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Research Article

H_∞ Guaranteed Cost Control for Networked Control Systems under Scheduling Policy Based on Predicted Error

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Scheduling policy based on model prediction error is presented to reduce energy consumption and network conflicts at the actuator node, where the characters of networked control systems are considered, such as limited network bandwidth, limited node energy, and high collision probability. The object model is introduced to predict the state of system at the sensor node. And scheduling threshold is set at the controller node. Control signal is transmitted only if the absolute value of prediction error is larger than the threshold value. Furthermore, the model of networked control systems under scheduling policy based on predicted error is established by taking uncertain parameters and long time delay into consideration. The design method of H_∞ guaranteed cost controller is presented by using the theory of Lyapunov and linear matrix inequality (LMI). Finally, simulations are included to demonstrate the theoretical results.

1. Introduction

Networked control systems (NCS) are frequently encountered in practice for widespread fields of applications due to their suitable and flexible structure [1, 2]. Nevertheless, networked control systems have various equipments, complicated structure, and large scale, and they require a high level for safety and reliability. Meanwhile, the characteristics of NCS, such as authorization of the spectrum, dynamic mobile, limited channels, and broadcast transmission, make themselves inevitably existing transmission delay and data packet loss, which could cause adverse effect to system and even lead to instability. Therefore, concerning how to reduce the negative influence on the system control performance, energy consumption of nodes has become one of the hot issues in the control field.

Literatures in the aspects of NCS have got plenty of achievements on stability analysis and controller design considering uncertain parameters, time delay, noise, and other factors [3–7]. All of the above have not involved the scheduling problems of networked control systems. For example, the problem of integrated design of controller and communication sequences is addressed for NCS with simultaneous consideration of medium access limitations and network-induced delays, packet dropouts, and measurement quantization in [6]. However, only relying on the controller design is difficult to improve the control performance of system effectively if a large number of data share the limited bandwidth. Reasonable network scheduling strategies to reduce the conflict and the energy consumption of controller nodes are introduced in [8–12]. In order to satisfy timeliness of messages and improve system's flexibility in NCS based on controller area network (CAN), a distributed dynamic message scheduling method based on deadline of message (DM) is proposed in [8]. A receding-horizon control and scheduling (RHCS) problem with a quadratic performance criterion is formulated and solved by (relaxed) dynamic programming in [9], but it is not considering the guaranteed cost problem. Zhao et al. [10] proposed a predictive control and scheduling codesign approach to deal with the controller and scheduler design for a set of networked control systems which are connected to a shared communication network. In [11], the scheduling of sensor information towards the controller is ruled by the classical Round-Robin protocol and the induced L_2-gain of NCS is analyzed, which is subject to time-varying transmission intervals, time-varying transmission delays, and communication constraints, but not referring to the affect of interference input and the guaranteed cost problem.

With the rapid development of computer technology, sensors sampling frequency and the processing speed of the controller are being improved continually; network conflict is becoming more and more serious at actuators side because of the limited channels of network during the transmission of information. So, it is important to explore a reasonable scheduling policy to reduce the network conflict at actuator node and to avoid the loss of important information. This motivates us to conduct the research work.

In this paper, scheduling policy based on model prediction error is presented to reduce energy consumption and network conflicts at the actuator node, in which the characters of NCS are considered, such as limited network bandwidth, limited node energy, and high collision probability. The prediction model is introduced to predict the state of system at the sensor node, and then referential value of control signal is obtained after the predicted value of state is calculated by the controller. And scheduling threshold is set at the controller node. Control signal is not transmitted if the absolute value of prediction error is lower than the threshold value. Moreover, the design method of H_∞ guaranteed cost controller is presented by using the theory of Lyapunov and linear matrix inequality (LMI) theory. Finally, simulations are included to demonstrate the theoretical results.

The paper is organized in 5 sections including the introduction. Section 2 presents models for NCS under scheduling policy based on predicted error and main assumptions. Section 3 presents the controller design of NCS under scheduling policy based on predicted error. There are some simulations to illustrate the results in Section 4. Section 5 summarized this paper.

2. Modeling for Networked Control Systems

The structure of networked control systems under scheduling policy based on predicted error is shown in Figure 1, where $x(k)$, $\overline{x}(k)$, and $\tilde{u}(k)$ represent state value sampled by sensors, state value predicted by model, and prediction error at time k separately, while k represents the kth sampling period.

Consider the NCS model with uncertain parameters as follows:

$$x(k+1) = \overline{A}x(k) + \overline{B}u(k) + \overline{H}\omega(k), \qquad y(k) = Cx(k), \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^r$, and $\omega(t) \in R^p$ represent state value, input, output, and interference input separately; $\overline{A} = A + \Delta A$, $\overline{B} = B + \Delta B$, $\overline{H} = H + \Delta H$, A, B, H, and C are matrices with appropriate dimensions; ΔA, ΔB, and ΔD are matrices with uncertain time-varying parameters, satisfying $[\Delta A \quad \Delta H \quad \Delta B] = \Omega F[E_1 \quad E_2 \quad E_3]$; F is an unknown matrix function with Legesgue measurable properties, satisfying $F^T F \le I$; Ω, E_1, E_2, and E_3 are constant matrices with appropriate dimensions.

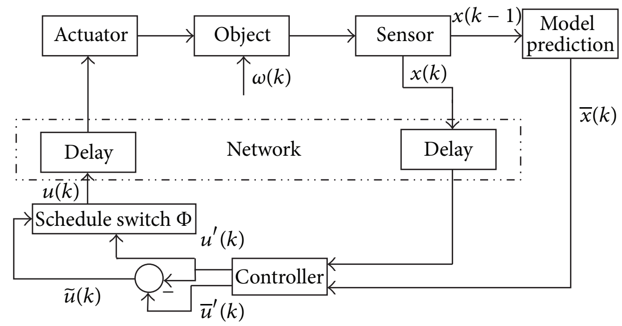


Figure 1: The structure of networked control systems under scheduling policy based on predicted error.

To facilitate discussion, some assumptions are employed as follows.

(A1) The state of NCS is completely measurable.

(A2) The cache devices are not used both at actuators end and at sensors end.

(A3) Sensors, controllers, and actuators are all time driving. Before the first controlled input reaches the actuator, controlled input always maintains $u(k) = 0$.

(A4) Forward channel delay caused by the network is denoted by l_1, while backward channel delay is denoted by l_2, and $d = l_1 + l_2$ is an integer in any case.

Remark 1. Based on assumption 2, system information obtained by controller is only the state of system object, namely, $x(k-\tau)$.

