Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 212469, 11 pages
http://dx.doi.org/10.1155/2013/212469

Research Article

Stability Analysis of Stochastic Markovian Jump Neural Networks with Different Time Scales and Randomly Occurred Nonlinearities Based on Delay-Partitioning Projection Approach

Jianmin Duan,[1] Manfeng Hu,[1,2] Yongqing Yang,[1,2] and Liuxiao Guo[1]

[1] *School of Science, Jiangnan University, Wuxi 214122, China*
[2] *Key Laboratory of Advanced Process Control for Light Industry (Jiangnan University), Ministry of Education, Wuxi 214122, China*

Correspondence should be addressed to Manfeng Hu; humanfeng@jiangnan.edu.cn

Received 27 June 2013; Accepted 2 October 2013

Academic Editor: Debora Amadori

In this paper, the mean square asymptotic stability of stochastic Markovian jump neural networks with different time scales and randomly occurred nonlinearities is investigated. In terms of linear matrix inequality (LMI) approach and delay-partitioning projection technique, delay-dependent stability criteria are derived for the considered neural networks for cases with or without the information of the delay rates via new Lyapunov-Krasovskii functionals. We also obtain that the thinner the delay is partitioned, the more obviously the conservatism can be reduced. An example with simulation results is given to show the effectiveness of the proposed approach.

1. Introduction

The human brain is made up of a large amount of neurons and their interconnections. An artificial neural network is an information processing system that has certain characteristics in common with biological neural networks. During the past decades, neural networks have been used for a wide variety of applications, for example, associative memories, pattern recognition, signal processing, and the other fields [1–4]. As is well known, the stability of neural networks plays an important role in modern control theories for these applications. However, time delays are often attributed as the major sources of instability in various engineering systems. Therefore, how to find sufficient conditions to guarantee the stability of neural networks with time delays is an important research topic [5–15].

Markovian jump system is an important class of stochastic models, which can be described by a set of *nonlinear systems with the transitions between models determined by a Markovian chain in a finite mode set. This kind of system has been extensively applied to study the stability of neural networks. In real life, neural networks have a phenomenon of information latching, and it is recognized that the best way for modeling this class of neural networks is Markovian jump system [16–18].* Obviously, the Markovian jump system is more complex and challenging than the system without Markovian jump parameters, in which many authors are interested [12, 19–24].

On the other hand, it should be pointed out that lots of practical systems are influenced by additive randomly occurred nonlinear disturbances which are caused by environmental circumstances. In today's networked environment, such nonlinear disturbances may be subject to random abrupt changes, which may result from abrupt phenomena, such as random failures and repairs of the components, environmental disturbance, and so forth. In other words, the nonlinear disturbances may occur in a probabilistic way, but they are randomly changeable in terms of their types and/or intensity. The stochastic nonlinearities, which are then named as randomly occurred nonlinearities (RONs), have recently attracted much attention [25–28].

Recently, by introducing free-weighting matrices [29, 30], model transformation method [31], linear matrix inequality (LMI) approach [32, 33], and adopting the concept of delay partitioning [9, 11, 34], stability criteria have been obtained *for some neural networks.* Sufficient conditions for the robust stability of uncertain stochastic system with interval time-varying delay were derived in [29] by employing the delay partitioning approach. Delay-dependent conditions on mean square asymptotic stability of stochastic neural networks with Markovian jumping parameters are presented by using the delay partitioning method which is different from the existing ones in the literature and convex combination method in [12]. The RONs model and the sensor failure model were introduced in [26]. In [9], better delay-dependent stability criteria for continuous systems with multiple delay components were established by utilizing a delay-partitioning projection approach. On the other hand, it is worth mentioning that neural networks on time scales have been presented and studied [35–37], which can unify the continuous and discrete situations. To the best of the authors' knowledge, the delay-partitioning projection approach to stability analysis of stochastic Markovian jump neural networks with different time scales and randomly occurred nonlinearities has never been tackled in the previous literature. This motivates our research.

In this paper, the problem of stability analysis of stochastic Markovian jump neural networks with different time scales and RONs is considered. The paper is organised as follows. Section 2 introduces model description and preliminaries. RONs are introduced to model a class of sector-like nonlinearities whose occurrence is governed by a Bernoulli distributed white sequence with a known conditional probability. In Section 3, we derive the stability results based on delay-partitioning projection approach for stochastic Markovian jump neural networks with RONs. The results include two cases, one with a specified delay rates and the other with arbitrary delay rates. In addition to delay dependence, the obtained conditions are also dependent on the partitioning size; we verify that the conservatism of the conditions is a nonincreasing function of the number of partitions. A numerical example is presented to illustrate the effectiveness of the obtained criteria in Section 4. And finally, conclusions are drawn in Section 5.

Notations. Throughout this paper, the notation is fairly standard. R^n denotes the n-dimensional Euclidean space. $R^{n\times m}$ stands for real matrix R of size $n\times m$ (simply abbreviated R_n when $m = n$). $P > (<)0$ is used to define a real symmetric positive definite (negative definite) matrix. For real symmetric matrices X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X - Y$ is positive semidefinite (resp., positive definite). The symmetric terms in a symmetric matrix are denoted by $*$ and diag$\{\cdots\}$ denotes a block-diagonal matrix. The superscripts A^T and A^{-1} stand for the transpose and inverse of matrix A. $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathscr{P})$ denotes a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$, where Ω is a sample space, $\mathscr{F}$ is the σ-algebra of subset of the sample space, and $\mathscr{P}$ is the probability measure on $\mathscr{F}$. $E\{x\}$ stands for the expectation of the stochastic variable x.

2. Model Description and Preliminaries

In this paper, the stochastic Markovian jump neural networks with different time scales RONs are considered:

$$\begin{aligned} \varepsilon\dot{x}(t) &= -A(r(t))x(t) + B(r(t))x(t-\tau(t)) \\ &\quad + \xi(t)Ef(x(t)) \\ &\quad + [C(r(t))x(t) + D(r(t))x(t-\tau(t))]W(t), \\ x(t) &= \phi(t), \quad \forall t \in [-d, 0], \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector. $W(t)$ is a scalar zero mean Gaussian white noise process. $\phi(t)$ is a real-valued initial condition. $\varepsilon > 0$ is the time scale. $\{r(t)\}$ is a right-continuous Markov chain on the probability space $(\Omega, \mathscr{F}, \mathscr{P})$ taking values in a given finite set $S = \{1, 2, \ldots, N\}$. The transition probability matrix of system (1) is given by

$$P\{r(t+\Delta t) = j \mid r(t) = i\} = \begin{cases} q_{ij}\Delta t + o(\Delta t) & \text{if } i \neq j, \\ 1 + q_{ii}\Delta t + o(\Delta t), & \text{if } i = j, \end{cases} \tag{2}$$

where $\Delta t > 0$ and $\lim_{\Delta t\to 0} o(\Delta t)/\Delta t = 0$, $q_{ij} \geq 0$ is the transition rate from mode i at time t to mode j at time $t + \Delta t$, and $q_{ii} = -\sum_{i\neq j} q_{ij}$. $A(r(t)) = \text{diag}\{a_{i1}, a_{i2}, \ldots, a_{in}\}$ is a positive diagonal matrix. $B(r(t))$, $C(r(t))$, $D(r(t))$, and E are known matrices. For the sake of simplicity, in the sequel, each possible value of $r(t)$ is denoted by i, $i \in S$ and $A(r(t))$, $B(r(t))$, $C(r(t))$, $D(r(t))$ are abbreviated as A_i, B_i, C_i, D_i, respectively. $d(t)$ denotes the time-varying delay satisfying

$$0 \leq d(t) \leq d, \quad \dot{d}(t) \leq \mu, \tag{3}$$

where d and μ are constant real numbers.

