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Research Article

Phase Velocity Estimation of a Microstrip Line in a Stoichiometric Periodically Domain-Inverted $LiTaO_3$ Modulator Using Electro-Optic Sampling Technique

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We estimate the phase velocity of a modulation microwave in a quasi-velocity-matched (QVM) electro-optic (EO) phase modulator (QVM-EOM) using EO sampling which is accurate and the most reliable technique for measuring voltage waveforms at an electrode. The substrate of the measured QVM-EOM is a stoichiometric periodically domain-inverted $LiTaO_3$ crystal. The electric field of a standing wave in a resonant microstrip line (width: 0.5 mm, height: 0.5 mm) is measured by employing a CdTe crystal as an EO sensor. The wavelength of the traveling microwave at 16.0801 GHz is determined as 3.33 mm by fitting the theoretical curve to the measured electric field distribution. The phase velocity is estimated as $v_m = 5.35 \times 10^7$ m/s, though there exists about 5% systematic error due to the perturbation by the EO sensor. Relative dielectric constant of $\epsilon_r = 41.5$ is led as the maximum likelihood value that derives the estimated phase velocity.

1. Introduction

Wide expansion of an optical spectrum by a deep phase modulation at a high-modulation frequency is the essential technique for many electro-optic (EO) light controlling fields, such as ultrashort pulse generation [1], comb generation [2], time-to-space mapping of an optical signal [3], and so on.

Quasi-velocity-matching (QVM) with periodic domain inversion of an $LiTaO_3$ (LT) or $LiNbO_3$ (LN) crystal in a traveling-wave EOM is one of the promising techniques to widely spread the optical spectrum. Especially, periodically domain-inverted stoichiometric LT (SLT) crystals are attractive QVM-EOMs because of their stronger EO effect and lower coercive fields than congruent-melt LT crystals. On the basis of the QVM technique, Morimoto et al. spread a 3-THz-wide modulation sidebands (comb) at a 16.25-GHz modulation frequency [4]. The QVM technique compensates for velocity mismatching between an optical group velocity v_g and a microwave phase velocity v_m in the EO crystal by periodically inverting the sign of the EO coefficient of the crystal. In the ideal QVM-EOM, the modulation index is almost proportional to the interaction length and as a result, large modulation index at the high modulation frequency can be achieved with long device.

Accurate values of v_g and v_m are needed in designing the QVM-EOM, because uncertainty of these values results in degradation of the modulation efficiency, which is achievable modulation index per unit interaction length per unit modulation electric field strength. The group velocity of the optical wave can be calculated with group refractive index $n_g(\lambda)$ of the EO crystal. Likewise, the phase velocity of the modulation microwave is calculated as $v_m = c/\sqrt{\epsilon_{\mathrm{eff}}}$, where ϵ_{eff} is the effective relative dielectric constant which is determined by the geometry of the transmission line and the relative dielectric constant ϵ_r of the substrate. The group refractive index of SLT crystals is estimated accurately using Sellmeier equation with constants listed in [5]. In contrast with the accurate value of $n_g(\lambda)$, very little data of ϵ_r for

LT crystal are available, which is in the range $38.9 \leq \epsilon_r \leq 44.5$ at microwave frequencies [6, 7]. To our knowledge, no accurate data of ϵ_r for SLT crystals at microwave frequencies are available.

In this paper, we estimate the phase velocity of the modulation microwave in SLT-based QVM-EOM on the basis of a direct observation of an amplitude of a standing-wave in a resonant microstrip line using EO sampling technique [8]. Because the EO sampling is accurate and most reliable technique for measuring voltage waveforms at an electrode, the phase velocity can be estimated accurately from the relation of $v_m = \lambda_m f_m$, where λ_m is the wavelength of the traveling microwave and f_m is the resonant frequency. In Section 2, the principle of the QVM is summarized. Section 3 discusses influence that the estimation error of the phase velocity gives to the modulation efficiency. In Section 4, experimental results are presented.

