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Research Article

Dynamical System and Nonlinear Regression for Estimate Host-Parasitoid Relationship

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The complex relationships of a crop with the pest, its natural enemies, and the climate factors exist in all the ecosystems, but the mathematic models has studied only some components to know the relation cause-effect. The most studied system has been concerned with the relationship pest-natural enemies such as prey-predator or host-parasitoid. The present paper shows a dynamical system for studying the relationship host-parasitoid (*Diaphorina citri*, *Tamarixia radiata*) and shows that a nonlinear model permits the estimation of the parasite nymphs using nymphs healthy as the known variable. The model showed the functional answer of the parasitoid, in which a point arrives that its density is not augmented although the number host increases, and it becomes necessary to intervene in the ecosystem. A simple algorithm is used to estimate the parasitoids level using the priori relationship between the host and the climate factors and then the nonlinear model.

1. Introduction

The study of the biological control is of obligatory inclusion in the knowledge of the relationships between ecosystem factors to maintain the equilibrium of the components. In agriculture, a variation of this relation carries on an inequilibrium favorable to the pest. In these cases, an additional meddling is necessary for the control [1].

For this, models of the relationship occurring in an agriculture ecosystem have been developed in order to find the moment and densities permitting an inequilibrium. The Lotka-Volterra model shows the dynamical relation [2] and other models have been constructed with the same objective. Some authors have used this type of model to simulate processes, but without explaining the relationships existing in the ecosystem [3, 4].

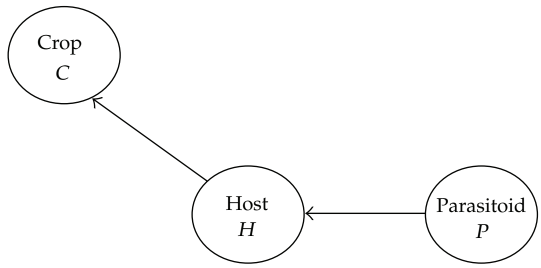

Figure 1: Three-trophic system crop-host-parasitoid. The direction of the arrows indicates a direct negative influence by one component on another. Here, C, H, and P represent young sprouts, host nymphs, and parasitoid adults, respectively.

Particularly, the Nicholson and Bailey model, or modifications of it, is used to describe the relation host-parasitoid [5]. They permit us to describe the influence of the host on the parasitoid dynamics [6]. But the models with exponential functions could be of chaotic behavior, even for a simple interaction [7]. Hence, statistical models can be used in some cases; however, the deterministic or stochastic models cannot be forgotten [8].

With the data of the samples, a statistical model can be found that proves the relationship host-parasitoid, and in the same way, the limit density for the control can be calculated.

In particular, the relation between *Diaphorina citri* Kuwayana (Hemiptera:Psyllidae) and *Tamarixia radaita* Waterston (Hymenoptera:Eulophidae) was described using a deterministic system of 11 differential equations [7]. For simulating this system, unknown biological parameters of this species were needed.

The present work is aimed at reducing the number of equations in the system guiding the study to the relation host-parasitoid and using the random sequential samples to do regression models to estimate the parasitoid percent knowing the density host in the crop.

2. Dynamical System

Even when the host has more than one natural enemy, the subsystem plant-host-parasitoid can be studied. This system is different from the system prey-predator [5], but when the parasitoid has an effect only on a specific host phase, the relation can be interpreted in the similar form because the host cannot become adult [9]. Then, for the relationship between nymphs of *D. citri* and adults of *T. radiata,* the three-trophic models represented in Figure 1 can be used.

T. radiata parasite nymphs from three to five instars present in the young sprouts of the plant and these populations can emigrate from other parts in the plants [10]. The system represented in the Figure 1 can be simulated by

$$\begin{aligned} \frac{dC}{dt} &= \alpha C(1-tC) - YCH, \\ \frac{dH}{dt} &= rCH - \mu HP, \\ \frac{dP}{dt} &= aHP - bP. \end{aligned} \tag{2.1}$$

The parameters in the models are: α and β: logistic growth of the plant young sprouts, γ: percent of damages produced in the crop by the interaction plant-pest, r: intrinsic rate of increase of the host, μ: death rate of the host, a: intrinsic rate of increase of the parasitoid, and b: death rate of the parasitoid.