Finally, $f(x(t))$ stands for the mismatched external nonlinearity. The stochastic variable $\xi(t) \in R$, which accounts for the phenomena of RONs, is a Bernoulli distributed white noise sequence specified by the following distribution laws:

$$\begin{aligned} \text{Prob}\{\xi(t) = 1\} &= E\{\xi(t)\} = \bar{\xi}, \\ \text{Prob}\{\xi(t) = 0\} &= 1 - E\{\xi(t)\} = 1 - \bar{\xi}. \end{aligned} \tag{4}$$

Furthermore, we can show that

$$E\{\xi(t) - \bar{\xi}\} = 0, \qquad E\{(\xi(t) - \bar{\xi})^2\} = \bar{\xi}(1 - \bar{\xi}), \tag{5}$$

where $\bar{\xi} \in [0, 1]$ is a constant.

Before proceeding, we make the following assumption.

Assumption 1. For all $x \in R^n$, the nonlinear function $f(x)$ is assumed to satisfy the following sector-bounded condition:

$$[f(x) - K_1x]^T [f(x) - K_2x] \leq 0, \tag{6}$$

where K_1 and K_2 are known real matrices of appropriate dimensions and $K_1 - K_2 \geq 0$.

Remark 2. The nonlinear function $f(x)$ satisfying (6) is customarily said to belong to sector $[K_1, K_2]$. Because such a nonlinear condition is quite general which includes the usual Lipschitz condition as a special case, the systems with sector-bounded nonlinearities have been intensively studied in [25, 26].

Recall that the time derivative of a Wiener process is a white noise [27, 28]. We establish $dw(t) = W(t)dt$, where $w(t)$ is a scalar Wiener process on a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$, which is independent from the Markov chain $\{r(t), t \geq 0\}$. It is further assumed that $w(t)$ and the stochastic variable $\xi(t)$ are mutually independent. Besides, $w(t)$ satisfies

$$E\{dw(t)\} = 0, \qquad E\left\{dw^2(t)\right\} = dt. \tag{7}$$

Hence, the network (1) is rewritten as the following stochastic differential equations:

$$\begin{aligned} dx(t) = \varepsilon^{-1} & \left[- A(r(t))\,x(t) + B(r(t))\,x(t-\tau(t)) \right. \\ & \left. + \xi(t)\,Ef(x(t)) \right] dt \\ & + \varepsilon^{-1} \left[C(r(t))\,x(t) + D(r(t))\,x(t-\tau(t)) \right] dw(t), \\ & x(t) = \phi(t), \quad \forall t \in [-d, 0]. \end{aligned} \tag{8}$$

Remark 3. The stochastic Markovian neural network with different time scales and RONs (8) is general enough to include many models as its special cases. For example, if we do not consider the RONs, time scales and remove the noise perturbations, then (8) is reduced to those studied in [9, 23, 38–41], whereas the authors in [9, 38–41] ignore the Markovian jump parameters. If we do not consider the RONs and time scales, then (8) is discussed in [29] and its references. If we do not take the noise perturbations and time scales into account and let $\text{Prob}\{\xi(t) = 1\} = 1$, $B(r(t)) = 0$, (8) is just the one in [42]. If we do not consider the RONs, the Markovian jump parameters, and the noise perturbations, then (8) is studied in [37]. *Furthermore, if we remove the time scales,* (8) *is the one in [43].* Thus, our model generalizes and improves greatly many previous works and is therefore very significant.

Remark 4. In [44], the delay-partitioning approach was introduced to study the discrete-time recurrent neural networks. As an extension of the approach, we use two different partitions to deal with the continuous Markovian jump neural networks with different time scales in this paper.

Note that it is easy to prove that there exists at least one equilibrium point for (8) by employing the well-known Brouwer's fixed point theorem. To end this section, the lemmas which are necessary for the proof of our main results are introduced as follows.

Lemma 5. *For any constant matrix $R > 0$, any scalars a and b with $a < b$, and a vector function $x(t) : [a, b] \to R^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left[\int_a^b x(s)ds\right]^T R \left[\int_a^b x(s)\,ds\right] \leq (b-a)\int_a^b x^T(s)\,Rx(s)\,ds. \tag{9}$$

Lemma 6 (see [9]). *Let $Z \in R^{n\times n}$ and the bidiagonal upper triangular block matrix*

$$J_K(Z) = \begin{pmatrix} I_n & -Z & & 0 \\ & \ddots & \ddots & \\ & & \ddots & -Z \\ 0 & & & I_n \end{pmatrix} \in R^{K_n \times K_n}. \tag{10}$$

If $Y = (J_K(Z)F) \in R^{K_n \times (K_n+m)}$ with $F = \begin{pmatrix} F_1 \\ \vdots \\ F_k \end{pmatrix} \in R^{K_n\times m}$, $F_j \in R^{n\times m}$ $(j = 1, \ldots, K)$, then

$$Y^{\perp} = \text{col}\left\{-\sum_{j=1}^{K} Z^{j-1}F_j, -\sum_{j=2}^{K} Z^{j-2}F_j, \ldots, -F_K, I_m\right\}. \tag{11}$$

Lemma 7 (Finsler's lemma). *Suppose that $x \in R^n$, $M = M^T \in R^{n\times n}$ and $B \in R^{m\times n}$ such that B has full row rank. Then, the following statements are equivalent:*

(1) *there exists a vector $x \in R^n$ such that $x^T Mx < 0$ and $Bx = 0$,*

(2) *there exists a scalar $\mu \in R$ such that $\mu B^T B - M > 0$,*

(3) *$\exists X \in R^{n\times m}$ such that $M + XB + B^T X^T < 0$,*

(4) *$B^{\perp T} MB^{\perp} < 0$, where $B^{\perp}$ is the orthogonal complement of B.*

Remark 8. It should be pointed out that various problems in control theory have been solved by combing Lyapunov control approach with Finsler's lemma. In lots of applications, Finsler's lemma is referred to Elimination lemma, which devotes to eliminate the redundant variables in matrix inequalities [45].

3. Main Results

In this section, we will give the mean square asymptotic stability for the system (8) in terms of LMI approach. The main results are stated as follows.

