Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 860760, 20 pages
doi:10.1155/2012/860760

Research Article

Statistical Analysis and Calculation Model of Flexibility Coefficient of Low- and Medium-Sized Arch Dam

Su Huaizhi,[1,2] You Ting,[2,3] and Wen Zhiping[4]

[1] *State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, No. 1 Xikang Road, Nanjing 210098, China*
[2] *College of Water Conservancy and Hydropower Engineering, Hohai University, No. 1 Xikang Road, Nanjing 210098, China*
[3] *National Engineering Research Center of Water Resources Efficient Utilization and Engineering Safety, No. 1 Xikang Road, Nanjing 210098, China*
[4] *Department of Computer Engineering, Nanjing Institute of Technology, No. 1 Hongjing Avenue, Nanjing 211167, China*

Correspondence should be addressed to Su Huaizhi, su_huaizhi@hhu.edu.cn

Received 8 June 2012; Accepted 20 July 2012

Academic Editor: Fei Kang

The flexibility coefficient is popularly used to implement the macroevaluation of shape, safety, and economy for arch dam. However, the description of flexibility coefficient has not drawn a widely consensus all the time. Based on a large number of relative instance data, the relationship between influencing factor and flexibility coefficient is analyzed by means of partial least-squares regression. The partial least-squares regression equation of flexibility coefficient in certain height range between 30 m and 70 m is established. Regressive precision and equation stability are further investigated. The analytical model of statistical flexibility coefficient is provided. The flexibility coefficient criterion is determined preliminarily to evaluate the shape of low- and medium-sized arch dam. A case study is finally presented to illustrate the potential engineering application. According to the analysis result of partial least-squares regression, it is shown that there is strong relationship between flexibility coefficient and average thickness of dam, thickness-height ratio of crown cantilever, arc height ratio, and dam height, but the effect of rise-span ratio is little relatively. The considered factors in the proposed model are more comprehensive, and the applied scope is clearer than that of the traditional calculation methods. It is more suitable for the analogy analysis in engineering design and the safety evaluation for arch dam.

1. Introduction

As a superior type, arch dam has been extensively used in dam construction. But its design and calculation methods are more complex than that of earth dam and gravity dam. There are the following problems. First of all, to implement the comparative analysis for different design schemes of arch dam, some shape data are lack of reference. Secondly, it is difficult to estimate the earthwork volume index of dam body which is used to determine the dam shape and assess the economy. The problem has an impact on selection of dam site and determination of project scale during engineering preplanning. With the help of flexibility coefficients, macroevaluation of arch dam's shape, security, and economy has recently become important research topic in the field of dam.

Lombardi [1], who is a famous dam expert in Swiss, first proposed the "flexibility coefficient" concept during researching the Kolnbrein dam heel cracking. The calculation equation of flexibility coefficient C was given as follows. $C = A^2/VH$, where A is the developed area of the arch dam in upstream face, m^2; V is the earthwork volume of dam, m^3; H is the arch dam height, m. And the above flexibility coefficient was used to assess the arch dam safety. Lombardi considered that in normal conditions, when the value of C is about 15, the arch is safe; in the higher concrete grouting technology and rational construction, the value of C can be up to 20. After that, many dam experts began to research the calculation models and functions. Many calculation models were built and its application scope got a great expansion. Lombardi damage line was proposed to distinguish empirically the cracking damage of the arch dam [2, 3]. The flexibility coefficient was introduced to estimate the reasonability on structure design of arch dam [4], implement the optimization design of arch dam shape [5], and assess the arch dam safety [6, 7].

On the whole, the existing definition and calculation method on the flexibility coefficient are accuracy and concision. They can embody the flexibility degree of arch dam at the horizontal direction. However, there are some questions to analyze and perfect. For example, the differences of flexibility coefficient between various canyon shapes are great which also have not some certain roles. In the condition of similar shape and height, a large difference in flexibility coefficient will affect engineering analogy analysis and arch shape design. Sometimes safety degree of the arch dam is unconscionable to reflect through Lombardi damage line building by flexibility coefficient.

Based on above problems in existing research, a large number of statistical data on low- and medium-sized arch dams are collected and implemented the regression analysis. The partial least-squares regression method is used to analyze the statistical data of the related factors on flexibility coefficient. The calculation model of flexibility coefficient is built. The statistical flexibility coefficient is proposed.

2. Analysis Method of Partial Least-Squares Regression

As a commonly multivariate statistical analysis method, PLSR (partial least-squares regression) combines the basic functions in multiple linear regression analysis, principal component analysis, and typical correlation analysis. It can be used to solve effectively the multicollinearity between the independent variables. After a partial least-squares regression analysis, the regression model between independent variable and dependent variable can be not only obtained but also the correlation between variables can be analyzed. It makes the analysis more richer and makes the interpretation of the regression model deeper.

(1) Basic Idea

A multiple linear regression model can be described as follows:

$$Y = XB + \varepsilon, \tag{2.1}$$

where Y is dependent-variable vector; X is independent variable matrix; B is regression coefficient vector; ε is residual vector.

The least-square estimation of regression coefficient vector B is

$$B = \left(X'X\right)^{-1}X'Y. \tag{2.2}$$

When multiple correlation is existed in factors belong to X, and $X'X$ is singular matrix or similar to singular, the least-square estimation will become invalid.

Partial least-squares regression extracts the principal component t_1 and u_1 from the X and Y. t_1 and u_1 as much as possible carry variability information from their own data table. At the same time, relevance of t_1 and u_1 reaches to maximum. After extraction, regression is carried out, respectively, through X to t_1 and Y to t_1. If the regression equation is accuracy, the algorithm is terminated; otherwise, the second round of extraction is conducted making use of the residual information that X is explained by t_1, Y by t_1. It is reciprocating until it can reach a satisfactory accuracy.

(2) Simplified Algorithm of Partial Least Squares for Unit-Dependent Variable

Assumed that dependent variable is $y \in R^n$, a set of the dependent variable is $X = [x_1, \ldots, x_p]$, $x_j \in R^n$, and F_0 is standardized variable of dependent variable y, it is found that $F_{0i} = (y_i - \overline{y})/s_y$, $i = 1, 2, \ldots, n$, in which $\overline{y}$ is mean value of y; s_y is a standard deviation of y; E_0 is standard matrix of a dependent-variable set X.