2. Quasi-Velocity-Matching

Figures 1(a) and 1(b) show a schematic of a normal traveling-wave EOM and a QVM-EOM, respectively. To avoid unnecessary complexities, we employ a one-dimensional analysis for our devices. For a traveling-wave EOM, if the y axis is the direction in which the optical and modulation wave propagate in a virgin (single-domain) EO crystal, the variation of the refractive index induced by an electric field $E_m \sin(\omega_m t)$ is obtained at y as

$$\Delta n(y; t_0) = \Delta n_m \sin\left(\omega_m t_0 - \frac{\pi}{L_0} y\right), \tag{1}$$

where $\Delta n_m = (1/2) n_e^3 \gamma_{33} E_m$ is the amplitude of index changes, n_e is the extraordinary refractive index of the crystal, γ_{33} is the EO coefficient of the crystal, and $\omega_m = 2\pi f_m$. A half-period L_0 of the domain inversion is given by

$$L_0 = \frac{1}{2 f_m (1/v_m - 1/v_g)}, \tag{2}$$

where v_m is the phase velocity of the modulation wave, and v_g is the group velocity of the optical wave. Here, we assume that the optical wave arrives at point $y = 0$ at time $t = t_0$.

For $v_g > v_m$, the phase retardation of optical wave at the position y is expressed as

$$\theta(y; t_0) = \frac{2\pi}{\lambda} \int_0^y \Delta n(y; t_0)\, dy = \Delta\phi \sin\left(\omega_m t_0 - \frac{\pi}{2L_0} y\right), \tag{3}$$

where

$$\Delta\phi = \frac{4L_0}{\lambda} \Delta n_m \sin\left(\frac{\pi}{2L_0} y\right). \tag{4}$$

If there is the so-called velocity mismatching between the modulation wave and the optical wave, the modulation index $|\Delta\phi|$ becomes a periodical function of y with period $2L_0$.

When a traveling-wave EOM has a suitable domain-inverted half-period of L_0, QVM occurs and accordingly a large modulation index is achieved. In such a situation, phase retardation $\theta(l; t_0)$ given to the optical wave passing through the length of l, $qL_0 \leq l < (q+1)L_0$, in a periodically domain-inverted crystal as shown in Figure 1(b) is expressed as

$$\begin{aligned} \theta(l; t_0) &= \frac{2\pi}{\lambda} \int_0^{L_0} \Delta n(y; t_0)\, dy - \frac{2\pi}{\lambda} \int_{L_0}^{2L_0} \Delta n(y; t_0)\, dy \cdots \\ &\quad + (-1)^q \frac{2\pi}{\lambda} \int_{qL_0}^{l} \Delta n(y; t_0)\, dy \\ &= \Delta\phi_{\mathrm{QVM}} \cos(\omega_m t_0 - \Phi). \end{aligned} \tag{5}$$

The modulation index $\Delta\phi_{\mathrm{QVM}}$ can be expressed by

$$\Delta\phi_{\mathrm{QVM}} = \begin{cases} \dfrac{4L_0 \Delta n_m}{\lambda} \sqrt{q^2 + (2q+1)\sin^2\left(\dfrac{\pi l}{2L_0}\right)} & (q : \text{even}), \\ \dfrac{4L_0 \Delta n_m}{\lambda} \sqrt{q^2 + (2q+1)\cos^2\left(\dfrac{\pi l}{2L_0}\right)} & (q : \text{odd}). \end{cases} \tag{6}$$

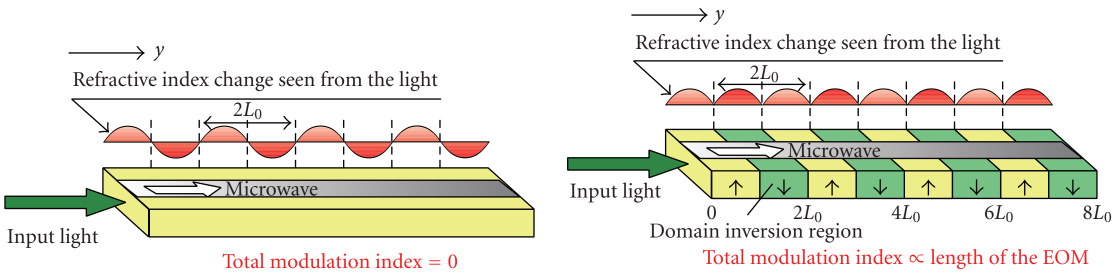
Figure 2 shows modulation indices of (a) the QVM-EOM, and (b) the typical traveling-wave EOM (velocity mismatching). The solid line of Figure 2(a) is calculated using (6). The modulation index achieved by QVM-EOM is almost proportional to the interaction length though it is lower than perfect velocity-matched condition by a factor of $2/\pi$. The upper limit of the modulation index for non-domain-inverted EOM is $\Delta\phi_0 = 8L_0 \Delta n_m / \lambda$.