2.1. Analysis on Time Delay for Networked Control Systems under Scheduling Policy Based on Predicted Error. Based on assumption 4, the feedback information received by system controller at time k is the state of system object at time $k - l_2$ without considering the data packet loss. Controlled input is obtained after controller calculates the information transmitted by sensor and then is transmitted to the actuator through network. The controlled input reaches actuator and work at time $k + l_1$. Hence, for the sensor node, the entire network delay becomes $d = l_1 + l_2$. System (1) can be written as

$$x(k+1) = \overline{A}x(k) + \overline{B}u(k-d) + \overline{H}\omega(k), \qquad y(k) = Cx(k). \tag{2}$$

Based on assumption 1 and Figure 1, state feedback is introduced as follows:

$$u'(k) = Kx(k). \tag{3}$$

2.2. The Description of Model Prediction Error. The controller nodes use the kth received data packets which are sampled and transmitted by sensor to predict the state of the model at next time, and then the predicted value of state is transmitted to controller. The predictive control signal is obtained after the predicted value of the state is calculated. After getting the prediction error by comparing predicted value of control signal with the true value at time $k+1$, the state value is updated spontaneously. In addition, to trace the state trajectory of system, we use certain parameters of the object model (1) to predict and calculate the referential value of control signal. The prediction model can be described as follows:

$$\overline{x}(k+1) = Ax(k) + Bu(k), \qquad \overline{u}'(k) = K\overline{x}(k), \tag{4}$$

where A, B, and K refer to (1) and (3) and $\overline{u}'$ is the referential value of control signal.

The predicted error of control signal produced by the model is shown as follows:

$$\tilde{u}(k) = u'(k) - \overline{u}'(k). \tag{5}$$

2.3. The Description of Scheduling Policy. With the rapid development of computer technology, sensors sampling frequency and the processing speed of the controller are being improved continually; network conflict is becoming more and more serious at actuators side because of the limited channels of network during the transmission of information. So it is important to introduce reasonable scheduling policy to reduce the network conflict at actuator node. Here we introduce the restrained condition of transmission as $|\tilde{u}_i(k)| \le \vartheta_i$ (ϑ_i represents scheduling threshold, $i = [1, 2, \ldots, m]$, and m is the dimension of the control signal). Controller will not send the control signal $u_i(k)$ taken as unimportant information to actuator and the actuator keeps the value of control signal at time $k-1$ if the restrained condition is satisfied, which helps to reduce the transmission frequency of unimportant information at actuator node.

According to the description above, piecewise function as follows is introduced:

$$u_i(k) = \begin{cases} u_i'(k-1), & |\tilde{u}_i(k)| \le \vartheta_i, \\ u_i'(k), & |\tilde{u}_i(k)| > \vartheta_i. \end{cases} \tag{6}$$

Moreover, we introduce

$$\delta_i(k) = \begin{cases} 0, & |\tilde{u}_i(k)| \le \vartheta_i, \\ 1, & |\tilde{u}_i(k)| > \vartheta_i, \end{cases} \qquad i = [1, 2, \ldots, m], \tag{7}$$

where $\delta_i = 0$ represents that u_i should not be transmitted, while $\delta_i = 1$ represents that u_i should be transmitted. We define $\Phi = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_m)$. Obviously, equality (6) is equivalent to

$$u(k) = \left[\Phi_j, I_{m\times m} - \Phi_j\right] \left[u'^T(k), u'^T(k-1)\right]^T. \tag{8}$$

Remark 2. According to the description about Φ above, the total number of cases that Φ could appear should be 2^m in the whole scheduling process; that is to say,

$$\Phi = \Phi_j \in \{\Phi_1, \Phi_2, \ldots, \Phi_{2^m}\}, \quad j = 1, 2, \ldots, 2^m. \tag{9}$$

Remark 3. Obviously, different from the previous method, such as [8], in which referential value of scheduling policy based on deadline of message is a fixed value. The referential value in Section 2.2 of scheduling policy based on predicted error obtained by using certain parameters of the object model (1) to predict the value of system is varied with the change of the system state trajectory. In this way, a bigger chance for losses of unimportant information in NCS is offered than the previous method as [8]; that is to say, scheduling policy based on predicted error is more effective to avoid the network conflict and save energy of nodes.

2.4. Augmented System Model of NCS under Scheduling Policy Based on Predicted Error. Based on the description of equalities (3) and (8) and Remark 2, it can be obtained that

$$u(k) = \Phi_j Kx(k) + \left(I - \Phi_j\right) u(k-1). \tag{10}$$

The augmented matrix is defined as

$$z = \left[x^T(k), x^T(k-1), \ldots, x^T(k-d-1), x^T(k-d), u^T(k-d-1)\right]^T. \tag{11}$$

Therefore, the augmented NCS model becomes

$$z(k+1) = \widehat{A}\, z(k) + \widehat{H}\, \omega(k), \qquad y(k) = \widehat{C}\, z(k), \tag{12}$$

where

$$\widehat{A} = \begin{bmatrix} \overline{A} & 0 & \cdots & 0 & 0 & \overline{B}\Phi_j K & \overline{B}(I-\Phi_j) \\ I & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & I & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & I & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \Phi_j K & I - \Phi_j \end{bmatrix}_{(d+2)\times(d+2)},$$

$$\widehat{H} = \left[\overline{H}^T \; 0 \; \cdots \; 0 \; 0 \; 0 \; 0\right]^T,$$

$$\widehat{C} = \left[C \; 0 \; \cdots \; 0 \; 0 \; 0 \; 0\right],$$

$$j = \left[1, 2, \ldots, 2^m\right]. \tag{13}$$

Obviously, (12) is a switching model; the number of switching modes is 2^m.

3. H_∞ Guaranteed Cost Control for NCS

For the system model established above, performance indicator is given as follows:

$$\begin{aligned} J_\infty = \sum_{i=0}^{\infty} \Bigg\{ & [Kx(i-d)]^T R [Kx(i-d)] \\ & + u^T(i-d-1) Q u(i-d-1) \\ & + \sum_{j=0}^{d} \left[x^T(i-j) Q x(i-j) \right] \Bigg\} \\ = \sum_{i=0}^{\infty} & \left[z^T(i) \widehat{Q} z(i) + \left(\widehat{K} z(i-d) \right)^T \widehat{R} \left(\widehat{K} z(i-d) \right) \right], \end{aligned} \tag{14}$$

where $\widehat{Q} = \operatorname{diag}(Q, Q, \dots, Q, Q, Q, Q)$, $\widehat{R} = \operatorname{diag}(R, R, \dots, R, R, R, R)$, $\widehat{K} = \operatorname{diag}(0, 0, \dots, 0, 0, K, 0)$, Q, and R are symmetric positive definite matrices.