Theorem 9. *For given constants d and μ, and two positive integers m and M, the stochastic system* (8) *is mean square asymptotically stable if there exist matrices $P_i > 0$, $Q_k > 0$ $(k = 1, \ldots, m)$, $R > 0$, $X_1 > 0$, $X_2 > 0$ and*

$$W = W^T = \begin{pmatrix} W_{11} & \cdots & W_{1M} \\ \vdots & \ddots & \vdots \\ W_{1M}^T & \cdots & W_{MM} \end{pmatrix} > 0, \tag{12}$$

and positive scalar $l > 0$ such that the following LMI holds:

$$B^{\perp T}\begin{pmatrix}\Xi_1+\Xi_2+\Xi_3 & 0\\ * & \Xi_4\end{pmatrix}B^{\perp}<0, \tag{13}$$

where $B^{\perp} \in R^{(2m+2M+3)n\times(m+M+3)n}$ is all arbitrary but fixed matrix whose columns form a basis of the kernel space of $B_{(m+M)n\times(2m+2M+3)n} = (J_{m+M}(I_n)F)$,

$$F=\begin{pmatrix}0 & 0 & 0 & -I_n & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \ddots & & \vdots\\ 0 & 0 & 0 & & \ddots & \ddots & 0\\ -I_n & 0 & 0 & 0 & \cdots & 0 & -I_n\end{pmatrix}\in R^{(m+M)n\times(m+M+3)n},$$

$$\Xi_1=\begin{pmatrix}\Omega_1 & 0 & \cdots & \varepsilon^{-1}P_iB_i & 0 & W_{1M} & \cdots & W_{12} & \Delta_1\\ * & \Omega_2 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ * & * & \cdots & \Omega_{m+1} & 0 & 0 & \cdots & 0 & 0\\ * & * & \cdots & * & \Gamma_{M+1} & -W_{(M-1)M}^T & \cdots & -W_{1M}^T & 0\\ * & * & \cdots & * & * & \Gamma_M & \cdots & W_{2M}^T-W_{1(M-1)}^T & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ * & * & \cdots & * & * & * & \cdots & \Gamma_2 & 0\\ * & * & \cdots & * & * & * & \cdots & * & \Delta_2\end{pmatrix},$$

$$\Omega_1=-2\varepsilon^{-1}P_iA_i+\sum_{j=1}^{N}q_{ij}P_j+W_{11}+\sum_{i=1}^{m}Q_i-l\widetilde{K}_1, \tag{14}$$

$$\Delta_1=-l\widetilde{K}_2+\bar{\xi}\varepsilon^{-1}P_iE, \qquad \Delta_2=-lI+R+E^T\left(\varepsilon^{-2}\left(\bar{\xi}-\bar{\xi}^2\right)h(d-h)X_1+\varepsilon^{-2}\left(\bar{\xi}-\bar{\xi}^2\right)d\sigma X_2\right)E,$$

$$\Omega_k=\left(\frac{(k-1)\mu}{m-1}\right)Q_{k-1}, \quad (k=2,\ldots,m+1),$$

$$\Gamma_k=W_{kk}-W_{(k-1)(k-1)}, \quad W_{(M+1)(M+1)}=0, \ (k=2,\ldots,M+1),$$

$$\Xi_2=\left(-A_i \ \ 0 \ \cdots \ 0 \ \ B_i \ \ 0 \ \cdots \ 0 \ \ \bar{\xi}E\right)^T\left(\varepsilon^{-2}h(d-h)X_1+\varepsilon^{-2}d\sigma X_2\right)\left(-A_i \ \ 0 \ \cdots \ 0 \ \ B_i \ \ 0 \ \cdots \ 0 \ \ \bar{\xi}E\right),$$

$$\Xi_3=\left(C_i \ \ 0 \ \cdots \ 0 \ \ D_i \ \ 0 \ \cdots \ 0\right)^T\left(\varepsilon^{-2}(d-h)X_1+\varepsilon^{-2}dX_2+\varepsilon^{-2}P_i\right)\left(C_i \ \ 0 \ \cdots \ 0 \ \ D_i \ \ 0 \ \cdots \ 0\right),$$

$$\Xi_4=\operatorname{diag}\left\{(\mu-1)R,-X_2,\ldots,-X_2,-m^{-1}X_2,-X_1,\ldots,-X_1\right\},$$

$$\widetilde{K}_1=\frac{\left(K_1^TK_2+K_2^TK_1\right)}{2}, \qquad \widetilde{K}_2=-\frac{\left(K_1^T+K_2^T\right)}{2}.$$

Proof. For presentation convenience, in the following, we denote

$$y(t)=-A(r(t))x(t)+B(r(t))x(t-d(t))+\xi(t)Ef(x(t)), \tag{15}$$
$$m(t)=C(r(t))x(t)+D(r(t))x(t-d(t)),$$

then, the system (8) becomes

$$dx(t)=y(t)\,dt+m(t)\,dw(t). \tag{16}$$

By employing the idea of delay partitioning approach to the time-delay $d(t)$, we choose the following Lyapunov-Krasovskii functional candidate:

$$V(x(t),t,i)=x^T(t)P_ix(t)+\sum_{k=1}^{m}\int_{t-\alpha_k(t)}^{t}x^T(s)Q_kx(s)\,ds+\int_{t-h}^{t}\gamma^T(s)W\gamma(s)\,ds+\int_{t-d(t)}^{t}f^T(x(s))Rf(x(s))\,ds$$

$$+h\int_{-d}^{-h}\int_{t+\theta}^{t}y^{T}(s)X_1y(s)\,ds\,d\theta$$
$$+\int_{-d}^{-h}\int_{t+\theta}^{t}m^{T}(s)X_1m(s)\,ds\,d\theta$$
$$+\sigma\int_{-d}^{0}\int_{t+\theta}^{t}y^{T}(s)X_2y(s)\,ds\,d\theta$$
$$+\int_{-d}^{0}\int_{t+\theta}^{t}m^{T}(s)X_2m(s)\,ds\,d\theta, \tag{17}$$

where

$$\gamma^{T}(t)=\left(x^{T}(t)\;\;x^{T}(t-h)\;\;\cdots\;\;x^{T}(t-(M-1)h)\right),$$
$$\sigma(t)=\frac{d(t)}{m},\qquad h=\frac{d}{M},\qquad \sigma=\frac{d}{m},$$
$$\alpha_k(t)=k\sigma(t),\quad(k=1,\ldots,m). \tag{18}$$

Let $\mathscr{L}$ be the weak infinitesimal generator of the random process $\{x(t),r(t),t\geq 0\}$ along the trajectory of the system (8). Then, by Itô differential formula, we have

$$\mathscr{L}V(x(t),t,i)$$
$$=2x^{T}(t)P_iy(t)+x^{T}(t)\sum_{j=1}^{N}q_{ij}P_jx(t)+m^{T}(t)P_im(t)$$
$$+\sum_{i=1}^{m}x^{T}(t)Q_ix(t)$$
$$-\sum_{i=1}^{m}\left(1-\dot{\alpha}_i(t)\right)x^{T}\left(t-\alpha_i(t)\right)Q_ix\left(t-\alpha_i(t)\right)$$
$$+\gamma^{T}(t)W\gamma(t)-\gamma^{T}(t-h)W\gamma(t-h)$$
$$+f^{T}(x(t))Rf(x(t))$$
$$-\left(1-\dot{d}(t)\right)f^{T}(x(t-d(t)))Rf(x(t-d(t)))$$
$$+h(d-h)y^{T}(t)X_1y(t)$$
$$-h\sum_{k=2}^{M}\int_{t-kh}^{t-(k-1)h}y^{T}(s)X_1y(s)\,ds$$
$$+(d-h)m^{T}(t)X_1m(t)$$
$$-\sum_{k=2}^{M}\int_{t-kh}^{t-(k-1)h}m^{T}(s)X_1m(s)\,ds$$
$$+\sigma dy^{T}(t)X_2y(t)$$
$$-\sigma\sum_{i=1}^{m}\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)}y^{T}(s)X_2y(s)\,ds$$
$$-\sigma\int_{t-d}^{t-d(t)}y^{T}(s)X_2y(s)\,ds+dm^{T}(t)X_2m(t)$$
$$-\sum_{i=1}^{m}\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)}m^{T}(s)X_2m(s)\,ds$$
$$-\int_{t-d}^{t-d(t)}m^{T}(s)X_2m(s)\,ds. \tag{19}$$

