.e., $t > 0$, $t \neq t_k$, $k \in Z_+$, $x_i(t_k^+) - x_i(t_k^-) = \mu c_{ik} x_i(t_k)$, $i = 1, 2, \ldots, n$, $k \in Z_+$, and $x_i'(t) = -a_i(t)x_i(t) + \lambda b_i(t) f_i(t, x(t), x(t-\tau_1(t)), \ldots, x(t-\tau_n(t))))$, a.e., $t > 0$, $t \neq t_k$, $k \in Z_+$, $x_i(t_k^+) - x_i(t_k^-) = \mu c_{ik} x_i(t_k)$, $i = 1, 2, \ldots, n$, $k \in Z_+$. As an application, we study some special cases of the previous systems, which have been studied extensively in the literature.

1. Introduction

Let $R = (-\infty, +\infty)$, $R_+ = [0, +\infty)$, $R_- = (-\infty, 0]$, $R^n = \{(x_1, \ldots, x_n)^T : x_i \geq 0, 1 \leq i \leq n\}$, $J \subset R$, and $Z_+ = \{1, 2, 3, \ldots\}$, respectively. Denote by $PC(J, R^n)$ the set of operators $\varphi : J \to R^n$ which are continuous for $t \in J$, $t \neq t_k$ and have discontinuities of the first kind at the points $t_k \in J\,(k \in Z_+)$ but are continuous from the left at these points. For each $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, the norm of x is defined as $|x| = \sum_{i=1}^n |x_i|$. Let $BC(R, R_+^n)$ denote the Banach space of bounded continuous functions $\psi : R \to R_+^n$ with the norm $\|\psi\| = \sup_{\xi \in R} \sum_{i=1}^n |\psi_i(\xi)|$, where $\psi = (\psi_1, \psi_2, \ldots, \psi_n)^T$. The matrix $A > B\,(A \leq B)$ means that each pair of corresponding elements of A and B satisfies the inequality " $>$ " (" $\leq$ "). In particular, A is called a positive matrix if $A > 0$.

Impulsive differential equations are suitable for the mathematical simulation of evolutionary process whose states are subject to sudden changes at certain moments. Equations of this kind are found in almost every domain of applied sciences, and numerous examples are given in [1–4]. In recent years, the existence theory of positive periodic solutions of delay differential equations with impulsive effects or without impulsive effects has been an object of active research, and we refer the reader to [5–17]. Recently, in [5], Jiang and Wei studied the following nonimpulsive delay differential equation:

$$\begin{aligned} x'(t) &= -a(t)x(t) \\ &\quad + f(t, x(t-\tau_0(t)), x(t-\tau_1(t)))), \ldots, x(t-\tau_n(t)). \end{aligned} \tag{1}$$

They obtained sufficient conditions for the existence of the positive periodic solutions of (1). Motivated by [5], in [6], Zhao et al. investigated the following impulsive delay differential equation:

$$\begin{aligned} x'(t) &= -a(t)x(t) \\ &\quad + f(t, x(t-\tau_0(t)), x(t-\tau_1(t)), \ldots, x(t-\tau_n(t))) \\ &\qquad\qquad \text{a.e., } t > 0,\ t \neq t_k; \\ x(t_k^+) - x(t_k) &= b_k x(t_k), \quad k = 1, 2, \ldots. \end{aligned} \tag{2}$$

They derived some sufficient conditions for the existence of the positive periodic solutions of (2). In [7], Huo et al. considered the following impulsive delay differential equation:

$$x'(t) + \alpha(t)x(t) = p(t)f(t, x(t-\sigma(t))) \quad \text{a.e., } t > 0,\ t \neq t_k; \qquad (3)$$
$$x(t_k^+) - x(t_k) = b_k x(t_k), \quad k = 1, 2, \ldots.$$

They got sufficient conditions for the existence and attractivity of the positive periodic solutions of (3). Motivated by [5–7], in [8], Zhang et al. studied the following impulsive delay differential equation:

$$\begin{aligned} x'(t) &= -a(t)x(t) + p(t)f(t, x(t-\tau_0(t)), x(t-\tau_1(t)), \ldots, \\ &\quad x(t-\tau_n(t))) \quad \text{a.e., } t > 0,\ t \neq t_k; \\ x(t_k^+) - x(t_k) &= b_k x(t_k), \quad k = 1, 2, \ldots. \end{aligned} \qquad (4)$$

They obtained some sufficient conditions for the existence of the positive periodic solutions of (4). However, to this day, only a little work has been done on the existence of positive periodic solutions to the high-dimension impulsive differential equations based on the theory in cones. Motivated by this, in this paper, we mainly consider the following two classes of impulsive functional differential equations with two parameters:

$$\begin{aligned} x'(t) &= A(t)x(t) - \lambda B(t)f(t, u(t)) \quad \text{a.e. , } t > 0,\ t \neq t_k, \\ \Delta x(t_k) &= \mu C_k x(t_k), \quad k \in Z_+, \end{aligned} \qquad (5)$$

$$\begin{aligned} x'(t) &= -A(t)x(t) + \lambda B(t)f(t, u(t)) \quad \text{a.e., } t > 0,\ t \neq t_k, \\ \Delta x(t_k) &= \mu C_k x(t_k), \quad k \in Z_+, \end{aligned} \qquad (6)$$

with initial conditions:

$$\begin{aligned} x_i(\theta) &= \phi_i(\theta), \quad \theta \in [-\tau, 0], \\ \phi_i(0) &> 0, \quad \phi_i \in C([-\tau, 0), [0, +\infty)), \\ i &= 1, 2 \ldots, n, \end{aligned} \qquad (7)$$

where

$$\begin{aligned} u(t) &= (x(t-\tau_1(t)), \ldots, x(t-\tau_n(t))) \\ &= (u_1(t), \ldots, u_n(t)), \\ \tau &= \max_{1\le i\le n} \sup \tau_i(t), \end{aligned} \qquad (8)$$

and $A(t) = \operatorname{diag}[a_1(t), a_2(t), \ldots, a_n(t)]$, $B(t) = \operatorname{diag}[b_1(t), b_2(t), \ldots, b_n(t)]$, $a_i, b_i \in C(R, R^+)$ $(i = 1, 2 \ldots, n)$ are ω-periodic; that is, $a_i(t+\omega) = a_i(t)$, $b_i(t+\omega) = b_i(t)$, $f = (f_1, \ldots, f_n)^T \in R \times BC(R, R_+^n)$ with $f_i(t+\omega, u_1, \ldots, u_n) = f_i(t, u_1, \ldots, u_n)$, $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k)$ $(i = 1, 2, \ldots, n)$ (here $x_i(t_k^+)$ representing the right limit of $x_i(t)$ at the point t_k). $C_k = (c_{1k}, c_{2k}, \ldots, c_{nk}) \in C(R_+^n, R_-^n)$; that is, $x_i(t)$ changes decreasingly suddenly at times t_k, and $\omega > 0$ is a constant. We assume that there exists an integer $q > 0$ such that $t_{k+q} = t_k + \omega$, $c_{i(k+q)} = c_{ik}$ $(i = 1, 2, \ldots, n)$, where $0 < t_1 < t_2 < \cdots < t_q < \omega$.

Throughout the paper, we make the following assumptions:

(P_1) $a_i, b_i, \tau_i : R_+ \to R_+$ are locally summable ω-periodic functions; that is, $a_i(t+\omega) = a_i(t)$, $b_i(t+\omega) = b_i(t)$, and $\tau_i(t+\omega) = \tau_i(t)$ for all $t \ge 0$, $\lambda > 0$, and $\mu > 0$ are two parameters;

(P_2) $f = (f_1, \ldots, f_n)^T \in R \times BC(R, R_+^n)$ and for all $(t, u_1, \ldots, u_n) \in R \times BC(R, R^n)$, $f_i(t+\omega, u_1, \ldots, u_n) = f_i(t, u_1, \ldots, u_n)$ such that $f_i(t, u_1, \ldots, u_n) \not\equiv 0$, $i = 1, 2, \ldots, n$;

(P_3) $\{t_k\}$, $k \in Z_+$ satisfies $0 < t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to+\infty} t_k = +\infty$. $C_k : R_+^n \to R$ $(k \in Z_+)$ satisfy Caratheodory conditions and are ω-periodic functions in t. Moreover, there exists a positive constant q such that $t_{k+q} = t_k + \omega$, $k \in Z_+$. Without loss of generality, we can assume that $t_k \neq 0$ and $[0, \omega] \cap \{t_k, k \in Z^+\} = \{t_1, t_2, \ldots, t_q\}$;

(P_4) $\{c_{ik}\}$ is a real sequence such that $\mu c_{ik} > -1$, $i = 1, 2, \ldots, n$, $k \in Z_+$ and $c_i(t) := \prod_{0<t_k<t}(1 + \mu c_{ik})$ satisfies $c_i(t+\omega) = c_i(t)$ for all $t \ge 0$.