Definition 4. For system (2) and system (12), it satisfies that (1) the closed-loop system is asymptotically stable if $\omega(k) = 0$; (2) under any zero initial condition, given $\gamma > 0$, for any nonzero vector $\omega(k) \in L_2[0, \infty)$, the output $y(k)$ satisfies $\|y(k)\|_2 \le \gamma \|\omega(k)\|_2$. It is called that system (2) and (12) is asymptotically stable with H_∞ norm bound γ.

Lemma 5 (Schur complement). *For a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where $S_{11} = S_{11}^T$, $S_{12}^T = S_{21}$, and $S_{22} = S_{22}^T$, the following three conditions are equivalent:*

(1) $S < 0$;

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 6 (see [3]). *W, M, N, and F are matrices with suitable dimensions, satisfying $F^T F \le I$, and W is symmetric matrix; then $W + MFN + N^T F^T M^T < 0$ is equivalent to*

$$W + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0, \tag{15}$$

where scalar $\varepsilon > 0$.

Lemma 7 (see [13]). *For matrices Ψ_c, M_c, and N_c with appropriate dimensions, where $N_c = \operatorname{diag}(N_{c1}, N_{c2}, \dots, N_{cm})$, satisfying $N_{ci}^T N_{ci} \le I$ $(i = 1, 2, \dots, m)$. If there exists $L = \operatorname{diag}(\sigma_1 I, \sigma_2 I, \dots, \sigma_m I) > 0$, where σ_i is a set of scalars $(i = 1, 2, \dots, m)$, it satisfies that*

$$\Psi_c N_c M_c + M_c^T N_c^T \Psi_c^T \le \Psi_c L \Psi_c^T + M_c^T L^{-1} M_c. \tag{16}$$

3.1. Stability Analysis of NCS under Scheduling Policy Based on Predicted Error

Theorem 8. *Given symmetric positive definite matrices Q and R, if there exist symmetric positive definite matrix P, the gain matrix K, and constant $\gamma > 0$, satisfying*

$$\begin{bmatrix} -\widehat{R}^{-1} & 0 & 0 & \widehat{K} & 0 \\ * & -\left(\widehat{C}^T \widehat{C} + \widehat{Q} \right)^{-1} & 0 & I & 0 \\ * & * & -P^{-1} & \widehat{A} & \widehat{H} \\ * & * & * & -P & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{17}$$

then it is called that system (2) and (12) is asymptotically stable with H_∞ norm bound γ. And its performance indicator satisfies $J_\infty < z^T(0) P z(0)$, where $*$ represents the symmetry blocks of matrix; $\widehat{A}$, $\widehat{C}$, and $\widehat{H}$ are shown as (12); and $\widehat{Q}$, $\widehat{R}$, and $\widehat{K}$ are shown as (14).

Proof. Because system (12) is the equivalent system of system (2), system (2) must satisfy the condition if and only if system (12) satisfies the asymptotically stable condition. Consider the following Lyapunov function:

$$V(k) = z^T(k) P z(k). \tag{18}$$

Conducting subtract operating along arbitrary trajectory of system (12) is given by

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= z^T(k+1) P z(k+1) - z^T(k) P z(k). \end{aligned} \tag{19}$$

Submitting (12) into (19) yields

$$\begin{aligned} \Delta V(k) & \\ = & \left[\widehat{A} z(k) + \widehat{H} \omega(k) \right]^T P \left[\widehat{A} z(k) + \widehat{H} \omega(k) \right] \\ & - z^T(k) P z(k) \\ = & z^T(k) \left(\widehat{A}^T P \widehat{A} - P \right) z(k) + z^T(k) \widehat{A}^T P \widehat{H} \omega(k) \\ & + \omega^T(k) \widehat{H}^T \widehat{A} z(k) + \omega^T(k) \widehat{H}^T P \widehat{H} \omega(k). \end{aligned} \tag{20}$$

Plus $y^T(k) y(k) - \gamma^2 \omega^T(k) \omega(k)$ at two ends of equality (20), we have

$$\begin{aligned} \Delta V(k) &+ y^T(k) y(k) - \gamma^2 \omega^T(k) \omega(k) \\ = & z^T(k) \left(\widehat{A}^T P \widehat{A} - P \right) z(k) + z^T(k) \widehat{A}^T P \widehat{H} \omega(k) \\ & + \omega^T(k) \widehat{H}^T \widehat{A} z(k) + \omega^T(k) \widehat{H}^T P \widehat{H} \omega(k) \\ & + y^T(k) y(k) - \gamma^2 \omega^T(k) \omega(k) \end{aligned}$$

$$= z^T(k)\left(\widehat{A}^T P\widehat{A} - P + \widehat{C}^T\widehat{C}\right)z(k) + z^T(k)\widehat{A}^T P\widehat{H}\omega(k) + \omega^T(k)\widehat{H}^T P\widehat{A}z(k) + \omega^T(k)\widehat{H}^T P\widehat{H}\omega(k) - \gamma^2\omega^T(k)\omega(k) = \left[z^T(k), \omega^T(k)\right]\Theta\left[z^T(k), \omega^T(k)\right]^T, \quad (21)$$

where

$$\Theta = \begin{bmatrix} \widehat{A}^T P\widehat{A} - P + \widehat{C}^T\widehat{C} & \widehat{A}^T P\widehat{H} \\ * & \widehat{H}^T P\widehat{H} - \gamma^2 I \end{bmatrix}. \quad (22)$$

If

$$\begin{bmatrix} \widehat{A}^T P\widehat{A} - P + \widehat{C}^T\widehat{C} + \widehat{Q} + \widehat{K}^T\widehat{R}\widehat{K} & \widehat{A}^T P\widehat{H} \\ * & \widehat{H}^T P\widehat{H} - \gamma^2 I \end{bmatrix} < 0 \quad (23)$$

it can be obtained that $\Theta < 0$. Based on equality (21), it is known that

$$\Delta V(k) + y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k) < 0. \quad (24)$$

If $\omega(k) \equiv 0$, obviously, there is $\Delta V(k) < 0$.