The data of F_0 and E_0 are known, due to the principal component of independent variable $u_1 = F_0$, it is gotten that

$$w_1 = \frac{E_0' F_0}{\left\| E_0' F_0 \right\|}, \qquad t_1 = E_0 w_1, \qquad p_1 = \frac{E_0' t_1}{\|t_1\|^2}, \qquad E_1 = E_0 - t_1 p_1'. \tag{2.3}$$

In the h step $(h = 2, \ldots, m)$, the data of E_{h-1}, F_0 is known, and it is gotten that

$$w_h = \frac{E'_{h-1} F_0}{\left\| E_{h-1}' F_0 \right\|}, \qquad t_h = E_{h-1} w_h, \qquad p_h = \frac{E_{h-1}' t_h}{\|t_h\|^2}, \qquad E_h = E_{h-1} - t_h p_h'. \tag{2.4}$$

At this moment, the m principal component $t_1, t_2, \ldots, t_m$ is obtained, the regression of F_0 on $t_1, t_2, \ldots, t_m$, is implemented, it is gotten that

$$\widehat{F}_0 = r_1 t_1 + \cdots + r_m t_m. \tag{2.5}$$

Considering that $t_1, t_2, \ldots, t_m$ is linear combination of E_0, that is,

$$t_h = E_{h-1} w_h = E_0 w_h^*, \tag{2.6}$$

where $w_h^* = E_0\prod_{j=1}^{h-1}(I - w_j p_j')w_h$, so $\widehat{F}_0$ could be written to linear combination style of E_0. That is,

$$\widehat{F}_0 = r_1 E_0 w_1^* + \cdots + r_m E_0 w_m^* = E_0\left[\sum_{h=1}^{m} r_h w_h^*\right]. \tag{2.7}$$

Finally, it can be converted to regression equation y for $x_1, x_2, \ldots, x_p$

$$\widehat{y} = \alpha_0 + \alpha_1 x_1 + \cdots + \alpha_p x_p. \tag{2.8}$$

(3) Cross-Validation (CV)

All the n sample points are divided into two parts adopting the working style similar to sampling test method. The first part is a set of the rest sample points (containing n-i sample points) removing a sample point i, and a regression equation is fitted by h principal components and these partial sample points; the second part is a sample point i which is substituted into regression equation, and a fitted value of regression equation $y_{h(-i)}$ has gotten. For each $i = 1, 2, \ldots, n$, the above steps are repeated, and the forecasting error square sum for y PRESS_h can be obtained

$$\mathrm{PRESS}_h = \sum_{i=1}^{n}\left(y_i - y_{h(-i)}\right)^2. \tag{2.9}$$

Adopting all sample points, regression model is established fetching h principal components. The i sample point is substituted into regression mode, and then a fitted value of y_{hi} can be obtained. If all of sample points are substituted successively, error square sum SS_h for y is defined

$$\mathrm{SS}_h = \sum_{i=1}^{n}\left(y_i - y_{hi}\right)^2. \tag{2.10}$$

For the principal component t_h, CV is defined as

$$Q_h^2 = 1 - \frac{\mathrm{PRESS}_h}{\mathrm{SS}_{(h-1)}}. \tag{2.11}$$

A great amount of research indicates that when $Q_h^2 \geq 0.0975$, the contribution of the principal component t_h on regression is outstanding, namely, the increase of the principal component t_h is beneficial; otherwise, it should stop introducing the principal component.

(4) Precision Analysis

In the partial least-squares regression, the principal component t_h extracted from independent variable not only represents variability information in X as much as possible but also associates with Y interpreting information in Y. In order to measure the t_h explanatory capacity $i = 1, 2, \ldots, n$, it is defined as the following equations.

The explanatory capacity of t_h to x_j:

$$Rd(x_j; t_h) = r^2(x_j, t_h). \tag{2.12}$$

The explanatory capacity of t_h to X:

$$Rd(X; t_h) = \frac{1}{p} \sum_{j=1}^{p} Rd(x_j; t_h). \tag{2.13}$$

The cumulate explanatory capacity of $t_1, t_2, \ldots, t_m$ to X:

$$Rd(X; t_1, t_2, \ldots, t_m) = \sum_{h=1}^{m} Rd(X; t_h). \tag{2.14}$$

The explanatory capacity of t_h to y:

$$Rd(y; t_h) = r^2(y, t_h). \tag{2.15}$$

The cumulate explanatory capacity of $t_1, t_2, \ldots, t_m$ to y:

$$Rd(y; t_1, t_2, \ldots, t_m) = \sum_{h=1}^{m} Rd(y; t_h). \tag{2.16}$$

(5) The Effect of Independent Variable x_j in the Interpretation of y

In order to analyze the relationship between independent variable X and dependent variable y, and to understand the role of each independent variable in the system analysis, it is needed that explanatory capacity is to be discussed when x_j explains y. This is a question of common interest in the regression analysis.

The explanatory capacity can be measured by variable importance in the projection VIP_j. The definition VIP_j is

$$\mathrm{VIP}_j^2 = \frac{p \sum_{h=1}^{m} Rd(y; t_h) w_{hj}^2}{\sum_{h=1}^{m} Rd(y; t_h)}, \tag{2.17}$$

where w_{hj} is the j component of the axis w_h; p is the number of independent variables.

It can be seen from the partial least-square principle that interpretation of x_j to y is transmitted by t_h. If the explanatory capacity that t_h to y is very capable and x_j plays an important role in the construction of t_h, it is believed to be more power. Accordingly, if a value of w_{hj} in t_h principal component of a larger value of $Rd(y; t_h)$ is larger, it plays a vital role that x_j explains all of y. The definition of VIP_j reflects this idea.

In addition, the square sum p of VIP_j can also be deduced for all factors. Therefore, if the function is similar for p independent variables as interpretation, all of the VIP_j are 1. The greater the value of VIP_j is, the deeper the function of interpretation is.