In addition, the parameters in this paper are assumed to be not identically equal to zero.

To conclude this section, we summarize in the following a few definitions and lemmas that will be needed in our arguments.

Definition 1 (see [1]). A function $x_i : R \to (0, +\infty)$ is said to be a positive solution of (5) and (6) if the following conditions are satisfied:

(a) $x_i(t)$ is absolutely continuous on each (t_k, t_{k+1});

(b) for each $k \in Z_+$, $x_i(t_k^+)$ and $x_i(t_k^-)$ exist, and $x_i(t_k^-) = x_i(t_k)$;

(c) $x_i(t)$ satisfies the first equation of (5) and (6) for almost everywhere (for short a.e.) in $[0, \infty) \setminus \{t_k\}$ and satisfies $x_i(t_k^+) = (1 + c_{ik})x_i(t_k)$ for $t = t_k$, $k \in Z_+ = \{1, 2, \ldots\}$.

Under the previous hypotheses (P_1)–(P_4), we consider the neutral nonimpulsive system:

$$\frac{dy}{dt} = A(t)y(t) - \lambda\overline{B}(t)f(t, v(t)), \quad \text{a.e., } t \ge 0, \qquad (9)$$

$$\frac{dy}{dt} = -A(t)y(t) + \lambda\overline{B}(t)f(t, v(t)), \quad \text{a.e., } t \ge 0, \qquad (10)$$

with initial conditions:

$$\begin{gathered} y_i(\xi) = \varphi_i(\xi), \quad \xi \in [-\tau, 0], \\ \varphi_i(0) > 0, \quad \varphi_i \in C([-\tau, 0), R^+), \\ i = 1, 2, 3, \ldots, n, \end{gathered} \tag{11}$$

where

$$\begin{aligned} v(t) &= (v_1(t), \ldots, v_n(t)) \\ &= (y(t-\tau_1(t)), \ldots, y(t-\tau_n(t))), \\ y(t-\tau_i(t)) &= c_i(t)\, x(t-\tau_i(t)), \\ \overline{B}(t) &= \operatorname{diag}\left[\overline{B_1}(t), \overline{B_2}(t), \ldots, \overline{B_n}(t)\right], \\ \overline{B_i}(t) &= \frac{b_i(t)}{c_i(t)}, \\ i &= 1, 2, \ldots, n. \end{aligned} \tag{12}$$

By a solution $y(t) = (y_1(t), \ldots, y_n(t))^T$ of (9) and (10), it means an absolutely continuous function $y(t) = (y_1(t), \ldots, y_n(t))^T$ defined on $[-\tau, 0]$ that satisfies (9) and (10), that is, for $t \geq 0$, and $y_i(\xi) = \varphi_i(\xi)$, $y_i'(\xi) = \varphi_i'(\xi)$ on $[-\tau, 0]$.

The following lemmas will be used in the proofs of our results. The proof of the first lemma is similar to that of Theorem 1 in [18].

Lemma 2. *Suppose that* (P_1)–(P_4) *hold. Then*

(i) *if* $y_i(t)$ $(i = 1, 2, \ldots, n)$ *are solutions of* (9) *and* (10) *on* $[-\tau, +\infty)$, *then* $x_i(t) = \prod_{0<t_k<t}(1+\mu c_{ik}) y_i(t)$ $(i = 1, 2, \ldots, n)$ *are solutions of* (5) *and* (6) *on* $[-\tau, +\infty)$;

(ii) *if* $x_i(t)$ $(i = 1, 2, \ldots, n)$ *are solutions of* (5) *and* (6) *on* $[-\tau, +\infty)$, *then* $y_i(t) = \prod_{0<t_k<t}(1+\mu c_{ik})^{-1} x_i(t)$ $(i = 1, 2, \ldots, n)$ *are solutions of* (9) *and* (10) *on* $[-\tau, +\infty)$.

Proof. (i) It is easy to see that $x_i(t) = \prod_{0<t_k<t}(1 + \mu c_{ik}) y_i(t)$ $(i = 1, 2, \ldots, n)$ are absolutely continuous on every interval $(t_k, t_{k+1}]$, $t \neq t_k$, $k = 1, 2, \ldots$,

$$\begin{aligned} & x_i'(t) - x_i(t)\, a_i(t) - \lambda b_i(t)\, f(t, u(t)) \\ &= \prod_{0<t_k<t}(1+\mu c_{ik})\, y_i'(t) \\ &\quad - \prod_{0<t_k<t}(1+\mu c_{ik})\, y_i(t)\, a_i(t) \\ &\quad - \lambda b_i(t)\, f\Biggl(t, \prod_{0<t_k<t}(1+\mu c_{1k})\, y(t-\tau_1(t)), \ldots, \\ &\qquad \prod_{0<t_k<t}(1+\mu c_{nk})\, y(t-\tau_n(t))\Biggr) \\ &= \prod_{0<t_k<t}(1+\mu c_{ik}) \\ &\quad \times \Biggl\{ y_i'(t) - y_i(t)\, a_i(t) \\ &\qquad - \lambda \prod_{0<t_k<t}(1+\mu c_{ik})^{-1} \\ &\qquad \times b_i(t)\, f \\ &\qquad \times \Biggl(t, \prod_{0<t_k<t}(1+\mu c_{1k}) \\ &\qquad \times y(t-\tau_1(t)), \ldots, \\ &\qquad \prod_{0<t_k<t}(1+\mu c_{nk}) \\ &\qquad \times y(t-\tau_n(t))\Biggr)\Biggr\} \\ &= \prod_{0<t_k<t}(1+\mu c_{ik}) \left\{ y_i'(t) - y_i(t)\, a_i(t) + \lambda \overline{B_i(t)}\, f(t, v(t)) \right\} \\ &= 0. \end{aligned} \tag{13}$$

On the other hand, for any $t = t_k$, $k = 1, 2, \ldots$,

$$\begin{gathered} x_i(t_k^+) = \lim_{t \to t_k^+} \prod_{0<t_j<t}(1+\mu c_{ik})\, y_i(t) = \prod_{0<t_j \leq t_k}(1+\mu c_{ik})\, y_i(t_k), \\ x_i(t_k) = \prod_{0<t_j<t_k}(1+\mu c_{ik})\, y_i(t_k), \quad i = 1, 2, \ldots, n, \end{gathered} \tag{14}$$

and thus

$$x_i(t_k^+) = (1+\mu c_{ik})\, x_i(t_k), \quad i = 1, 2, \ldots, n, \; k = 1, 2, \ldots. \tag{15}$$

It follows from (13)–(15) that $x_i(t)$ $(i = 1, 2, \ldots, n)$ are solutions of (5). Similarly, if $y_i(t)$ $(i = 1, 2, \ldots, n)$ are solutions of (10), we can prove that $x_i(t)$ $(i = 1, 2, \ldots, n)$ are solutions of (6).

(ii) Since $x_i(t) = \prod_{0<t_k<t}(1 + \mu c_{ik}) y_i(t)$ is absolutely continuous on every interval $(t_k, t_{k+1}]$, $t \neq t_k$, $k = 1, 2, \ldots$, and in view of (14), it follows that, for any $k = 1, 2, \ldots$,

$$\begin{aligned} y_i(t_k^+) &= \prod_{0<t_j \leq t_k}(1+\mu c_{ik})^{-1} x_i(t_k^+) \\ &= \prod_{0<t_j<t_k}(1+\mu c_{ik})^{-1} x_i(t_k) = y_i(t_k), \end{aligned}$$

$$\begin{aligned} y_i(t_k^-) &= \prod_{0<t_j<t_k} (1+\mu c_{ik})^{-1} x_i(t_k^-) \\ &= \prod_{0<t_j\le t_k^-} (1+\mu c_{ik})^{-1} x_i(t_k^-) = y_i(t_k), \end{aligned} \tag{16}$$

which implies that $y_i(t)$ $(i = 1,2,\ldots,n)$ are continuous on $[-\tau,+\infty)$. It is easy to prove that $y_i(t)$ $(i = 1,2,\ldots,n)$ are absolutely continuous on $[-\tau,+\infty)$. Similar to the proof of (i), we can check that $y_i(t) = \prod_{0<t_k<t}(1+\mu c_{ik})^{-1}x_i(t)$ $(i = 1,2,\ldots,n)$ are solutions of (9) on $[-\tau,+\infty)$. Similarly, if $x_i(t)$ $(i = 1,2,\ldots,n)$ are solutions of (6), we can prove that $y_i(t)$ $(i = 1,2,\ldots,n)$ are solutions of (10). The proof of Lemma 2 is completed. □

In the following section, we only discuss the existence of a periodic solution for (9) and (10).