From zero initial condition, we know $V(0) = 0$. And it can be obtained that $V(\infty) \geq 0$. Therefore,

$$\sum_{k=0}^{\infty}\left[\Delta V(k) + y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k)\right] = V(\infty) - V(0) + \sum_{k=0}^{\infty}\left[y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k)\right] < 0. \quad (25)$$

Moreover,

$$\sum_{k=0}^{\infty}\left[y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k)\right] < -V(\infty) \leq 0. \quad (26)$$

Therefore, we have

$$\sum_{k=0}^{\infty} y^T(k)y(k) < \sum_{k=0}^{\infty}\gamma^2\omega^T(k)\omega(k). \quad (27)$$

Namely, $\|y(k)\|_2 < \gamma\|w(k)\|_2$. Based on Definition 4, it is known that system (2) and (12) is asymptotically stable with H_∞ norm bound γ. Inequality (23) can be written as

$$\begin{bmatrix} \widehat{A}^T P\widehat{A} - P + \widehat{C}^T\widehat{C} & \widehat{A}^T P\widehat{H} \\ * & \widehat{H}^T P\widehat{H} - \gamma^2 I \end{bmatrix} < -\begin{bmatrix} \widehat{Q} + \widehat{K}^T\widehat{R}\widehat{K} & 0 \\ * & 0 \end{bmatrix}. \quad (28)$$

Namely,

$$\Delta V(k) = \left[z^T(k), \omega^T(k)\right]\begin{bmatrix} \widehat{A}^T P\widehat{A} - P + \widehat{C}^T\widehat{C} & \widehat{A}^T P\widehat{H} \\ * & \widehat{H}^T P\widehat{H} - \gamma^2 I \end{bmatrix} \times\left[z^T(k), \omega^T(k)\right]^T < -\left[z^T(k), \omega^T(k)\right]\begin{bmatrix} \widehat{Q} + \widehat{K}^T\widehat{R}\widehat{K} & 0 \\ * & 0 \end{bmatrix}\left[z^T(k), \omega^T(k)\right]^T. \quad (29)$$

It can be obtained that

$$\sum_0^{\infty}\Delta V(k) < \sum_0^{\infty} -\left[z^T(k), \omega^T(k)\right]\begin{bmatrix} \widehat{Q} + \widehat{K}^T\widehat{R}\widehat{K} & 0 \\ * & 0 \end{bmatrix}\left[z^T(k), \omega^T(k)\right]^T = -J_\infty. \quad (30)$$

Namely, $J_\infty < -\sum_0^\infty \Delta V(k) = V(0) - [V(0) + \sum_0^\infty \Delta V(k)] = V(0) - V(\infty)$.

Because the process above elucidated that the system is asymptotically stable, we have $V(\infty) = 0$. Therefore, $J_\infty < V(0) = z^T(0)Pz(0)$ is verified.

Inequality (23) can be written as

$$\begin{bmatrix} -P + \widehat{C}^T\widehat{C} + \widehat{Q} + \widehat{K}^T\widehat{R}\widehat{K} & 0 \\ * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \widehat{A}^T \\ \widehat{H}^T \end{bmatrix} P\left[\widehat{A} \quad \widehat{H}\right] < 0. \quad (31)$$

Using Lemma 5, we have

$$\begin{bmatrix} -P^{-1} & \widehat{A} & \widehat{H} \\ * & -P + \widehat{C}^T\widehat{C} + \widehat{Q} + \widehat{K}^T\widehat{R}\widehat{K} & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0. \quad (32)$$

Using Lemma 5, inequality (32) is equivalent to inequality (17); thus, Theorem 8 is verified. □

Remark 9. Uncertain system forms like ΔA are contained in matrix inequality (17), so the problem cannot be solved by using LMI toolbox. The next work is conducting appropriate deformation to eliminate the uncertainties in the matrix and convert it to linear matrix inequality (LMI), in which variable parameters are contained.

3.2. The Controller Design of NCS under Scheduling Policy Based on Predicted Error

Theorem 10. *Given symmetric positive definite matrices Q and R, if there exist a set of symmetric positive definite matrices*

X_i $(i = 1, 2, \ldots, d+2)$, matrix Y, and a set of constants $\sigma_1 > 0$, $\sigma_2 > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and $\mu > 0$, satisfying

$$\begin{bmatrix} -\varepsilon_1 I + \sigma_1 BB^T + \sigma_2 E_3 E_3^T & 0 & 0 & \Delta_1 & \Delta_2 & 0 & 0 & 0 & 0 \\ * & -\widehat{R}^{-1} & 0 & 0 & \overleftrightarrow{Y} & 0 & 0 & 0 & 0 \\ * & * & -\left(\widehat{C}^T \widehat{C} + \widehat{Q}\right)^{-1} & 0 & X & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_1 + (\varepsilon_1 + \varepsilon_2) \overleftrightarrow{\Omega} \overleftrightarrow{\Omega}^T & \Pi_2 & \overleftrightarrow{H} & 0 & 0 & 0 \\ * & * & * & * & -X & 0 & 0 & \Delta_3^T & \Delta_4^T \\ * & * & * & * & * & -\mu I & E_2^T & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & -\sigma_1 I & 0 \\ * & * & * & * & * & * & * & * & -\sigma_2 I \end{bmatrix} < 0, \quad (33)$$

where

$$\overleftrightarrow{Y} = \operatorname{diag}\begin{pmatrix} 0 & 0 & \cdots & 0 & 0 & Y & 0 \end{pmatrix}, \qquad X = \operatorname{diag}\left(X_1, X_2, \ldots, X_{d-1}, X_d, X_{d+1}, X_{d+2}\right),$$

$$\Delta_1 = \begin{bmatrix} \sigma_1 BB^T + \sigma_2 E_3 B^T & 0 & \cdots & 0 & 0 & 0 & \sigma_1 B + \sigma_2 E_3 \end{bmatrix}, \qquad \Delta_2 = \begin{bmatrix} E_1 X_1 & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Delta_3 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & Y & 0 \end{bmatrix}, \qquad \Delta_4 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & X_{d+2} \end{bmatrix},$$

$$\Pi_1 = \begin{bmatrix} -X_1 + (\sigma_1 + \sigma_2) BB^T & 0 & \cdots & 0 & 0 & 0 & (\sigma_1 + \sigma_2) B \\ 0 & -X_2 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -X_{d-1} & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & -X_d & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & -X_{d+1} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & -X_{d+2} + (\sigma_1 + \sigma_2) I \end{bmatrix},$$

$$\Pi_2 = \begin{bmatrix} AX_1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ X_1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & X_2 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & X_{d-1} & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & X_d & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \overleftrightarrow{\Omega} = \begin{bmatrix} \Omega \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \overleftrightarrow{H} = \begin{bmatrix} H \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (34)$$

$\widehat{C}$ and $\widehat{H}$ are shown as (12) *and $\widehat{Q}$ and $\widehat{R}$ are shown as* (14).