By virtue of (3), (7), and (9), it can be verified that

$$E\{\mathscr{L}V(x(t),t,i)\}$$
$$\leq E\left\{2x^{T}(t)P_iy(t)+x^{T}(t)\sum_{j=1}^{N}q_{ij}P_jx(t)+m^{T}(t)P_im(t)\right.$$
$$-\sum_{i=1}^{m}\left(1-\frac{i}{m}\mu\right)x^{T}\left(t-\alpha_i(t)\right)Q_ix\left(t-\alpha_i(t)\right)$$
$$+\sum_{i=1}^{m}x^{T}(t)Q_ix(t)+\gamma^{T}(t)W\gamma(t)$$
$$-\gamma^{T}(t-h)W\gamma(t-h)+f^{T}(x(t))Rf(x(t))$$
$$-(1-\mu)f^{T}(x(t-d(t)))$$
$$\times Rf(x(t-d(t)))$$
$$+h(d-h)y^{T}(t)X_1y(t)$$
$$+(d-h)m^{T}(t)X_1m(t)$$
$$+\sigma dy^{T}(t)X_2y(t)+dm^{T}(t)X_2m(t)$$
$$-\sum_{k=2}^{M}\left(\int_{t-kh}^{t-(k-1)h}y(s)\,ds\right)^{T}X_1$$
$$\times\left(\int_{t-kh}^{t-(k-1)h}y(s)\,ds\right)$$
$$-\sum_{k=2}^{M}\left(\int_{t-kh}^{t-(k-1)h}m(s)\,d\omega(s)\right)^{T}X_1$$
$$\times\left(\int_{t-kh}^{t-(k-1)h}m(s)\,d\omega(s)\right)$$
$$-\sum_{i=1}^{m}\left(\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)}y(s)\,ds\right)^{T}X_2$$

$$\times\left(\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)} y(s)\,ds\right)$$
$$-\frac{1}{m}\left(\int_{t-d}^{t-d(t)} y(s)\,ds\right)^T X_2$$
$$\times\left(\int_{t-d}^{t-d(t)} y(s)\,ds\right)$$
$$-\sum_{i=1}^{m}\left(\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)} m(s)\,d\omega(s)\right)^T X_2$$
$$\times\left(\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)} m(s)\,d\omega(s)\right)$$
$$-\frac{1}{m}\left(\int_{t-d}^{t-d(t)} m(s)\,d\omega(s)\right)^T X_2$$
$$\times\left(\int_{t-d}^{t-d(t)} m(s)\,d\omega(s)\right)\Bigg\}. \tag{20}$$

On the other hand, note that (6) is equivalent to

$$\begin{pmatrix} x \\ f(x) \end{pmatrix}^T \begin{pmatrix} \widetilde{K}_1 & \widetilde{K}_2 \\ * & I \end{pmatrix} \begin{pmatrix} x \\ f(x) \end{pmatrix} \le 0, \tag{21}$$

which implies

$$\begin{aligned} &-l\left[f^T(x(t))\,f(x(t)) + f^T(x(t))\,\widetilde{K}_2^T x(t)\right.\\ &\quad\left. +\, x^T(t)\,\widetilde{K}_2 f(x(t)) + x^T(t)\,\widetilde{K}_1 x(t)\right] \ge 0, \end{aligned} \tag{22}$$

where l is a positive constant.

Substituting (5), (7), (15), and (22) into (20), we can obtain

$$E\{\mathscr{L}V(x(t),t,i)\}$$
$$\le E\Bigg\{2x^T(t)\,P_i\left(-A_i x(t) + B_i x(t-d(t))\right)$$
$$-\,l x^T(t)\,\widetilde{K}_1 x(t) + 2x^T(t)\left(\varepsilon^{-1}\overline{\xi}P_iE - l\widetilde{K}_2\right)$$
$$\times f(x(t)) + x^T(t)\sum_{j=1}^{N} q_{ij}P_j x(t)$$
$$+\,h^T(t)\,P_i h(t) - \sum_{i=1}^{m}\left(1-\frac{i}{m}\mu\right)x^T$$
$$\times\left(t-\alpha_i(t)\right)Q_i x\left(t-\alpha_i(t)\right)$$
$$+\sum_{i=1}^{m} x^T(t)\,Q_i x(t) + \gamma^T(t)\,W\gamma(t)$$
$$-\,\gamma^T(t-h)\,W\gamma(t-h)$$
$$+\,f^T(x(t))\,(R-lI)\,f(x(t))$$
$$+\,h(d-h)\,y^T(t)\,X_1 y(t) - (1-\mu)\,f^T$$
$$\times\left(x(t-d(t))\right)Rf\left(x(t-d(t))\right)$$
$$+\,(d-h)\,m^T(t)\,X_1 m(t) + \sigma d y^T(t)\,X_2 y(t)$$
$$+\,d m^T(t)\,X_2 m(t)$$
$$-\sum_{k=2}^{M}\left(\int_{t-kh}^{t-(k-1)h} y(s)\,ds\right)^T X_1$$
$$\times\left(\int_{t-kh}^{t-(k-1)h} y(s)\,ds\right)$$
$$-\sum_{k=2}^{M}\left(\int_{t-kh}^{t-(k-1)h} m(s)\,d\omega(s)\right)^T X_1$$
$$\times\left(\int_{t-kh}^{t-(k-1)h} m(s)\,d\omega(s)\right)$$
$$-\sum_{i=1}^{m}\left(\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)} y(s)\,ds\right)^T X_2$$
$$\times\left(\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)} y(s)\,ds\right)$$
$$-\frac{1}{m}\left(\int_{t-d}^{t-d(t)} y(s)\,ds\right)^T X_2$$
$$\times\left(\int_{t-d}^{t-d(t)} y(s)\,ds\right)$$
$$-\sum_{i=1}^{m}\left(\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)} m(s)\,d\omega(s)\right)^T X_2$$
$$\times\left(\int_{t-\alpha_i(t)}^{t-\alpha_{i-1}(t)} m(s)\,d\omega(s)\right)$$
$$-\frac{1}{m}\left(\int_{t-d}^{t-d(t)} m(s)\,d\omega(s)\right)^T X_2$$
$$\times\left(\int_{t-d}^{t-d(t)} m(s)\,d\omega(s)\right)\Bigg\}$$
$$= E\left\{\xi^T(t)\begin{pmatrix} \Xi_1+\Xi_2+\Xi_3 & 0 \\ * & \Xi_4 \end{pmatrix}\xi(t)\right\}, \tag{23}$$

where

$$\xi(t) = \left(\zeta_1^T(t) \quad \zeta_2^T(t) \quad f^T(x(t)) \quad f^T(x(t-d(t))) \quad \zeta^T(t)\right)^T,$$
$$\zeta_1(t) = \left(x^T(t) \quad x^T(t-\alpha_1(t)) \quad \cdots \quad x^T(t-\alpha_m(t))\right)^T,$$
$$\zeta_2(t) = \left(x^T(t-d) \quad x^T(t-(M-1)h) \quad \cdots \quad x^T(t-h)\right)^T,$$
$$\zeta(t) = \left(\delta_1 \quad \cdots \quad \delta_m \quad \int_{t-d}^{t-d(t)} y^T(s)\,ds + \int_{t-d}^{t-d(t)} m^T(s)\,d\omega(s) \quad \theta_{M-1} \quad \theta_1\right)^T,$$
$$\delta_k = \int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} y^T(s)\,ds + \int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} m^T(s)\,d\omega(s), \quad (k=1,\ldots,m),$$
$$\theta_k = \int_{t-(k+1)h}^{t-kh} y^T(s)\,ds + \int_{t-(k+1)h}^{t-kh} m^T(s)\,d\omega(s), \quad (k=1,\ldots,M-1). \tag{24}$$