Definition 3 (see [19]). Let X be a real Banach space, and let E be a closed, nonempty subset of X. E is said to be a cone if

(1) $\alpha x + \beta y \in E$ for all $x, y \in E$, and $\alpha, \beta > 0$, and

(2) $x, -x \in E$ imply $x = 0$.

Lemma 4 (Krasnoselskii fixed point theorem see [20–22]). *Let E be a cone in a real Banach space X. Assume that Ω_1 and Ω_2 are open subsets of X with $0 \in \Omega_1 \subset \overline{\Omega_1} \subset \Omega_2$, where $\Omega_i = \{x \in X : \|x\| < r_i\}$ $(i = 1,2)$. Let $T : E \cap (\Omega_2 \setminus \overline{\Omega_1}) \to E$ be a completely continuous operator and satisfy either*

(1) *$\|Tx\| \ge \|x\|$, for any $x \in E \cap \partial\Omega_1$ and $\|Tx\| \le \|x\|$, for any $x \in E \cap \partial\Omega_2$*

or

(3) *$\|Tx\| \le \|x\|$, for any $x \in E \cap \partial\Omega_1$ and $\|Tx\| \ge \|x\|$, for any $x \in E \cap \partial\Omega_2$.*

Then T has a fixed points in $E \cap (\Omega_2 \setminus \overline{\Omega_1})$.

The paper is organized as follows. In next section, firstly, we give some definitions and lemmas. Secondly, we derive some existence theorems for one or two positive periodic solutions of system (5) by using Krasnoselskii fixed point theorem under some conditions. In Section 3, we also get some existence theorems for one or two positive periodic solutions of system (6) that are also established by applying Krasnoselskii fixed point theorem under some conditions. Finally, as an application, we give two examples to show our results.

2. Existence of Periodic Solutions of System (5)

We establish the existence of positive periodic solutions of (5) by applying the Krasnoselskii fixed point theorem on cones. We will first make some preparations and list a few preliminary results. For $(t,s) \in R^2$, $1 \le i \le n$, we define

$$G_i(t,s) = \frac{e^{-\int_t^s a_i(\xi)}d\xi}{1-e^{-\int_0^\omega a_i(\xi)}d\xi} = \frac{e^{\int_s^{t+\omega} a_i(\xi)}d\xi}{e^{\int_0^\omega a_i(\xi)}d\xi - 1}, \tag{17}$$

$$G(t,s) = \operatorname{diag}[G_1(t,s), G_2(t,s), \ldots, G_n(t,s)].$$

It is clear that $G_i(t+\omega, s+\omega) = G_i(t,s)$, $\partial G_i(t,s)/\partial t = a_i(t)G_i(t,s)$, $G_i(t,t) - G_i(t,t+\omega) = 1$. In view of (P_1), we also define for $1 \le i \le n$,

$$\begin{aligned} \alpha_i &:= \min_{0\le t\le s\le\omega} |G_i(t,s)| = \frac{1}{e^{\int_0^\omega a_i(\xi)d\xi} - 1}, \\ \beta_i &:= \max_{0\le t\le s\le\omega} |G_i(t,s)| = \frac{e^{\int_0^\omega a_i(\xi)d\xi}}{e^{\int_0^\omega a_i(\xi)d\xi} - 1}, \end{aligned} \tag{18}$$

$$\alpha = \min_{1\le i\le n} \alpha_i, \qquad \beta = \max_{1\le i\le n} \beta_i, \qquad \sigma = \frac{\alpha}{\beta} \in (0,1).$$

Let $X = \{y = (y_1(t), y_2(t), \ldots, y_n(t))^T \in C(R, R^n) \mid y(t+\omega) = y(t)\}$ with the norm $\|y\| = \sum_{i=1}^n |y_i|_0$, where $|y_i|_0 = \sup_{t\in[0,\omega]}|y_i(t)|$, and it is easy to verify that $(X, \|\cdot\|)$ is a Banach space. Define E to be a cone in X by

$$\begin{aligned} E = \{ & y = (y_1(t), y_2(t), \ldots, y_n(t))^T \\ & \in X : y_i(t) \ge \sigma|y_i|_0,\ t \in [0,\omega]\}, \end{aligned} \tag{19}$$

and we easily verify that E is a cone in X.

We define an operator $\psi : X \to X$ as follows:

$$(\psi y)(t) = ((\psi_1 y)(t), (\psi_2 y)(t), \ldots, (\psi_n y)(t))^T, \tag{20}$$

where

$$\begin{aligned} (\psi_i y)(t) = \lambda \int_t^{t+\omega} & G_i(t,s)\overline{b}_i(s) \\ & \times f_i(s, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s)))\,ds. \end{aligned} \tag{21}$$

For convenience in the following discussion, we introduce the following notations:

$$\begin{aligned} b^M &= \max_{1\le i\le n}\left\{\sup_{t\in[0,\omega]} \overline{b}_i(t)\right\}, \\ b^L &= \min_{1\le i\le n}\left\{\inf_{t\in[0,\omega]} \overline{b}_i(t)\right\}, \\ \overline{g} &= \frac{1}{\omega}\int_0^\omega g(t)\,dt, \end{aligned} \tag{22}$$

$$f^a = \limsup_{u\in E, \|u\|\to a} \frac{\int_0^\omega |f(t,u(t))|\,dt}{\|u\|},$$

$$f_a = \liminf_{u\in E, \|u\|\to a} \frac{\int_0^\omega |f(t,u(t))|\,dt}{\|u\|},$$

where a denotes either 0 or ∞, $g(t) \in C(R, R)$. Moreover, we list several assumptions:

$(H_1): f_0 = \infty$;

$(H_2): f_\infty = \infty$;

$(H_3): f^0 = 0$;

$(H_4): f^\infty = 0$;

$(H_5): f^0 = \theta_1 \in [0, 1/\lambda\beta b^M)$;

$(H_6): f^\infty = \gamma_1 \in [0, 1/\lambda\beta b^M)$;

$(H_7): f_0 = \theta_2 \in (1/\lambda\alpha\sigma b^L, \infty)$;

$(H_8): f_\infty = \gamma_2 \in (1/\lambda\alpha\sigma b^L, \infty)$;

(H_9): there exists $R > 0$, such that $\int_0^\omega |f(t, v(t))|dt > R/\lambda b^L\alpha$, for any $\|y\| \in [\sigma R, R]$;

(H_{10}): there exists $r > 0$, such that $\int_0^\omega |f(t, v(t))|dt < r/\lambda b^M\beta$, for any $\|y\| \leq r$.

The proofs of the main results in this paper are based on an application of Krasnoselskii fixed point theorem in cones. To make use of fixed point theorem in cones, firstly, we need to introduce some definitions and lemmas.