It is called that system (2) *and* (12) *is asymptotically stable with H_∞ norm bound γ; the gain matrix of feedback control is $K = YX_{d+1}^{-T}$. And its performance indicator satisfies $J_\infty < z^T(0)Pz(0)$. $*$ represents the symmetry blocks of matrix.*

Proof. The proof is based on a suitable congruence transformation and a change of variables allowing us to obtain inequality (17). $\widehat{A}$ and $\widehat{H}$ can be written as $\widehat{A} = \Psi + \overleftrightarrow{\Omega} F \overleftrightarrow{E}$, $\widehat{H} = \overleftrightarrow{H} + \overleftrightarrow{\Omega} F E_2$, where

$$\Psi = \begin{bmatrix} A & 0 & \cdots & 0 & 0 & B\Phi_j K & B\left(I - \Phi_j\right) \\ I & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & I & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & I & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \Phi_j K & I - \Phi_j \end{bmatrix}, \quad (35)$$

$$\overleftrightarrow{E} = \begin{bmatrix} E_1 & 0 & \cdots & 0 & 0 & E_3 \Phi_j K & E_3\left(I - \Phi_j\right) \end{bmatrix}.$$

Inequality (17) can be written as

$$\begin{bmatrix} -\widehat{R}^{-1} & 0 & 0 & \widehat{K} & 0 \\ * & -\left(\widehat{C}^T\widehat{C}+\widehat{Q}\right)^{-1} & 0 & I & 0 \\ * & * & -P^{-1} & \Psi & \widehat{H} \\ * & * & * & -P & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \overleftrightarrow{\Omega} \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overleftrightarrow{E}^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overleftrightarrow{E}^T \\ 0 \end{bmatrix} F^T \begin{bmatrix} 0 \\ 0 \\ \overleftrightarrow{\Omega} \\ 0 \\ 0 \end{bmatrix}^T < 0. \tag{36}$$

Based on Lemma 6, we have

$$\begin{bmatrix} -\widehat{R}^{-1} & 0 & 0 & \widehat{K} & 0 \\ * & -\left(\widehat{C}^T\widehat{C}+\widehat{Q}\right)^{-1} & 0 & I & 0 \\ * & * & -P^{-1} & \Psi & \widehat{H} \\ * & * & * & -P & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} + \varepsilon_1 \begin{bmatrix} 0 \\ 0 \\ \overleftrightarrow{\Omega} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \overleftrightarrow{\Omega} \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon_1^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overleftrightarrow{E}^T \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overleftrightarrow{E}^T \\ 0 \end{bmatrix}^T < 0. \tag{37}$$

Based on Lemma 5, inequality (37) is equivalent to

$$\begin{bmatrix} -\varepsilon_1 I & 0 & 0 & 0 & \overleftrightarrow{E} & 0 \\ * & -\widehat{R}^{-1} & 0 & 0 & \widehat{K} & 0 \\ * & * & -\left(\widehat{C}^T\widehat{C}+\widehat{Q}\right)^{-1} & 0 & I & 0 \\ * & * & * & -P^{-1}+\varepsilon_1\overleftrightarrow{\Omega}\,\overleftrightarrow{\Omega}^T & \Psi & \widehat{H} \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{38}$$

And it can be written as

$$\begin{bmatrix} -\varepsilon_1 I & 0 & 0 & 0 & \overleftrightarrow{E} & 0 \\ * & -\widehat{R}^{-1} & 0 & 0 & \widehat{K} & 0 \\ * & * & -\left(\widehat{C}^T\widehat{C}+\widehat{Q}\right)^{-1} & 0 & I & 0 \\ * & * & * & -P^{-1}+\varepsilon_1\overleftrightarrow{\Omega}\,\overleftrightarrow{\Omega}^T & \Psi & \overleftrightarrow{H} \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overleftrightarrow{\Omega} \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_2^T \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_2^T \end{bmatrix} F^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overleftrightarrow{\Omega} \\ 0 \\ 0 \end{bmatrix}^T < 0. \tag{39}$$

Using Lemma 6 again, it can be obtained that

$$\begin{bmatrix} -\varepsilon_1 I & 0 & 0 & 0 & \overleftrightarrow{E} & 0 \\ * & -\widehat{R}^{-1} & 0 & 0 & \widehat{K} & 0 \\ * & * & -\left(\widehat{C}^T\widehat{C}+\widehat{Q}\right)^{-1} & 0 & I & 0 \\ * & * & * & -P^{-1}+\varepsilon_1\overleftrightarrow{\Omega}\,\overleftrightarrow{\Omega}^T & \Psi & \overleftrightarrow{H} \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} + \varepsilon_2 \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overleftrightarrow{\Omega} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \overleftrightarrow{\Omega} \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon_2^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_2^T \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_2^T \end{bmatrix} < 0. \tag{40}$$

Using Lemma 5 again, inequality (40) is equivalent to

$$\begin{bmatrix} -\varepsilon_1 I & 0 & 0 & 0 & \overleftrightarrow{E} & 0 & 0 \\ * & -\widehat{R}^{-1} & 0 & 0 & \widehat{K} & 0 & 0 \\ * & * & -\left(\widehat{C}^T\widehat{C}+\widehat{Q}\right)^{-1} & 0 & I & 0 & 0 \\ * & * & * & -P^{-1}+(\varepsilon_1+\varepsilon_2)\overleftrightarrow{\Omega}\,\overleftrightarrow{\Omega}^T & \Psi & \overleftrightarrow{H} & 0 \\ * & * & * & * & -P & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & E_2^T \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0. \tag{41}$$

By denoting $P = \mathrm{diag}(P_1, P_2, \ldots, P_{d-1}, P_d, P_{d+1}, P_{d+2})$, naturally, we have $P^{-1} = \mathrm{diag}(P_1^{-1}, P_2^{-1}, \ldots, P_{d-1}^{-1}, P_d^{-1}, P_{d+1}^{-1}, P_{d+2}^{-1})$. By premultiplying and postmultiplying inequality (41) by $\mathrm{diag}(I, I, I, I, P^{-1}, I, I)$, with the change of variables $P_i^{-1} = X_i$ $(i = 1, 2, \ldots, d+2)$, $X = \mathrm{diag}(X_1, X_2, \ldots, X_{d-1}, X_d, X_{d+1}, X_{d+2})$, $Y = KX_{d+1}$, $\mu = \gamma^2$, it can be obtained that