3. Statistical Calculation Model of Flexibility Coefficient for Arch Dam Based on the Project Cases

3.1. Dependent Variable Selection of Flexibility Coefficient and Project Data

According to the definition and existing research results of flexibility coefficient, dependent variable factor set of flexibility coefficient is selected as follows: $X = \{x_1, x_2, \ldots, x_{12}\}$ = (dam height, temperature drop, concrete volume of the dam, central plane area, average thickness of dam, thickness-height ratio of crown cantilever, arc-height ratio, chord length-height ratio, rise-span ratio, Top boom-bottom chord ratio of downstream face (this is also called valley shape factor), upstream face area of normal water level, dam water thrust of normal water level). The actual project data collected is shown in Figure 1. Data used in this paper are derived from actual dam projects. Data of the actual project cases above have the following characteristics.

(1) Dam height is between 30 m–70 m.

(2) According to the design code of concrete arch dam (SL282-2003), arch dam thickness is divided. The ratio of above arch dams is thin arch dam : medium arch dam : thick arch dam = 38% : 61% : 1%.

(3) According to arch ring type, various types of arch dams above in the ratio are parabolic variable-thickness double-curved arch : double-curvature constant thickness arch dam with single-centered arc : others (such as mixture-type arch dam and circular variable thickness arch dam with five-centered arc) = 68% : 9% : 23%.

(4) Dams of discharging through crest orifice, which account for 66% of the total, are usually used.

3.2. The PLSR Analysis of Flexibility Coefficient

3.2.1. Establishment of PLSR Equation for Flexibility Coefficient

According to (2.9)–(2.11), the principal component cross-validation of data is

$$\left\{Q_1^2, Q_2^2, Q_3^2, Q_4^2, Q_5^2\right\} = \{0.3581, 0.1965, 0.1496, 0.2161, -0.0802\}. \tag{3.1}$$

According to cross validation principle, when $Q_h^2 \geq 0.0975$, the contribution of principal component t_n to regression equation is significant. Then introducing t_n and the first four principal components are necessary.

Table 1: The explanatory capacity of main components to dependent variables and independent variables.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
t_1	0.631	0.223	0.308	0.226	0.400	0.001	0.181
t_2	0.258	0.029	0.546	0.729	0.248	0.012	0.230
t_3	0.038	0.004	0.117	0.022	0.303	0.869	0.447
t_4	0.012	0.165	0.003	0.000	0.005	0.032	0.103
	x_8	x_9	x_{10}	x_{11}	x_{12}	y	
t_1	0.156	0.061	0.190	0.191	0.276	0.488	
t_2	0.230	0.000	0.058	0.767	0.677	0.204	
t_3	0.441	0.003	0.011	0.011	0.000	0.119	
t_4	0.152	0.293	0.138	0.000	0.004	0.066	

Through calculating and analyzing, the PLSR equations of standardized data and raw data are (3.2) and (3.3), respectively,

$$\begin{aligned}\widehat{F}_0 = & -8.307\times10^{-2}E_{01}+2.874\times10^{-2}E_{02}-0.168E_{03}+0.260E_{04}-0.753E_{05}\\ & -0.507E_{06}+0.378E_{07}+0.416E_{08}-0.186E_{09}+0.220E_{10}+0.280E_{11}+0.178E_{12},\end{aligned} \tag{3.2}$$

$$\begin{aligned}\widehat{y} = {} & 21.770-3.610\times10^{-2}x_1+0.102x_2-4.362\times10^{-5}x_3+6.664\times10^{-4}x_4-1.872x_5\\ & -46.043x_6+2.186x_7+2.761x_8-6.863x_9+7.548x_{10}+7.574\times10^{-4}x_{11}+1.900\times10^{-5}x_{12}.\end{aligned} \tag{3.3}$$

Multiple correlation coefficients R and F of raw data PLSR equation are 0.936 and 21.384, respectively.

3.2.2. *The Precision Analysis*

According to (2.12)–(2.16), the explanatory capacity and the accumulative explanatory capacity of main components to dependent variables and independent variables are calculated. The results are shown in Tables 1 and 2.

(1) As can be seen from Table 1, explanatory capacity of each component to independent variables is the capacity that how many variation information can be used in the analysis process. Sometimes the capacity is little, even nothing. This is mainly because that (1) the PLSR requires the covariance between main components and dependent variables be maximum. However, when it is maximal, explaining capacity of some main components to independent variables is low. (2) The contribution of certain independent variables to some main components is little or nothing.

(2) As can be seen from Table 2, the total explaining capacity of dependent variable is 87.7% and that of independent variable is 81.7%.

(3) The explaining capacity of t_1 to variation information in y is 48.8%, and the linear correlation coefficient between them is 0.7. There is a good linear correlation.

Based on the above analysis, the data have relatively good linear trend and the PLSR equation has high precision. They can well reflect the average law between X and y.

Table 2: The accumulative explanatory capacity of main components to dependent variables and independent variables.

	t_1	t_2	t_3	t_4
The explanatory capacity on X	0.237	0.315	0.189	0.076
The accumulative explanatory capacity on X	0.237	0.552	0.741	0.817
The accumulative explanatory capacity on Y	0.488	0.692	0.811	0.877

3.2.3. The Explanatory Role Analysis of Independent Variable to Dependent Variable

According to (2.17), variable importance in the projection VIP can be calculated, and the histogram can be drawn in Figure 2. From Figure 2, it shows the following.

(1) The VIP values of x_5, x_6, x_7, x_8 are greater than 1 and that of x_1 is near 1. From the explanatory capability of independent variable x_j to dependent variable y, it can be known that all the VIP_j is 1 when the explanatory role of them to y is the same aiming at p independent variables. When the VIP_j value is bigger than 1, the capability of explaining y is bigger. It can be seen that these five factors (average thickness of the dam, thickness-height ratio, arc length-height ratio, chord length-height ratio, dam height) are significant in explaining y.

(2) The VIP value of x_9 is smallest. It shows that the explanatory capability of x_9 to y is weakest.