Lemma 5. *Assume that (P_1)–(P_4) hold. The existence of positive ω-periodic solution of (9) is equivalent to that of nonzero fixed point of ψ in E.*

Proof. Assume that $y = (y_1(t), y_2(t), \ldots, y_n(t))^T \in X$ is a periodic solution of (9). Then, we have

$$\left[y_i(t)\, e^{-\int_0^t a_i(\xi)d\xi}\right]' = -\lambda e^{-\int_0^t a_i(\xi)d\xi}\overline{b_i}(t)\, f_i(t, v(t)), \quad i = 1, 2, \ldots, n. \tag{23}$$

Integrating the above equation over $[t, t+\omega]$, we can have

$$y_i(s)\, e^{-\int_0^s a_i(\xi)d\xi}\Big|_t^{t+\omega} = -\lambda \int_t^{t+\omega} e^{-\int_0^s a_i(\xi)d\xi}\overline{b_i}(s) \times f_i(s, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s)))\, ds. \tag{24}$$

Therefore, we have

$$y_i(t)\, e^{-\int_0^t a_i(\xi)d\xi}\left[1 - e^{-\int_0^\omega a_i(\xi)d\xi}\right] = \lambda \int_t^{t+\omega} e^{-\int_0^s a_i(\xi)d\xi}\overline{b_i}(s) \times f_i(s, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s)))\, ds, \tag{25}$$

which can be transformed into

$$\begin{aligned} y_i(t) &= \lambda \int_t^{t+\omega} \frac{e^{-\int_0^s a_i(\xi)d\xi}}{e^{-\int_0^t a_i(\xi)d\xi}\left[1 - e^{-\int_0^\omega a_i(\xi)d\xi}\right]} \times \overline{b_i}(s)\, f_i(s, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s)))\, ds \\ &= \lambda \int_t^{t+\omega} \frac{e^{\int_s^{t+\omega} a_i(\xi)d\xi}}{e^{\int_0^\omega a_i(\xi)d\xi} - 1} \times \overline{b_i}(s)\, f_i(s, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s)))\, ds \\ &= \lambda \int_t^{t+\omega} G_i(t,s)\, \overline{b_i}(s)\, f_i \times (s, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s)))\, ds, \\ &\qquad i = 1, 2, \ldots, n. \end{aligned} \tag{26}$$

Thus, $y_i(t)$ $(i = 1, 2, \ldots, n)$ are periodic solutions for (21).

If $y = (y_1(t), y_2(t), \ldots, y_n(t))^T \in X$ and $\psi y = (\psi_1 y, \psi_2 y, \ldots, \psi_n y)^T = y$ with $y \neq 0$, then for any $t = t_k$, derivative the two sides of (21) about t,

$$\begin{aligned} &(\psi_i y)'(t) \\ &= \frac{d}{dt}\left[\lambda \int_t^{t+\omega} G_i(t,s)\, \overline{b_i}(s) \times f_i(s, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s)))\, ds\right] \\ &= \lambda\left[G_i(t, t+\omega)\, b_i(t+\omega)\, f_i \times (t+\omega, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s))) - G_i(t,t)\, b_i(t) \times f_i(t, y(s-\tau_1(s)), \ldots, y(s-\tau_n(s)))\right] + a_i(t)\, y_i(t) \\ &= a_i(t)\, y_i(t) - \lambda b_i(t)\, f_i(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t))) \\ &= y_i'(t). \end{aligned} \tag{27}$$

Hence $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in X$ is a positive ω-periodic solution of (9). Thus we complete the proof of Lemma 5. □

Lemma 6. *Assume that (P_1)–(P_4) hold. Then the solutions of (5) are defined on $[-\tau, \infty)$ and are positive.*

Proof. By Lemma 2, we only need to prove that the solutions $y_i(t)$ $(i = 1, 2, \ldots, n)$ of (9) are defined on $[-\tau, \infty)$ and are positive on $[0, \infty)$. From (9), we have that, for any $\varphi_i \in C([-\tau, 0), R^+)$ $(i = 1, 2, \ldots, n)$ and $t > 0$,

$$\begin{aligned} &y_i(t) \\ &= \varphi_i(0) \\ &\times \exp\left\{\int_0^t \left[a_i(\xi)\right.\right. \end{aligned}$$

$$- \lambda \Big(b_i(\xi) f_i(\xi, y(\xi - \tau_1(\xi)), \ldots, y(\xi - \tau_n(\xi)))(y_i(\xi))^{-1}\Big)\Big] d\xi \Big\}. \tag{28}$$

Therefore, $y_i(t)$ $(i = 1, 2, \ldots, n)$ are defined on $[-\tau, \infty)$ and are positive on $[0, \infty)$. The proof of Lemma 6 is complete. □

Lemma 7. *Assume that* (P_1)–(P_4) *hold. Then* $\psi : E \rightarrow E$ *is well defined.*

Proof. From (21), for any $y \in E$,

$$\begin{aligned}
(\psi y)(t + \omega) &= \lambda \int_{t+\omega}^{t+2\omega} G(t + \omega, s)\overline{B}(s) \times f(s, y(s - \tau_1(s)), \ldots, y(s - \tau_n(s)))\, ds \\
&= \lambda \int_{t}^{t+\omega} G(t + \omega, \eta + \omega)\overline{B}(s + \omega) \times f(s + \omega, y(s + \omega - \tau_1(s + \omega)), \ldots, y(s + \omega - \tau_n(s + \omega)))\, ds \\
&= \lambda \int_{t}^{t+\omega} G(t, s)\overline{B}(s) f(s, y(s - \tau_1(s)), \ldots, y(s - \tau_n(s)))\, ds \\
&= (\psi y)(t).
\end{aligned} \tag{29}$$

Therefore, $(Ty) \in X$. From (21), we have

$$|\psi_i y|_0 \leq \beta_i \left[\lambda \int_0^\omega \left|\overline{b_i}(s) f_i(s, y(s - \tau_1(s)), \ldots, y(s- \tau_n(s)))\right| ds\right]. \tag{30}$$

On the other hand, we obtain

$$\begin{aligned}
(\psi_i y)(t) &\geq \alpha_i \left[\lambda \int_0^\omega \left|\overline{b_i}(s) f_i(s, y(s - \tau_1(s)), \ldots, y(s - \tau_n(s)))\right| ds\right] \\
&\geq \frac{\alpha_i}{\beta_i} |\psi_i y|_0 \geq \sigma |\psi_i y|_0.
\end{aligned} \tag{31}$$

Therefore, $\psi y \in E$. The proof of Lemma 7 is complete. □

Lemma 8. *Assume that* (P_1)–(P_4) *hold, and there exists* $\eta > 0$ *such that*

$$\int_0^\omega |f(t, y(t - \tau_1(t)), \ldots, y(t - \tau_n(t)))|\, dt \geq \eta \|y\|, \quad \textit{for any } y \in E, \tag{32}$$

and then

$$\|\psi y\| \geq \lambda b^L \alpha\eta \|y\|, \quad \textit{for any } y \in E. \tag{33}$$

Proof. For any $y \in E$, then

$$\begin{aligned}
|(\psi_i y)(t)| &\geq \alpha_i \left[\lambda \int_0^\omega \left|\overline{b_i}(s) f_i(s, y(s - \tau_1(s)), \ldots, y(s - \tau_n(s)))\right| ds\right] \\
&\geq b^L \lambda\alpha \int_0^\omega |f_i(t, y(t - \tau_1(t)), \ldots, y(t - \tau_n(t)))|\, dt.
\end{aligned} \tag{34}$$

Thus, we have

$$\begin{aligned}
\|\psi y\| &= \sum_{i=1}^n \sup_{t\in[0,\omega]} |(\psi_i y)(t)| \\
&\geq \lambda b^L \alpha \sum_{i=1}^n \int_0^\omega |f_i(s, y(s - \tau_1(s)), \ldots, y(s - \tau_n(s)))|\, ds \\
&\geq \lambda b^L \alpha \int_0^\omega |f(t, y(t - \tau_1(t)), \ldots, y(t - \tau_n(t)))|\, dt \\
&\geq \lambda b^L \alpha\eta \|y\|.
\end{aligned} \tag{35}$$

The proof of Lemma 8 is complete. □

Lemma 9. *Assume that* (P_1)–(P_4) *hold, and let* $r > 0$. *If there exists a sufficiently small* $\epsilon > 0$ *such that*

$$\int_0^\omega |f(t, y(t - \tau_1(t)), \ldots, y(t - \tau_n(t)))|\, dt \leq \epsilon \|y\|, \quad \textit{for any } y \in E \cap \partial\Omega_r, \tag{36}$$

and then

$$\|\psi y\| \leq \lambda b^M \beta\epsilon \|y\|, \quad \textit{for any } y \in E \cap \partial\Omega_r. \tag{37}$$

Proof. For any $y \in E \cap \partial\Omega_r$, we have

$$\begin{aligned}
\|\psi y\| &= \sum_{i=1}^n \sup_{t\in[0,\omega]} |(\psi_i y)(t)| \\
&\leq \lambda\beta \sum_{i=1}^n \int_0^\omega \left|\overline{b_i}(s) f_i(s, y(s - \tau_1(s)), \ldots, y(s - \tau_n(s)))\right| ds \\
&\leq \lambda b^M \beta \int_0^\omega |f(t, y(t - \tau_1(t)), \ldots, y(t - \tau_n(t)))|\, dt \\
&\leq \lambda b^M \beta\epsilon \|y\|.
\end{aligned} \tag{38}$$

The proof of Lemma 9 is complete. □

Our main results of this paper are as follows.