3. Efficiency Analysis for the QVM-EOM

The analysis presented in the former section was ideal case in which there is no error in the period of the domain inversion. Uncertainties in the value of the phase velocity and group velocity degrade the modulation efficiency through the length error of the domain-inversion period for the QVM. In this section, we will discuss degradation of the modulation efficiency in the presence of length error in the period of the domain inversion.

Figure 3 show the refractive index changes seen from the optical wave in periodically domain-inverted EO crystal. The half period of the domain inversion in Figures 3(a) and 3(b) is $L = L_0$ and $L = L_0 + \Delta L$, where $\Delta L / L_0 = 10\%$, respectively. The QVM occurs in the case of Figure 3(a), and as a result the traveling optical wave of $\omega_m t_0 = 0$ sees the positive refractive index change throughout the interaction. The case of 10% error in the domain-inversion length is shown in Figure 3(b). If there is an error in length of domain inversion, the optical wave of $\omega_m t_0 = 0$ sees not only positive but also a negative refractive index change throughout the interaction.

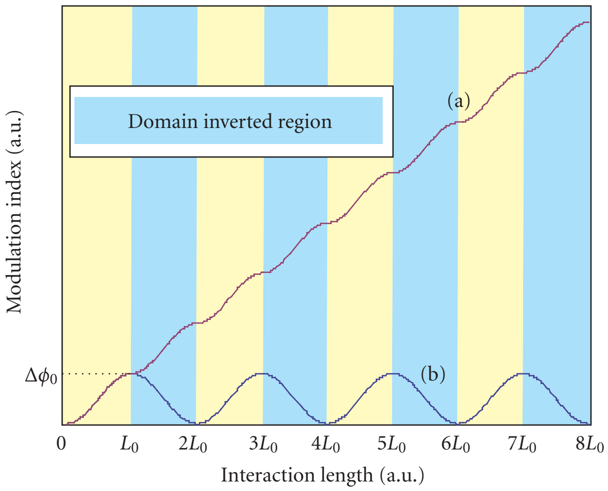
Figure 4(a) shows the normalized modulation index calculated for the interaction length of $l = 12L_0$. The modulation efficiency almost proportionally decreases with the increase of $|\Delta L|$. The modulation efficiency decreases to about 50% in the case of $\Delta L / L_0 = 10\%$. Figure 4(b) shows the modulation index as a function of the interaction length. The calculation is carried out for the length error in the half period of the domain inversion of (i) $\Delta L / L_0 = 0\%$, (ii) $\Delta L / L_0 = 10\%$, and (iii) $\Delta L / L_0 = 15\%$. An upper limit of the modulation index exists if $\Delta L \neq 0$. Because longer



(a) Traveling-wave EOM

(b) QVM-EOM

FIGURE 1: (a) Traveling-wave EOM, (b) QVM-EOM with periodic domain inversion.



(a) Quasi-velocity-matching
(b) Velocity mismatching

FIGURE 2: Modulation indices of EOMs. (a) QVM-EOM with periodic domain inversion, and (b) typical traveling-wave EOM (velocity mismatching).

interaction length is essential for larger modulation index, it is important to estimate the length of the half period of the domain inversion in high accuracy.

Propagation of uncertainties of v_m and v_g can be evaluated by

$$\frac{\Delta L}{L_0} = \frac{v_g}{v_g - v_m}\frac{\Delta v_m}{v_m} \approx 1.8 \times \frac{\Delta v_m}{v_m},$$
$$\frac{\Delta L}{L_0} = \frac{v_m}{v_g - v_m}\frac{\Delta v_g}{v_g} \approx 0.8 \times \frac{\Delta v_g}{v_g}. \quad (7)$$

The influence on the modulation efficiency of uncertainty of v_g is lower than half of the influence on the modulation efficiency of uncertainty of v_m. Moreover, v_g can be estimated more accurately than v_m by using Sellmeier equation with constants for SLT crystal listed in [5]. Using Sellmeier equation, $n_g = 2.41$ at $\lambda = 514.5$ nm is derived. From this reliable value of n_g, the group velocity is derived as $v_g = 1.24 \times 10^8$ m/s.