3. Stability of the System

Using Derive 6.0, the equilibrium points $\overrightarrow{P_i} = (C; H; P)\; i : 1,\ 2,\ 3$ were calculated. C, H, and P represent the three variables of interest, young sprouts number, host nymphs density and parasitoid adults' density, respectively

$$P_1 : \left(0; \frac{\alpha}{\gamma}; 0\right), \tag{3.1}$$

$$P_2 : \left(\frac{1}{\beta}; 0; 0\right), \tag{3.2}$$

$$P_3 : \left(\frac{a\alpha - b\gamma}{a\alpha\beta}; \frac{b}{a}; \frac{r(a\alpha - b\gamma)}{a\alpha\beta\mu}\right). \tag{3.3}$$

The Jacobian matrix

$$\begin{bmatrix} \alpha - 2\alpha\beta C - \gamma H & -\gamma C & 0 \\ rH & rC & -\mu H \\ 0 & aP & aH - b \end{bmatrix}. \tag{3.4}$$

For (3.1), we obtain the matrix:

$$\begin{bmatrix} 0 & 0 & 0 \\ \dfrac{r\alpha}{\gamma} & 0 & -\dfrac{\alpha\mu}{\gamma} \\ 0 & 0 & \dfrac{a\alpha}{\gamma} - b \end{bmatrix}. \tag{3.5}$$

The characteristic equation is

$$P(\lambda) = \lambda^3 + \left[\frac{b\gamma - a\alpha}{\gamma}\right]\lambda^2. \tag{3.6}$$

And then, the eigenvalues for the system are

$$\lambda_1 = 0, \qquad \lambda_2 = \frac{a\alpha - b\gamma}{\gamma}. \tag{3.7}$$

As λ_1 is zero, then all the eigenvalues has not negative real part. This means that the equilibrium position is not stable.

For (3.2), we obtain the following matrix:

$$\begin{bmatrix} -\alpha & -\dfrac{\gamma}{\beta} & 0 \\ 0 & \dfrac{r}{\beta} & 0 \\ 0 & 0 & -b \end{bmatrix}. \tag{3.8}$$

The characteristic equation is

$$P(\lambda) = \beta\lambda^3 + (b\beta - r + \alpha\beta)\lambda^2 - (br - b\alpha\beta + r\alpha)\lambda - br\alpha. \tag{3.9}$$

The eigenvalues are $\lambda_1 = -b$, $\lambda_2 = r/\beta$, $\lambda_3 = -\alpha$.

As always, r/β is not negative, then the equilibrium point is not stable.

The most important point is (3.3) because all the factors of the ecosystem are present. The jacobian matrix for this point is

$$\begin{bmatrix} \dfrac{b\gamma - a\alpha}{a} & \dfrac{\gamma(b\gamma - a\alpha)}{a\alpha\beta} & 0 \\ \dfrac{br}{a} & \dfrac{r(a\alpha - b\gamma)}{a\alpha\beta} & -\dfrac{b\mu}{a} \\ 0 & \dfrac{r(a\alpha - b\gamma)}{\alpha\beta\mu} & 0 \end{bmatrix}. \tag{3.10}$$

The characteristic equation is

$$\begin{aligned} P(\lambda) = {} & a\alpha\beta\lambda^3 - \left[(r - \alpha\beta)(a\alpha - b\gamma)\right]\lambda^2 \\ & - \left[\frac{r(a^2\alpha b - a^2\alpha^2 + 3a\alpha b\gamma - ab^2\gamma - 2b^2\gamma^2}{a}\right]\lambda \\ & + \frac{br(a^2\alpha^2 - 2ab\alpha\gamma + b^2\gamma^2)}{a}. \end{aligned} \tag{3.11}$$

The eigenvalues are

$$\begin{aligned} \lambda_1 &= -\sqrt{3}Q_1\cos(Q_2) - Q_1\operatorname{sen}(Q_2) + Q_3, \\ \lambda_2 &= \sqrt{3}Q_1\cos(Q_2) - Q_1\operatorname{sen}(Q_2) + Q_3, \\ \lambda_3 &= 2Q_1\operatorname{sen}(Q_2) + Q_3, \end{aligned} \tag{3.12}$$

Q_1, Q_2 and Q_3: nonlinear combinations of the system parameters (2.1).