Theorem 10. *In addition to* (P_1)–(P_4)*, if* (H_1) *and* (H_4) *hold, then system* (5) *has at least one positive* ω*-periodic solution.*

Proof. By (H_1), there exists $r_1 > 0$ such that

$$\int_0^{\omega} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \geq \eta \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_1}, \tag{39}$$

where the constant $\eta > 0$ satisfies $\eta\lambda\beta b^L > 1$. Then by Lemma 8, we have

$$\|\psi y\| \geq \lambda b^L \beta\eta \|y\| \geq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_1}. \tag{40}$$

On the other hand, by (H_4), for any $0 < \epsilon \leq (1/2\lambda B^M \beta)$, there exists $N_1 > r_1 > 0$ such that

$$\int_0^{\omega} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \leq \epsilon \|y\|, \quad \text{for any } y \in E,\ \|y\| \geq N_1. \tag{41}$$

We choose

$$r_2 > N_1 + 1 + 2\lambda B^M \beta \times \sup_{\|y\| < N_1, y \in E} \int_0^{\omega} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt. \tag{42}$$

If $y \in E \cap \partial\Omega_{r_2}$, then

$$\begin{aligned} \|\psi y\| &= \sum_{i=1}^{n} \sup_{t\in[0,\omega]} |(\psi_i y)(t)| \\ &\leq \lambda b^M \beta \int_0^{\omega} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \\ &= \lambda b^M \beta \left[\int_{I_1} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \right. \\ &\quad \left. + \int_{I_2} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \right] \\ &\leq \frac{r_2}{2} + \frac{\|y\|}{2} = \|y\|, \end{aligned} \tag{43}$$

where

$$I_1 = \{y \in E : \|y\| \leq N_1\}, \qquad I_2 = \{y \in E : \|y\| > N_1\}. \tag{44}$$

This implies that

$$\|\psi y\| \leq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_2}. \tag{45}$$

In conclusion, under the assumptions (H_1) and (H_4), ψ satisfies the conditions in Lemma 4, and then ψ has a fixed point in $E \cap (\Omega_{r_2} \setminus \overline{\Omega_{r_1}})$. By Lemma 5, system (5) has at least one positive ω-periodic solution. The proof of Theorem 10 is complete. □

Theorem 11. *In addition to* (P_1)–(P_4)*, if* (H_2) *and* (H_3) *hold, then system* (5) *has at least one positive* ω*-periodic solution.*

Proof. By (H_3), for any $0 < \epsilon \leq 1/\lambda B^M \beta$, there exists $R_1 > 0$ such that

$$\int_0^{\omega} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \leq \epsilon \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{R_1}. \tag{46}$$

Then by Lemma 9, we have

$$\|\psi y\| \leq \lambda b^M \beta\epsilon \|y\| \leq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{R_1}. \tag{47}$$

On the other hand, by (H_3), there exists $R_2 > R_1 > 0$ such that

$$\int_0^{\omega} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \geq \eta \|y\|, \quad \text{for any } y \in E,\ \|y\| > R_2, \tag{48}$$

where the constant $\eta > 0$ satisfies $\eta\lambda\beta b^L > 1$. Then by Lemma 8, we have

$$\|\psi y\| \geq \lambda b^L \beta\eta \|y\| \geq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{R_2}. \tag{49}$$

In conclusion, under the assumptions (H_2) and (H_3), ψ satisfies the conditions in Lemma 4, then ψ has a fixed point in $E \cap (\Omega_{R_2} \setminus \overline{\Omega_{R_1}})$. By Lemma 4, the system (5) has at least one positive ω-periodic solution. The proof of Theorem 11 is complete. □

Theorem 12. *In addition to* (P_1)–(P_4)*, if* (H_1)*,* (H_2)*, and* (H_{10}) *hold, then system* (5) *has two positive* ω*-periodic solutions* x^1 *and* x^2 *satisfying* $0 < \|x^1\| < r < \|x^2\|$*, where* r *is defined in* (H_{10}).

Proof. By (H_1), there exists $0 < r_1 < r$ such that

$$\int_0^{\omega} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \geq \eta \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_1}, \tag{50}$$

where the constant $\eta > 0$ satisfies $\eta\lambda\beta b^L > 1$. Then by Lemma 8, we have

$$\|\psi y\| \geq \lambda b^L \beta\eta \|y\| \geq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_1}. \tag{51}$$

Likewise, from (H_2), there exists $r_2 > r > 0$ such that

$$\int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \geq \eta \|y\|, \quad \text{for any } y \in E,\ \|y\| > r_2, \tag{52}$$

where the constant $\eta > 0$ satisfies $\eta\lambda\beta b^L > 1$. Then by Lemma 8, we have

$$\|\psi y\| \geq \lambda b^L \beta \eta \|y\| \geq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_2}. \tag{53}$$

Define $\Omega_r = \{y \in X : \|y\| \geq r\}$. Then from (H_{10}), for any $y \in E$, $\|y\| > r$, we obtain

$$\|\psi y\| \leq \lambda b^M \beta \int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt < r \leq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_r, \tag{54}$$

which yields

$$\|\psi y\| \leq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_r. \tag{55}$$

In conclusion, under the assumptions (H_1) and (H_{10}), ψ satisfies the conditions in Lemma 4, and then ψ has a fixed point $x^1 \in E \cap (\Omega_r \setminus \overline{\Omega_{r_1}})$. Likewise, under the assumptions (H_2) and (H_{10}), ψ satisfies all the conditions in Lemma 4, and then ψ has a fixed point $x^2 \in E \cap (\Omega_{r_2} \setminus \overline{\Omega_r})$. By Lemma 5, the system (5) has at two positive ω-periodic solutions x^1 and x^2 satisfying $0 < \|x^1\| < r < \|x^2\|$. The proof of Theorem 12 is complete. □

Theorem 13. *In addition to* (P_1)–(P_4)*, if* (H_3), (H_4)*, and* (H_9) *hold, then system* (5) *has two positive* ω*-periodic solutions* x^1 *and* x^2 *satisfying* $0 < \|x^1\| < R < \|x^2\|$*, where* R *is defined in* (H_9).

Proof. By (H_3), for any $0 < \epsilon \leq 1/\lambda B^M \beta$, there exists $R > R_1 > 0$ such that

$$\int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \leq \epsilon \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{R_1}. \tag{56}$$

Then by Lemma 9, we have

$$\|\psi y\| \leq \lambda b^M \beta \epsilon \|y\| \leq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{R_1}. \tag{57}$$

Likewise, by (H_4), for any $0 < \epsilon \leq 1/2\lambda B^M \beta$, there exists $N_2 > R > 0$ such that

$$\int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \leq \epsilon \|y\|, \quad \text{for any } y \in E,\ \|y\| \geq N_2. \tag{58}$$

We choose

$$R_2 > N_2 + 1 + 2\lambda B^M \beta \times \sup_{\|y\| < N_2, y \in E} \int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt. \tag{59}$$

If $y \in E \bigcap \partial\Omega_{R_2}$, then

$$\begin{aligned} \|\psi y\| &= \sum_{i=1}^n \sup_{t\in[0,\omega]} |(\psi_i y)(t)| \\ &\leq \lambda b^M \beta \int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \\ &= \lambda b^M \beta \left[\int_{I_1} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \right. \\ &\quad \left. + \int_{I_2} |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt \right] \\ &\leq \frac{R_2}{2} + \frac{\|y\|}{2} = \|y\|, \end{aligned} \tag{60}$$

where

$$I_1 = \{y \in E : \|y\| \leq N_2\}, \qquad I_2 = \{y \in E : \|y\| > N_2\}. \tag{61}$$

This implies that

$$\|\psi y\| \leq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{R_2}. \tag{62}$$