From the dam structure, clearly, the average dam thickness and dam height have a great influence on the flexibility; the thickness-height ratio and the arc length-height ratio of crown cantilever, which reflect the thickness of arch dam and valley shape, have a major impact on the shape of arch dam, then affect the flexibility coefficient. The ratio of arc and chord of the dam crest is x_9, which just reflects the bending degree of horizontal arch of dam crest and has a limited impact on the overall dam.

3.2.4. The Evaluation of Regression Equation Stability

According to the complexity of flexibility coefficient factors and the requirement of sample data, the method of stability in this paper is that after extracting a certain amount of the date, build the model by remaining data, make a coefficient compared, and judge stability of the equation. The specific implementation is to remove five sample points by three times and build the model with the remaining data.

After removing the extracted sample points and judging the main ingredients number of remaining data, the PLSR model can be made. In order to compare easily, the coefficient, result of regression model of the standardized data, can be used to be compared. The calculated specific factors are shown in Table 3. From the table it can be seen that the change of coefficient is within 5% excepting x_1, x_2 (dam height factor and temperature drop factor) by means of comparing each sampling factors and original factors. Stability of the whole coefficient is relatively good.

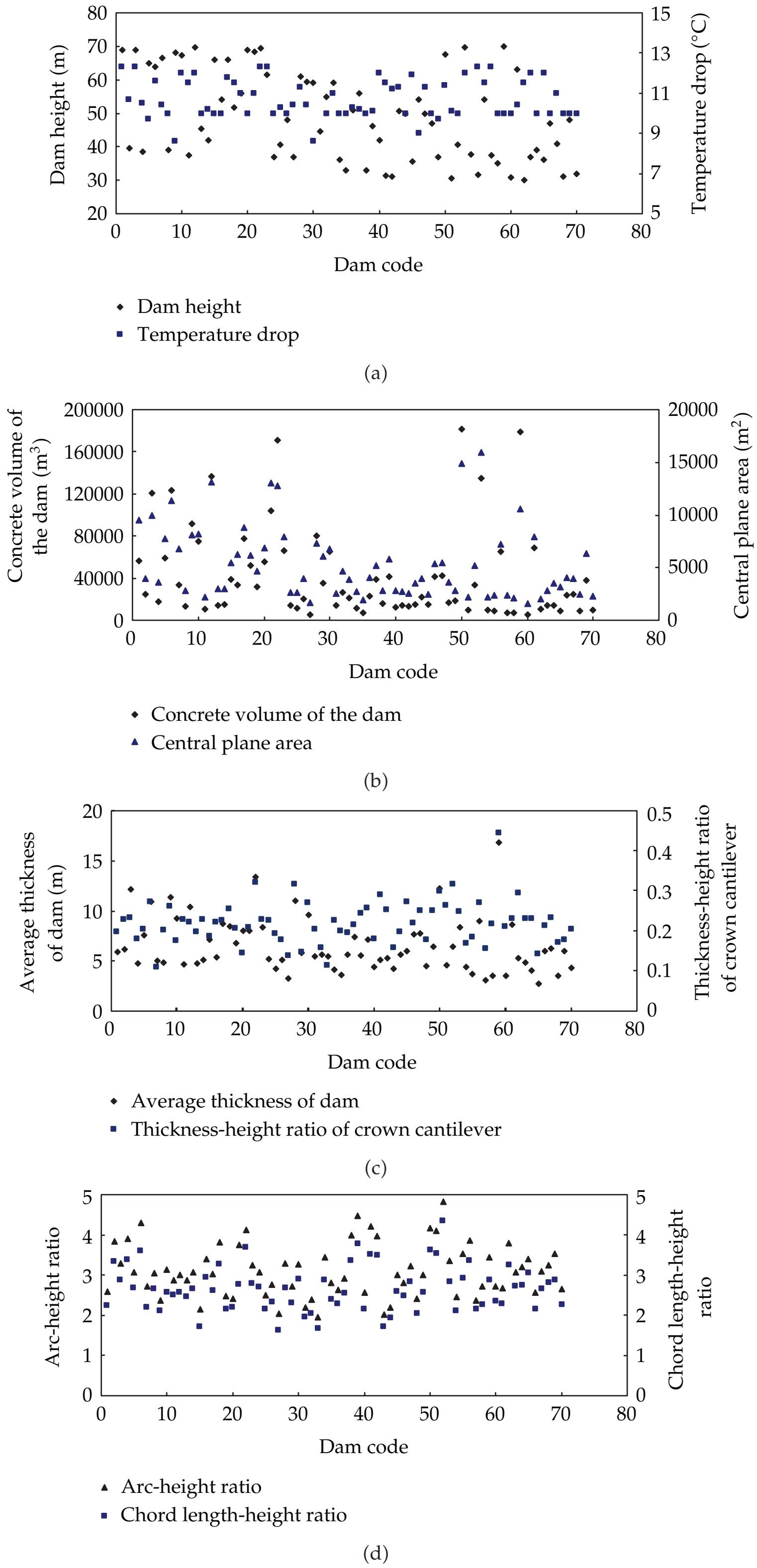


Figure 1: Continued.

