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Research Article

Globally Exponential Stability of Impulsive Neural Networks with Given Convergence Rate

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This paper deals with the stability problem for a class of impulsive neural networks. Some sufficient conditions which can guarantee the globally exponential stability of the addressed models with given convergence rate are derived by using Lyapunov function and impulsive analysis techniques. Finally, an example is given to show the effectiveness of the obtained results.

1. Introduction

Recently, special interest has been devoted to the dynamics analysis of neural networks due to their potential applications in different areas of science. Particularly, there has been a significant development in the theory of neural networks with impulsive effects [1–9], since such neural networks with impulsive effect can be used as an appropriate description of the phenomena of abrupt qualitative dynamical changes of essential continuous time systems. Based on the theory of impulsive differential equations [10–17], some sufficient conditions guaranteeing the exponential stability are derived [18–24]. For example, in [8], the author has obtained a criterion of exponential stability for a Hopfield neural network with periodic coefficients; in [18], by constructing the extended impulsive delayed Halanay inequality and Lyapunov functional methods, authors have got some sufficient conditions ensuring exponential stability of the unique equilibrium point of impulsive Hopfield neural networks with time delays. They all have obtained exponential stability for some kinds of neural networks through different methods. However, most of the existing results about the exponential stability of impulsive neural networks have a common feature that the exponential convergence rate cannot be derived, or derived but not the given one [8, 18, 23, 24]. The purpose of this paper is to establish some criteria which can guarantee the globally exponential stability of impulsive neural networks with the given convergence rate by using Lyapunov function and impulsive analysis techniques. This work is organized as follows. In Section 2, we introduce some basic definitions and notations. In Section 3, the main results are presented. In Section 4, an example is discussed to illustrate the results.

2. Preliminaries

Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ denote the set of nonnegative real numbers, $\mathbb{Z}_+$ denote the set of positive integers and $\mathbb{R}^n$ denote the n-dimensional real space equipped with the Euclidean norm $\|\cdot\|$.

Consider the following impulsive neural networks:

$$\begin{aligned} \dot{x}_i(t) &= -a_i(t)\,x_i(t) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t)\,f_j\left(x_j(t)\right) + I_i(t), \quad t \ge t_0,\ t \ne t_k, \\ \Delta x_i|_{t=t_k} &= x_i(t_k) - x_i(t_k^-), \quad k \in \mathbb{Z}_+,\ i \in \Lambda, \end{aligned} \tag{1}$$

where $\Lambda = \{1, 2, \ldots, n\}$. $n \ge 2$ corresponds to the number of units in a neural network; the impulse times t_k satisfy $0 \le t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$; x_i corresponds to the state of the neurons, f_j denotes the measures of response to its incoming potentials of the unit j at time t; $I_i(t)$ is the input of the unit i at time t. $PC[I, \mathbb{R}] \triangleq \{\varphi : I \to \mathbb{R} \mid \varphi(t^+) = \varphi(t)$ for $t \in I$, $\varphi(t^-)$ exists for $t \in I$, $\varphi(t^-) = \varphi(t)$ for all but points $t_k \in I\}$, where $I \subset \mathbb{R}$ is an interval, $\varphi(t^+)$ and $\varphi(t^-)$ denote the left limit and right limit of function $\varphi(t)$, respectively. $a_i(t) > 0$, $b_{ij}(t), I_i(t) \in PC[[t_0, +\infty), \mathbb{R}]$. For given $t_0, x(t_0) = x_0 = (x_1^0, x_2^0, \ldots, x_n^0) \in \mathbb{R}^n$, we denote by $x(t)$ the solution of system (1) with initial value $(t_0, x(t_0))$.

In this paper, we assume that some conditions are satisfied so that the equilibrium point of system (1) does exist, see [16, 17]. Assume that $x^\star = (x_1^\star, x_2^\star, \ldots, x_n^\star)^T$ is an equilibrium point of system (1). Impulsive operator is viewed as perturbation of the point $x^\star$ of such system without impulsive effects. We assume that the following impulsive condition holds.