The phase velocity of the modulation microwave can be calculated as $v_m = c/\sqrt{\epsilon_{\text{eff}}}$. The effective relative dielectric constant for microstrip line can be calculated using Kobayashi's formula [9, 10]. Kobayashi's formula is claimed to predict dispersion better than 0.6% in the range $1 \le \epsilon_r \le 128$ and $0.1 \le w/h \le 10$, where w and h are width and height of the microstrip line, respectively. However, uncertainty of the estimated v_m is comparatively large, because the reported value of ϵ_r for LT crystal is inaccurate and ranging $38.9 \le \epsilon_r \le 44.5$ [7]. If we employ values of $38.9 \le \epsilon_r \le 44.5$, v_m of the modulation microwave for 16.25 GHz can be calculated as 5.14×10^7 m/s $\le v_m \le 5.53 \times 10^7$ m/s using Kobayashi's formula. In that case, the half period of the domain inversion for QVM at 16.25 GHz is calculated as 2.7 mm $\le L_0 \le$ 3.1 mm with the group velocity of $v_g = 1.24 \times 10^8$ m/s. There is 14% of length uncertainty in the half period of the domain inversion. From Figure 4(b), $\Delta L/L_0 = 14\%$ results in the degradation of the modulation efficiency to about 10%.

4. Estimation of the Phase Velocity of the Modulation Wave Using EO Sampling Technique

Figure 5 shows a schematic of our QVM-EOM. An SLT crystal is used for an EO substrate. The modulation electrode structure is a microstrip line. The width of the microstrip line is $w = 0.5$ mm and the height of the substrate is $h = 0.5$ mm. The strip line is open-terminated for resonance.

Figure 6(a) shows an experimental setup. A pulsed fiber laser (repetition frequency: $f_{\text{rep}} = 40$ MHz) was used for probe pulses. The repetition frequency of the pulsed laser and the frequency of the microwave signal source are synchronized with each other. The frequency of the microwave should be set to be $f_m = N \times f_{\text{rep}} + \Delta f$, where N is an integer and Δf is an offset frequency. In the experiment, the frequency of the microwave is set to be $f_m = 16.0801$ GHz, where $N = 402$ and $\Delta f = 100$ kHz.

Figure 6(b) shows a cross-section of the microstrip line under the measurement. A CdTe crystal is used for the EO sensor. The aperture size of the EO sensor is 3 mm × 3 mm. The probe beam is focused on the CdTe crystal by the object lens (×10). The spot size is about 20 μm. The probe beam is reflected by a dielectric mirror attached to the EO

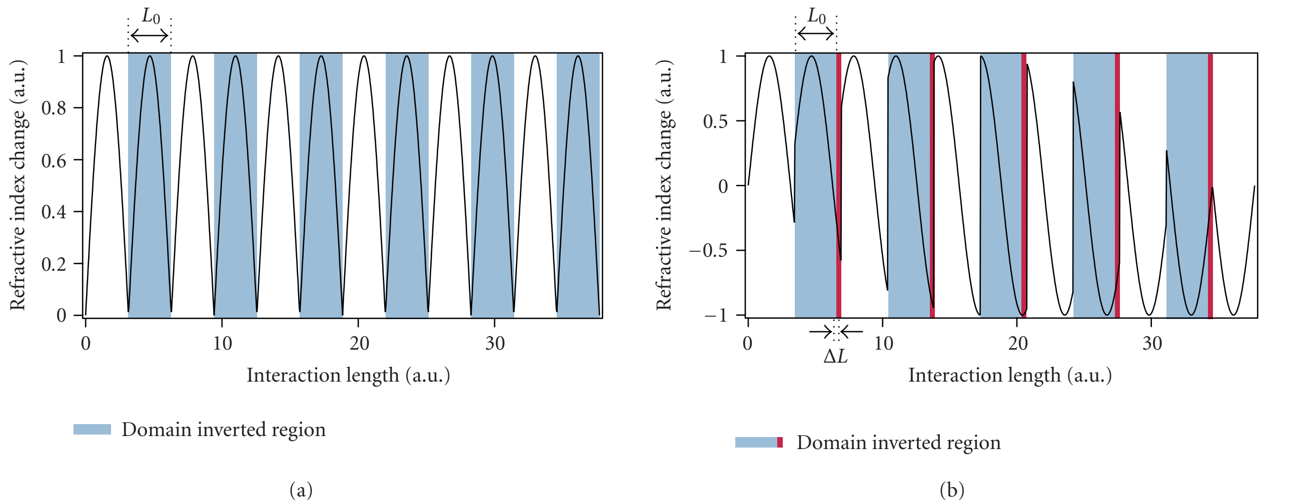


Figure 3: Refractive index change seen from the optical wave in periodically domain-inverted EO crystals. (a) Perfectly quasi-velocity-matched condition. Half period of the domain inversion is $L = L_0$. (b) Half period of the domain inversion is $L = L_0 + \Delta L$, where $\Delta L/L_0 = 10\%$.