$$\begin{bmatrix} -\varepsilon_1 I & 0 & 0 & 0 & \overleftrightarrow{E}X & 0 & 0 \\ * & -\widehat{R}^{-1} & 0 & 0 & \overleftrightarrow{Y} & 0 & 0 \\ * & * & -\left(\widehat{C}^T \widehat{C} + \widehat{Q}\right)^{-1} & 0 & X & 0 & 0 \\ * & * & * & -P^{-1} + (\varepsilon_1 + \varepsilon_2) \overleftrightarrow{\Omega} \overleftrightarrow{\Omega}^T & \Psi' & \overleftrightarrow{H} & 0 \\ * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & -\mu I & E_2^T \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{42}$$

where

$$\overleftrightarrow{E}X = \left[E_1 X_1 \;\; 0 \;\; \cdots \;\; 0 \;\; 0 \;\; E_3 \Phi_j Y \;\; E_3 \left(I - \Phi_j\right) X_{d+2}\right],$$

$$\Psi' = \begin{bmatrix} AX_1 & 0 & \cdots & 0 & 0 & B\Phi_j Y & B\left(I - \Phi_j\right) X_{d+2} \\ X_1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & X_2 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & X_{d-1} & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & X_d & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \Phi_j Y & \left(I - \Phi_j\right) X_{d+2} \end{bmatrix}. \tag{43}$$

Inequality (42) can be written as

$$\begin{aligned} &\Xi + \mathfrak{I}_1 \Phi \aleph_1 + \aleph_1^T \Phi^T \mathfrak{I}_1^T + \mathfrak{I}_2 \left(I - \Phi_j\right) \aleph_2 \\ &\quad + \aleph_2^T \left(I - \Phi_j\right)^T \mathfrak{I}_2^T < 0, \end{aligned} \tag{44}$$

where

$$\Xi = \begin{bmatrix} -\varepsilon_1 I & 0 & 0 & 0 & \Delta_2 & 0 & 0 \\ * & -\widehat{R}^{-1} & 0 & 0 & \overleftrightarrow{Y} & 0 & 0 \\ * & * & -\left(\widehat{C}^T \widehat{C} + \widehat{Q}\right)^{-1} & 0 & X & 0 & 0 \\ * & * & * & -P^{-1} + (\varepsilon_1 + \varepsilon_2) \overleftrightarrow{\Omega} \overleftrightarrow{\Omega}^T & \Psi'' & \overleftrightarrow{H} & 0 \\ * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & -\mu I & E_2^T \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix},$$

$$\mathfrak{I}_1 = \begin{bmatrix} B \\ 0 \\ 0 \\ \Gamma_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \aleph_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Delta_3^T \\ 0 \\ 0 \end{bmatrix}^T, \quad \mathfrak{I}_2 = \begin{bmatrix} E_3 \\ 0 \\ 0 \\ \Gamma_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \aleph_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Delta_4^T \\ 0 \\ 0 \end{bmatrix}^T, \quad \Gamma_1 = \Gamma_2 = \begin{bmatrix} B \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ I \end{bmatrix}, \tag{45}$$

$$\Psi'' = \begin{bmatrix} AX_1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ X_1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & X_2 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & X_{d-1} & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & X_d & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix}.$$

From Lemma 7, it can be obtained that

$$\begin{aligned} &\Xi + \mathfrak{I}_1 \Phi_j \aleph_1 + \aleph_1^T \Phi_j^T \mathfrak{I}_1^T + \mathfrak{I}_2 \left(I - \Phi_j\right) \aleph_2 + \aleph_2^T \left(I - \Phi_j\right)^T \mathfrak{I}_2^T \\ &\quad \leq \Xi + \sigma_1 \mathfrak{I}_1 \mathfrak{I}_1^T + \sigma_1^{-1} \aleph_1^T \aleph_1 + \sigma_2 \mathfrak{I}_2 \mathfrak{I}_1^T + \sigma_2^{-1} \aleph_2^T \aleph_2. \end{aligned} \tag{46}$$

Namely, if

$$\Xi + \sigma_1 \mathfrak{I}_1 \mathfrak{I}_1^T + \sigma_1^{-1} \aleph_1^T \aleph_1 + \sigma_2 \mathfrak{I}_2 \mathfrak{I}_1^T + \sigma_2^{-1} \aleph_2^T \aleph_2 < 0 \tag{47}$$

there is

$$\begin{aligned} &\Xi + \mathfrak{I}_1 \Phi_j \aleph_1 + \aleph_1^T \Phi_j^T \mathfrak{I}_1^T + \mathfrak{I}_2 \left(I - \Phi_j\right) \aleph_2 \\ &\quad + \aleph_2^T \left(I - \Phi_j\right)^T \mathfrak{I}_2^T < 0. \end{aligned} \tag{48}$$

Inequality (47) can be written as

$$\begin{bmatrix} -\varepsilon_1 I + \sigma_1 BB^T + \sigma_2 E_3 E_3^T & 0 & 0 & \Delta_1 & \Delta_2 & 0 & 0 \\ * & -\widehat{R}^{-1} & 0 & 0 & \overleftrightarrow{Y} & 0 & 0 \\ * & * & -\left(\widehat{C}^T \widehat{C} + \widehat{Q}\right)^{-1} & 0 & X & 0 & 0 \\ * & * & * & \Pi_1 + (\varepsilon_1 + \varepsilon_2) \overleftrightarrow{\Omega} \overleftrightarrow{\Omega}^T & \Pi_2 & \overleftrightarrow{H} & 0 \\ * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & -\mu I & E_2^T \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix}$$
$$+ \sigma_1^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Delta_3^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Delta_3^T \\ 0 \\ 0 \end{bmatrix} + \sigma_2^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Delta_4^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Delta_4^T \\ 0 \\ 0 \end{bmatrix} < 0. \quad (49)$$

Based on Lemma 5, inequality (49) is equivalent to inequality (33).

From $Y = KX_{d+1}$, we have $K = YX_{d+1}^{-1}$; thus, Theorem 8 is verified. □

$\gamma > 0$ exists not only in Definition 4 but also in Theorem 10; this paper gets the value of γ by solving the following optimization problem:

$$\begin{aligned} &\min \quad \mu \\ &\text{s.t.} \quad \text{Inequality (33)}. \end{aligned} \quad (50)$$

4. Simulations

Consider the parameters of system (2) as follows:

$$A = \begin{bmatrix} -0.079 & 0.1 \\ 0.1 & -1.01 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.08 & -0.3 \\ -0.7 & 1.7 \end{bmatrix},$$
$$\Omega = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \qquad C = \begin{bmatrix} -1.0860 & 0 \\ -0.0053 & 0 \end{bmatrix},$$
$$H = \begin{bmatrix} -0.0400 & 0.4100 \\ -0.3000 & 0.6390 \end{bmatrix}, \qquad F = \begin{bmatrix} \sin k & 0 \\ 0 & \sin k \end{bmatrix},$$
$$\Delta A = \begin{bmatrix} 0.01 \sin k & 0 \\ 0.01 \sin k & 0.012 \sin k \end{bmatrix},$$
$$\Delta B = \begin{bmatrix} 0.05 \sin k & 0 \\ 0.05 \sin k & 0.03 \sin k \end{bmatrix},$$
$$\Delta H = \begin{bmatrix} 0.04 \sin k & 0 \\ 0.04 \sin k & 0.02 \sin k \end{bmatrix},$$
$$\omega(t) = \begin{cases} [1.7, 1.8]^T, & 15 \le k \le 30, \\ 0, & \text{others}. \end{cases} \quad (51)$$

Here select the sampling period $T = 0.1$ ms, $d = 2T$.

It can be obtained that

$$E_1 = \begin{bmatrix} 0.1000 & 0 \\ 0 & 0.1200 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} -0.4000 & 0 \\ 0 & 0.2000 \end{bmatrix},$$
$$E_3 = \begin{bmatrix} 0.5000 & 0 \\ 0 & 0.3000 \end{bmatrix}. \quad (52)$$

Consider

$$\sigma_1 = 2.1000 \times 10^{-5}, \qquad \sigma_2 = 4.1000 \times 10^{-5},$$
$$Q = \begin{bmatrix} 0.0250 & 0 \\ 0 & 0.0560 \end{bmatrix}, \qquad R = \begin{bmatrix} 0.0500 & 0.0100 \\ 0.0100 & 0.0200 \end{bmatrix}. \quad (53)$$

By taking advantage of inequality (33) in Section 3, it can be obtained that

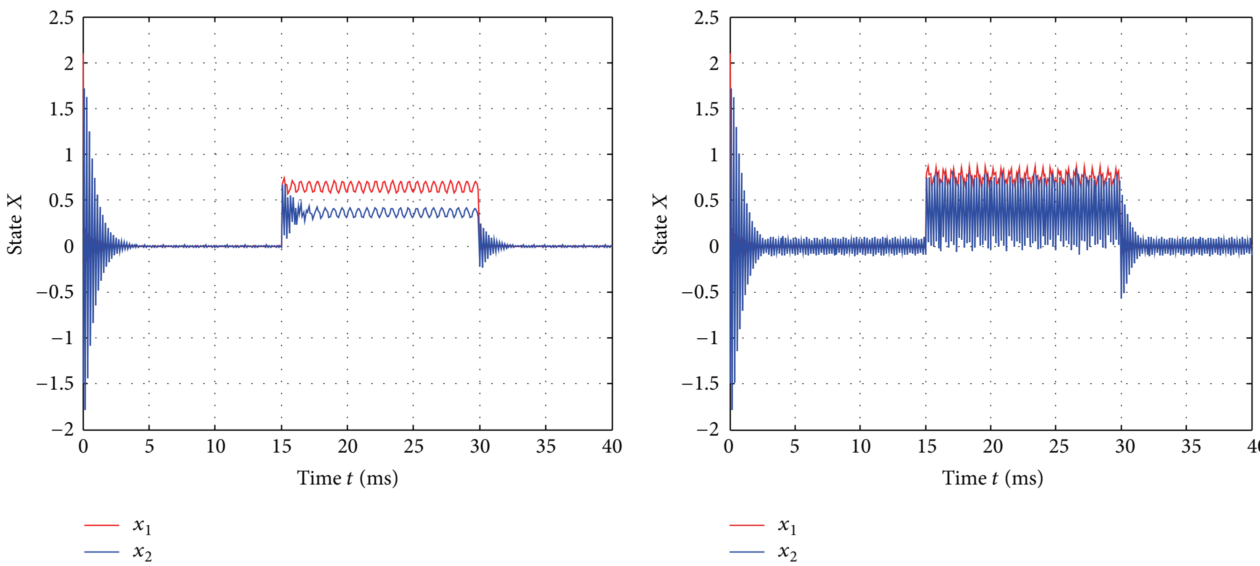
$$\gamma = \sqrt{\mu} = 90.8413, \qquad K = YX_3^{-T} = \begin{bmatrix} 0.0273 & -0.0001 \\ 0 & 0.0656 \end{bmatrix}. \quad (54)$$

The following two group thresholds of restrained transmission will be selected for simulation experiments.

(1) Select $\delta_1 = 0.0062$, and $\delta_2 = 0.0012$, in other words, when the prediction error produced by model (4) meets the following:

(a) if $|\tilde{u}_1(k)| < 0.0062$, the controller does not send $u_1(k)$ to the actuator;

(b) if $|\tilde{u}_2(k)| < 0.0012$, the controller does not send $u_2(k)$ to the actuator.



(a) State response of NCS in the threshold condition (1)

(b) State response of NCS in the threshold condition (2)

Figure 2: The state response curves of NCS.

(2) Select $\delta_1 = 0.0162$, and $\delta_2 = 0.0112$, in other words, when the prediction error produced by model (4) meets the following:

(a) if $|\tilde{u}_1(k)| < 0.0162$, the controller does not send $u_1(k)$ to the actuator;

(b) if $|\tilde{u}_2(k)| < 0.0112$, the controller does not send $u_2(k)$ to the actuator.

The initial states of system are as follows: $x(0) = \begin{bmatrix} 9.8 \\ -5 \end{bmatrix}$, $x(k) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $(k < 0)$; it can be obtained that

(1) the performance index of the system in the threshold condition (1) is $J = 9.7135$;

(2) the performance index of the system in the threshold condition (2) is $J = 7.5000$.

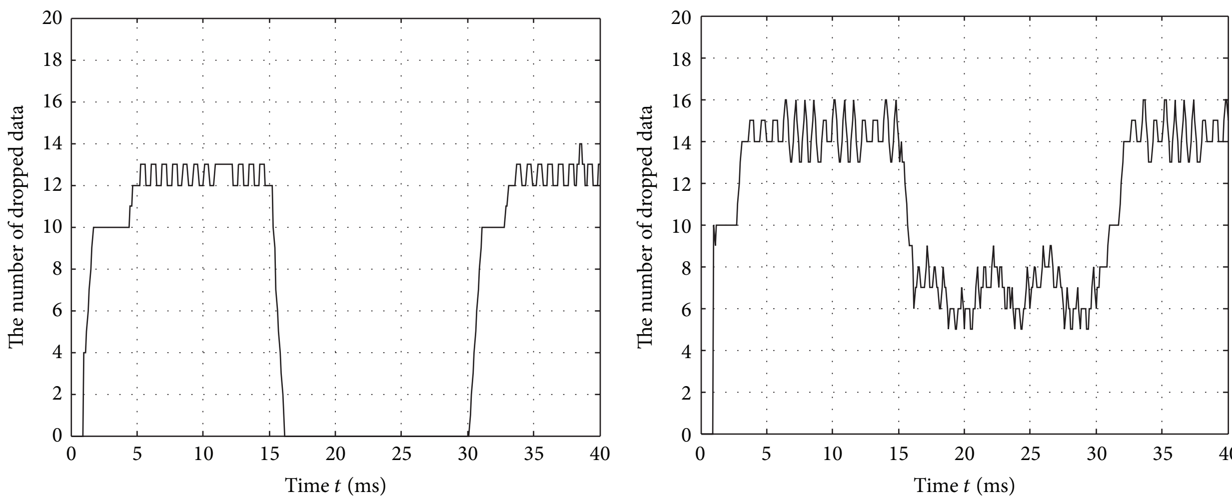
The system under scheduling policy based on predicted error is stable according to the state response curve shown in Figure 2. However, due to the characteristics of the scheduling model based on the prediction error and the system's uncertainties, the state is not keeping zero all the time but in dynamic equilibrium. The simulation results show that the changeable range of system in a state of equilibrium in the threshold condition (1) is lower than that in the threshold value condition (2). Therefore, the stability of system in the threshold condition (1) is better. It manifests that the stability of system under scheduling policy based on predicted error relates to restrained transmission threshold.

After entering the steady state, data will stop being transmitted and calculated unless interference makes the value of prediction error surpass the value of threshold. In order to facilitate comparison, the number of data packet losses at time k is obtained by calculating the total number of packet losses from time $k-19$ to time k as Figure 3. Obviously, at the beginning stage, very few packets are dropped. With the system getting closer to steady state, data transmission and calculating are terminated gradually. By the calculation, the average packet loss rate of NCS is 35.45% in the threshold value condition (1) during the whole simulation, while it can achieve 54.08% in the threshold value condition (2). Actually, because of larger value of threshold, the packet loss probability of system in the threshold value condition (2) is larger than that in the threshold value condition (1), which manifests that setting a bigger threshold value is more helpful to save energy at the actuator nodes.

In addition, we apply the method proposed by Longo et al. [12] into the same problem. The average packet loss rate of NCS is 4.72% in the threshold value condition (1) and is 6.53% in the threshold value condition (2). And the design of control fails with $K = \begin{bmatrix} -0.1375 & 0.0401 \\ 0.1646 & 0.0243 \end{bmatrix}$ shown in Figure 4. Thus, it sufficiently demonstrates the effectiveness and feasibility of this paper.

5. Conclusions

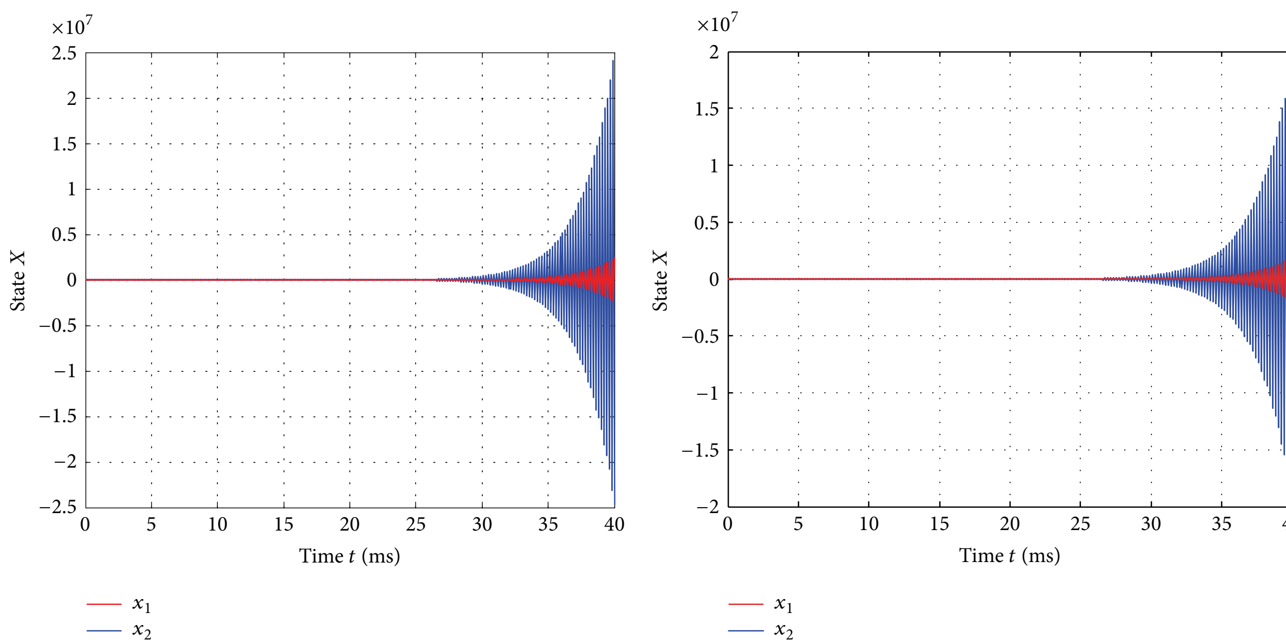
In this paper, scheduling policy based on model prediction error is presented to reduce energy consumption and network conflicts at the actuator node, where the characters of NCS are considered, such as limited network bandwidth, limited node energy, and high collision probability. The object model is introduced to predict the state of system at the sensor node. And scheduling threshold is set at the controller node. Control signal is transmitted only if the absolute value of prediction error is larger than the threshold value. And the model of NCS under scheduling policy based on predicted error is established by taking uncertain parameters and long time delay into consideration. The design method of H_∞ guaranteed cost controller is presented by using the theory of Lyapunov and linear matrix inequality (LMI). The stability of



(a) Packet losses of NCS in the threshold condition (1)

(b) Packet losses of NCS in the threshold condition (2)

Figure 3: The packet losses curves of NCS.



(a) State response of NCS in the threshold condition (1)

(b) State response of NCS in the threshold condition (2)

Figure 4: The state response curves of NCS.

NCS under scheduling policy based on predicted error relates to restrained transmission threshold. And setting different restrained transmission threshold, the number of dropped packets is obviously different. After all, the feasibility and effectiveness of method in this paper are demonstrated. The next research task will be choosing reasonable parameters σ_i $(i = 1, 2)$ to reduce the conservative.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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