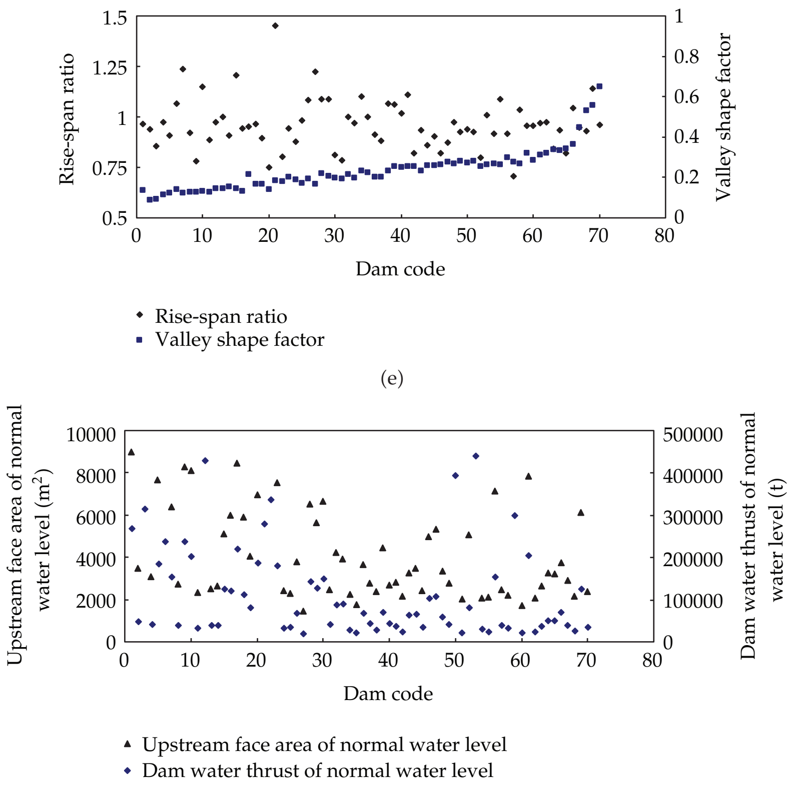


Figure 1: The actual project data of dependent variables for flexibility coefficient.

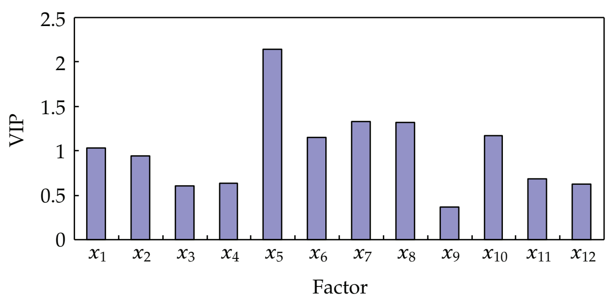


Figure 2: Variable importance in the projection.

3.2.5. The Statistical Calculation Model of Flexibility Coefficient

The above regression analysis is implemented to obtain the PLSR equation of flexibility coefficient in certain dam height range between 30 m and 70 m. From stability and regression

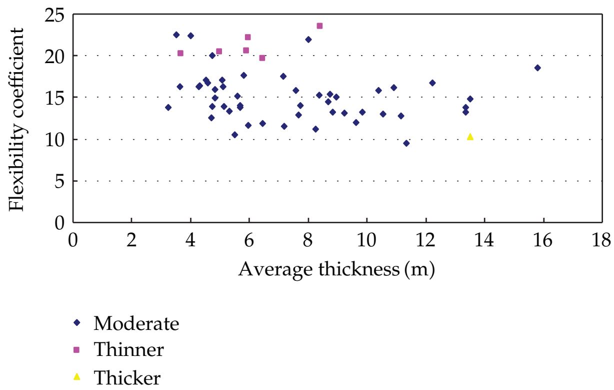


Figure 3: Scatter diagrams of flexibility coefficient and average thickness.

Table 3: The coefficients of regression equation based on original data and extracted sample data.

Variable	Group			
	Original coefficient	Coefficient of sample 1	Coefficient of sample 2	Coefficient of sample 3
x_1	−0.0831	−0.0887	−0.0976	−0.1084
x_2	0.0287	0.0352	0.0212	0.0271
x_3	−0.1681	−0.1656	−0.1690	−0.1518
x_4	0.2603	0.2560	0.2503	0.2426
x_5	−0.7530	−0.7343	−0.7346	−0.7157
x_6	−0.5071	−0.5159	−0.5185	−0.4716
x_7	0.3780	0.3668	0.3704	0.3498
x_8	0.4155	0.4030	0.4067	0.3851
x_9	−0.1859	−0.2082	−0.1933	−0.1900
x_{10}	0.2196	0.2250	0.2043	0.2144
x_{11}	0.2799	0.2771	0.2730	0.2672
x_{12}	0.1776	0.1726	0.1684	0.1730

accuracy, it can be seen that the PLSR equation is rational. Accordingly, calculation model of the flexibility coefficient C is proposed

$$\begin{aligned} C = {} & 21.770 - 3.610 \times 10^{-2} x_1 + 0.102 x_2 - 4.362 \times 10^{-5} x_3 + 6.664 \times 10^{-4} x_4 - 1.872 x_5 \\ & - 46.043 x_6 + 2.186 x_7 + 2.761 x_8 - 6.863 x_9 + 7.548 x_{10} + 7.574 \times 10^{-4} x_{11} + 1.900 \times 10^{-5} x_{12}. \end{aligned} \tag{3.4}$$

4. Examples

From the analysis of interpretation, it can be seen that explaining function of the average thickness of dam to the flexibility coefficient is strongest. According to [8], the scatter diagrams of flexibility coefficient and average thickness of dam are shown in Figure 3. It shows that flexibility coefficient of sample points, whose thickness for dam body is moderate, is almost distributed from 10 to 20. Therefore, this preliminary view is that the dam thickness

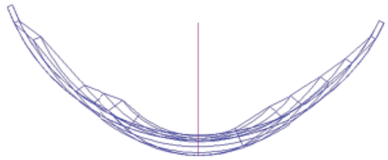


Figure 4: Arch dam plan.

Table 4: The calculated results of statistical factors for one arch dam.

x_1	x_2	x_3	x_4	x_5	x_6
34	12.1	13768	3824.11	3.6	0.206
x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
5.129	3.965	1.336	0.263	3607.94	48453.13

is moderate, whose flexibility coefficient is from 10 to 20. When it is more than 20, the dam is thinner; when less than 20, the dam is thicker.

Based on the above evaluation criteria and the calculation model (3.4), the structural safety of an arch dam project is studied. With the help of the stress analysis results of this arch dam, the feasibility and reliability of the calculation model of flexibility coefficient and its criterion is verified.