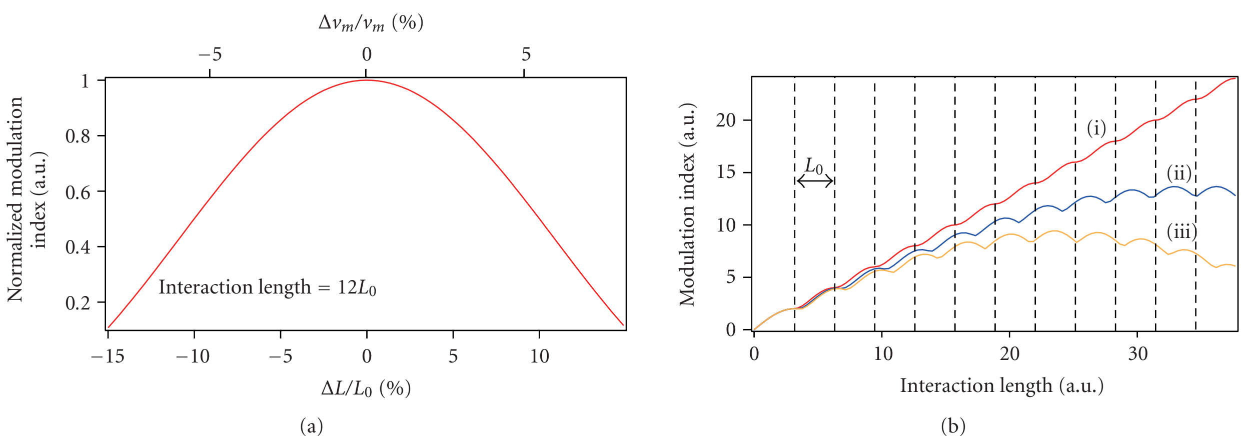


Figure 4: (a) Modulation index as a function of $\Delta L/L_0$. The interaction length is $12L_0$. (b) Modulation index as a function of the interaction length. (i) $\Delta L/L_0 = 0\%$, (ii) $\Delta L/L_0 = 10\%$, and (iii) $\Delta L/L_0 = 15\%$.

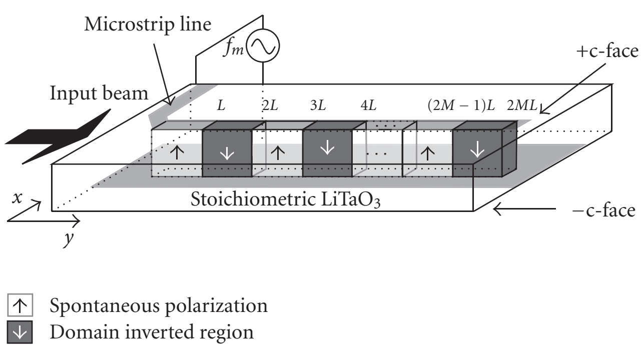


Figure 5: A schematic of our QVM-EOM. The substrate of the QVM-EOM is the stoichiometric $LiTaO_3$ crystal. The domain inversion is performed with the half period of L. The modulation electrode structure is the microstrip line. The width of the microstrip line is 0.5 mm. The height of the substrate is 0.5 mm.

sensor. Polarization of the reflected beam is modulated by the electric field of the microwave. The polarization-modulated beam propagates along the same path of the incident beam path. Two orthogonal polarization components of the modulated beam are differentially detected by two photodiodes. The detected signal component of $A\cos(2\pi\Delta f t + \phi)$ passes through a bandpass filter whose center frequency is 100 kHz. The amplitude of the signal, which is proportional to the amplitude of the electric field of the standing wave, is measured by a spectrum analyzer.

