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On Hermitian separability of the next-to-leading order BFKL kernel for the adjoint representation of the gauge group in the planar $N = 4$ SYM

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Abstract We analyze a modification of the BFKL kernel for the adjoint representation of the color group in the maximally supersymmetric ($N = 4$) Yang–Mills theory in the limit of a large number of colors, related to the modification of the eigenvalues of the kernel suggested by Bondarenko and Prygarin in order to obtain Hermitian separability of the eigenvalues. We restore the modified kernel in the momentum space. It turns out that the modification is related only to the real part of the kernel and that the correction to the kernel cannot be presented by a single analytic function in the entire momentum region, which contradicts the known properties of the kernel.

1 Introduction

The kernel of the BFKL (Balitsky–Fadin–Kuraev–Lipatov) equation [1–6] contains the so-called real and virtual parts. The virtual part is determined by the gluon Regge trajectory and is the same for all representations of the color group in the t-channel. In the next-to-leading order (NLO) the calculation of the trajectory in QCD was carried out in Refs. [7–13] and was confirmed in Refs. [14,15]. The supersymmetric Yang–Mills theories contain, in addition to the gauge bosons and fermions, also scalar particles. Their contribution to the trajectory was obtained in Refs. [16,17]. The real part of the kernel comes from the real particle production. In QCD at the NLO these particles are gluons and quark–antiquark pairs. Their contributions to the kernel for the adjoint representation of the color group were calculated in Refs. [18–21] and Ref. [22], respectively. The scalar particle contribution to the real part of the kernel was obtained in Refs. [17,23,24].

It is necessary to note here that the NLO corrections to the BFKL kernel are scheme dependent because of the possibility to redistribute corrections to scattering amplitudes between the kernel and impact factors of the scattered particles [25]. The calculations in Refs. [18–24] were performed in the scheme introduced in Ref. [26], which we call the standard one. It turns out, however, that in the $N = 4$ supersymmetric Yang–Mills theory ($N = 4$ SYM) in the planar limit another scheme, which we call conformal, is more convenient. It is associated with the modified kernel $\mathcal{K}_m$, introduced in Ref. [27], which is obtained from the usual BFKL kernel in the adjoint representation by subtraction of the gluon trajectory depending on the total t-channel momentum. One of advantages of this kernel is its infrared safety, which permits to consider this kernel at physical transverse dimension $D - 2 = 2$. This advantage is manifested in all Yang–Mills theories. Another important advantage, manifested in the $N = 4$ SYM, is the dual conformal invariance, i.e. invariance under Möbius transformations in the space of dual two-dimensional transverse momenta. In the leading order (LO) the invariance of $\mathcal{K}_m$ is easily seen [27]. However, in the NLO in the standard scheme, in which the kernel was initially calculated, $\mathcal{K}_m$ is not Möbius invariant. The existence of the scheme where the modified kernel is Möbius invariant (Möbius scheme) was conjectured in Ref. [28] and then proved in Ref. [29], where the transformation of the kernel from the standard form to the conformal (Möbius invariant) form $\mathcal{K}_c$ was found explicitly.

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The eigenvalues $\omega(t)$ of the kernel $\mathcal{K}_m$ calculated in the NLO in [28] are written as

$$\omega(\nu,n) = -a\,(E_{\nu n} + a\,\epsilon_{\nu n})\,, \quad a = \frac{g^2 N_c}{8\pi^2}, \tag{1}$$

where $E_{\nu n}$ is the "energy" in the leading approximation [27], given by

$$E_{\nu n} = -\frac{1}{2}\,\frac{|n|}{\nu^2+\frac{n^2}{4}} + \psi\left(1+i\nu+\frac{|n|}{2}\right) + \psi\left(1-i\nu+\frac{|n|}{2}\right) - 2\psi(1), \quad \psi(x) = (\ln\Gamma(x))', \tag{2}$$

and the next-to-leading correction $\epsilon_{\nu n}$ can be written as follows:

$$\epsilon_{\nu n} = -\frac{1}{4}\Bigg[\psi''\left(1+i\nu+\frac{|n|}{2}\right) + \psi''\left(1-i\nu+\frac{|n|}{2}\right) + \frac{2i\nu\left(\psi'(1-i\nu+\frac{|n|}{2}) - \psi'(1+i\nu+\frac{|n|}{2})\right)}{\nu^2+\frac{n^2}{4}}\Bigg] - \zeta(2)\,E_{\nu n} - 3\zeta(3) - \frac{1}{4}\,\frac{|n|\,(\nu^2-\frac{n^2}{4})}{\left(\nu^2+\frac{n^2}{4}\right)^3}\,. \tag{3}$$

Here the $\zeta(n)$ is the Riemann zeta-function.

Recently in Ref. [30] the modification of the eigenvalues (3) was suggested so that they acquired the property of Hermitian separability present for the singlet BFKL kernel [31]. After this modification the adjoint NLO BFKL eigenvalues are expressed through holomorphic and antiholomorphic parts of the leading order eigenvalue and their derivatives. It was argued that the proposed choice of the modified NLO expression is supported by the fact that it is possible to obtain the same result in a relatively straightforward way directly from the singlet NLO BFKL eigenvalue replacing the alternating series by a series of constant sign.

2 The modification of the kernel

The proposed modification of the eigenvalues (1)–(3) is

$$\omega(\nu,n) \to \omega(\nu,n) + \Delta\omega(\nu,n),$$

$$\Delta\omega(\nu,n) = \frac{a^2}{2}\left(\frac{i\nu|n|}{(\nu^2+\frac{n^2}{4})^2} - \psi'\left(1+i\nu+\frac{|n|}{2}\right) + \psi'\left(1-i\nu+\frac{|n|}{2}\right)\right) \times\left(\psi\left(1+i\nu+\frac{|n|}{2}\right) - \psi\left(1-i\nu+\frac{|n|}{2}\right)\right). \tag{4}$$

Evidently, the difference in the eigenvalues means the difference in the kernels:

$$\hat{\mathcal{K}} \to \hat{\mathcal{K}} + \Delta\hat{\mathcal{K}}\,.$$

Formally, one can write

$$\Delta\hat{\mathcal{K}} = \sum_{n=-\infty}^{n=+\infty}\int_{-\infty}^{+\infty} d\nu\,\Delta\omega(\nu,n)|\nu,n\rangle\langle\nu,n|,$$

where $|\nu,n\rangle$ are the eigenstates of the kernel normalized as

$$\langle\nu',n'|\nu,n\rangle = \delta_{nn'}\delta(\nu'-\nu).$$

In the Möbius scheme, the eigenfunctions $\langle\vec{q}_1,\vec{q}_2|\nu,n\rangle = \phi_{\nu,n}(\vec{q}_1,\vec{q}_2)$ in the momentum space can be taken as in Refs. [28] and [29], i.e. as

$$\phi_{\nu,n}(q_1,q_2) = f_{\nu,n}\left(\frac{q_1}{q_2}\right) = \frac{1}{\sqrt{2\pi^2}}\left(\frac{q_1}{q_2}\right)^{\frac{n}{2}+i\nu}\left(\frac{q_1^*}{q_2^*}\right)^{-\frac{n}{2}+i\nu}, \quad q_1+q_2 = q, \tag{5}$$

with the normalization

$$\int\frac{\vec{q}^{\,2}d\vec{q}_1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left(\phi_{\nu,n}(q_1,q_2)\right)^*\phi_{\mu,m}(q_1,q_2) = \int\frac{d^2z}{|z|^2}\,f^*_{\nu,n}(z)\,f_{\mu,m}(z) = \delta(\mu-\nu)\,\delta_{mn}\,. \tag{6}$$

Here we use the complex notations $q = q_x + iq_y$ and $q^* = q_x - iq_y$. Then we can represent the difference in the kernel as follows:

$$\Delta K_c(\vec{q}_1,\vec{q}_1^{\,\prime};\vec{q}) = \sum_{n=-\infty}^{n=+\infty}\int_{-\infty}^{+\infty} d\nu\ \Delta\omega(\nu,n)\phi_{\nu n}(q_1,q_2)\left(\phi_{\nu n}(q_1',q_2')\right)^*\,. \tag{7}$$

Let us define

$$f_1(z) = \sum_{n=-\infty}^{n=+\infty}\int_{-\infty}^{+\infty} d\nu\ \frac{1}{2\pi^2}|z^2|^{i\nu}\left(\frac{z}{z^*}\right)^{\frac{n}{2}}\frac{i\nu|n|}{(\nu^2+\frac{n^2}{4})^2} \times\left(\psi\left(1+i\nu+\frac{|n|}{2}\right) - \psi\left(1-i\nu+\frac{|n|}{2}\right)\right) \tag{8}$$

and

$$f_2(z) = \sum_{n=-\infty}^{n=+\infty}\int_{-\infty}^{+\infty} d\nu\ \frac{1}{2\pi^2}|z^2|^{i\nu}\left(\frac{z}{z^*}\right)^{\frac{n}{2}}\left(\psi'\left(1-i\nu+\frac{|n|}{2}\right) - \psi'\left(1+i\nu+\frac{|n|}{2}\right)\right) \times\left(\psi\left(1+i\nu+\frac{|n|}{2}\right) - \psi\left(1-i\nu+\frac{|n|}{2}\right)\right), \tag{9}$$

so that

$$\Delta K_c(\vec{q}_1, \vec{q}_1'; \vec{q}) = \frac{a^2}{2} F(z), \quad F(z) = f_1(z) + f_2(z)\,, \quad (10)$$

where $z = q_1 q_2'/(q_2 q_1')$.

At $|z| < 1$ the integrals over ν in Eqs. (8) and (9) can be calculated by taking residues in the lower half-plane of ν. Taking into account that $\psi(x)$ is an analytical function of x having only poles with residues equal to -1 at $x = -l$, l being a natural number, we obtain for $f_1(z)$

$$f_1(z) = \frac{1}{2\pi} \sum_{n=1}^{\infty} z^n \Big[\ln|z^2| (\psi(1) - \psi(1+n)) - \psi'(1+n) - \psi'(1) + \sum_{l=0}^{\infty} |z^2|^{l+1} \left(\frac{1}{(l+1)^2} - \frac{1}{(l+n+1)^2} \right) \Big] + c.c.\,, \quad (11)$$

where $c.c.$ means complex conjugate. Using the relations

$$\sum_{n=1}^{\infty} a^n (\psi(1+n) - \psi(1)) = -\frac{\ln(1-a)}{1-a},$$
$$\sum_{n=1}^{\infty} a^n (\psi'(1+n) + \psi'(1)) = \frac{2a\zeta(2) - Li_2(a)}{1-a}\,,$$
$$\sum_{l=0}^{\infty} \frac{a^{l+1}}{(l+1)^2} = Li_2(a),$$
$$\sum_{n=0}^{\infty} z^n \sum_{l=0}^{\infty} |z^2|^{l+1} \frac{1}{(l+n+1)^2} = z^* \frac{Li_2(z) - Li_2(|z^2|)}{1-z^*}, \quad (12)$$

where

$$Li_2(x) = -\int_0^1 \frac{dy}{y} \ln(1-xy), \quad Li_2(1) = \zeta(2)\,, \quad (13)$$

we obtain

$$f_1(z) = \frac{1}{2\pi} \Big[\ln|z^2| \left(\frac{\ln(1-z)}{1-z} + \frac{\ln(1-z^*)}{1-z^*} \right) + 2\frac{1-|z^2|}{|1-z|^2} Li_2(|z^2|) + \frac{1-2z^*+|z^2|}{|1-z|^2} Li_2(z) + \frac{1-2z+|z^2|}{|1-z|^2} Li_2(z^*) - 2\frac{z+z^*-2|z^2|}{|1-z|^2}\zeta(2) \Big]. \quad (14)$$

For the function $f_2(z)$, taking into account that

$$\psi(x)\psi'(x)|_{x\to -l} = -\frac{1}{(x+l)^3} + \frac{\psi(1+l)}{(x+l)^2} + \text{constant}, \quad (15)$$

where l is a natural number, we have at $|z| < 1$

$$f_2(z) = \frac{1}{2\pi} \sum_{n=0}^{\infty} \left(1 - \frac{1}{2}\delta_{n,0}\right) z^n \times \sum_{l=0}^{\infty} |z^2|^{l+1} \Big[\ln^2|z^2| + 2\ln|z^2| (\psi(1+l) - \psi(2+n+l)) - 4\psi'(2+n+l) \Big] + c.c.. \quad (16)$$

The equalities

$$\sum_{l=0}^{\infty} a^{1+l} \psi(2+l) = \frac{a\psi(1) - \ln(1-a)}{1-a},$$
$$\sum_{l=0}^{\infty} a^{1+l} \psi'(2+l) = \frac{a\zeta(2) - Li_2(a)}{1-a},$$
$$\sum_{n=1}^{\infty} z^n \sum_{l=0}^{\infty} |z^2|^{l+1} \psi(2+n+l) = \frac{1}{1-z^*} \times \left[\frac{\ln(1-|z^2|) - |z^4|\psi(1)}{1-|z^2|} - z^* \frac{\ln(1-z) - z^2\psi(1)}{1-z} \right],$$
$$\sum_{n=1}^{\infty} z^n \sum_{l=0}^{\infty} |z^2|^{l+1} \psi'(2+n+l) = \frac{1}{1-z^*} \times \left[\frac{Li_2(|z^2|) - |z^2|\zeta_2}{1-|z^2|} - z^* \frac{Li_2(z) - z\zeta(2)}{1-z^2} \right] \quad (17)$$

give us

$$f_2(z) = \frac{1}{2\pi|1-z|^2} \Big[|z^2| \ln^2|z^2| - 2\ln|z^2| \Big((1+|z^2|) \times \ln(1-|z^2|) - z^* \ln(1-z) - z\ln(1-z^*) \Big) - 4Li_2(|z^2|) + 4z^* Li_2(z) + 4z Li_2(z^*) - 4|z^2|\zeta(2) \Big]. \quad (18)$$

For the sum $F(z) = f_1(z) + f_1(z)$ we obtain at $|z| < 1$

$$F(z) = \frac{1}{2\pi(|1-z|^2)} \Big[|z^2| \ln^2|z^2| - \ln|z^2| \times \Big(2(1+|z^2|)\ln(1-|z^2|) - (1+z^*)\ln(1-z) - (1+z)\ln(1-z^*) \Big) - 2(1+|z^2|) Li_2(|z^2|) + (1+2z^*+|z^2|) Li_2(z) + (1+2z+|z^2|) Li_2(z^*) - 2(z^*+z)\zeta(2) \Big]. \quad (19)$$

Here it should be noted that the functions $f_1(z)$ and $f_2(z)$ (and hence their sum $F(z)$) are defined by Eqs. (8) and (9) both for $|z| < 1$ and for $|z| > 1$; moreover, due to the property $\Delta\omega(\nu, n)$ (see Eq. (4))

$$\Delta\omega(-\nu, -n) = \Delta\omega(\nu, n), \quad (20)$$

we must have

$$F(z) = F\left(\frac{1}{z}\right). \tag{21}$$

Together with Eq. (19), which gives the function $F(z)$ in the region $|z| < 1$, Eq. (21) determines $F(z)$ in the region $|z| > 1$. On the other hand, the right side of Eq. (19) gives the function $\mathcal{F}(z)$ in the whole plane of z. It turns out, however, that at $|z| > 1$ the function $\mathcal{F}(z)$ does not coincide with $F(z)$ determined by Eq. (21). Indeed, it is seen from Eq. (19) that the function $\mathcal{F}(z)$ has a cut starting at $z = 1$ and is not a single valued function. To see this clearly one can rewrite $\mathcal{F}(z)$, using the relation

$$Li_2(x) + Li_2(1-x) = \zeta(2) - \ln(x)\ln(1-x), \tag{22}$$

in the form

$$\begin{aligned}\mathcal{F}(z) = \frac{1}{2\pi(|1-z|^2)}\Bigg[&|z^2|\ln^2|z^2| + 2(1+|z^2|)Li_2(1-|z^2|)\\
&-(1+2z^*+|z^2|)Li_2(1-z)\\
&-(1+2z+|z^2|)Li_2(1-z^*)\\
&+\frac{1}{2}(1-|z^2|)\ln|z^2|\ln|1-z|^2\\
&+\frac{z-z^*}{2}\ln\frac{z}{z^*}\ln|1-z|^2\\
&-\frac{|1+z|^2}{2}\ln\frac{z}{z^*}\ln\frac{1-z}{1-z^*}\Bigg].\end{aligned} \tag{23}$$

It is easy to see that all terms besides the last one are single valued around the point $z = 1$, but the last one does not have such a property. Of course, $\mathcal{F}(z)$ is a single valued function at $|z| < 1$; but this property is lost in the whole z-plane. It means, in particular, that $\mathcal{F}(z) \neq \mathcal{F}(\frac{1}{z})$. This can be shown explicitly from Eq. (23) using the relation

$$Li_2(1-x) + Li_2(1-1/x) = -\frac{1}{2}\ln^2(x)\ . \tag{24}$$

It gives

$$\begin{aligned}\mathcal{F}\left(\frac{1}{z}\right) = \frac{1}{2\pi|1-z|^2}\Bigg[&-\frac{|1-z|^2}{4}\ln^2|z^2|\\
&-2(1+|z^2|)Li_2(1-|z^2|)\\
&+(1+2z+|z^2|)Li_2(1-z)\\
&+(1+2z^*+|z^2|)Li_2(1-z^*)\\
&+\frac{1}{2}(1-|z^2|)\ln|z^2|\ln|1-z|^2\\
&+\frac{z-z^*}{2}\ln\frac{z}{z^*}\ln|1-z|^2 - \frac{|1+z|^2}{4}\ln^2\frac{z}{z^*}\\
&+\frac{|1+z|^2}{2}\ln\frac{z}{z^*}\ln\frac{z-1}{z^*-1}\Bigg].\end{aligned} \tag{25}$$

Note that the point $z = 1$ is the only singular point of the function $\mathcal{F}(z)$ in the closed circle $|z| \leq 1$. Moreover, it is easily seen from Eq. (23) that the singularity of $\mathcal{F}(z)$ in this point is an integrable one. It means that the modification of the eigenvalues (4) is related only with the real part of the kernel. Thus, we see that the modification of the BFKL kernel corresponding to the modification of the eigenvalues suggested in Ref. [30] is written as

$$\Delta K_c(\vec{q}_1, \vec{q}_1,{}'; q) = \begin{cases} \frac{a^2}{2}F\left(\frac{q_1 q_2'}{q_2 q_1'}\right) \text{ if } \left|\frac{q_1 q_2'}{q_2 q_1'}\right| \leq 1 \\ \frac{a^2}{2}F\left(\frac{q_2 q_1'}{q_1 q_2'}\right) \text{ if } \left|\frac{q_1 q_2'}{q_2 q_1'}\right| \geq 1 \end{cases}, \tag{26}$$

where $F(z)$ is defined in Eq. (19), $q_1 + q_2 = q_1' + q_2' = q$, and it cannot be presented by a single analytic function in the entire domain.

3 Conclusion

We found the correction (26) to the BFKL kernel for the adjoint representation of the color group in the planar $N = 4$ SYM corresponding to the modification of the eigenvalues of the kernel suggested in Ref. [30]. It turned out that this correction is related only to the real part of the kernel. However, it cannot be presented by one analytic function in the entire region of transverse momenta, contrary to the real parts of the kernel in the Möbius [29] and standard [18–24] schemes. Note that the real part in the standard scheme was found for arbitrary space-time dimension, therefore the argument of Ref. [30] in favor of the modification, based on removal of the infrared divergences, seems untenable.

In our opinion, the other arguments of Ref. [30] in favor of the modification are also inconsistent. The ambiguity of the NLO kernel because of the possibility to redistribute NLO corrections between the kernel and impact factors is irrelevant, because transformations of the kernel admitting to this ambiguity do not change their eigenvalues. This is clearly seen from the fact that a change of eigenvalues means a change of the dependence on energy, whereas impact factors are energy independent by definition. It was argued also in Ref. [30] that the modification is supported by the fact that it is possible to obtain the same result in a relatively straightforward way directly from the singlet NLO BFKL eigenvalue, replacing the alternating series by a series of constant sign. But this cannot be a serious argument because there is no simple relation between singlet and adjoint kernels.

Thus, the modification of the eigenvalues of the BFKL kernel suggested in Ref. [30] contradicts the known properties of the kernel, and the main motivation for this modification—the Hermitian separability of the eigenvalues—does not have serious grounds.

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References

1. V.S. Fadin, E.A. Kuraev, L.N. Lipatov, Phys. Lett. B **60**, 50 (1975)
2. E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Z. Eksp. Teor. Fiz. **71**, 840 (1976)
3. E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP **44**, 443 (1976)
4. E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Z. Eksp. Teor. Fiz. **72**, 377 (1977)
5. E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP **45**, 199 (1977)
6. I.I. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978)
7. V.S. Fadin, Pisma. Zh. Eksp. Teor. Fiz. **61**, 342 (1995)
8. V.S. Fadin, JETP Lett. **61**, 346 (1995)
9. V.S. Fadin, M.I. Kotsky, R. Fiore, Phys. Lett. B **359**, 181 (1995)
10. V.S. Fadin, R. Fiore, A. Quartarolo, Phys. Rev. D **53**, 2729 (1996). arXiv:hep-ph/9506432
11. M.I. Kotsky, V.S. Fadin, Yad. Fiz. **59N6**, 1080 (1996)
12. M.I. Kotsky, V.S. Fadin, Phys. Atom. Nucl. **59**, 1035 (1996)
13. V.S. Fadin, R. Fiore, M.I. Kotsky, Phys. Lett. B **387**, 593 (1996). arXiv:hep-ph/9605357
14. J. Blumlein, V. Ravindran, W.L. van Neerven, Phys. Rev. D **58**, 091502 (1998). arXiv:hep-ph/9806357
15. V. Del Duca, E.W.N. Glover, JHEP **0110**, 035 (2001). arXiv:hep-ph/0109028
16. A.V. Kotikov, L.N. Lipatov, Nucl. Phys. B **582**, 19 (2000). arXiv:hep-ph/0004008
17. V.S. Fadin, R. Fiore, Phys. Lett. B **661**, 139 (2008). arXiv:0712.3901 [hep-ph]
18. V.S. Fadin, D.A. Gorbachev, JETP Lett. **71**, 222 (2000)
19. V.S. Fadin, D.A. Gorbachev, Pisma Zh. Eksp. Teor. Fiz. **71**, 322 (2000)
20. V.S. Fadin, D.A. Gorbachev, Phys. Atom. Nucl. **63**, 2157 (2000)
21. V.S. Fadin, D.A. Gorbachev, Yad. Fiz. **63**, 2253 (2000)
22. V.S. Fadin, R. Fiore, A. Papa, Phys. Rev. D **60**, 074025 (1999). arXiv:hep-ph/9812456
23. R.E. Gerasimov, V.S. Fadin, Phys. Atom. Nucl. **73**, 1214 (2010)
24. R.E. Gerasimov, V.S. Fadin, Yad. Fiz. **73**, 1254 (2010)
25. V.S. Fadin, R. Fiore, A. Papa, Nucl. Phys. B **769**, 108 (2007). arXiv:hep-ph/0612284
26. V.S. Fadin, R. Fiore, Phys. Lett. B **440**, 359 (1998). arXiv:hep-ph/9807472
27. J. Bartels, L.N. Lipatov, A. Sabio-Vera, Eur. Phys. J. C **65**, 587 (2010). arXiv:0807.0894 [hep-th]
28. V.S. Fadin, L.N. Lipatov, Phys. Lett. B **706**, 470 (2012). arXiv:1111.0782 [hep-th]
29. V.S. Fadin, R. Fiore, L.N. Lipatov, A. Papa, Nucl. Phys. B **874**, 230 (2013)
30. S. Bondarenko and A. Prygarin, arXiv:1510.00589 [hep-th]
31. A.V. Kotikov, L.N. Lipatov, Nucl. Phys. B **661**, 19 (2003). arXiv:hep-ph/0208220. Erratum: Nucl. Phys. B **685**, 405 (2004)