4.1. Project Introduction

The arch dam project began in 1974 and basically completed in 1979. The dam is a concrete double-curvature masonry arch dam, whose total storage capacity is 120.5 ten thousand m^3, crest elevation is 121.0 m, bottom elevation is 86.0 m, the maximum dam height is 35 m, thickness of the dam crest is 2 m, thickness of the dam bottom is 7 m, thickness-height ratio is 0.2, chord length of dam crest is 128.2 m, width-height ratio is 3.66, central angle of dam crest is 120°, and central angle of dam bottom is 60°. At the corresponding dam height of 4 m, setting a horizontal fracture, cutting beam-based, and using bridge-type rubber to stop water are to be done. The spill way whose net width is 30 m is arranged on the dam crest. Curved form of free jump is used to overflow.

4.2. Safety Evaluation of Arch Dam Structure Based on the Proposed Model and Criteria

The shape calculation of above arch dam is implemented. The results are shown in Table 4.

The values of factors in Table 4 are substituted into calculation model of flexibility coefficient (see (3.4)). The flexibility coefficient of the arch dam is 26.13, larger than 20. Therefore, the dam is deemed to be relatively thin and the structure safety is lower.

4.3. Safety Check for Arch Dam Based on the Calculation Results of Dam Stress

Dam stress is calculated and analyzed to check for the rationality of the above results.

Table 5: Characteristic elevation and water level (m).

Maximum dam height	35.0	Check water level	121.2
Crest elevation	121.0	Normal water level	119.5
Bottom elevation	86.0	Lowest operating water level	90.2
Sediment elevation	95	Design water level	120.95

Table 6: Physical and mechanical parameters of dam body and foundation.

	Dam foundation	Dam body
Deformation modulus	7.02×10^6 MPa	10^7 MPa
Poisson's ratio	0.28	0.225
Density	$2.4\,\mathrm{t/m^3}$	$2.3\,\mathrm{t/m^3}$
Concrete thermal diffusivity	/	$3.0\,\mathrm{m^2/month}$
Coefficient of thermal expansion	/	0.000008/°C

4.3.1. Calculation Conditions

(1) *Arch Outline*

The arch ring is a circular and single-centered ring with constant thickness. The dam is divided into 6 arches and 13 beams. Three of arches are located in the river bed. Analysis planar graph can be seen from Figure 4.

(2) Characteristic elevation and water level are shown in Table 5.

(3) Physical and mechanical parameters are given in Table 6.

(4) *Temperature Parameters*

The temperature considering perennial mean temperature and sunshine effects is 16°C; the temperature considering annual temperature amplitude (temperature rise) and sunshine effects is 11.7°C; the temperature considering surface temperature of reservoir water and sunshine effects is 16°C; the temperature considering surface temperature of reservoir water (temperature drop) and sunshine effects is 11°C; the temperature considering surface temperature of reservoir water (temperature rise) and sunshine effects is 10.6°C; the water temperature of reservoir bottom is 11°C.

(5) *Operating Conditions*

The following six operational conditions are selected.

Case 1. Normal water level, sediment, and dead weight.

Case 2. Check water level, sediment, dead weight, and temperature rise.

Case 3. Lowest operating water level, sediment, dead weight, and temperature drop.

Case 4. Lowest operating water level, sediment, dead weight, and temperature drop.

Case 5. Normal water level, sediment, dead weight, and temperature rise.

Case 6. Design water level, sediment, dead weight, and temperature rise.

4.3.2. Calculation Results

(1) Maximum Dam Surface Stress

Maximum dam surface stress of every condition can be seen in Table 7.

(2) Contour Map of Principal Tensile Stress on the Dam Surface

For low and medium arch dam dominated by tensile stress, contour map of principal tensile stress is only given (see Figure 5).

4.3.3. Stress Analysis

The following can be seen from stress distributions.

(1) For the first, second, and sixth conditions, the upstream tensile stress exceeds the allowed value. The stress value is largest at the first condition, which is at the normal water level, and is distributed approximately in the whole river bed of the upstream dam bottom. The downstream tensile stresses of three conditions meet the standard value.

(2) For the third and fourth conditions, the upstream tensile stresses meet the standard value. But the downstream tensile stresses at the middle-lower part of the abutment exceed the standard value. The maximum tensile stress occurred at the fourth condition, up to 3.11 MPa, which is far more than the norms.

(3) For the fifth condition, the upstream tensile stress at the bottom of dam exceeds the standard value, and the downstream tensile stress at the abutment exceeds the standard value.

In addition, the maximum radial displacement to the downstream is 17.2 mm, to the upstream is 8.47 mm, which is a little high for arch dams whose heights are 30 m–35 m. That shows that the overall stiffness of the arch dam is not high and the ability of deformation resistance is finite.

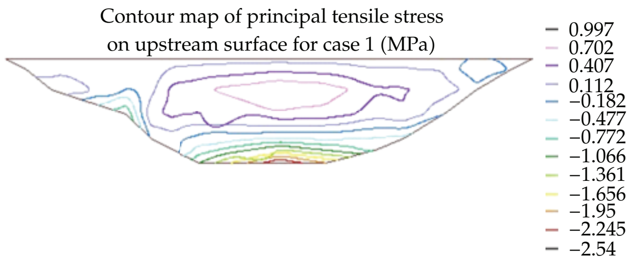
From the above analysis, they can be seen that stress distributions of this arch dam are not good, and the radial displacement is relatively large. It is reason that there are unreasonable shape design, relatively small average thickness of arch dam, and relatively small overall stiffness. They also have been proved from comparisons of arch dams that are the same height range to the case.

4.4. Results Comparison

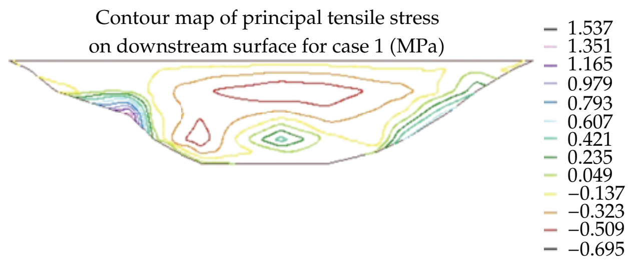
From the stress analysis, it can be known that the dam is relatively thin and lower structure safety keeping in step with the introduced model. In addition, during dam safety evaluation, experts also think that the dam is a little thin and has a limited overload capacity, which showed that it is feasible to evaluate arch dam safety with the introduced model once more.

