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New complex exact travelling wave solutions for the generalized-Zakharov equation with complex structures

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Abstract. In this paper, we apply the sine-Gordon expansion method which is one of the powerful methods to the generalized-Zakharov equation with complex structure. This algorithm yields new complex hyperbolic function solutions to the generalized-Zakharov equation with complex structure. Wolfram Mathematica 9 has been used throughout the paper for plotting two- and three-dimensional surface of travelling wave solutions obtained.

Keywords: The sine-Gordon expansion method; generalized-Zakharov equation with complex structure; complex hyperbolic function solution; dark soliton solutions.
AMS Classification: 35Axx; 35Cxx; 34Mxx

1. Introduction

The new complex exact travelling wave solutions of nonlinear partial differential equations plays an important role in various fields such as engineering, plasma physics, solid state physics, optical fibers, quantum field theory, hydrodynamics, fluid dynamics and applied sciences. They submit to the literature new reviews in terms of better understanding of mathematical models of physical problems. Especially various type travelling wavesolutions such as dark, complex, elliptic, Jacobi elliptic, exponential, rational, hyperbolic and trigonometric function solutions means that they have new properties of physical problems. In the process many powerful methods such as sumudu transform method, Riccati-Bernoulli sub-ODE method, G'/G-expansion method, Exp-function method, Fitted finite difference method, extended jacobi elliptic function expansion method, modified simple equation method and Generalized Bernoulli Sub-ODE method, functional variable method, variational iteration method, improved Bernoulli sub-equation function method, Laplace-variationaliteration method, finite difference method, generalized Kudryashov method and so on have been used to find new solutions of nonlinear evolution equations [1-14,27-50]. In the rest of this paper, we present the general properties of the sine-Gordon expansion method(SGEM) in comprehensive manner in section 2. In section 3, we obtain the complex travelling wavesolutions to the generalized- Zakharov equation with complex structure which reads as following [15]:

$$\begin{aligned} &iu_t + u_{xx} - 2a|u|^2 u + 2uv = 0, \\ &v_{tt} - v_{xx} + \left(|u|^2\right)_{xx} = 0, \end{aligned} \tag{1}$$

where a is real constants and non-zero. In the last section of manuscript, a comprahensive conclusion has been submitted by mentioning significant properties of $u(x,t)$ and $v(x,t)$.

Shi Jin, P. A. Markowich and C. Zheng have applied the time-splitting spectral method for obtaining numerical solutions of Eq.(1) [24]. Yuhuai Sun et al. have considered the first integral method for finding exact explicit solutions of Eq.(1) [25]. Malomed B. et al. have investigated

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the Dynamics of Solitary Waves of Eq.(1) [26].

2. General facts of the SGEM

Let's consider the following sine-Gordon equation [16-18, 51];

$$u_{xx}-u_{tt}=m^2\sin(u), \tag{2}$$

where $u=u(x,t)$, and m is real constant. When we apply the wave transform $\xi=\mu(x-ct)$ to Eq.(2), we obtain the nonlinear ordinary differential equation (NODE) as following;

$$U''=\frac{m^2}{\mu^2(1-c^2)}\sin(U), \tag{3}$$

where $U=U(\xi)$, and, ξ is the amplitude of the travelling wave, c is the velocity of the travelling wave. If we reconsider Eq.(3), we can write in the full simplified version as following;

$$\left[\left(\frac{U}{2}\right)'\right]^2=\frac{m^2}{\mu^2(1-c^2)}\sin^2\left(\frac{U}{2}\right)+K, \tag{4}$$

where K is the integration constant. When we resubmit as $K=0,\ w(\xi)=\frac{U}{2},$ and $a^2=\frac{m^2}{\mu^2(1-c^2)}$ in Eq.(4), we can obtain following equation;

$$w'=a\sin(w). \tag{5}$$

If we put as $a=1$ in Eq.(5)($a=1$, for convenience [16]), we can obtain following equation;

$$w'=\sin(w). \tag{6}$$

If we solve Eq.(6) by using separation of variables, we find the following two significant equations;

$$\sin(w)=\sin(w(\xi))=\left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1}, \quad =\sec h(\xi), \tag{7}$$

or

$$\cos(w)=\cos(w(\xi))=\left.\frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\right|_{p=1}, \quad =\tanh(\xi), \tag{8}$$

where p is the integral constant and non-zero. For obtaining the solution of following nonlinear partial differential equation;

$$P(u,u_x,u_t,\cdots)=0, \tag{9}$$

let's consider as

$$U(\xi)=\sum_{i=1}^{n}\tanh^{i-1}(\xi)[B_i\sec h(\xi)+A_i\tanh(\xi)]+A_0. \tag{10}$$

We can rewrite Eq.(10) according to Eqs.(7,8) as following;

$$U(w)=\sum_{i=1}^{n}\cos^{i-1}(w)[B_i\sin(w)+A_i\cos(w)]+A_0. \tag{11}$$

Under the terms of homogenous balance technique, we can determine the values of n under the terms of $NODE$. Let the coefficients of $\sin^i(w)\cos^j(w)$ all be zero, it yields a system of equations. Solving this system by using Wolfram Mathematica 9 give the values of A_i, B_i, μ, c. Finally, substituting the values of A_i, B_i, μ, c in Eq.(10), we can find the new travelling wave solutions to the Eq.(9).

3. Implementations of proposed method

In this subsection of this paper, we provide some experimental results to illustrate the performance of the travelling wave algorithm proposed.

Example: We consider the traveling wave transformation defined by

$$u(x,t)=e^{i\theta}U(\xi),\ \theta=\alpha x+\beta t, \quad v(x,t)=V(\xi),\ \xi=x-2\alpha t, \tag{12}$$

where α,β are real constant and non-zero. When we can apply Eq.(12) to the Eq.(1), we can find the following $NODE$ under the some simplifications [15];

$$V(\xi)=\frac{c-U^2(\xi)}{4\alpha^2-1}, \tag{13}$$

where c is second integration constant and the first one is taken to zero. Considering Eq.(13), we rewrite the following ODE [15];

$$RU''+SU+TU^3=0, \tag{14}$$

where $R=-1,\ S=\beta+\alpha^2-\frac{2c}{4\alpha^2-1},$ $T=2\left(a+\frac{1}{4\alpha^2-1}\right)$. When we reconsider the Eq.(11) for homogenous balance method between U'' and U^3, we obtain the value of n as following;

$$n=1. \tag{15}$$

If we put Eq.(15) in Eq.(11), we obtain follows;

$$U(w)=B_1\sin(w)+A_1\cos(w)+A_0, \tag{16}$$

$$U'(w)=B_1\cos(w)\sin(w)-A_1\sin^2(w), \tag{17}$$

$$U''(w)=B_1\left[\cos^2(w)\sin(w)-\sin^3(w)\right] -2A_1\sin^2(w)\cos(w). \tag{18}$$

Substituting Eqs.(16,18) in Eq.(14) by using Wolfram Mathematica 9, we can obtain following equation;

$$\begin{aligned}&SA_0+TA_0^3+S\cos(w)A_1+3TA_0^2A_1\cos(w)\\&-2RA_1\cos(w)\sin^2(w)+3TA_1^2A_0\cos^2(w)\\&+TA_1^3\cos^3(w)+SB_1\sin(w)+TB_1^3\sin^3(w)\\&+B_1R\sin(w)\cos^2(w)-RB_1\sin^3(w)\\&+3TA_0^2B_1\sin(w)+6TA_0A_1B_1\sin(w)\cos(w)\\&+3TA_1^2B_1\sin(w)\cos^2(w)+3TB_1^2A_0\sin^2(w)\\&+3TB_1^2A_1\cos(w)\sin^2(w)=0.\end{aligned} \tag{19}$$

When we equal to zero all the same power of trigonometric terms, we find the following equations;

$$\begin{aligned}&Cons\tan t\ :SA_0+TA_0^3+3TA_1^2A_0=0,\\&\sin(w)\quad :SB_1+3TA_0^2B_1=0,\\&\cos(w)\quad :SA_1-2RA_1+3TA_0^2A_1+3TB_1^2A_1=0,\\&\sin^2(w)\quad :3TB_1^2A_0-3TA_1^2A_0=0,\\&\sin(w)\cos(w)\ :6TA_0A_1B_1=0,\\&\cos^2(w)\sin(w):B_1R+3TA_1^2B_1=0,\\&\sin^3(w)\quad :TB_1^3-RB_1=0,\\&\cos^3(w)\quad :2RA_1+TA_1^3-3TB_1^2A_1=0.\end{aligned} \tag{20}$$

Solving the system of equations Eq.(20) yields the following coefficients:

$$\begin{aligned}&A_0=0, A_1=\frac{-\sqrt{-1+4\alpha^2}}{\sqrt{1+a\left(-1+4\alpha^2\right)}},\\&\beta=\frac{2+2c-7\alpha^2-4\alpha^4}{-1+4\alpha^2}, B_1=0.\end{aligned} \tag{21}$$

$$\begin{aligned}&A_0=0, A_1=\frac{\sqrt{-1+4\alpha^2}}{\sqrt{1+a\left(-1+4\alpha^2\right)}},\\&\beta=\frac{2+2c-7\alpha^2-4\alpha^4}{-1+4\alpha^2}, B_1=0.\end{aligned} \tag{22}$$

$$\begin{aligned}&A_0=0, A_1=A_1, B_1=0,\\&c=\frac{1}{2}\left(-1+4\alpha^2\right)\left(2+\alpha^2+\beta\right),\\&a=\frac{1}{1-4\alpha^2}+\frac{1}{A_1^2}.\end{aligned} \tag{23}$$

Substituting Eq.(21) coefficients in Eq.(12) along with Eq.(16) for $u(x,t)$ and in Eq.(13) for $v(x,t)$, we obtain the complex hyperbolic function solution to the Eq.(1) as following;

$$\begin{aligned}&u_1(x,t)=re^{i(\alpha x+wt)}\tanh(x-2t\alpha),\\&v_1(x,t)=\frac{c}{-1+4\alpha^2}-\frac{1}{\vartheta}e^{2i(\alpha x+wt)}\tanh^2(x-2t\alpha),\end{aligned} \tag{24}$$

where

$$r=\frac{-\sqrt{-1+4\alpha^2}}{\sqrt{1+a\left(-1+4\alpha^2\right)}}, w=\frac{2+2c-7\alpha^2-4\alpha^4}{-1+4\alpha^2},$$

$$\vartheta=1+a\left(-1+4\alpha^2\right).$$

When we consider the Eq.(22) coefficients in Eq.(12) along with Eq.(16) for $u(x,t)$ and in Eq.(13) for $v(x,t)$, we find another complex hyperbolic function solution to the Eq.(1) as following;

$$\begin{aligned}&u_2(x,t)=pe^{i(\alpha x+kt)}\tanh(x-2\alpha t),\\&v_2(x,t)=\frac{c}{-1+4\alpha^2}-\frac{e^{2i(\alpha x+kt)}}{\varpi}\tanh^2(x-2\alpha t),\end{aligned} \tag{25}$$

where

$$p=\frac{\sqrt{-1+4\alpha^2}}{\sqrt{1+a\left(-1+4\alpha^2\right)}}, \varpi=1+a\left(-1+4\alpha^2\right),$$

$$k=\frac{2+2c-7\alpha^2-4\alpha^4}{-1+4\alpha^2}.$$

Substituting the Eq.(23) coefficients in Eq.(12)

along with Eq.(16) for $u(x,t)$ and in Eq.(13) for $v(x,t)$, we find another hyperbolic function solution to the Eq.(1) as following;

$$
\begin{aligned}
u_3(x,t) &= A_1 e^{i(\alpha x+\beta t)} \tanh(x-2\alpha t),\\
v_3(x,t) &= \frac{\nu - 2e^{2i(\alpha x+\beta t)} A_1^2 \tanh^2(x-2\alpha t)}{-2+8\alpha^2},
\end{aligned} \tag{26}
$$

where $\nu = \left(-1+4\alpha^2\right)\left(2+\alpha^2+\beta\right)$.

4. Tables and Figures

In this subsection of paper, we have plotted two- and three-dimensional surfaces of travelling wave solutions obtained in this paper under the suitable values of parameters by using SGEM as follows.

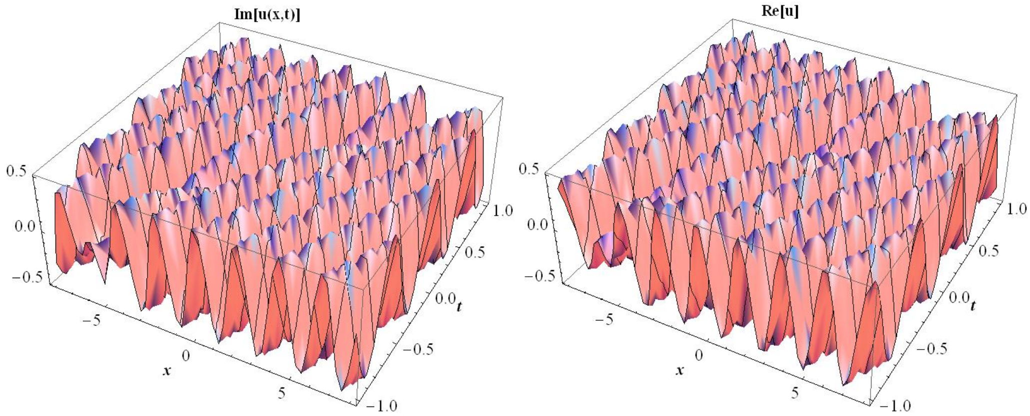

Figure 1. The 3D surfaces of u_1 of Eq.(24) under the terms of considering the values $c=5,\ a=4, \alpha=3, -8<x<8, -1<t<1.$

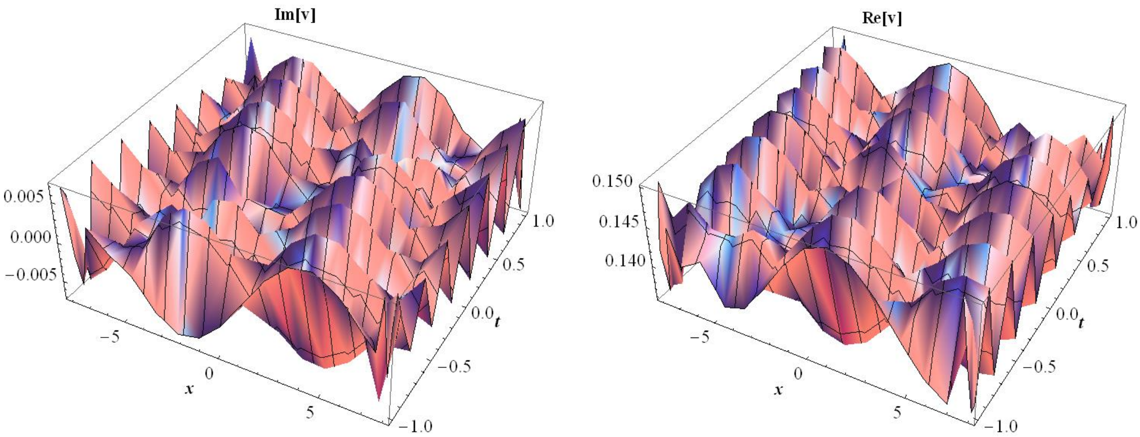

Figure 2. The 3D surfaces of v_1 of Eq.(24) under the terms of considering the values $c=5,\ a=4, \alpha=3, -8<x<8, -1<t<1.$

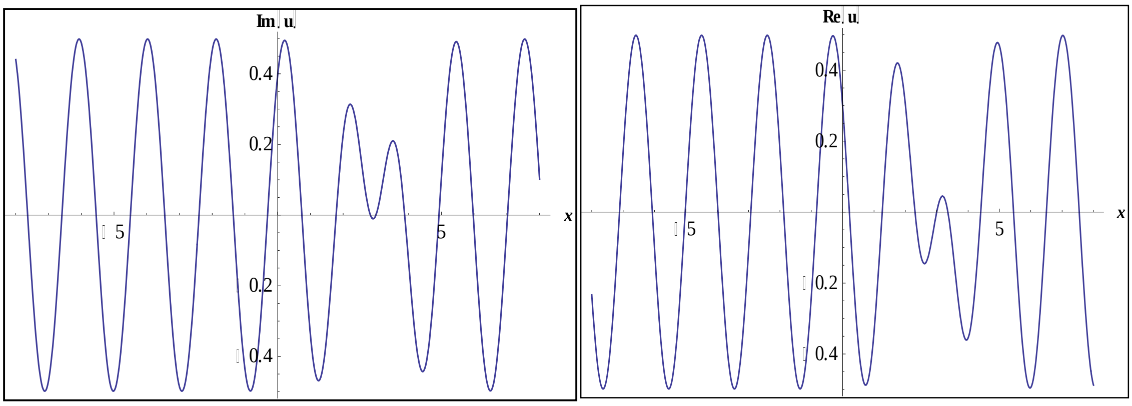

Figure 3. The 2D surfaces of u_1 of Eq.(24) under the terms of considering the values $c=5,\ a=4, \alpha=3, t=0.5, -8<x<8.$

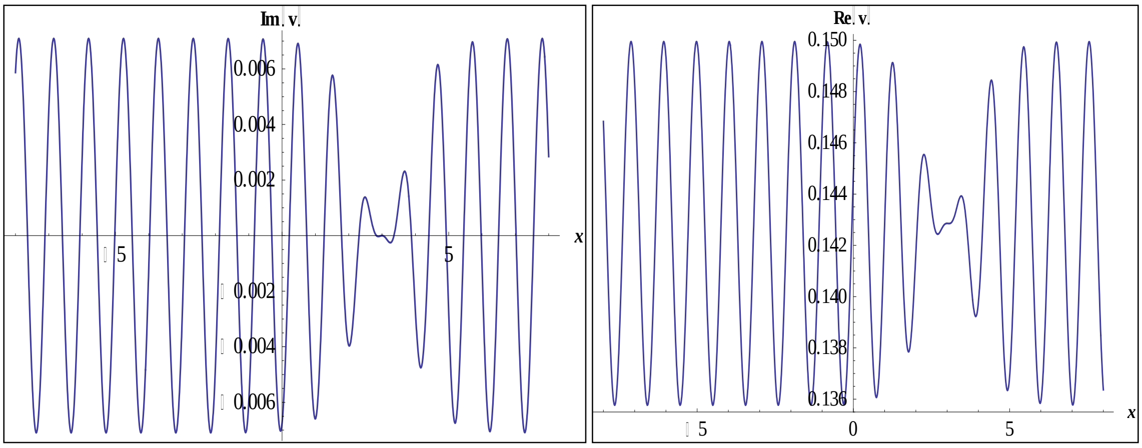

Figure 4. The 2D surfaces of v_1 of Eq.(24) under the terms of considering the values $c = 5,\ a = 4, \alpha = 3, t = 0.5, -8 < x < 8.$

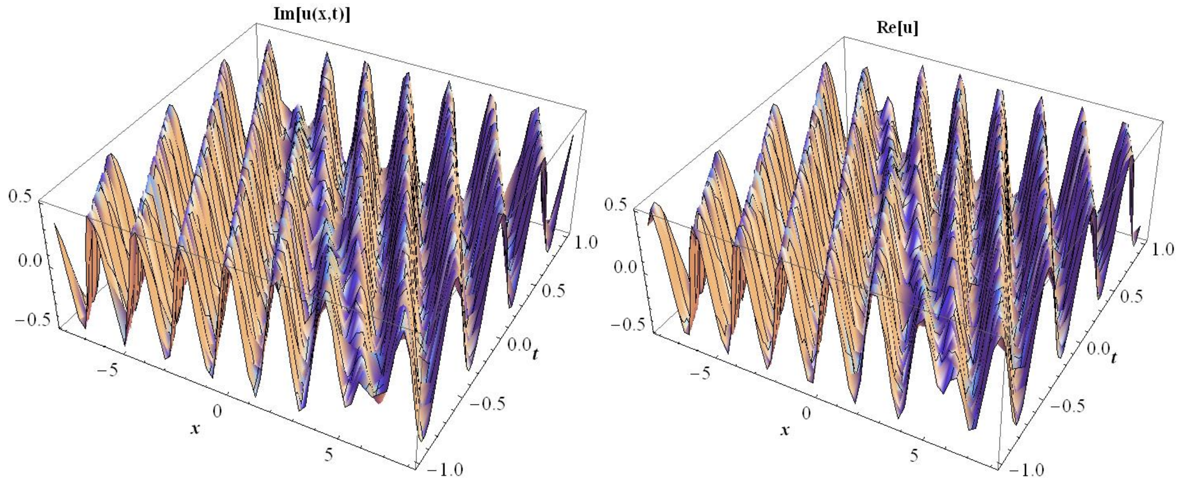

Figure 5. The 3D surfaces of u_2 of Eq.(25) under the terms of considering the values $c = -5,\ a = 4, \alpha = -3, -8 < x < 8, -1 < t < 1.$

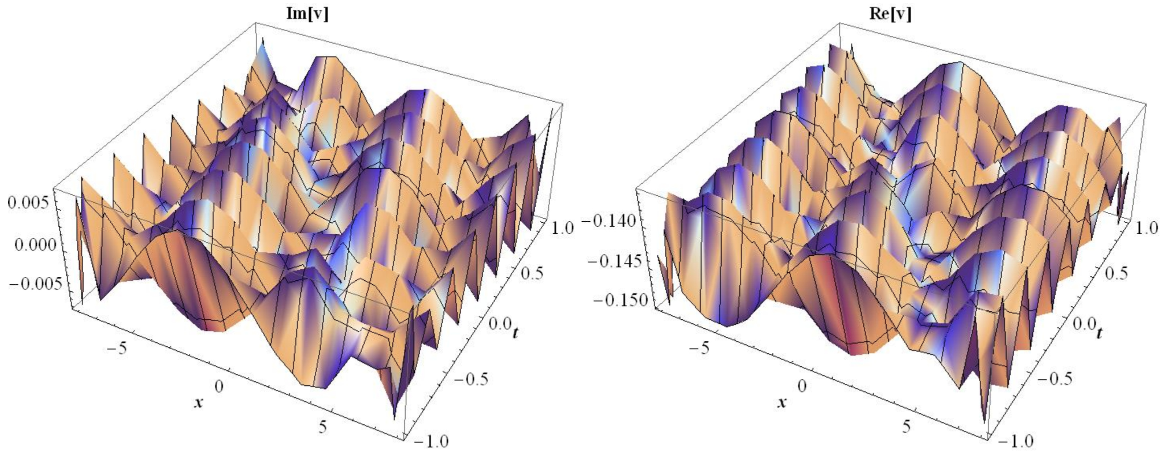

Figure 6. The 3D surfaces of v_2 of Eq.(25) under the terms of considering the values $c = -5,\ a = 4, \alpha = -3, -8 < x < 8, -1 < t < 1.$

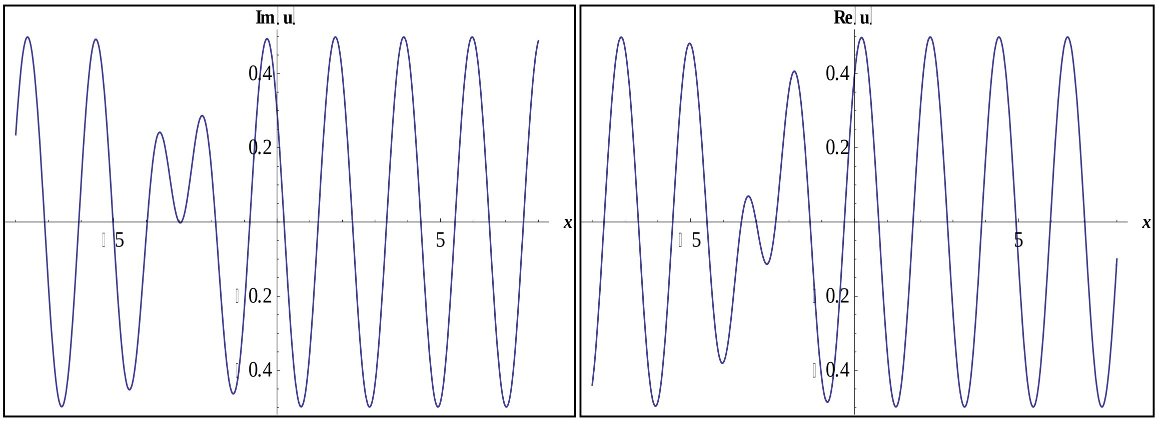

Figure 7. The 2D surfaces of u_2 of Eq.(25) under the terms of considering the values $c=-5,\ a=4, \alpha=-3, t=0.5, -8<x<8.$

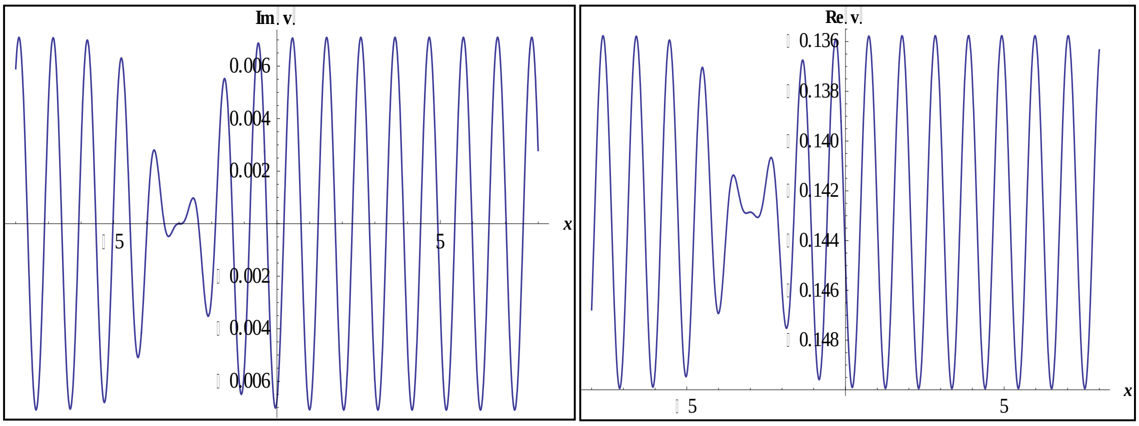

Figure 8. The 2D surfaces of v_2 of Eq.(25) under the terms of considering the values $c=-5,\ a=4, \alpha=-3, t=0.5, -8<x<8.$

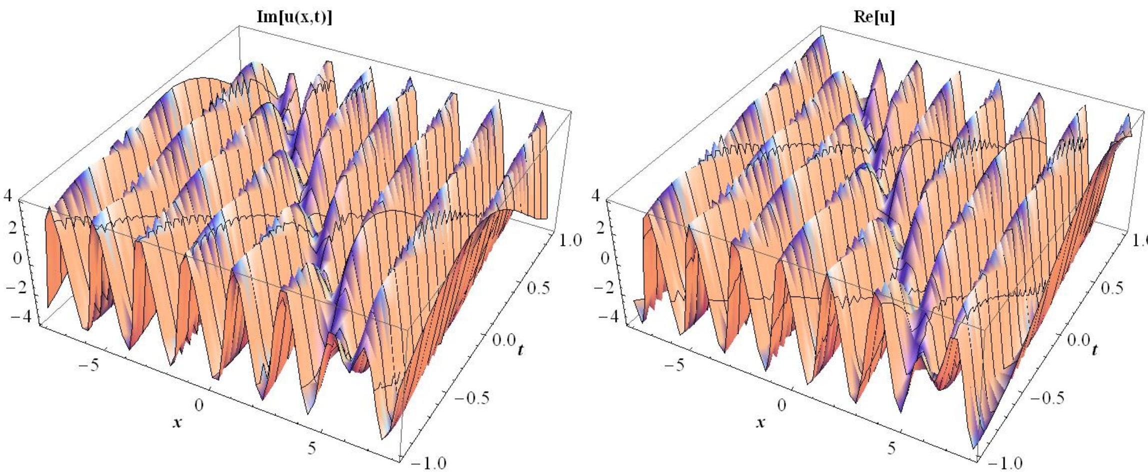

Figure 9. The 3D surfaces of u_3 of Eq.(26) under the terms of considering the values $\beta=-2,\ A_1=4, \alpha=-3, -8<x<8, -1<t<1.$

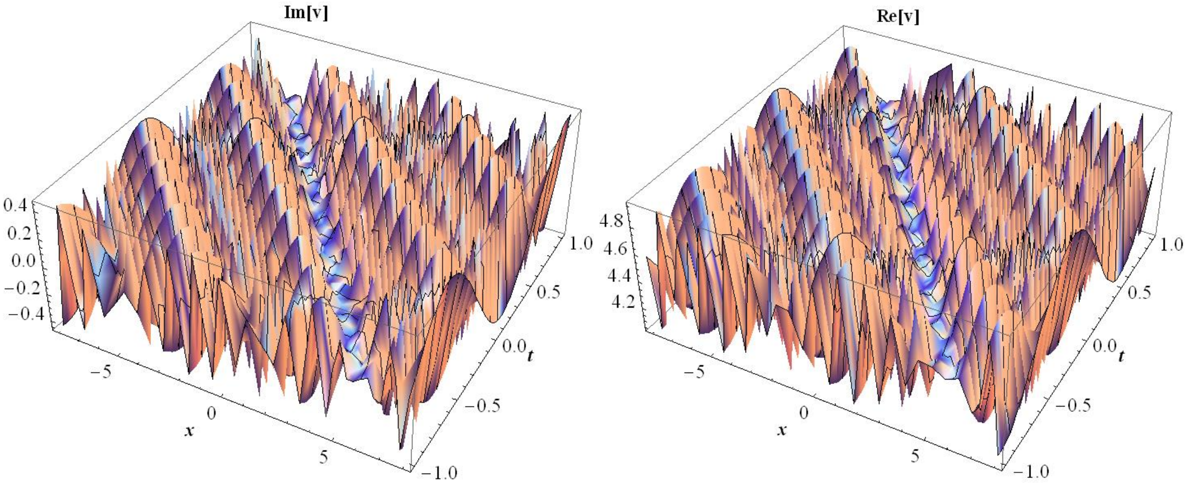

Figure 10. The 3D surfaces of v_3 of Eq.(26) under the terms of considering the values

$$\beta = -2,\ A_1 = 4, \alpha = -3, -8 < x < 8, -1 < t < 1.$$

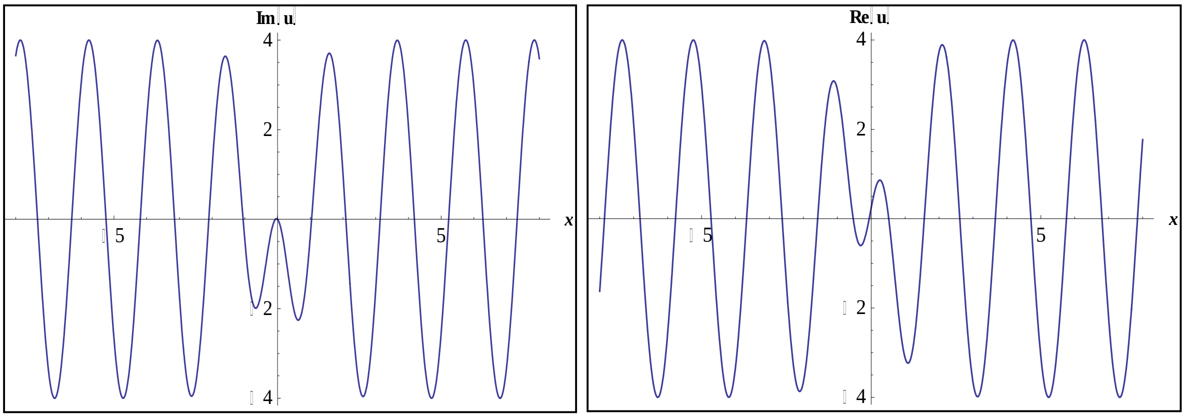

Figure 11. The 2D surfaces of u_3 of Eq.(26) under the terms of considering the values

$$\beta = -2,\ A_1 = 4, \alpha = -3, t = 0.01, -8 < x < 8.$$

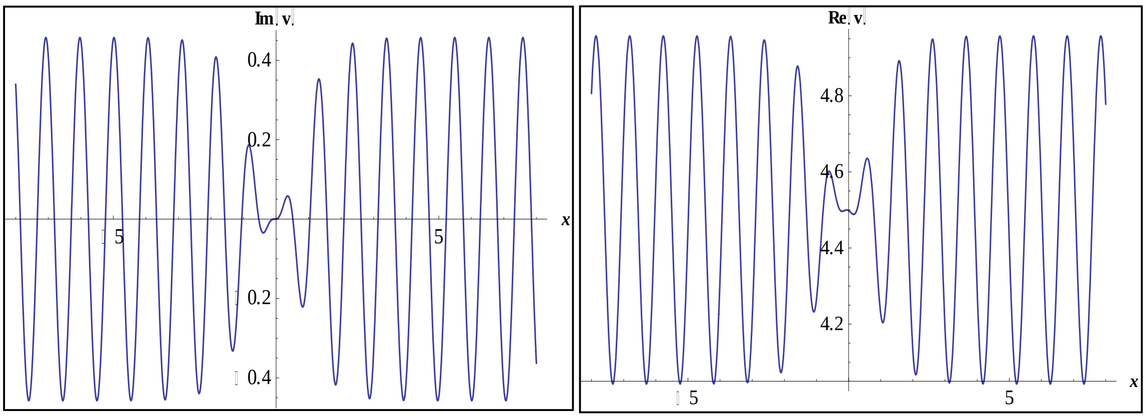

Figure 12. The 2D surfaces of v_3 of Eq.(26) under the terms of considering the values

$$\beta = -2,\ A_1 = 4, \alpha = -3, t = 0.01, -8 < x < 8.$$

5. Discussion and remark

In fact, the coefficients found in this paper such as Eqs.(21,22,23) belong to Eq.(11) defined as

$$U(w)=\sum_{i=1}^{n}\cos^{i-1}(w)\left[B_i\sin(w)+A_i\cos(w)\right]$$
$$+A_0.$$

According to fundamental properties of SGEM which includes the interesting equations such as Eq.(7) and Eq.(8), we have used the Eq.(10) because Eq.(11) equal to Eq.(10) defined by

$$U(\xi)=\sum_{i=1}^{n}\tanh^{i-1}(\xi)\left[B_i\,\mathrm{sec}h(\xi)+A_i\tanh(\xi)\right]$$
$$+A_0,$$

for finding the hyperbolic function solutions to the Eq.(1).

6. Conclusion

To be brief, SGEM has been successfully applied to the generalized-Zakharov equation with complex structures for obtaining the complex travelling wavesolutions. We have plotted two- and three-dimensional surfaces for the Eq.(1) under the suitable values of parameters.

When we consider all the results and Figures (1-12), we can say that this method is efficient and suitable for obtaining new travelling wave solutions to the ordinary differential equations with powerful nonlinearity. These hyperbolic function solutions have been introduced to the literature with important physical meaning about the generalized-Zakharov equation. Moreover, travelling wave solutions Eqs.(24,25,26) are dark soliton solutions to the Generalized-Zakharov equation with complex structures [19-21]. It has been observed that they are related to physical features of hyperbolic functions [22, 23]. It is estimated that they are related to the physical properties of dark soliton solutions.

We think that this method play an important role for finding travelling wave solutions to such models. To the best of our knowledge, the application of SGEM to the Eq.(1) has not been submitted to literature in advance.

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