In the experiment, we scanned the probe beam in the y-direction with 50 μm step and measured electric field profile of the resonant standing wave. Figure 7 shows an experimental result. The theoretical curve of $a + 20\mathrm{Log}\,[V(y;\gamma)]$ (solid line) is fitted to the experimental data. Fitting parameters are offset power a and $\gamma = \alpha + j\beta$. From the least square fitting, a, α, and β are estimated as $a = -64.7$ dBm, $\alpha = -0.0057$,

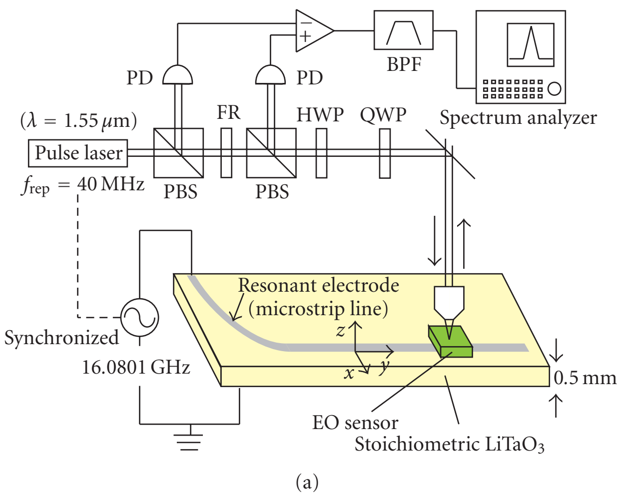


Figure 6: (a) Experimental setup. (b) Cross-section of the microstrip line under the measurement.

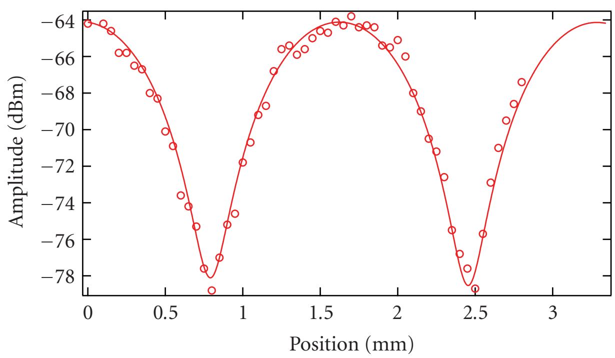


Figure 7: Magnitude of the voltage standing wave on an open-terminated microstrip line at $f_m = 16.0801$ GHz.

and $\beta = 1.888$, respectively. As a result, the wavelength of the traveling microwave is estimated as $\lambda_m = 2\pi/\beta = 3.33$ mm. The phase velocity at the modulation frequency of $f_m = 16.0801$ GHz is estimated as $f_m \times \lambda_m = 5.35 \times 10^7$ m/s. From this phase velocity, relative dielectric constant of $\epsilon_r = 41.5$ is derived using Kobayashi's formula. The half period of the domain inversion at the modulation frequency of near 16 GHz is derived as $L = 2.94$ mm with the phase velocity of $v_m = 5.35 \times 10^7$ m/s and the group velocity of 1.24×10^9 m/s.

The estimated phase velocity is slower than the real-phase velocity because of the perturbation of the EO sensor. In our case, this systematic error is estimated by numerical analysis (moment method) to be 5% or less. The 5% error in the phase velocity corresponds to the domain-inversion length error of $\Delta L/L_0 = 9\%$ which results in the degradation of the modulation efficiency to about 50%.

The systematic error depends mainly on (1) the difference of the dielectric constant between the device under test (DUT) and the EO sensor, (2) the distance between the DUT and the EO sensor, and (3) the thickness of the EO sensor. Using low-dielectric constant materials such as a polymer [11] as a sensor, the perturbation can be reduced. By loading a low-dielectric material between the DUT and the sensor to keep a distance, the perturbation can also be reduced, though the sensitivity is reduced at the same time. The use of a thinner sensor is an adequate plan to reduce the perturbation, however it also reduces the sensitivity. For a specific case, the tradeoff relationship between the perturbation strength and the sensitivity has been analyzed in the literature [12].

By using SLT substrate of the EOM as the EO sensor, it is also possible to measure the standing wave without using an external superstrate sensor [13]. In this case, special geometry should be constructed to transmit the probe beam between the microstrip line and the ground plane. This type of EOS system is now under construction and the results including the comparison with current technique will appear elsewhere.

5. Conclusion

We have estimated the phase velocity of the modulation microwave of 16 GHz in the periodically domain-inverted SLT phase modulator. The theoretical curve of the amplitude distribution of the standing wave in the microstrip line was fitted to the experimental data measured by EO sampling technique. From the curve fitting, the phase velocity of $v_m = 5.35 \times 10^7$ m/s was estimated though there is about 5% systematic error due to the perturbation by the EO sensor. Relative dielectric constant of $\epsilon_r = 41.5$ was led as the maximum likelihood value that derives the estimated phase velocity.

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