Table 7: Maximum dam surface stress of every condition (MPa).

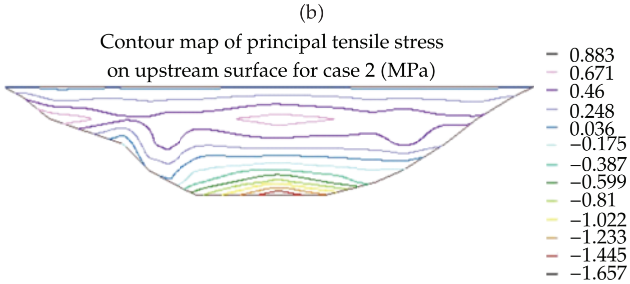
Case	Upstream surface				Downstream surface			
	Maximum principal tensile stress	Position	Maximum principal pressure stress	Position	Maximum principal pressure stress	Position	Maximum principal pressure stress	Position
Case 1	2.54	7R 0C	1.88	3R 0C	0.69	3R 0C	3.22	7R 0C
Case 2	1.66	7R 0C	2.00	1R-7C	0.74	3R-5C	2.49	7R 0C
Case 3	0.66	1R-7C	2.24	6R-2C	1.42	6R-2C	1.09	7R 0C
Case 4	0.42	4R 0C	3.70	6R-2C	3.11	6R-2C	1.28	1R-7C
Case 5	1.34	7R 0C	1.77	1R-7C	1.16	3R-5C	2.19	7R 0C
Case 6	1.60	7R 0C	1.96	1R-7C	0.82	3R-5C	2.44	7R 0C



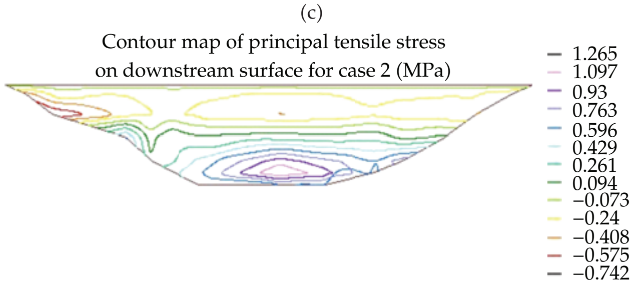
(a)



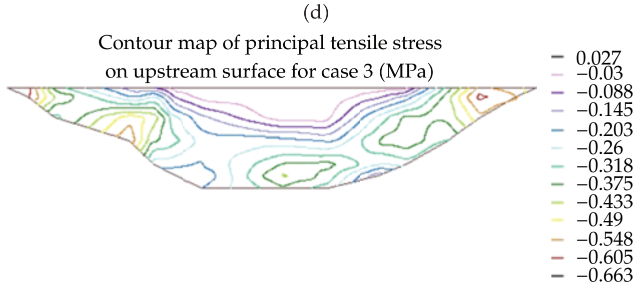
(b)



(c)

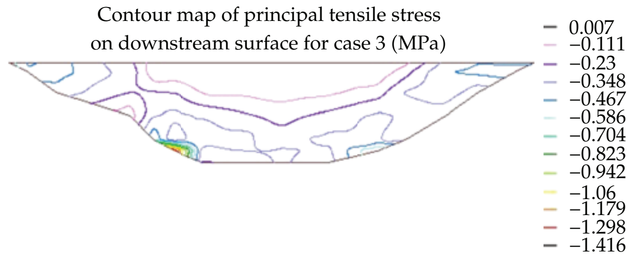


(d)

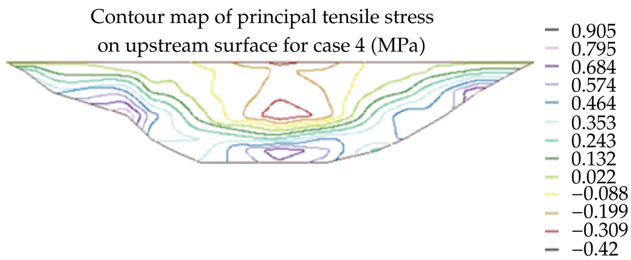


(e)

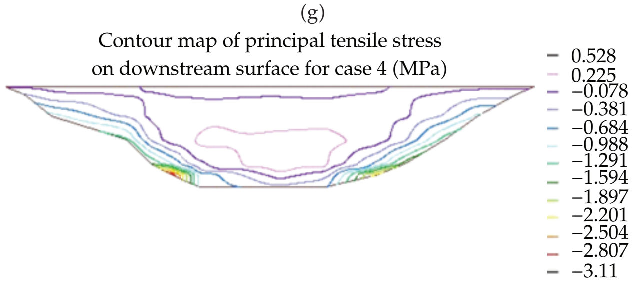
Figure 5: Continued.



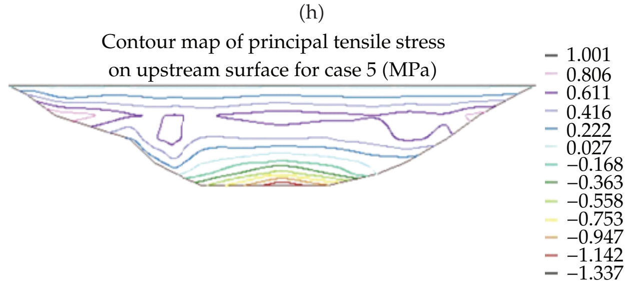
(f)



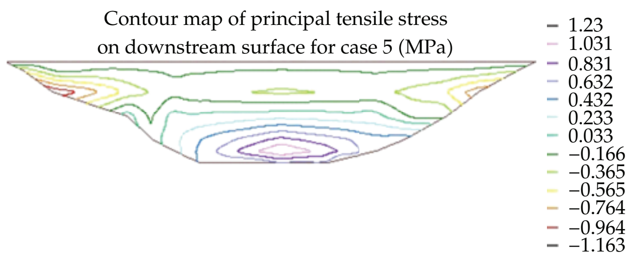
(g)



(h)