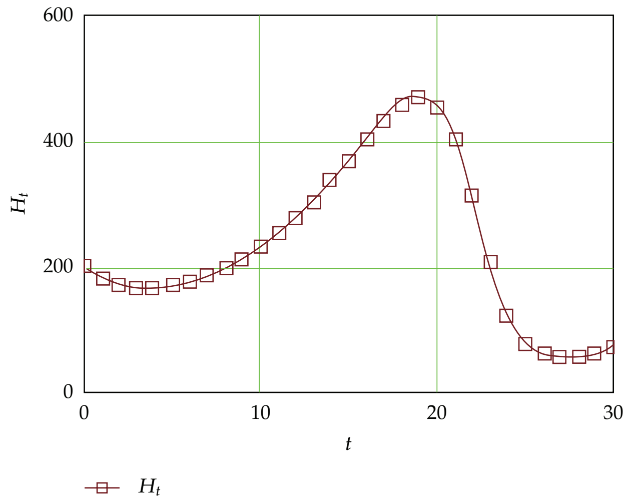

Figure 2: Stable solid line cycle for $\alpha = 0.05$, $\beta = 0.01$, $\gamma = 0.0001$, $r = 0.099$, $\mu = 0.008$, $a = 0.002$, $b = 0.5$.

Conditions to obtain a stable equilibrium can occur but, in this case, all the eigenvalues must be negative It is important to know the conditions in which all the positive coordinates of (3.3) are obtained, this condition is $a\alpha > b\gamma$. The result of multiplying the crop growth by the parasitoid birth must be higher than the loss or mortality, in a way that if the parasitoid mortality is high, the losses produced by the pest must be minimal, and otherwise the equilibrium point does not exist.

4. Numerical Result

We used Matlab to solve (2.1), the parameter values were $\alpha = 0.05$, $\beta = 0.01$, $\gamma = 0.0001$, $r = 0.099$, $\mu = 0.008$, $a = 0.002$, and $b = 0.5$, the Euler method was used, and a stable solid line cycle was obtained (Figure 2).

Figure 3 shows that a stable cycle exists, the parasitoid can control the host during a specific period of time.

A simple change in the parameter b can change the dynamics; for example, if the parameter b is very much lower than a, the host density will decrease rapidly (Figure 4). It shows that the parasitoid can be an excellent control of the host if birth is favored by the system.

5. Nonlinear Regression

To find a model to estimate the parasitoid percent knowing the pest density, weekly samples were taken during two years (from April 2007 to March 2009). The crop sampled was *Muraya paniculata* in the municipality of Playa, Havana City. The total of samplings was 82. In each sampling, they were selected five plants at random that were taken to the laboratory of entomology of the "Centro Nacional de Sanidad Agropecuaria", where it were revised to the stereo counting the total of healthy and parasite nymphs.

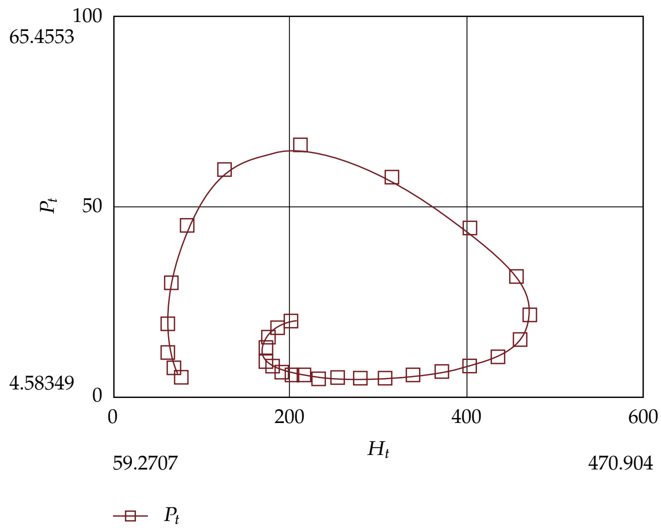

Figure 3: The stable limit cycle for the host-parasitoid relation.