Define $\Omega_R = \{y \in X : \|y\| \geq R\}$. Then from (H_9), for any $y \in E \cap \partial\Omega_R$, we obtain

$$\|\psi y\| \geq \lambda b^L \alpha \int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\, dt > \lambda b^L \alpha \frac{R}{\lambda b^L \alpha} = R = \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_2}, \tag{63}$$

which yields

$$\|\psi y\| > \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_R. \tag{64}$$

In conclusion, under the assumptions (H_3) and (H_9), ψ satisfies the conditions in Lemma 4, and then ψ has a fixed point $x^1 \in E \cap (\Omega_R \setminus \overline{\Omega_{R_1}})$. Likewise, under the assumptions (H_2) and (H_{10}), ψ satisfies all the conditions in Lemma 4, and then ψ has a fixed point $x^2 \in E \cap (\Omega_{R_2} \setminus \overline{\Omega_R})$. By Lemma 4, the system (5) has at two positive ω-periodic solutions x^1 and x^2 satisfying $0 < \|x^1\| < R < \|x^2\|$. The proof of Theorem 13 is complete. □

Theorem 14. *In addition to* (P_1)–(P_4)*, if* (H_9) *and* (H_{10}) *hold, then system* (5) *has at least one positive* ω*-periodic solution* x *satisfying* $r < \|x\| < R$*, where* r *and* R *are defined in* (H_9) *and* (H_{10})*, respectively.*

Proof. Without loss of generality, we may assume that $0 < r < R$. If $y \in E \cap \partial\Omega_r$, then by (H_{10}), one can get

$$\begin{aligned}\|\psi y\| &= \sum_{i=1}^{n} \sup_{t\in[0,\omega]} \left|(\psi_i y)(t)\right| \\ &\leq \lambda b^M \beta \int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt \\ &< \lambda b^M \beta \frac{r}{\lambda b^M \beta} = r = \|y\|,\end{aligned} \tag{65}$$

which yields

$$\|\psi y\| < \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_r. \tag{66}$$

Likewise, for $y \in E \cap \partial\Omega_R$, then from (H_8), we can get

$$\begin{aligned}\|\psi y\| &= \sum_{i=1}^{n} \sup_{t\in[0,\omega]} \left|(\psi_i y)(t)\right| \\ &\geq \lambda b^L \alpha \int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt \\ &> \lambda b^L \alpha \frac{R}{\lambda b^L \alpha} = R = \|y\|,\end{aligned} \tag{67}$$

which yields

$$\|\psi y\| > \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_R. \tag{68}$$

In conclusion, under the assumptions (H_8) and (H_{10}), ψ satisfies the conditions in Lemma 4, and then ψ has a fixed point $x^1 \in E \cap (\Omega_R \setminus \overline{\Omega_r})$. By Lemma 4, the system (5) has at least one positive ω-periodic solution x^1 satisfying $r < \|x^1\| < R$, where r and R are defined in (H_9) and (H_{10}), respectively. The proof of Theorem 14 is complete. □

Theorem 15. *In addition to (P_1)–(P_4), if (H_5) and (H_8) hold, then system* (5) *has at least one positive ω-periodic solution.*

Proof. By (H_5), for any $\epsilon = 1/\lambda\beta b^M - \theta_1 > 0$, there exists a sufficiently small $r > 0$ such that

$$\begin{aligned}&\frac{\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt}{\|y\|} \\ &\quad < \theta_1 + \epsilon = \frac{1}{\lambda\beta b^M}, \quad \text{for } \|y\| \leq r;\end{aligned} \tag{69}$$

that is

$$\begin{aligned}&\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt \\ &\quad < \frac{\|y\|}{\lambda\beta b^M} \leq \frac{r}{\lambda\beta b^M}, \quad \text{for } \|y\| \leq r,\end{aligned} \tag{70}$$

which implies that (H_{10}) is satisfied.

Likewise, by (H_8), for any $\epsilon = \gamma_2 - 1/\lambda\alpha\sigma b^L > 0$, there exists a sufficiently large $R > 0$ such that

$$\begin{aligned}&\frac{\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt}{\|y\|} \\ &\quad > \gamma_2 + \epsilon = \frac{1}{\lambda\alpha\sigma b^L}, \quad \text{for } \|y\| \geq \sigma R;\end{aligned} \tag{71}$$

that is

$$\begin{aligned}&\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt \\ &\quad > \frac{\|y\|}{\lambda\alpha\sigma b^L} \leq \frac{\sigma R}{\lambda\alpha\sigma b^L} = \frac{R}{\lambda\alpha b^L}, \quad \text{for } \|y\| \leq R,\end{aligned} \tag{72}$$

which implies that (H_9) is satisfied. Therefore, by Theorem 14, we complete the proof. □

Theorem 16. *In addition to (P_1)–(P_4), if (H_6) and (H_7) hold, then system* (5) *has at least one positive ω-periodic solution.*

Proof. By (H_7), for any $\epsilon = \theta_2 - 1/\lambda\alpha\sigma b^L > 0$, there exists a sufficiently small $R > 0$ such that

$$\begin{aligned}&\frac{\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt}{\|y\|} \\ &\quad > \theta_2 + \epsilon = \frac{1}{\lambda\alpha\sigma b^L}, \quad \text{for } 0 < \|y\| \leq R;\end{aligned} \tag{73}$$

that is

$$\begin{aligned}&\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt \\ &\quad > \frac{\|y\|}{\lambda\alpha\sigma b^L} \geq \frac{\sigma R}{\lambda\alpha\sigma b^L} = \frac{R}{\lambda\alpha b^L}, \quad \text{for } \sigma R \leq \|y\| \leq R,\end{aligned} \tag{74}$$

which implies that (H_9) is satisfied. On the other hand, by (H_6), for any $\epsilon = 1/\lambda\beta b^M - \gamma_1 > 0$, there exists a sufficiently large $r^* > 0$ such that

$$\begin{aligned}&\frac{\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt}{\|y\|} \\ &\quad < \gamma_1 + \epsilon = \frac{1}{\lambda\beta b^M}, \quad \text{for } \|y\| \geq r^*.\end{aligned} \tag{75}$$

In the following, we consider two cases to prove (H_6) to be satisfied: $\int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|dt$ are bounded and unbounded. The bounded case is clear. If $\int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|dt$ are unbounded, then there exist $y^* \in R_+^n, r = \|y^*\| \geq r^*$ and $t_0 \in [0, \omega]$ such that

$$\begin{aligned}&\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt \\ &\quad \leq \int_0^\omega \left|f\left(t, y^*\left(t-\tau_1(t)\right), \ldots, y^*\left(t-\tau_n(t)\right)\right)\right| dt, \\ &\qquad \text{for any } \|y\| \leq \|y^*\| = r.\end{aligned} \tag{76}$$

Since $r = \|y\| \geq \|y^*\| \geq r^*$, then we get

$$\begin{aligned}&\int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\,dt\\&\quad\leq \int_0^\omega |f(t, y^*(t-\tau_1(t)), \ldots, y^*(t-\tau_n(t)))|\,dt\\&\quad< \frac{\|y^*\|}{\lambda\beta b^M} = \frac{r}{\lambda\beta b^M}, \quad \text{for any } 0 < \|y\| \leq r,\end{aligned} \tag{77}$$

which implies that the condition (H_6) holds. By Theorem 14, we complete the proof. □

Theorem 17. *In addition to* (P_1)–(P_4), *if* (H_5), (H_6), *and* (H_9) *hold, then system* (5) *has at two positive* ω*-periodic solutions* x^1 *and* x^2 *satisfying* $0 < \|x^1\| < R < \|x^2\|$, *where* R *is defined in* (H_9).

Proof. By (H_5) and the proof of Theorem 15, there exists a sufficiently small $r_1 \in (0, r)$ such that

$$\begin{aligned}&\int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\,dt\\&\quad< \frac{\|y\|}{\lambda\beta b^M} \leq \frac{r_1}{\lambda\beta b^M}, \quad \text{for } 0 < \|y\| \leq r_1.\end{aligned} \tag{78}$$

On the other hand, from (H_6) and the proof of Theorem 16, there exists a sufficiently large $r_2 \in (r, \infty)$ such that

$$\begin{aligned}&\int_0^\omega |f(t, y(t-\tau_1(t)), \ldots, y(t-\tau_n(t)))|\,dt < \frac{r}{\lambda\beta b^M},\\&\qquad\qquad \text{for } 0 < \|y\| \leq r_2.\end{aligned} \tag{79}$$

Therefore, from the proof of Theorem 14, there exist two positive solutions y^1 and y^2 satisfying $r < \|y^1\| < r < \|y^2\| < r_2$, where r is defined in (H_9); the proof of Theorem 17 is complete. □

Theorem 18. *In addition to* (P_1)–(P_4), *if* (H_7), (H_8), *and* (H_{10}) *hold, then system* (5) *has at least two positive* ω*-periodic solutions* x^1 *and* x^2 *satisfying* $0 < \|x^1\| < r < \|x^2\|$, *where* R *is defined in* (H_{10}).