In addition, it follows from the Newton-Leibniz formula that

$$\begin{aligned}
&x(t) - x(t-\alpha_1(t)) - \int_{t-\alpha_1(t)}^{t} y(s)\,ds - \int_{t-\alpha_1(t)}^{t} m(s)\,d\omega(s) = 0,\\
&\vdots\\
&x(t-\alpha_{m-1}(t)) - x(t-\alpha_m(t)) - \int_{t-\alpha_m(t)}^{t-\alpha_{m-1}(t)} y(s)\,ds - \int_{t-\alpha_m(t)}^{t-\alpha_{m-1}(t)} m(s)\,d\omega(s) = 0,\\
&x(t-\alpha_m(t)) - x(t-d) - \int_{t-d}^{t-\alpha_m(t)} y(s)\,ds - \int_{t-d}^{t-\alpha_m(t)} m(s)\,d\omega(s) = 0,\\
&x(t-d) - x(t-(M-1)h) + \int_{t-d}^{t-(M-1)h} y(s)\,ds + \int_{t-d}^{t-(M-1)h} m(s)\,d\omega(s) = 0,\\
&\vdots\\
&x(t-2h) - x(t-h) + \int_{t-2h}^{t-h} y(s)\,ds + \int_{t-2h}^{t-h} m(s)\,d\omega(s) = 0
\end{aligned} \tag{25}$$

which are equivalent to

$$\begin{aligned}
B\xi(t) &= \left(J_{m+1}(I_n) \quad F\right)\xi(t)\\
&= \begin{pmatrix}
I_n & -I_n & 0 & \cdots & 0 & 0 & 0 & -I_n & 0 & \cdots & 0\\
0 & I_n & -I_n & \cdots & 0 & 0 & 0 & 0 & -I_n & \cdots & 0\\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & I_n & -I_n & 0 & 0 & 0 & 0 & \cdots & -I_n
\end{pmatrix}\xi(t)\\
&= 0.
\end{aligned} \tag{26}$$

The full column rank matrix representation of the right orthogonal complement of B is denoted by $B^{\perp}$, and Lemma 6 offers a computation method with $Z = I_n$, $B^{\perp} = \text{col}\{-\sum_{j=1}^{K} F_j, -\sum_{j=2}^{K} F_j, \ldots, -F_K, I_m\}$. According to Lemma 7, $E\{\mathscr{L}V(x(t),t,i)\}$ is negative as long as

$$\xi^T(t)\left(B^{\perp}B - \begin{pmatrix} \Xi_1 + \Xi_2 + \Xi_3 & 0\\ * & \Xi_4 \end{pmatrix}\right)\xi(t) > 0 \tag{27}$$

holds, which, in other words, can be expressed as (13). Hence, the stochastic system (8) is asymptotical stable in the mean square, and it completes the proof. □

Remark 10. The delay-partitioning projection technique employed in this paper constitutes the major improvement from most existing results in the literature. Firstly, it should be pointed out that such technique is very rational. The reasons are twofold. (1) The properties of subinterval delays may be sharply different in many practical situations. Thus, it is not reasonable to combine them together. (2) When $d(t)$ reaches its upper bound, we do not necessarily have every subinterval delay reaches its maximum at the same time. That is to say, if we use an upper bound to bound the delay $d(t)$ we have to use the sum of the maxima of subinterval delays; however, $d(t)$ does not achieve this maximum value usually. Therefore, by adopting the delay-partitioning projection approach, less conservative conditions can be proposed. Secondly, in [46], the central point of variation of the delay was introduced to study the stability for time-delay systems, which is called the DCP method. As an extension of the method, we divide the delay into several subintervals which permits employing one slightly different function for different subintervals. This treatment makes us utilize more information on the time delay and thus may present better criteria with less conservatism. Thirdly, two different partitions are made to deal with the time-varying delay in this paper, which are different from the approach in [43]. The parameters m and M refer to the number of delay partitioning, and it indicates that the solution can be searched in a wider space, which leads to the reduction of conservatism. Finally, using the similar analysis method of [7], it is easy to verify that the conservatism of the conditions is a nonincreasing function of the number of delay partitions. However, as we all know, the computational complexity will be increased as the partitioning becomes thinner. Therefore, the delay-partitioning projection approach can provide the flexibility that allows us to trade off between complexity and performance of the stability analysis.

We now consider that the time-varying delay is nondifferentiable or the bound of the delay is unknown, which means that the restriction on the delay is removed. For this goal, we modify the Lyapunov-Krasovskii functional as

$$\begin{aligned}\widehat{V}(x(t),t,i) &= x^T(t)P_ix(t) + \int_{t-h}^{t} y^T(s)Wy(s)\,ds \\ &+ \int_{t-d}^{t} f^T(x(s))Rf(x(s))\,ds \\ &+ h\int_{-d}^{-h}\int_{t+\theta}^{t} y^T(s)X_1y(s)\,ds\,d\theta \\ &+ \int_{-d}^{-h}\int_{t+\theta}^{t} m^T(s)X_1m(s)\,ds\,d\theta \\ &+ \sigma\int_{-d}^{0}\int_{t+\theta}^{t} y^T(s)X_2y(s)\,ds\,d\theta \\ &+ \int_{-d}^{0}\int_{t+\theta}^{t} m^T(s)X_2m(s)\,ds\,d\theta.\end{aligned} \tag{28}$$

The proof can then be derived by following a similar line of arguments as that in Theorem 9. Besides, $f(x(t-d(t)))$ in $\xi(t)$ will turn out to be $f(x(t-d))$.

Theorem 11. *For given constant d and two positive integers m and M, the stochastic system (8) is mean square asymptotically stable if there exist matrices $P_i > 0$, $R > 0$, $X_1 > 0$, $X_2 > 0$, and*

$$W = W^T = \begin{pmatrix} W_{11} & \cdots & W_{1M} \\ \vdots & \ddots & \vdots \\ W_{1M}^T & \cdots & W_{MM} \end{pmatrix} > 0, \tag{29}$$

and positive scalar $l > 0$ such that the following LMI holds:

$$B^{\perp T}\begin{pmatrix} \widehat{\Xi}_1 + \Xi_2 + \Xi_3 & 0 \\ * & \widehat{\Xi}_4 \end{pmatrix} B^{\perp} < 0, \tag{30}$$

where

$$\widehat{\Xi}_1 = \begin{pmatrix} \widehat{\Omega}_1 & 0 & \cdots & \varepsilon^{-1}P_iB_i & 0 & W_{1M} & \cdots & W_{12} & \Delta_1 \\ * & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & \cdots & * & \Gamma_{M+1} & -W_{(M-1)M}^T & \cdots & -W_{1M}^T & 0 \\ * & * & \cdots & * & * & \Gamma_M & \cdots & W_{2M}^T - W_{1(M-1)}^T & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & * & * & * & \cdots & \Gamma_2 & 0 \\ * & * & \cdots & * & * & * & \cdots & * & \Delta_2 \end{pmatrix},$$

$$\widehat{\Omega}_1 = -2\varepsilon^{-1}P_iA_i + \sum_{j=1}^{N} q_{ij}P_j + W_{11} - l\widetilde{K}_1, \qquad \widehat{\Xi}_4 = \operatorname{diag}\left\{-R, -X_2, \ldots, -X_2, -m^{-1}X_2, -X_1, \ldots, -X_1\right\}, \tag{31}$$

and $B^{\perp}$, Δ_1, Δ_2, Γ_k, Ξ_2, Ξ_3, $\widetilde{K}_1$, $\widetilde{K}_2$ are the same as in Theorem 9.