(H_0) $\Delta x_i|_{t=t_k} = x_i(t_k) - x_i(t_k^-) = \sigma_{ik}(x_i(t_k^-) - x_i^\star)$, $\sigma_{ik} \in \mathbb{R}$, $i \in \Lambda$, $k \in \mathbb{Z}_+$.

Furthermore, we will assume that the response function f_i satisfies the following condition.

(H_1) f_i is globally Lipschizian with Lipschitz constant $l_i > 0$, that is, $|f_i(s_1) - f_i(s_2)| \le l_i|s_1 - s_2|$, for all $s_1, s_2 \in \mathbb{R}$, $i \in \Lambda$.

Note that $x^\star$ is an equilibrium point of system (1), one can derive from system (1) that the transformation $z_i = x_i - x_i^\star$, $i \in \Lambda$ transforms such system into the following system:

$$\dot{z}_i(t) = -a_i(t) z_i(t) + \sum_{j=1}^{n} b_{ij}(t) F_j(z_j(t)), \quad t \ge t_0,\ t \ne t_k, \qquad (2)$$
$$z_i(t_k) = (1 + \sigma_{ik}) z_i(t_k^-), \quad i \in \Lambda,\ k \in \mathbb{Z}_+,$$

where $F_j(z_j(t)) = f_j(x_j^\star + z_j(t)) - f_j(x_j^\star)$, and from the condition (H_1), it holds that $\|F_j(z_j(t))\| \le l_j \|z_j(t)\|$, $j \in \Lambda$.

Furthermore, let $y_i(t) = z_i(t) e^{\alpha(t-t_0)}$, $i \in \Lambda$, then system (2) becomes as follows:

$$\dot{y}_i(t) = (\alpha - a_i(t)) y_i(t) + e^{\alpha(t-t_0)} \sum_{j=1}^{n} b_{ij}(t) F_j(y_j(t) e^{-\alpha(t-t_0)}), \quad t \ge t_0,\ t \ne t_k,$$
$$y_i(t_k) = (1 + \sigma_{ik}) y_i(t_k^-), \quad i \in \Lambda,\ k \in \mathbb{Z}_+. \qquad (3)$$

To prove the stability of $x^\star$ of system (1), it is equal to prove the stability of the zero solution of system (2), and also equal to the boundedness of system (3).

In the following, the notion A^T means the transpose of a square matrix A. We will use the notation $A > 0$ (or $A < 0$, $A \ge 0$, $A \le 0$) to denote that the matrix A is a positive definite (negtive definite, positive semidefinite, an negative semidefinite) marix.

Let $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$, $A(t) = \mathrm{diag}[a_1(t), a_2(t), \ldots, a_n(t)]$, $B(t) = (b_{ij}(t))_{n\times n}$, $L = \mathrm{diag}[l_1, l_2, \ldots, l_n]$, $I = \mathrm{diag}[1, 1, \ldots, 1]$, $D_k = \mathrm{diag}[1 + \sigma_{1k}, 1 + \sigma_{2k}, \ldots, 1 + \sigma_{nk}]$, $F(y) = (F_1(y_1), F_2(y_2), \ldots,$ and $F_n(y_n))^T$ then system (3) with initial condition becomes as follows:

$$\dot{y}(t) = (\alpha I - A(t)) y(t) + e^{-\alpha(t-t_0)} B(t) F(y(t) e^{-\alpha(t-t_0)}), \quad t \ge t_0,\ t \ne t_k,$$
$$y(t_k) = D_k y(t_k^-), \quad i \in \Lambda,\ k \in \mathbb{Z}_+. \qquad (4)$$

We introduce a definition as follows.

Definition 1. Assume $x^\star = (x_1^\star, x_2^\star, \ldots, x_n^\star)^T \in \mathbb{R}^n$ is the equilibrium point of system (1), then the equilibrium point $x^\star$ of system (1) is said to be globally exponential stable with given convergence rate $\alpha > 0$. If for any initial data $x(t_0) = x_0 \in \mathbb{R}$, there exists a constant $M \geqslant 1$, such that

$$\|x(t, t_0, x_0) - x^\star\| \leqslant \|x_0 - x^\star\| M e^{-\alpha(t-t_0)}, \quad t \geqslant t_0. \qquad (5)$$

From the transformation $z_i = x_i - x_i^\star$, $i \in \Lambda$, and $z(t_0) = z_0 = x_0 - x^\star$, the globally exponential stability of the equilibrium point $x^\star$ of system (1) can be transformed into the globally exponential stability of trivial solution of system (2), so (5) can be rewritten as follows:

$$\|z(t, t_0, z_0)\| \le \|z(t_0)\| M e^{-\alpha(t-t_0)}, \quad t \geqslant t_0. \qquad (6)$$

Furthermore, form the transformation $y(t) = z(t, t_0, z_0) e^{\alpha(t-t_0)}$, $i \in \Lambda$, the globally exponential stability of trivial solution of system (2) can be transformed into the boundedness of the solution of system (4) and it can be rewritten as follows:

$$\|y(t)\| \le \|y(t_0)\| M, \quad t \geqslant t_0. \qquad (7)$$

3. Main Results

Theorem 2. *Given constant $\alpha > 0$. The equilibrium point of the system (1) is globally exponentially stable with the given convergence rate α, if the conditions (H_0) and (H_1) are fulfilled; moreover, suppose that*

(i) $\alpha I - A(t) + B(t) L \le 0$, *for all* $t > t_0$,

(ii) $\prod_{k=1}^{\infty} \max_{i\in\Lambda}(1 + \sigma_{ik}) < \infty$, *and* $\sigma_{ik} \ge 0$, $i \in \Lambda$, $k \in \mathbb{Z}_+$.

Proof. We only need to prove $y(t) = z(t) e^{\alpha(t-t_0)}$ is bounded when $t \ge t_0$, where $z(t) = z(t, t_0, z_0)$ is a solution of (3) through (t_0, z_0).

Consider the Lyapunov function as follows:

$$V(t) = y(t)^T y(t) = \sum_{i=1}^{n} y_i^2(t). \qquad (8)$$

Particularly, $V(t_0) = \sum_{i=1}^{n} y_i^2(t_0)$.

Then from conditions (H_0)-(H_1) and (i), we get the upper right-hand derivative of $V(t)$ along the solutions of system (3), for $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$

$$D^+V(t) = 2\sum_{i=1}^{n} y_i(t) \dot{y}_i(t)$$
$$= 2\sum_{i=1}^{n} z_i(t) e^{2\alpha(t-t_0)} [\dot{z}_i(t) + \alpha z_i(t)]$$
$$= 2\sum_{i=1}^{n} z_i(t) e^{2\alpha(t-t_0)} \times \left[-a_i(t) z_i(t) + \sum_{j=1}^{n} b_{ij}(t) F_j(z_j(t)) + \alpha z_i(t)\right]$$

$$\begin{aligned}
&= 2\sum_{i=1}^{n} e^{2\alpha(t-t_0)} \\
&\quad \times \left[\left(\alpha - a_i(t)\right) z_i^2(t) + z_i(t) \sum_{j=1}^{n} b_{ij}(t) F_j\left(z_j(t)\right) \right] \\
&\le 2\sum_{i=1}^{n} e^{2\alpha(t-t_0)} \\
&\quad \times \left[\left(\alpha - a_i(t)\right) z_i(t)^2 + z_i(t) \sum_{j=1}^{n} b_{ij}(t) l_j \left\| z_j(t) \right\| \right] \\
&= 2\sum_{i=1}^{n} e^{2\alpha(t-t_0)} z_i^2(t) \left(\alpha - a_i(t)\right) \\
&\quad + 2\sum_{i=1}^{n} e^{2\alpha(t-t_0)} z_i(t) \sum_{j=1}^{n} b_{ij}(t) l_j \left\| z_j(t) \right\| \\
&= 2y(t)^T \left[\alpha I - A(t) + B(t) L\right] y(t) \\
&\le 0,
\end{aligned} \tag{9}$$