Proof. The proof is similar to that of Theorem 17, and we omit the details here. □

Theorem 19. *In addition to* (P_1)–(P_4), *if* (H_1) *and* (H_6) *hold, then system* (5) *has at least one positive* ω*-periodic solution.*

Proof. Let $\Omega_r = \{y \in X : \|y\| < r\}$. By (H_1) and the proof of Theorem 10, there exists a sufficiently small $r_1 \in (0, r)$ such that

$$\|\psi y\| \geq \lambda b^L \beta\eta \|y\| \geq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_1}. \tag{80}$$

Likewise, by (H_6) and the proof of Theorem 16, there exists a sufficiently large $r_2 \in (r, \infty)$ such that

$$\|\psi y\| < \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_2}. \tag{81}$$

In conclusion, under the assumptions (H_1) and (H_6), ψ satisfies the conditions in Lemma 4, and then ψ has a fixed point in $E \cap (\Omega_{r_2} \backslash \overline{\Omega_{r_1}})$. By Lemma 4, the system (5) has at least one positive ω-periodic solution. The proof of Theorem 19 is complete. □

Similar to Theorem 19, we can get the following consequences.

Theorem 20. *In addition to* (P_1)–(P_4), *if* (H_2) *and* (H_5) *hold, then system* (5) *has at least one positive* ω*-periodic solution.*

Theorem 21. *In addition to* (P_1)–(P_4), *if* (H_3) *and* (H_8) *hold, then system* (5) *has at least one positive* ω*-periodic solution.*

Theorem 22. *In addition to* (P_1)–(P_4), *if* (H_4) *and* (H_7) *hold, then system* (5) *has at least one positive* ω*-periodic solution.*

Theorem 23. *In addition to* (P_1)–(P_4), *if* (H_1), (H_8), *and* (H_{10}) *hold, then system* (5) *has two positive* ω*-periodic solutions* x^1 *and* x^2 *satisfying* $0 < \|x^1\| < r < \|x^2\|$, *where* r *is defined in* (H_{10}).

Proof. Let $\Omega_r = \{y \in X : \|y\| < r\}$. By (H_1) and the proof of Theorem 10, there exists a sufficiently small $r_1 \in (0, r)$ such that

$$\|\psi y\| \geq \lambda b^L \beta\eta \|y\| \geq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_1}. \tag{82}$$

Likewise, by (H_8) and the proof of Theorem 15, there exists a sufficiently large $r_2 \in (0, r)$ such that

$$\|\psi y\| \geq \lambda b^L \beta\eta \|y\| \geq \|y\|, \quad \text{for any } y \in E \cap \partial\Omega_{r_2}. \tag{83}$$

Incorporating (H_{10}) and the proof of Theorem 14, we know that there exist two positive ω-periodic solutions x^1 and x^2 satisfying $r_1 < \|x^1\| < r < \|x^2\| < r_2$, where r is defined in (H_{10}). The proof of Theorem 23 is complete. □

Similar to Theorem 23, one immediately has the following consequences.

Theorem 24. *In addition to* (P_1)–(P_4), *if* (H_2), (H_7), *and* (H_{10}) *hold, then system* (5) *has two positive* ω*-periodic solutions* x^1 *and* x^2 *satisfying* $0 < \|x^1\| < r < \|x^2\|$, *where* r *is defined in* (H_{10}).

Theorem 25. *In addition to* (P_1)–(P_4), *if* (H_3), (H_6), *and* (H_9) *hold, then system* (5) *has two positive* ω*-periodic solutions* x^1 *and* x^2 *satisfying* $0 < \|x^1\| < R < \|x^2\|$, *where* R *is defined in* (H_9).

Theorem 26. *In addition to* (P_1)–(P_4), *if* (H_4), (H_5), *and* (H_9) *hold, then system* (5) *has two positive* ω*-periodic solutions* x^1 *and* x^2 *satisfying* $0 < \|x^1\| < R < \|x^2\|$, *where* R *is defined in* (H_9).

3. Existence of Periodic Solutions of System (6)

Now, we are at the position to study the existence of positive periodic solutions of system (6). By carrying out similar arguments as in Section 2, it is not difficult to derive sufficient criteria for the existence of positive periodic solutions of system (6). For simplicity, we prefer to list below the corresponding criteria for system (6) without proof, since the proofs are very similar to those in Section 2.

For $(t,s) \in R^2$, $1 \le i \le n$, we define

$$G_i^*(t,s) = \frac{e^{\int_t^s a_i(\xi)d\xi}}{e^{\int_0^\omega a_i(\xi)d\xi} - 1},$$
$$G^*(t,s) = \operatorname{diag}\left[G_1^*(t,s), G_2^*(t,s), \ldots, G_n^*(t,s)\right], \tag{84}$$

and it is clear that $G_i^*(t,t) \le G_i^*(t,s) \le G^*(t,t+\omega)$, $\partial G_i^*(t,s)/\partial t = a_i(t)G_i^*(t,s)$, $G_i^*(t,t+\omega) - G_i^*(t,t) = 1$. In view of (P_1), we also define for $1 \le i \le n$

$$\alpha_i^* := \min_{0\le t\le s\le\omega} \left|G_i^*(t,s)\right| = \frac{1}{e^{\int_0^\omega a_i(\xi)d\xi} - 1} = \alpha_i,$$
$$\beta_i^* := \max_{0\le t\le s\le\omega} \left|G_i^*(t,s)\right| = \frac{e^{\int_0^\omega a_i(\xi)d\xi}}{e^{\int_0^\omega a_i(\xi)d\xi} - 1} = \beta_i,$$
$$\alpha^* = \min_{1\le i\le n} \alpha_i^* = \alpha, \qquad \beta^* = \max_{1\le i\le n} \beta_i^* = \beta,$$
$$\delta = \frac{\alpha^*}{\beta^*} \in (0,1) = \sigma, \tag{85}$$
$$B_i(t) = \max\left\{\left|b_{1i}(t)\right|, \left|b_{2i}(t)\right|\right\},$$
$$B_i'(t) = \min\left\{\left|b_{1i}(t)\right|, \left|b_{2i}(t)\right|\right\},$$
$$B(t) = \max_{1\le i\le n}\left\{B_i(t)\right\}, \qquad B'(t) = \min_{1\le i\le n}\left\{B_i'(t)\right\}.$$

Let $X = \{y = (y_1(t), y_2(t), \ldots, y_n(t))^T \in PC(R, R^n) \mid y(t+\omega) = y(t)\}$ with the norm $\|y\| = \sum_{i=1}^n |y_i|_0$, $|y_i|_0 = \sup_{t\in[0,\omega]}|y_i(t)|$, and it is easy to verify that $(X, \|\cdot\|)$ is a Banach space. Define P to be a cone in X by

$$P = \left\{y = \left(y_1(t), y_2(t), \ldots, y_n(t)\right)^T \in X : y_i(t) \ge \delta\left\|y_i\right\|_0, t \in [0,\omega]\right\}. \tag{86}$$

We easily verify that P is a cone in X. We define an operator $A : X \to X$ as follows:

$$(Ay)(t) = \left((A_1y)(t), (A_2y)(t), \ldots, (A_ny)(t)\right)^T, \tag{87}$$

where

$$(A_iy)(t) = \lambda \int_t^{t+\omega} G_i^*(t,s)\, b_i(s)\, f_i \times \left(s, y\left(s-\tau_1(s)\right), \ldots, y\left(s-\tau_n(s)\right)\right) ds. \tag{88}$$

The proof of the following lemmas and theorems is similar to those in Section 2, and we all omit the details here.