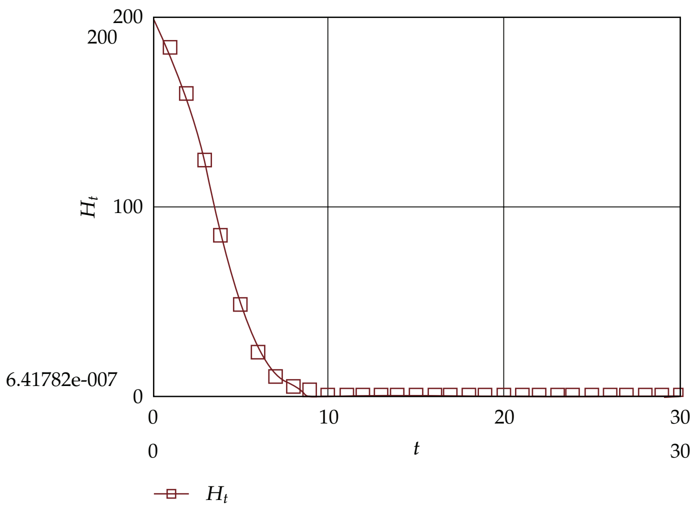

Figure 4: Host dynamics with $b \ll a$ for the (2.1).

Linear and nonlinear models were proved to find a logical relation host healthy-parasite host. The models used were: logistic, saturate growth, exponential, Weibull's model, geometric, logarithmic, potential, sinusoidal, Gauss' model, MMF model, and all the models of first, second, third, fourth, and fifth degree polynomials. The analyses were done for one year, the best model was selected using the determination coefficient, and the standard error obtained three models finally. The parameters in the polynomials models were estimated using least-square method, and in the nonlinear models, the Gauss-Newton method was used. The host density whose increment does not produce changes in the parasitism percent was calculated; in this moment the liberations of parasitoid is necessary.

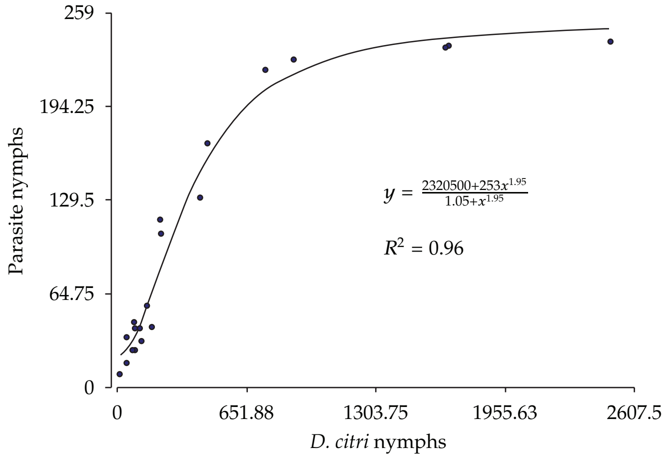

Figure 5: Relation between *D. citri* nymphs and parasitoid nymphs in *M. paniculata* in urban zone of Havana city.

For this analysis, the software CurveExpert 4.0, Derive 6.0, and the Statistical System SAS 9.0 were used.

In 2007 and 2008, the best model was the lineal model because the parasitoid percent grows while the *D. citri* density increases. In 2009, the *D. citri* nymphs number is more than other years the function response of parasitoid is slow, the parasitoid level increases until host density is very high, and the control is not possible (Figure 5).

The model

$$Y = \left(2320500 + 253x^{1.95} / \left(1.05 + x^{1.95}\right)\right) \tag{5.1}$$

as the form of the MMF model:

$$Y = \left(ab + cx^{d} / \left(b + x^{d}\right)\right) \tag{5.2}$$

with a, b, c, and d parameters show that when the population in the ecosystem is 36 nymphs or less, there exists adequate control superior to 70 percent of parasitism ($Y/X > 70$). When the nymphs overcome 1200 individuals, the parasitism is no longer effective, it remains without big variations, and the pest density increases ($Y(x + 1) \cong Y(x)$).

6. Estimation Algorithm

It is possible that the host samples are not available, but if the relationship between the populations and the climate factors is known, an algorithm can be done to predict the host level, and using (5.1), the parasitoid host density can be estimated. For this algorithm, the daily humidity and temperature values are recorded, and then a model is searched to relate the host density with these climate factors.

In Cuba, the climate has cyclical periods, and its curve can be estimated using time series. Establishment of Fourier series [11] in order to predict the temperature and humidity is possible. The algorithm to find the points (T_i, Hr_i, H_i, HP_i), where, T_i: average of the halfway temperatures found 10 days before the date of sampling i, Hr_i: average of the halfway humidity found 10 days before the date of sampling i, H_i: density of the host in the sampling i, HP_i: density of the parasitoid host in the sampling i, is the following.