Remark 12. Obviously, the delay rate is not specified in Theorem 11, which includes the constant time delay as a special case. Therefore, Theorem 11 is applicable to the special case to derive some results.

Remark 13. In previous work as [47], the authors have investigated the stability of neutral type neural networks with discrete and distributed delays. The reason why we do not take these neural networks into account is to make our idea more lucid and to avoid complicated notations. However, it is not difficult to extend our results to the neutral type neural networks with mixed time-varying delays. The results will appear in our following study.

4. A Numerical Example

One illustrative example is presented to show the effectiveness of the theoretical results. For simplicity, the system (8) with Markovian switching between two modes is taken into consideration. In addition, the state vector of each node is of dimension two. In other words, $N = 2$, $n = 2$. The mode switching is governed by the rate matrix $\begin{pmatrix} -0.4 & 0.4 \\ 0.3 & -0.3 \end{pmatrix}$, and the other parameters are assumed as

$$A_1 = \begin{pmatrix} 3.3 & 0 \\ 0 & 3.4 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 4.4 & 0 \\ 0 & 4.5 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} -0.3 & -0.6 \\ -0.5 & 0.5 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} -2.2 & -0.5 \\ -1.2 & 1.2 \end{pmatrix},$$

$$E=\begin{pmatrix}-1 & 0\\ 0 & -0.1\end{pmatrix},\qquad C_1=\begin{pmatrix}0.1 & 0.2\\ 0.1 & 0.5\end{pmatrix},$$
$$C_2=\begin{pmatrix}0.5 & 1\\ 0.2 & 0.1\end{pmatrix},\qquad D_1=\begin{pmatrix}0.1 & -0.2\\ -0.5 & 0.4\end{pmatrix},$$
$$D_2=\begin{pmatrix}0.2 & -0.5\\ -0.1 & 0.2\end{pmatrix},$$
$$\overline{\xi}=0.56,\qquad \varepsilon=1,\qquad d(t)=0.8+0.5\sin\frac{t\pi}{2}\quad (t\in Z). \tag{32}$$

The sector-bounded nonlinear function $f(x(t))$ is given as follows:

$$f(x(t)) = \frac{1}{2}\begin{pmatrix}\dfrac{0.3(x_1(t)+x_2(t))}{1+x_1^2(t)+x_2^2(t)}+0.1x_1(t)+0.1x_2(t) & 0.3x_1(t)+0.3x_2(t)\end{pmatrix}^T, \tag{33}$$

which can be bounded by

$$K_1=\begin{pmatrix}0.2 & 0.1\\ 0 & 0.2\end{pmatrix},\qquad K_2=\begin{pmatrix}-0.1 & 0\\ -0.1 & 0.1\end{pmatrix}. \tag{34}$$

We choose the parameters $m = 2$ and $M = 2$. It is easy to know $d = 1.3$ and $\mu = 0.5$.

By using Matlab LMI control toolbox, we can find the feasible solution of the LMI (13) as follows:

$$\begin{gathered}
P_1=\begin{pmatrix}1.8392 & -0.8471\\ -0.8471 & 4.5444\end{pmatrix},\\
P_2=\begin{pmatrix}2.0534 & -0.5930\\ -0.5930 & 3.9335\end{pmatrix},\\
Q_1=\begin{pmatrix}0.4422 & -0.3769\\ -0.3769 & 1.8266\end{pmatrix},\\
Q_2=\begin{pmatrix}7.6178 & -2.8997\\ -2.8997 & 11.9753\end{pmatrix},\\
X_1=\begin{pmatrix}0.0594 & -0.1562\\ -0.1562 & 0.4677\end{pmatrix},\\
X_2=\begin{pmatrix}0.0663 & -0.1841\\ -0.1841 & 0.5600\end{pmatrix},\\
R=\begin{pmatrix}0.2487 & -0.0084\\ -0.0084 & 0.5947\end{pmatrix},\\
W_{11}=\begin{pmatrix}1.1837 & -0.4007\\ -0.4007 & 3.6472\end{pmatrix},\\
W_{12}=\begin{pmatrix}0.0111 & -0.1045\\ -0.1045 & 0.4755\end{pmatrix},\\
W_{22}=\begin{pmatrix}0.7856 & -0.4878\\ -0.4878 & 2.4023\end{pmatrix}.
\end{gathered} \tag{35}$$

The simulation result is shown in Figure 1, which implies that all the expected system performance requirements are well achieved.

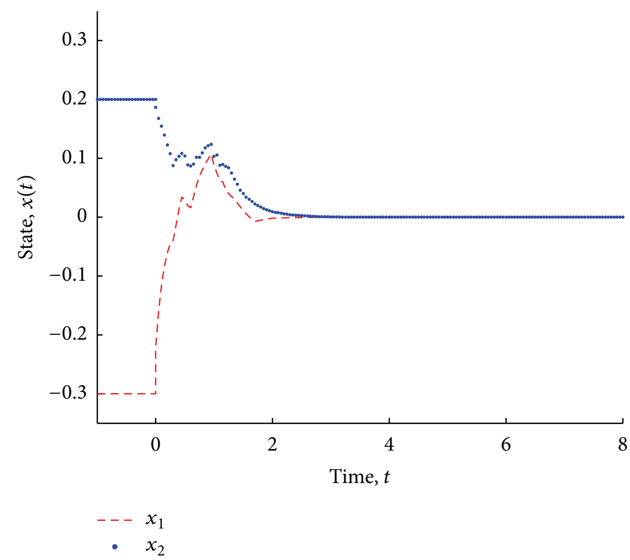


Figure 1: State development of system (8) with parameters (32).

Remark 14. It is worth pointing out that if $\text{Prob}\{\xi(t) = 1\} = 1$, then the function $f(x(t))$ in (8) will be the neuron activation function. According to Assumption 1, $f(x(t))$ satisfies the sector-bounded condition which is more general than the Lipschitz condition. Therefore, the stability criteria in [42] can not apply to our example.

Remark 15. In our example, many factors such as noise perturbations, Markovian jump parameters, RONs, and different time scales are considered and the delay-partitioning projection approach is employed. Therefore, the results reported in [9, 23, 38–41] do not hold in our example. Moreover, our results are expressed by LMIs, which can be easily checked by using the powerful Matlab LMI Toolbox. Thus, our stability criteria are more computationally efficient than those given in [13, 40, 41].

5. Conclusions

In this paper, we have dealt with the mean square asymptotic stability problem for stochastic Markovian jump neural networks with different time scales and RONs. By using new Lyapunov-Krasovskii functionals and delay-partitioning projection approach, the LMI-based criteria have been developed to guarantee stability of such systems. It is also shown that the delay-partitioning projection approach can achieve the aim of reducing conservativeness. An numerical example is exploited to illustrate the effectiveness of the theoretical results.

Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (JUSRP51317B) and the National Natural Science Foundation of China under Grant 10901073 and 11002061.