Lemma 27. *Assume that (P_1)–(P_4) hold. The existence of positive ω-periodic solution of system (6) is equivalent to that of nonzero fixed point of A in P.*

Lemma 28. *Assume that (P_1)–(P_4) hold. Then the solutions of system (6) are defined on $[-\tau,\infty)$ and are positive.*

Lemma 29. *Assume that (P_1)–(P_4) hold. Then $A : P \to P$ is well defined.*

Lemma 30. *Assume that (P_1)–(P_4) hold, and there exists $\eta > 0$ such that*

$$\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt \ge \eta \|y\|, \quad \text{for any } y \in P, \tag{89}$$

and then

$$\|\psi y\| \ge \lambda b^L \alpha\eta \|y\|, \quad \text{for any } y \in P. \tag{90}$$

Lemma 31. *Assume that (P_1)–(P_4) hold, and let $r > 0$. If there exists a sufficiently small $\epsilon > 0$ such that*

$$\int_0^\omega \left|f\left(t, y\left(t-\tau_1(t)\right), \ldots, y\left(t-\tau_n(t)\right)\right)\right| dt \le \epsilon \|y\|, \quad \text{for any } y \in P \cap \partial\Omega_r, \tag{91}$$

and then

$$\|Ay\| \le \lambda b^M \beta\epsilon \|y\|, \quad \text{for any } y \in P \cap \partial\Omega_r. \tag{92}$$

Theorem 32. *Assume that (P_1)–(P_4) and (H_9) hold. Moreover, if one of the following conditions holds:*

(H_3) and (H_4); (H_3) and (H_6); (H_4) and (H_5); (H_5) and (H_6),

then system (6) has two positive ω-periodic solutions x^1 and x^2 satisfying $0 < \|x^1\| < R < \|x^2\|$, where R is defined in (H_9).

Theorem 33. *Assume that (P_1)–(P_4), and (H_{10}) hold. Moreover, if one of the following conditions holds:*

(H_1) and (H_2); (H_1) and (H_8); (H_2) and (H_7); (H_7) and (H_8),

then system (6) has two positive ω-periodic solutions x^1 and x^2 satisfying $0 < \|x^1\| < r < \|x^2\|$, where r is defined in (H_{10}).

Theorem 34. *Assume that (P_1)–(P_4) hold. Moreover, if one of the following conditions holds:*

(H_1) and (H_4); (H_1) and (H_6); (H_2) and (H_3); (H_2) and (H_5); (H_3) and (H_8); (H_4) and (H_9); (H_5) and (H_8); (H_6) and (H_7); (H_9) and (H_{10}),

then system (6) has at least one positive ω-periodic solution.

4. Examples

In order to illustrate our results, we take the following examples.

Example 35. We consider the following generalized so-called Michaelis-Menton type single species growth model with impulse:

$$y'(t) = y(t)\left[a(t) - \lambda\sum_{i=1}^{n}\frac{\alpha_i(t)\,y(t-\tau_i(t))}{1+\beta_i(t)\,y(t-\tau_i(t))}\right], \quad t \in R,\ t \neq t_k,$$
$$\Delta y(t_k) = \mu c_k y(t_k), \quad k \in Z_+, \tag{93}$$

which is a special case of system (5), and where $a(t), \alpha_i(t), \beta_i(t), \tau_i(t) \in C(R, R_+)$ $(i = 1, 2, \ldots, n)$ are ω-periodic, and $\lambda > 0$, $\mu > 0$ are two parameters.

Theorem 36. *Assume that* (P_1)*–*(P_4) *hold. Moreover, if the following condition holds:*

$$\sum_{i=1}^{n}\int_0^{\omega}\alpha_i(t)\,dt > \frac{\beta_i^M}{\lambda\alpha\sigma^3 b^L}, \tag{94}$$

then system (93) *has at least one positive* ω*-periodic solution.*

Proof. Note that

$$f(t, y(t-\tau_i(t))) = y(t)\sum_{i=1}^{n}\frac{\alpha_i(t)\,y(t-\tau_i(t))}{1+\beta_i(t)\,y(t-\tau_i(t))}. \tag{95}$$

We can construct the same Banach space X and cone E as in Section 2. Then for any $y \in E$, we have

$$\begin{aligned}\int_0^{\omega} f(t, y(t-\tau_i(t)))\,dt &= \sum_{i=1}^{n}\int_0^{\omega} y(t)\frac{\alpha_i(t)\,y(t-\tau_i(t))}{1+\beta_i(t)\,y(t-\tau_i(t))}dt \\ &\geq \frac{\sigma^2\|y\|^2}{1+c_i^M\|y\|}\sum_{i=1}^{n}\int_0^{\omega}\alpha_i(t)\,dt.\end{aligned} \tag{96}$$

This can lead to

$$\frac{\int_0^{\omega} f(t, y(t-\tau_i(t)))\,dt}{\|y\|} \geq \frac{\sigma^2\|y\|}{1+\beta_i^M\|y\|}\sum_{i=1}^{n}\int_0^{\omega}\alpha_i(t)\,dt. \tag{97}$$

Then we can have

$$f_\infty \geq \frac{\sigma^2}{\beta_i^M}\sum_{i=1}^{n}\int_0^{\omega}\alpha_i(t)\,dt > \frac{1}{\lambda\alpha\sigma b^L}. \tag{98}$$

On the other hand, we have

$$\begin{aligned}\int_0^{\omega} f(t, y(t-\tau_i(t)))\,dt &= \sum_{i=1}^{n}\int_0^{\omega} y(t)\frac{\alpha_i(t)\,y(t-\tau_i(t))}{1+\beta_i(t)\,y(t-\tau_i(t))}dt \\ &\leq \|y\|^2\sum_{i=1}^{n}\int_0^{\omega}\alpha_i(t)\,dt.\end{aligned} \tag{99}$$

This can lead to

$$\frac{\int_0^{\omega} f(t, y(t-\tau_i(t)))\,dt}{\|y\|} \leq \|y\|\sum_{i=1}^{n}\int_0^{\omega}\alpha_i(t)\,dt \longrightarrow 0, \quad \|y\| \longrightarrow 0. \tag{100}$$

That is

$$f^0 = 0. \tag{101}$$

By Theorem 21, it follows that system (93) has at least one positive ω-periodic solution. The proof of Theorem 36 is complete. □

Example 37. We consider the following generalized hematopoiesis model with impulse:

$$y'(t) = -\alpha(t)\,y(t) + \lambda\beta(t)\exp\{-\gamma(t)\,y(t-\tau(t))\}, \quad t \in R,\ t \neq t_k,$$
$$\Delta y(t_k) = \mu c_k y(t_k), \quad k \in Z_+, \tag{102}$$

which is a special case of system (6), and where $x(t)$ is the number of red blood cells at time t, $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $\tau(t) \in C(R, R_+)$ are ω-periodic and $\lambda > 0$, $\mu > 0$ are two parameters.

Theorem 38. *Assume that* (P_1)*–*(P_4) *hold. Moreover, if the following condition holds:*

$$\sum_{i=1}^{n}\int_0^{\omega}\alpha_i(t)\,dt > \frac{\beta_i^M}{\lambda\alpha\sigma^3 b^L}, \tag{103}$$

then system (102) *has at least one positive* ω*-periodic solution.*

Proof. Note that

$$f(t, y(t-\tau(t))) = \exp\{-\gamma(t)\,y(t-\tau(t))\}. \tag{104}$$

We can construct the same Banach space X and cone E as in Section 2. Then for any $y \in E$, we have

$$\begin{aligned}\frac{\int_0^{\omega} f(t, y(t-\tau(t)))\,dt}{\|y\|} &\geq \frac{\omega}{\exp\{\gamma^M\|y\|\}\|y\|}; \\ \frac{\int_0^{\omega} f(t, y(t-\tau(t)))\,dt}{\|y\|} &\leq \frac{\omega}{\exp\{\gamma^L\|y\|\}\|y\|}.\end{aligned} \tag{105}$$

This can lead to

$$f_{\infty} = 0, \qquad \|y\| \longrightarrow \infty;$$
$$f^{0} = \infty, \qquad \|y\| \longrightarrow 0. \tag{106}$$

By Theorem 34, it follows that system (102) has at least one positive ω-periodic solution. The proof of Theorem 38 is complete. □

Remark 39. We apply the main results obtained in the previous sections to study some examples which have some biological implications. A very basic and important ecological problem associated with the study of population is that of the existence of positive periodic solutions which play the role played by the equilibrium of the autonomous models and means that the species is in an equilibrium state. From Theorems 36 and 38, we see that under the appropriate conditions, the impulsive perturbations do not affect the existence of periodic solution of the systems.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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