(1) Obtaining of the estimated temperature using Fourier series

$$T(t) = \frac{a_0}{2} + \sum_{n=1}^{k} a_n \cos\left(\frac{n\pi t}{A}\right) + b_n \operatorname{sen}\left(\frac{n\pi t}{A}\right). \tag{6.1}$$

(2) Obtaining of the estimated relative humidity using Fourier series

$$Hr(t) = \frac{c_0}{2} + \sum_{n=1}^{k} c_n \cos\left(\frac{n\pi t}{A}\right) + d_n \operatorname{sen}\left(\frac{n\pi t}{A}\right). \tag{6.2}$$

The values of the parameters a_i, b_i, c_i, and d_i are estimated using a nonlinear regression.

The value of k is determined according to the predictive level of the models; k values of 2 and 3 are excellent levels for a good prediction [12]. In (6.1) and (6.2), the value of A is calculated by the relation $2A = P$.

(3) Finding of a function that relates the host density with temperature and relative humidity

$$H(t) = f(T(t), Hr(t)). \tag{6.3}$$

(4) Estimation of the parasitoid host density using (5.1).

7. Discussion

The dynamical system with trophic relation is an excellent estimate of the relationships in the ecosystem. Scott and Lawrence [13] used a similar dynamical system to analyze a five-dimensional trophic food. In this case, the interaction between the components was represented in a compartmental model too.

Particularly, a regression model could be used to predict a parasitoid host level for a number of hosts. An MMF model was the best model and more than 1200 *D. citri* nymphs are not controled by the parasitoid *T. radiata* of natural manner.

D. citri populations are susceptible to changes of temperature and humidity [14]. The longevity of *T. radiata* adults could be modified by temperature changes [10]. Another environment factor can provoke changes in the population density, but including the temperature and humidity in a model could be sufficient to simulate dynamic processes with a good level of prediction.

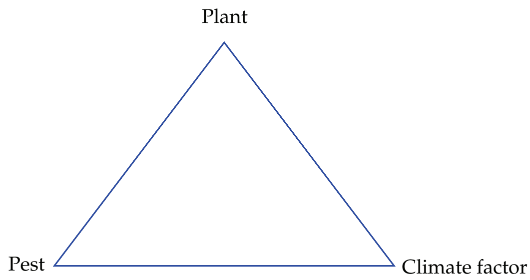

Figure 6: Cause-effect relation present in the ecosystem by De Dios et al. [15].

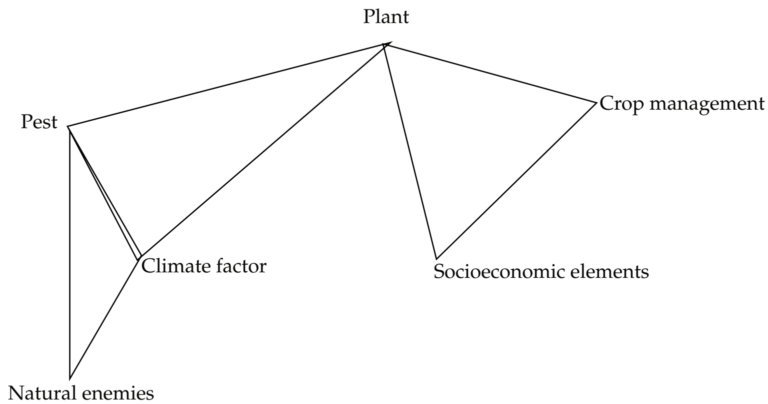

Figure 7: New proposal of nonequidistant cause-effect relation present in the ecosystem.

De Dios et al. [15] proved the existence of one interaction in triangular form plant-pest-climate (Figure 6), but this and other works [9, 16, 17] showed that the relation pest –natural enemies is the most important to explain the dynamics of populations in the ecosystem. The crop management, the agro ecological techniques, social economic factors and other aspects will influence too, and this is a new proposal of nonequidistant cause-effect relation present in the ecosystem (Figure 7). In the present work, a model to simulate the interaction pest-natural enemies-climate is developed, but all the interactions can be considered.

It is concluded that the dynamical system can be used to predict the behavior of the host, and the nonlinear regression shows that 1200 nymphs by plant indicate that is necessary applied a control method.

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