References

[1] S. Arik, "Global asymptotic stability of a class of dynamical neural networks," *IEEE Transactions on Circuits and Systems I*, vol. 47, no. 4, pp. 568–571, 2000.

[2] X. S. Yang, J. D. Cao, and J. Q. Lu, "Synchronization of delayed complex dynamical networks with impulsive and stochastic effects," *Nonlinear Analysis: Real World Applications*, vol. 12, no. 4, pp. 2252–2266, 2011.

[3] J. D. Cao, G. R. Chen, and P. Li, "Global synchronization in an array of delayed neural networks with hybrid coupling," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 38, no. 2, pp. 488–498, 2008.

[4] J. D. Cao, Z. D. Wang, and Y. H. Sun, "Synchronization in an array of linearly stochastically coupled networks with time delays," *Physica A*, vol. 385, no. 2, pp. 718–728, 2007.

[5] Z. Orman and S. Arik, "An analysis of stability of a class of neutral-type neural networks with discrete time delays," *Abstract and Applied Analysis*, vol. 2013, Article ID 143585, 9 pages, 2013.

[6] Q. Luo, X. J. Miao, Q. Wei, and Z. X. Zhou, "Stability of impulsive neural networks with time-varying and distributed delays," *Abstract and Applied Analysis*, vol. 2013, Article ID 325310, 10 pages, 2013.

[7] X. Y. Meng, J. Lam, B. Z. Du, and H. J. Gao, "A delay-partitioning approach to the stability analysis of discrete-time systems," *Automatica*, vol. 46, no. 3, pp. 610–614, 2010.

[8] O. M. Kwon, M. J. Park, J. H. Park, S. M. Lee, and E. J. Cha, "Improved criteria on delay-dependent stability for discrete-time neural networks with interval time-varying delays," *Abstract and Applied Analysis*, vol. 2012, Article ID 285931, 16 pages, 2012.

[9] B. Du, J. Lam, Z. Shu, and Z. Wang, "A delay-partitioning projection approach to stability analysis of continuous systems with multiple delay components," *IET Control Theory and Applications*, vol. 3, no. 4, pp. 383–390, 2009.

[10] H. B. Zeng, Y. He, M. Wu, and C. Zhang, "Complete delay-decomposing approach to asymptotic stability for neural networks with time-varying delays," *IEEE Transactions on Neural Networks*, vol. 22, no. 5, pp. 806–812, 2011.

[11] B. Z. Du and J. Lam, "Stability analysis of static recurrent neural networks using delay-partitioning and projection," *Neural Networks*, vol. 22, no. 4, pp. 343–347, 2009.

[12] W. M. Chen, Q. Ma, G. Y. Miao, and Y. J. Zhang, "Stability analysis of stochastic neural networks with Markovian jump parameters using delaypartitioning approach," *Neurocomputing*, vol. 103, pp. 22–28, 2013.

[13] L. Wan and J. H. Sun, "Mean square exponential stability of stochastic delayed Hopfield neural networks," *Physics Letters A*, vol. 343, no. 4, pp. 306–318, 2005.

[14] H. Y. Shao and Q. L. Han, "New stability criteria for linear discrete-time systems with interval-like time-varying delays," *IEEE Transactions on Automatic Control*, vol. 56, no. 3, pp. 619–625, 2011.

[15] S. Arik, "Stability analysis of delayed neural networks," *IEEE Transactions on Circuits and Systems I*, vol. 47, no. 7, pp. 1089–1092, 2000.

[16] Y. Chen and W. X. Zheng, "Stochastic state estimation for neural networks with distributed delays and Markovian jump," *Neural Networks*, vol. 25, pp. 14–20, 2012.

[17] S. P. Wen, Z. G. Zeng, and T. W. Huang, "Reliable H filter design for a class of mixed-delay Markovian jump systems with stochastic nonlinearities and multiplicative noises via delay-partitiong method," *International Journal of Control, Automation and Systems*, vol. 10, no. 4, pp. 711–720, 2012.

[18] Y. Kang, W. K. Shang, and H. S. Xi, "Estimating the delay-time for the stability of Markovian jump bilinear systems with saturating actuators," *Acta Automatica Sinica*, vol. 36, no. 5, pp. 762–767, 2010.

[19] L. L. Xiong, J. K. Tian, and X. Z. Liu, "Stability analysis for neutral Markovian jump systems with partially unknown transition probabilities," *Journal of the Franklin Institute*, vol. 349, no. 6, pp. 2193–2214, 2012.

[20] Q. X. Zhu and J. D. Cao, "Stability of Markovian jump neural networks with impulse control and time varying delays," *Nonlinear Analysis: Real World Applications*, vol. 13, no. 5, pp. 2259–2270, 2012.

[21] Q. Ma, S. Y. Xu, Y. Zou, and J. J. Lu, "Stability of stochastic Markovian jump neural networks with mode-dependent delays," *Neurocomputing*, vol. 74, no. 12-13, pp. 2157–2163, 2011.

[22] Q. X. Zhu and J. D. Cao, "Stability analysis of Markovian jump stochastic BAM neural networks with impulse control and mixed time delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 3, pp. 467–479, 2010.

[23] H. Y. Li, Q. Zhou, B. Chen, and H. H. Liu, "Parameter-dependent robust stability for uncertain Markovian jump systems with time delay," *Journal of the Franklin Institute*, vol. 348, no. 4, pp. 738–748, 2011.

[24] Y. F. Guo and Z. J. Wang, "Stability of Markovian jump systems with generally uncertain transition rates," *Nonlinear Dynamics*, vol. 350, pp. 2826–2836, 2013.

[25] Z. D. Wang, Y. Wang, and Y. R. Liu, "Global synchronization for discrete-time stochastic complex networks with randomly occurred nonlinearities and mixed time delays," *IEEE Transactions on Neural Networks*, vol. 21, no. 1, pp. 11–25, 2010.

[26] Y. S. Liu, Z. D. Wang, and W. Wang, "Reliable H_∞ filtering for discrete time-delay systems with randomly occurred nonlinearities via delay-partitioning method," *Signal Processing*, vol. 91, no. 4, pp. 713–727, 2011.

[27] L. Arnold, *Stochastic Differential Equations: Theory and Applications*, Wiley-Interscience, London, UK, 1974.

[28] B. Øksendal, *Stochastic Differential Equations: An Introduction with Applications*, Springer, Amsterdam, The Netherlands, 2006.

[29] C. Wang and Y. Shen, "Delay partitioning approach to robust stability analysis for uncertain stochastic systems with interval time-varying delay," *IET Control Theory and Applications*, vol. 6, no. 7, pp. 875–883, 2012.

[30] J. Yu, K. Zhang, and S. Fei, "Exponential stability criteria for discrete-time recurrent neural networks with time-varying delay," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 1, pp. 207–216, 2010.

[31] Y. He, J. H. She, and M. Wu, "New delay-dependent stability criteria and stabilizing method for neutral systems," *IEEE Transactions on Automatic Control*, vol. 49, no. 12, pp. 2266–2271, 2004.

[32] C. G. Li, L. N. Chen, and K. Aihara, "Stability of genetic networks with SUM regulatory logic: Lur'e system and LMI approach," *IEEE Transactions on Circuits and Systems I*, vol. 53, no. 11, pp. 2451–2458, 2006.

[33] H. Huang and G. Feng, "State estimation of recurrent neural networks with time-varying delay: a novel delay partition approach," *Neurocomputing*, vol. 74, no. 5, pp. 792–796, 2011.