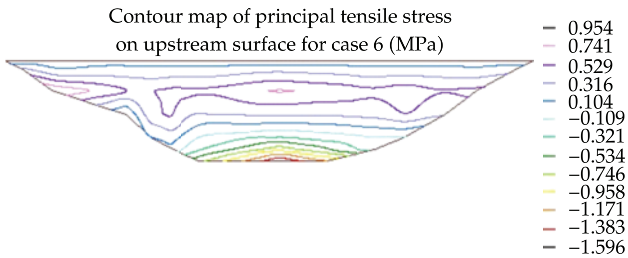


(i)

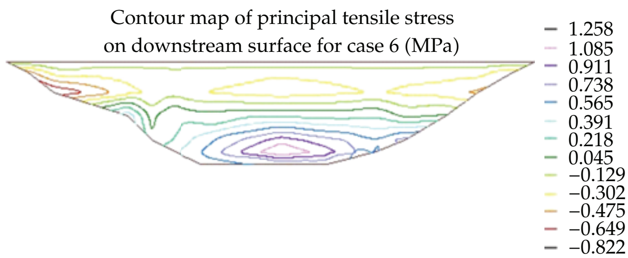


(j)

Figure 5: Continued.



(k)



(l)

Figure 5: Contour map of principal tensile stress on the dam surface.

5. Conclusions

In recent years, flexibility coefficient, which is an objective index, is put forward to deal with problems, such as much subjective evaluation to the body safety of arch dam and lack of criteria of determining shape parameters of shape design. Flexibility coefficient has a unique advantage on the macroevaluation of arch dam shape, safety, and economy. According to large numbers of projects data, statistic rules of flexibility coefficient of arch dam are studied from the perspective of regression. The regressive equation of flexibility coefficient in certain height range, which is based on partial least-squares method, is established. Further, regressive precision and equation stability is analyzed deeply. And the calculation model of statistical flexibility coefficient is presented. A case application shows that the model has certain application value.

(1) After analyzing explanatory capacity of factors to dependent variable, the result shows that average thickness of dam, thickness-height ratio of crown cantilever, arc-height ratio, and dam height have the higher explanation ability than others. The relation between them should be focused mainly when calculating flexibility coefficient.

(2) Compared to traditional methods calculate of flexibility coefficient, the model in this paper has a comprehensive consideration, such as the valley shape coefficient that reflects the valley shape, thickness-height ratio that reflects arch dam thickness and thinness, and temperature-lowering load that has an important influence to arch dam stress, and the force of water and areas of upstream face in normal water level in the dam working. There is a wider application in calculating dam volume inversely. Traditional models do not distinguish dam height. But the rationality is worth to be discussed. It is because that the low- and medium-sized and the

high-sized arch dam have different stress conditions and methods. While statistical flexibility coefficient presented by this paper has a clear operating range, it is more suitable to analogy analysis and study on variation of flexibility coefficient.

(3) Because the designing method of arch dam is more complex than gravity dam and earth dam. For specific valley conditions, project quantity can not be estimated quickly, which bring many difficulties to choose dam site and determine engineer scales during the preliminary planning. Now, we can apply calculation model introduced by this paper to select the specific flexibility coefficient. In coupled with the fitting values of other factors, the volume of dam body is inversely calculated. Choosing dam site and determine engineer scales preliminarily also provides certain references to shape data estimation.

(4) For the arch dam shape which is designing and optimizing, after calculating shape data and values of corresponding factors, its flexibility coefficient can be gotten by the introduced calculation model of flexibility coefficient. Then, considering dam volume and dam safety, its reasonable shape can be chosen based on the flexibility coefficient.

(5) The calculation model of statistical flexibility coefficient, which is based on PLSR, not only provides the more reasonable method to ascertain flexibility coefficient but also accomplishes some study related to principle component regression (PCR) and canonical correlation analysis (CCR). It can supply the better regressive equation that contains rich and deep data information. Moreover, it is studied that quadratic term and cubic term of related factors of flexibility coefficient affect the regressive equation on the basis of linear analysis. The result shows nonlinear parts of added factors have an unapparent influence to improve the precision of regressive analysis.

Acknowledgments

This research has been partially supported by National Natural Science Foundation of China (SN: 51179066, 51139001), Jiangsu Natural Science Foundation (SN: BK2012036), the Program for New Century Excellent Talents in University (SN: NCET-10-0359), the Special Fund of State Key Laboratory of China (SN: 2009586912), the Fundamental Research Funds for the Central Universities (Grant No. 2012B06614), Jiangsu Province “333 High-Level Personnel Training Project” (Grant No. BRA2011179) and the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD) (SN: YS11001).

References

[1] G. Lombardi, “Koelnbrein dam: an unusual solution for an unusual problem,” *International Water Power and Dam Construction*, vol. 43, no. 6, pp. 31–34, 1991.
[2] Q. W. Ren and B. L. Wang, “Discussion on slenderness coefficient of arch dams,” *Journal of Hohai University (Natural Sciences)*, vol. 31, no. 1, pp. 1–4, 2003.
[3] F. W. Xu and B. Tian, “Investigation on flexibility coefficient of arch dam,” *Yangtze River*, vol. 38, no. 11, pp. 41–42, 2007.
[4] H. L. Rao, “Arch dam configuration parameters statistic analysis,” *Design of Hydroelectric Power Station*, vol. 20, no. 3, article 22, pp. 1–5, 2004.

[5] Y. Jiang, X. T. Ding, and C. S. Gu, "Buckling analysis and modification of stress control index for super high and thin arc dam body," *Journal of Hohai University (Natural Sciences)*, vol. 36, no. 1, pp. 54–57, 2008.
[6] B. F. Zhu, "Stress level coefficient and safety level coefficient for arch dams," *Dam Engineering*, vol. 11, no. 3, pp. 133–142, 2000.
[7] H. Su, Z. Wu, Y. Gu, J. Hu, and Z. Wen, "Game model of safety monitoring for arch dam deformation," *Science in China E*, vol. 51, no. 2, pp. 76–81, 2008.
[8] Y. C. Ma and S. C. Lou, "Discussion on flexibility and volume of arch dams," *Dam and Safety*, no. 6, pp. 1–3, 2005.