which implies the functional $V(t)$ is nonincreasing for $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$. By condition (ii), it holds that

$$\begin{aligned}
V(t_k) = \sum_{i=1}^{n} y_i^2(t_k) &= \sum_{i=1}^{n} z_i^2(t_k) e^{2\alpha(t_k-t_0)} \\
&= \sum_{i=1}^{n} e^{2\alpha(t-t_0)} \left[z_i(t_k^-) + J_i(z_i(t_k^-))\right]^2 \\
&= \sum_{i=1}^{n} e^{2\alpha(t-t_0)} (1+\sigma_{ik})^2 z_i^2(t_k^-) \\
&\le \max_{i\in\Lambda} (1+\sigma_{ik})^2 V(t_k^-).
\end{aligned} \tag{10}$$

For any $t \in [t_0, t_1)$, since $V(t)$ is nonincreasing, it holds that $V(t) \le V(t_0)$; moreover,

$$V(t_1) \le \max_{i\in\Lambda} (1+\sigma_{i1})^2 V(t_1^-) \le \max_{i\in\Lambda} (1+\sigma_{i1})^2 V(t_0). \tag{11}$$

Similarly, for any $t \in [t_1, t_2)$, it holds that $V(t) \le V(t_1) \le \max_{i\in\Lambda}(1+\sigma_{i1})^2 V(t_0)$, and

$$\begin{aligned}
V(t_2) &\le \max_{i\in\Lambda} (1+\sigma_{i2})^2 V(t_2^-) \\
&\le \max_{i\in\Lambda} (1+\sigma_{i2})^2 \max_{i\in\Lambda} (1+\sigma_{i1})^2 V(t_0).
\end{aligned} \tag{12}$$

Thus, it can be deduced that for $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$,

$$V(t) \le \prod_{j=1}^{k} \max_{i\in\Lambda} \left(1+\sigma_{ij}\right)^2 V(t_0). \tag{13}$$

Hence, we obtain that for any $t \ge t_0$,

$$V(t) \le \prod_{j=1}^{\infty} \max_{i\in\Lambda} \left(1+\sigma_{ij}\right)^2 V(t_0), \tag{14}$$

which implies that

$$\left\| y(t) \right\| \le M \left\| y(t_0) \right\|, \quad t \ge t_0, \tag{15}$$

where $M = \prod_{k=1}^{\infty} \max_{i\in\Lambda}(1+\sigma_{ik}) < \infty$.

The proof of Theorem 2 is complete. □

Remark 3. Most of the existing results about the exponential stability of impulsive neural networks cannot effectively control the convergence rate. It is interesting to see that Theorem 2 can guarantee the globally exponential stability of impulsive neural networks with the given convergence rate.

Remark 4. In particular, if $A(t) \equiv A, B(t) \equiv B$ in Theorem 2, where A, B are constant matrices, then condition $\alpha I - A + BL < 0$ can be easily checked via Matlab.

Theorem 5. *Given constant $\alpha > 0$. The equilibrium point of the system* (1) *is globally exponentially stable with the given convergence rate α, if the conditions (H_0)-(H_1) are fulfilled; moreover, suppose that*

(i) *there exists a constant $\lambda > 0$, such that $(\alpha+\lambda)I - A(t) + B(t)L < 0$, for all $t > t_0$,*

(ii) $\tau \triangleq \min_{k\in\mathbb{Z}_+}\{t_{k+1} - t_k\}$, $\max_{i\in\Lambda}(1+\sigma_{ik}) \le M_k e^{\lambda\tau}$, *where $1 \le M_k < \infty$ and $\prod_{k=1}^{\infty} M_k < \infty$, $i \in \Lambda$, $k \in \mathbb{Z}_+$.*

Proof. We only need to prove that $y(t) = z(t)e^{\alpha(t-t_0)}$ is bounded when $t \ge t_0$, where $z(t) = z(t, t_0, z_0)$ is a solution of (3) through (t_0, z_0).

Consider the Lyapunov function as follows:

$$V(t) = y(t)^T y(t) = \sum_{i=1}^{n} y_i^2(t). \tag{16}$$

In particular, $V(t_0) = \sum_{i=1}^{n} y_i^2(t_0)$.

Then from conditions (H_0)-(H_1) and (i), we get the upper right-hand derivative of $V(t)$ along the solutions of system (3), for $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$

$$\begin{aligned}
D^+V(t) &= 2\sum_{i=1}^{n} y_i(t) \dot{y}_i(t) \\
&\le 2y(t)^T \left(\alpha I - A(t) + B(t) L\right) y(t) < -2\lambda V(t).
\end{aligned} \tag{17}$$

Thus,

$$V(t) < V(t_k) e^{-2\lambda(t-t_k)} < V(t_k) e^{-2\lambda\tau}, \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{Z}_+. \tag{18}$$

By condition (ii), it holds that

$$V(t_k) = \sum_{i=1}^{n} y_i^2(t_k) = \sum_{i=1}^{n} e^{2\alpha(t-t_0)}(1+\sigma_{ik})^2 z_i^2(t_k^-) \le M_k^2 e^{2\lambda\tau}\sum_{i=1}^{n} e^{2\alpha(t-t_0)} z_i^2(t_k^-) \le M_k^2 e^{2\lambda\tau} V(t_k^-). \tag{19}$$

For any $t \in [t_0, t_1)$, it holds that $V(t) \le V(t_0)e^{-2\lambda\tau}$, moreover

$$V(t_1) \le M_1^2 e^{2\lambda\tau} V(t_1^-) \le M_1^2 V(t_0). \tag{20}$$

Similarly, for any $t \in [t_1, t_2)$, it holds that $V(t) \le V(t_1)e^{-2\lambda\tau} \le M_1^2 V(t_0)e^{-2\lambda\tau}$, and

$$V(t_2) \le M_2 e^{2\lambda\tau} V(t_2^-) \le M_2^2 M_1^2 V(t_0). \tag{21}$$

Without loss of generality, when $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$, it can be deduced that

$$V(t) \le \prod_{j=1}^{k} M_j^2 e^{-2\lambda\tau} V(t_0). \tag{22}$$

Hence, we obtain that for any $t \ge t_0$,

$$V(t) \le \prod_{j=1}^{\infty} M_j^2 V(t_0), \tag{23}$$

which implies that

$$\|y(t)\| \le M\|y(t_0)\|, \quad t \ge t_0, \tag{24}$$

where $M = (\prod_{j=1}^{\infty} M_j) < \infty$.

The proof of Theorem 5 is complete. □

Remark 6. Although Theorem 5 enhances the restriction on condition (i), the impulsive restriction in (ii) is weaker; that is, σ_{ik} is not necessary to converge to 0 as k is large enough, provided that the impulsive intervals are not too small.

Theorem 7. *Given constant $\alpha > 0$. The equilibrium point of the system* (1) *is globally exponentially stable with the given exponential convergence rate α, if the conditions (H_0)-(H_1) are fulfilled; moreover, suppose that*

(i) *there exists a constant $\lambda > 0$, such that $(\alpha-\lambda)I - A(t) + B(t)L < 0$, for all $t > t_0$,*

(ii) *$\tau \triangleq \max_{k\in\mathbb{Z}_+}\{t_{k+1} - t_k\}$ and $\beta_k e^{\lambda\tau} \le 1$, where $\beta_k = \max_{i\in\Lambda}(1+\sigma_{ik})$, $-1 \le \sigma_{ik} \le 0$, $i \in \Lambda$, $k \in \mathbb{Z}_+$.*

Proof. We only need to prove that $y(t) = z(t)e^{\alpha(t-t_0)}$ is bounded when $t \ge t_0$, where $z(t) = z(t, t_0, z_0)$ is a solution of (3) through (t_0, z_0).

Consider the Lyapunov functional as follows:

$$V(t) = y(t)^T y(t) = \sum_{i=1}^{n} y_i^2(t). \tag{25}$$

Particularly, $V(t_0) = \sum_{i=1}^{n} y_i^2(t_0)$.

Then from conditions (H_0)-(H_1) and (i), we get the upper right-hand derivative of $V(t)$ along the solutions of system (1), for $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$

$$D^+V(t) = 2\sum_{i=1}^{n} y_i(t)\dot{y}_i(t) = 2y(t)^T(\alpha I - A(t) + B(t)L)y(t) < 2\lambda V(t). \tag{26}$$

Thus,

$$V(t) < V(t_k)e^{2\lambda(t-t_k)} < V(t_k)e^{2\lambda\tau}, \quad t \in [t_k, t_{k+1}), \ k \in \mathbb{Z}_+. \tag{27}$$

By condition (ii), it holds that

$$V(t_k) = \sum_{i=1}^{n} y_i^2(t_k) \le \max_{i\in\Lambda}(1+\sigma_{ik})^2 V(t_k^-) = \beta_k^2 V(t_k^-), \quad k \in \mathbb{Z}_+. \tag{28}$$

By simple induction, we can prove that for any $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}_+$,

$$V(t) \le \prod_{j=1}^{k} \beta_j^2 e^{2(k+1)\lambda\tau} V(t_0) \le e^{2\lambda\tau} V(t_0), \tag{29}$$

which implies that

$$\|y(t)\| \le M\|y(t_0)\|, \quad t \ge t_0, \tag{30}$$

where $M = e^{\lambda\tau}$.

The proof of Theorem 7 is complete. □

4. Applications

The following illustrative example will demonstrate the effectiveness of our results.

Example 8. Consider the following impulsive neural networks:

$$\begin{aligned}
\dot{x}_1(t) &= -a_1(t)x_1(t) + b_{11}(t)f_1(x_1(t)) + b_{12}(t)f_2(x_2(t)) + I_1(t), \\
\dot{x}_2(t) &= -a_2(t)x_2(t) + b_{21}(t)f_1(x_1(t)) + b_{22}(t)f_2(x_2(t)) + I_2(t), \\
x_1(t_k) &= (1+\sigma_{1k})x_1(t_k^-) - \sigma_{1k}, \quad k \in \mathbb{Z}_+, \\
x_2(t_k) &= (1+\sigma_{2k})x_2(t_k^-) - \sigma_{2k}, \quad k \in \mathbb{Z}_+,
\end{aligned} \tag{31}$$

where $f_1(u) = f_2(u) = (|u+1| - |u-1|)/2$.

It is easy to see that f_j, $j = 1, 2$ satisfying hypothesis (H_1) with $l_i = l_2 = 1$. We have $a_1(t) = 3$, $a_2(t) = 2 - (3/2)\cos(2t)$, $b_{11}(t) = |\sin t|/2$, $b_{12}(t) = 1/2$, $b_{21}(t) = 2$, $b_{22}(t) = -1 - (\cos(2t)/2)$, $I_1 = (5/2) - (|\sin t|/2)$, $I_2 = 1 - \cos(2t)$, $\sigma_{1k} = \sqrt{1 + (1/5k^2)} - 1$, $\sigma_{2k} = \sqrt{1 + (1/6k^2)} - 1$.

Let $\alpha = 3$. It can be deduced that $\alpha I - A(t) + B(t)L \leq 0$ and $\prod_{k=1}^{\infty} \max_{i=1,2} |1 + \sigma_{ik}| < \infty$. Hence, all the conditions of Theorem 2 are satisfied; then the equilibrium point $x^{\star} = (1, 1)$ of the above system (31) is globally exponentially stable with the given convergence rate $\alpha = 3$.

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