[34] G. F. Song and Z. Wang, "A delay partitioning approach to output feedback control for uncertain discrete time-delay systems with actuator saturation," *Nonlinear Dynamics*, vol. 74, no. 1-2, pp. 189–202, 2013.

[35] Q. T. Gan, "Synchronization of competitive neural networks with diffetent time scales and time-varying delay based on delay partitioning approach," *International Journal of Machine Learning and Cybernetics*, vol. 4, no. 4, pp. 327–337, 2013.

[36] Q. X. Cheng and J. D. Cao, "Global synchronization of complex networks with discrete time delays on time scales," *Discrete Dynamics in Nature and Society*, vol. 2011, Article ID 287670, 19 pages, 2011.

[37] Y. P. Ren and Y. K. Li, "Stability and existence of periodic solutions for cellular neural networks with state dependent delays on time scales," *Discrete Dynamics in Nature and Society*, vol. 2012, Article ID 386706, 14 pages, 2012.

[38] Y. Zhao, H. J. Gao, J. Lam, and B. Z. Du, "Stability and stabilization of delayed T-S fuzzy systems: a delay partitioning approach," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 4, pp. 750–762, 2009.

[39] J. Lam, H. J. Gao, and C. H. Wang, "Stability analysis for continuous systems with two additive time-varying delay components," *Systems and Control Letters*, vol. 56, no. 1, pp. 16–24, 2007.

[40] Q. L. Han and K. Q. Gu, "Stability of linear systems with time-varying delay: a generalized discretized lyapunov functional approach," *Asian Journal of Control*, vol. 3, no. 3, pp. 170–180, 2001.

[41] Q. S. Liu, J. Wang, and J. D. Cao, "A delayed lagrangian network for solving quadratic programming problems with equality constraints," in *Advances in Neural Networks—ISNN 2006*, vol. 3971 of *Lecture Notes in Computer Science*, pp. 369–378, Springer, New York, NY, USA, 2006.

[42] H. Y. Liu, L. Zhao, Z. X. Zhang, and Y. Ou, "Stochastic stability of Markovian jumping Hopfield neural networks with constant and distributed delays," *Neurocomputing*, vol. 72, no. 16–18, pp. 3669–3674, 2009.

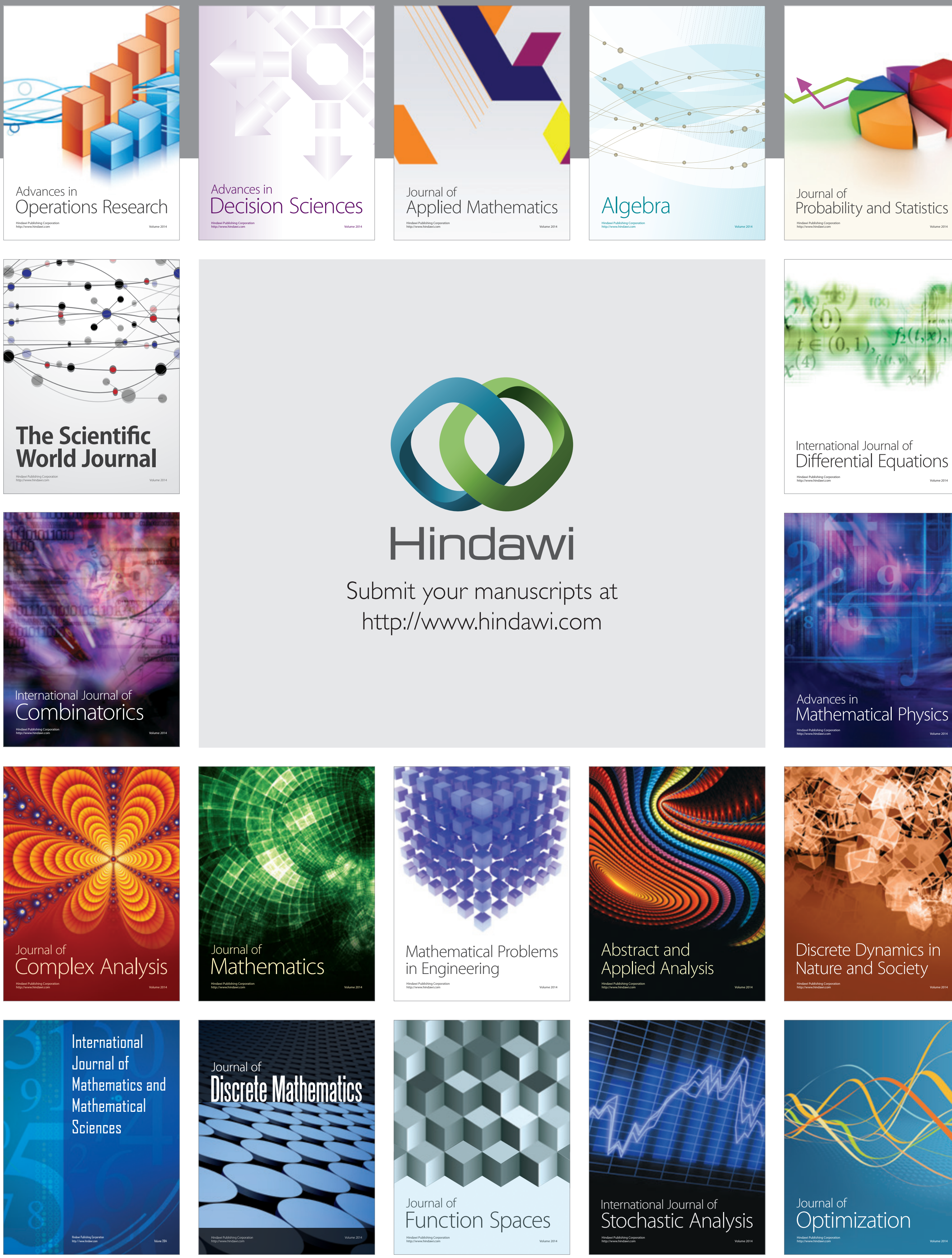
[43] J. M. Duan, M. F. Hu, Y. Q. Yang, and L. X. Liu, "A delay partitioning projection approach to stability analysis of stochastic Markovian jump neural networks with randomly occurred nonlinearities," *Neurocomputing*. In press.

[44] J. M. Duan, M. F. Hu, and Y. Q. Yang, "A delay-partitioning approach to stability analysis of discrete-time recurrent neural networks with randomly occurred nonlinearities," in *Advances in Neural Networks—ISNN 2013*, vol. 7951, pp. 197–204, Springer, New York, NY, USA, 2013.

[45] Y. Q. Xia, M. Y. Fu, and P. Shi, *Analysis and Synthesis of Dynamical Systems with Time-Delays*, vol. 387 of *Lecture Notes in Control and Information Sciences*, Springer, New York, NY, USA, 2009.

[46] C. Peng and Y. C. Tian, "Improved delay-dependent robust stability criteria for uncertain systems with interval time-varying delay," *IET Control Theory and Applications*, vol. 2, no. 9, pp. 752–761, 2008.

[47] S. Lakshmanan, J. H. Park, H. Y. Jung, O. M. Kwon, and R. Rakkiyappan, "A delay partitioning approach to delay-dependent stability analysis for neutral type neural networks with discrete and distributed delays," *Neurocomputing*, vol. 111, pp. 81–89, 2013.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization