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Competitive Microcredit Markets: Differentiation and ex-ante Incentives for Multiple Borrowing*

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Abstract

We analyze an oligopolistic microcredit market characterized by asymmetric information and institutions that can offer only one type of contract. We study the effects of competition on contract choice when small entrepreneurs can borrow from more than one institution due to the absence of credit bureaus. We show that appropriate contract design can eliminate the ex-ante incentives for multiple borrowing. Moreover, when the market is still largely unserved and particularly risky, a screening strategy leading to contract differentiation and credit rationing is unambiguously the most effective to avoid multiple borrowing.

Keywords: Microfinance, Competition, Credit Bureaus, Multiple Borrowing, Credit Rationing
JEL Classification: G21, L13, L31, O16

1 Introduction

Competition is increasingly a cause for concern in microcredit markets. A growing number of institutions enter the market, motivated by goals spanning from poverty reduction to profit maximization. Economists generally

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welcome competition as a positive phenomenon, especially in terms of consumer welfare, but some of the special features of microcredit raise some doubts regarding this conventional wisdom.

Whenever borrowers and lenders are tied in a reciprocal relationship, lending money without incurring important financial losses is relatively easy. Lenders need borrowers to repay their loans in order to avoid losses. Borrowers need lenders to finance their businesses and their daily activities. When microcredit was still at its origin, this relation was quite balanced since the supply of credit was largely insufficient, and the demand side was still limited, mainly because of distrust toward microfinance institutions. This was enough to discipline the involved parties. But the increase of competition is destabilizing the relation in favor of borrowers: when there are different Micro Finance Institutions (MFI) to which borrowers can apply for credit, the link borrower-lender becomes weaker. This creates incentive for borrowers to engage in potentially harmful behavior like, for instance, multiple borrowing.

Practitioners report that the presence of competitors in the market weakens MFIs in two respects.[1] First, it reduces the borrowers' incentives for repayment. These incentives, in fact, depend importantly on the threat of being denied access to further credit in case of default. Second, due to the lack of well functioning credit bureaus, borrowers might take multiple loans. In these cases, the level of indebtedness can become so large to render repayments extremely unlikely.

This paper focuses on multiple borrowing. We analyze it in relation to the strategic behavior of competing MFIs. Our goal is to understand how the contracts chosen by competing MFIs can affect borrowers' incentives for multiple borrowing and how this, in turn, modifies the strategies of MFIs.

Technically, allowing borrowers to take out more than one loan is equivalent to assuming that MFIs cannot share information about the borrowers they are serving, and that borrowers do want to take multiple loans. Both assumptions must be considered carefully. Some Microfinance markets, especially the ones characterized by a higher degree of competition, do show a certain level of information sharing. Indeed, there are more and more attempts to set up credit bureaus, as well as different examples of bilateral agreements between MFIs to share the most relevant information. Nonetheless, practitioners report that in most markets borrowers do take multiple loans and hide their real level of indebtedness. As a consequence, making reliable assessments of credit risk becomes more difficult and, thus, important

[1] See McIntosh et al. [15], or Armendariz and Murdoch [1]

financial losses are more likely to hit MFIs.

The literature has proposed mainly two different explanations for multiple borrowing (see, for instance, McIntosh et al. [15], McIntosh and Wydick [16], de Janvry et al [6]). The first is that *ex-post*, i.e. after the loan is taken and invested, some unexpected negative shocks can hurt borrowers and their businesses. This can make it impossible for them to repay the loan. Thus, borrowers might decide to take a second loan in order to repay the first, increasing dangerously their level of indebtedness.

A second motivation for multiple borrowing comes from the fact that micro-loans can be too small to cover the borrowers' needs for a specific investment. In order to obtain the missing capital, they might find it convenient to hide their real level of indebtedness and ask for additional loans at different institutions.

However, even ruling out negative shocks, and assuming that loans are optimally sized, borrowers might have incentives to take multiple loans. In fact, they might desire a second loan to invest in a different and possibly riskier use. We take this into account by allowing borrowers to choose on whether to undertake one or more investments. This clearly provides *ex-ante* incentives for multiple borrowing.

An additional empirical motivation to justify our modeling strategy comes from the fact that, although micro-loans are typically made to individuals, profits, burdens and responsibilities of the investments are typically shared within households. Many MFIs, for instance, make loans primarily to women since they are considered safer. But empirical evidence shows that although women are the members of the family officially taking out the loan, often men are the ones controlling the relevant investment decisions and taking mostly care of the business.[2] Independently of that, within households it is likely to find a certain level of solidarity. Thus, if more than one member of the household has a loan, the probability of repayment depends on the success of both investments. This creates an artificial correlation between the probabilities of default and makes loans riskier. Our model can also be interpreted as a way to take into account these circumstances, shifting the focus from individuals to households.

The experience of several MFIs all over the world has shown that the poor are reliable borrowers. The default rate is extremely low, and in particular *total* default is considered as a particularly rare event. This is due to the fact that the repayment schedule of most micro-loans is characterized by very frequent repayment installments, starting very early after the start of

[2]See for instance Goetz and Sen Gupta (1996).

the contract. In our model we take this feature into account and show its extreme importance to mitigate the incentives to take multiple loans.

We use a simple credit rationing model with adverse selection in which two MFIs compete simultaneously for a pool of heterogeneous borrowers. We assume that MFIs can offer only one type of contract, as it typically happens in microcredit markets. Borrowers have access to two investment opportunities, and therefore take two different decisions: first, how many loans to take out and, second, from which MFI to take them. Multiple borrowing leads to a social loss in our model because of decreasing returns to scale. For ease of exposition we first solve the model assuming that multiple borrowing is impossible (for instance, because of the existence of an information sharing mechanism). Then we check how the possibility to take multiple loans (or, in other words, the absence of information sharing), changes the predictions of the model.

Adverse selection plays an important role in our model. In fact it prevents MFIs from extracting rent from borrowers, and as a consequence it makes the incentives to take multiple loans much stronger than in a model of perfect information. Moreover, even when multiple borrowing is assumed away, it leads to separating equilibria, characterized by credit rationing, in which MFIs specialize in one type of borrower only. That allows us to mimic some stylized fact typical of microcredit markets.

To the best of our knowledge, the only theoretical paper tackling the problem of multiple borrowing in microcredit markets is McIntosh and Wydick (2005). They also focus on microfinance, but their approach is different in at least two respects: (i) they consider dynamic incentives, (ii) the incentives to multiple borrow depend solely on an exogenous parameter measuring the borrowers' impatience. In other words, borrowers trade off the utility from borrowing more today with the risk of being denied credit access tomorrow. The choice is not influenced by the contract design. Our paper is based on a static model and, as such, considers ex-ante incentives only. The added value of this approach is that it allows to study how the incentives for multiple borrowing can be controlled by appropriate contract design. These incentives are, in fact, endogenously determined by the contracts chosen by MFIs.

Bennardo, Pagano and Piccolo (2009) consider the problem of multiple-bank lending by considering a market in which borrowers decide on whether to invest in a small or a big project and can appropriate part of the revenues. They analyze the effects of introducing an information sharing mechanism and show how it would reduce interest rates and rationing. In our paper we rather focus on those situation in which such mechanism is not implementable, as it is often the case in development countries.

Fluet and Garella (2007), consider banks' incentives to reschedule loans to borrowers in financial distress. They assume that borrowers are indebted with many lenders. Each lender cannot observe the performance of the borrowers with the other lenders. In their model, borrowers lend from multiple sources by assumption, since they want to finance a big scale project. Each lender finance only a share of the whole, unique project.

Other papers study the effects of the presence of a credit bureau on borrowers in terms of reputation building (see for instance Vercammen (1995)). In this branch of the literature, credit bureaus are an important disciplining device. De Janvry, McIntos and Sadoulet (2006), study the impact of the implementation of a credit bureau on both demand and supply side using a natural experiment.

The organization of the paper is the following: in the next section we introduce the model and analyze the incentives for multiple borrowing when a credit bureau is not at work. In Section 3 we describe the strategic behavior of two competing MFIs, first assuming the existence of a perfectly functioning credit bureau and then allowing borrowers to take multiple loans. We explain how the strategic behavior of MFIs influences the borrower incentives to take out more than one loan. In section 4 we conclude.

2 The model

We model a market characterized by asymmetric information and oligopolistic competition. We assume that, due to high management costs, each MFI can only offer one contract.[3]. Contracts are chosen simultaneously. We assume that MFIs are not perfectly symmetric in that they have different capacities.[4]

This assumption can be interpreted in different ways. For instance the high capacity MFI could be a firm that entered the market beforehand, and had therefore more time to accumulate capital. Alternatively the high capacity institution could be a 'normal' bank downscaling her business into

[3] Empirical evidence shows that micronance contracts are very standardized at the firm level. Some big and viable MFIs consider standardization as one of the main factors of their success. For instance, ASA, in Bangladesh denes its organization as the Ford Motor Model of Micronance. Grameen Bank, probably the most celebrated Micronance Institution in the world, offers loans with a unique interest rate, namely 16%. In general, MFIs operating in competive markets offer extremely few contract types, and often only one. See Casini (2008) [4] for a wider discussion

[4] This asymmetry allows to avoid the use of mixed strategies.

a market that has previously been pioneered by a small NGO.

More formally, we consider a market in which two MFIs, say a and b, are operating. We assume that each MFI is endowed with a capacity α^j, $j \in \{a, b\}$. Without loss of generality let $\alpha^a > \alpha^b$. Finally, let $\alpha^a + \alpha^b \leq 1$, so that the market is not necessarily fully covered.[5] There is a unit measure of borrowers demanding a loan, whose size is, for simplicity, set to one. Each borrower can be interpreted as a single individual or as a household. There are two potential investment opportunities in the market, available to everybody. Both investments give the same return to a given borrower, but we assume that only one of them can be given priority. In other words, we assume that borrowers exert a bigger effort in one investment, that is successful with probability p, and only residual effort in the second one, that is successful with probability p', where $p' \leq p$. This is equivalent to assuming decreasing returns to scale. The level of effort is exogenously given.

There is fraction β of Safe borrowers, characterized by a return R_s and a probability of success p_s for the first investment and p'_s for the second, with $p'_s \leq p_s$. The remaining $1 - \beta$ borrowers are Risky and are characterized by a return R_r and probabilities of success p_r and p'_r, $p'_r \leq p_r$, on the first and second investment respectively. We also set $p_s R_s = p_r R_r = m$, $p_s > p_r$ and $p'_s > p'_r$. Hence, $R_s < R_r$. This makes sure that all borrowers have the same expected return, so that MFIs are ex-ante indifferent between them. Note that, under our assumptions, multiple borrowing is inefficient since a part of the scarce financial resources is allocated to project with a lower probability of success. Nonetheless, MFIs could prefer serving twice the safe borrowers when $p'_s > p_r$.

Let $x^i \in [0, 1]$ denote the fraction of the demand MFI i is willing to serve or, equivalently, the probability for each borrower to obtain the scarce funds. We can define a contract as a pair $C^i = (x^i, D^i)$, in which MFIs specify the repayment D^i, inclusive of principal and interests, and the probability x^i for a borrower to be served. Each MFI offers only one contract. The borrower type is private information. We use two tie-breaking rules: first, we assume that if a contract leaves the borrowers with no rent, they still prefer borrowing to not borrowing; second, we assume that MFIs prefer serving both types of borrowers rather than targeting the residual demand if that gives them the same profit.[6]

[5] This assumption is not necessary to prove the existence of screening equilibria. Nonetheless it is useful for the exposition since it ensures that an equilibrium in pure strategies of the PM model, be it with or without screening, *always* exists.

[6] This rule is only relevant for non-screening equilibria.

2.1 Incentives for Multiple Borrowing

Most of the credit rationing models that followed Stiglitz and Weiss (1981)' seminal contribution assume that borrowers can take out one loan only. This is equivalent to assuming that either MFIs can share information about the borrowers they are serving, or that borrowers do not want to take multiple loans. Both assumptions must be considered carefully when examining microcredit markets. Although there exist examples of information sharing through the creation of credit bureaus, the amount of information available to MFIs is generally scarce.[7] In countries like India, for instance, people are not even registered at birth, so that most of the inhabitants of rural areas are not identifiable through an ID. In this situations MFIs can only rely on informal sources of information (like personal knowledge) to asses on the credit history of potential borrowers. As a consequence, in many markets borrowers do take multiple loans by hiding their real level of indebtedness.[8] This can lead to incorrect risk assessment by MFIs and, as a consequence, to important financial losses.

In what follows we formalize the behavior of borrowers when, due to lack of information sharing, multiple borrowing is possible. In order to do it we assume that borrowers take out at most one loan from each MFI[9]. As standard in similar models, we exclude strategic default, that is we assume that borrowers repay their loan whenever they can. On top of that, we assume that borrowers repay their loans as much as they can, even when they cannot pay back the whole capital. In other words, partial reimbursements are allowed[10].

Each loan is invested in a distinct and independent business. The return on investments is strictly related to types: a Risky borrower gets the same return on all the investment she makes. But we assume that one of the two investments has a lower probability of being reimbursed. This can be either interpreted as excessive level of investment by the borrowers, or as inability to properly manage two projects at the same time. A different way to read this assumption is to interpret borrowers as members of a household. Each household has a primary business, in which much of the efforts and resources are invested, and a secondary one to which only the residual assets

[7] see for instance de Janvry, McIntosh and Sadoulet (2008) [6].

[8] see for instance McIntosh, de Janvry and Sadoulet (2005) [15].

[9] Later we show that in equilibrium MFIs do not want to multi-lend

[10] Evidence shows that borrowers almost never totally default on their loans. This is mainly a consequence of the fact that most MFIs offer loans whose repayment is done by very frequent instalments, starting almost immediately after the issue. Thus total default is considered a rare event. See, for instance, Armendariz & Morduch page 31 and ss)

are dedicated.

We keep the implicit assumption that the loan size offered by the MFI is the optimal one, so that no borrower wants to invest more money in the same project. In other words, investing more resources in the same business does not increase the probability of success. This clearly rules out the incentives to multi-borrow arising from imperfect contract design, allowing us to identify the pure effects generated by competition and adverse selection. For the time being, suppose that applications for credit are committing: if a borrower applies for a loan and the application is accepted, she cannot decline the contract. We also assume that $R_r < \frac{2}{p_r}$, so that being successful in only one investment is not enough to repay two loans.[11]

If a borrower applies for only one loan from MFI i, she enjoys the following *ex-ante* utility:

$$U_j(C^i) = x^i p_j(R_j - D^i) \quad \text{with} \quad j = s, r \quad \text{and} \quad i = a, b$$

since she attains the loan with probability x^i, earns R_j with probability p_j, in which case she repays D^i.

Suppose now that a Risky borrower applies for credit at both MFIs simultaneously. Since we allow for partial reimbursements, the *ex-ante* utility she gets from applying for two different loans is given by the weighed sum of the utility she gets in four mutually exclusive cases:

1- The borrower applies at both MFIs and both applications are accepted:

$$x^a x^b[p_r(1-p'_r)R_r + p'_r(1-p_r)R_r + 2p_r p'_r R_r +$$
$$-p_r(1-p'_r)R_r - p'_r(1-p_r)R_r - p'_r p_r(D^a + D^b)]$$

2- She applies at both MFIs but only a accepts the application:

$$x^a(1-x^b)p_r(R_r - D^a)$$

3- She applies at both MFIs but only b accepts the application:

$$x^b(1-x^a)p_r(R_r - D^b)$$

4- She applies at both MFIs and none of the application is accepted: in this case the expected utility is simply zero.

[11] Note that setting $D^i = \frac{1}{p_r}$, an MFI serving the Risky borrowers only would make zero profit. Thus, our assumption makes sure that being successful in one investment only is not enough to repay two loans.

Summing up the equations above we get:

$$p_r p'_r x^a x^b (2R_r - D^b - D^a) + x^b(1-x^a)p_r(R_r - D^b) + x^a(1-x^b)p_r(R_r - D^a) \quad (1)$$

We can compare this equation with the expected utility a Risky borrower enjoys by applying at one MFI only. Suppose, without loss of generality, that the Risky borrowers prefer the contract offered by a. Then equation (1) must be compared to $x^a p_r(R_r - D^a)$. Rearranging the formulas, it is easy to see that the condition for the Risky borrowers *not* to prefer to multi-borrow is given by:

$$(R_r - D^b)[1 - x^a(1 - p'_r)] < x^a(R_r - D^a)(1 - p'_r) \quad (2)$$

Similar calculations can be made for the Safe types assuming, without loss of generality, that they prefer the contract offered by MFI b. This leads to the analogous condition:

$$(R_s - D^a)[1 - x^b(1 - p'_s)] < x^b(R_s - D^b)(1 - p'_s) \quad (3)$$

Note that for p'_s and p'_r small enough, the conditions are jointly satisfied when x^a and x^b are high. In other words, a higher level of rationing can increase the borrower incentives to apply for credit at different MFIs simultaneously.

The calculations above hinge on the assumption that borrowers repay their loans as much as they can. That is, even if they do not have money enough to repay both loans, they refund the MFIs at least partially. As discussed above, this is a very important feature of microfinance markets.

Note that we did not assume any criterion to establish which MFI has priority in case of partial reimbursement. In general, the ranking can be made dependent on the borrowers preferences. But the conditions stated above do *not* depend on borrower preferences about which MFI to give priority to. There could obviously be several motivations for a borrower to prefer repaying first one MFI rather than the other (different dynamic incentives, different enforcement power etc.). But this is immaterial for this part of the analysis. In our static set-up, any assumption in this respect would influence MFIs' profits rather than borrower utility.

Multiple borrowing produces a considerable reduction of the total welfare. From the MFIs point of view, the loss is determined by the higher probability of defaults. From the borrowers point of view, the most apparent consequence of multiple lending is the exclusion of a higher number of borrowers. In fact, given the capacity constraint of the MFIs, if borrowers take more than one loan, then less individuals can be served. This loss outweighs the gain in terms of utility of the borrowers that do access credit. In fact, the low probability of repaying the second loan ensures that the same amount of money gives in the aggregate more utility if it is invested by two different individuals (or households).

The conditions stated above depend on the contract chosen by both MFIs. That allows us to investigate whether there exist competitive equilibria in which MFIs offer contracts that eliminate the incentives to multi-borrowing. We consider the case in which two *profit maximizing* MFIs compete in the market. This set-up describes a mature microcredit market like the ones, for instance, in Bangladesh or Bolivia.[12]

3 The Equilibria

Some of the most celebrated and imitated MFIs are profit maximizing or, at least, so they claim. In large part, Microfinance has become famous because of its promise of being able to effectively reduce poverty while running a profitable business. But very few MFIs actually manage to earn profit. Still, profit seeking is considered by many practitioners as a 'best practice' for the success of a microfinance program. For this reason, we assume that both MFIs are profit maximizing.The solution of this model provides a useful benchmark allowing us to draw some interesting policy conclusions.

For ease of exposition, we first solve the model by assuming that multiple borrowing is not possible, because of perfect information sharing between MFIs. We then relax this hypothesis to show the existence of equilibria in which borrowers do not want to take multiple loans.

We prove the existence of two different types of equilibria. The first type is characterised by screening whereas the second one is a pooling equilibrium in which no screening takes place. We will not consider equilibria in mixed strategies.

3.1 No Multiple Borrowing

Define a function $B^i(\cdot,\cdot) : (\mathbb{R}_+ \times [0,1]) \times (\mathbb{R}_+ \times [0,1]) \to [0,1]$, assigning to each combination of contracts the mass of borrowers preferring MFI i. Let $P^i(\cdot,\cdot) : (\mathbb{R}_+ \times [0,1]) \times (\mathbb{R}_+ \times [0,1]) \to [0,1]$ be the mapping assigning to each combination of contracts the probability of repayment of MFI i's

[12] For a detailed analysis of the case in which an altruistic MFI is in the market, see Casini (2009), [4] and [5].

pool of clients. It takes value p_r, p_s or $p_b := \beta p_s + (1-\beta)p_r$ when the MFI serves respectively the Risky, the Safe or Both types of borrowers. Finally, let $X^i(C^a, C^b, \alpha^i) := \min\{x^i B^i(C^a, C^b), \alpha^i\}$ denote the mass of borrowers served by i. MFI i faces the following maximization problem:

$$\max_{C^i} \Pi^i = X^i(C^a, C^b, \alpha^i)\left[P(C^a, C^b)D^i - 1\right]$$

Since by assumption $\alpha^i < 1$, for any given strategy of i, her competitor can always target the residual demand $(1 - X^i(C^a, C^b, \alpha^i))$, and impose on it a monopoly price. For the sequel, it is useful to calculate the profit MFI a earns serving the residual demand of the Risky types, when b faces a demand $B^b(C^a, C^b) = 1$ and serves both types. a optimally sets $D^a = R_r$, extracting the whole surplus from the residual Risky borrowers. Since by assumption $\alpha^a < (1 - \alpha^b)$, she earns:

$$\Pi^a_{ResR} = \alpha^a(1-\beta)(m-1). \quad (4)$$

In the same way we can define the profit a earns serving the residual demand of both types. She sets $D^a = R_s$, extracting all the Safe borrower's surplus and leaving the Risky ones a rent. She earns:

$$\Pi^a_{ResB} = \alpha^a[\beta(m-1) + (1-\beta)(p_r R_s - 1)] \quad (5)$$

Whether Π^a_{ResR} or Π^a_{ResB} is bigger depends on the particular values of the parameters. Π^b_{ResR} and Π^b_{ResB} are analogously defined.

We can now describe the borrowers' reaction functions. For any given contract chosen by the competitor, an MFI has four different choices: (i) Offer a contract that attracts *all* the borrowers of a specific type and only them (i.e. a screening contract); (ii) Undercut the competitor's contract; (iii) Target the residual demand of the chosen type(s); (iv) Offer a contract that attract both types. Given the definition of Π^i_{ResB} and the assumption $\alpha^a + \alpha^b \leq 1$, the last option gives the same profit as serving the residual demand of both types. In what follows we state the conditions supporting the first choice, i.e. we describe under which conditions the best reaction of an MFI is to offer a contract that allows screening.

Lemma 1. *If i chooses a contract such that $D^i \leq R_s$ and $x^i \leq \hat{x}(D^i) < 1$ where $\hat{x}(D^i)$ is defined as:*

$$\begin{array}{lll} \dfrac{(1-\alpha^j)(m-1)}{m - p_r D^i} & \textit{if} & \Pi^j_{ResR} \geq \Pi^j_{ResB} \\[2ex] \dfrac{(1-\beta)(m-1) - \Pi^j_{ResB}}{(1-\beta)(m - p_r D^i)} & \textit{if} & \Pi^j_{ResB} \geq \Pi^j_{ResR} \end{array}$$

then j's optimal reaction is to offer a contract $C^j = (1, R_r - \frac{x^i}{x_j}(R_r - D^j))$*, so that screening takes place with* i *serving the Safe borrowers and* j *serving the Risky ones.*

Proof. See Appendix A. □

$\hat{x}^i(D_i)$ is the value of x^i making MFI j indifferent between engaging in a screening strategy (serving the Risky borrowers only) and the best of her outside options. The intuition behind this result is standard: if i wants to serve only the Safe borrowers, she must ration some of them. What is less standard is that the number of excluded borrowers depends on the prevailing j's outside option. A similar intuition is at the basis of the next lemma.

Lemma 2. *If* i *offers a contract* (x^i, D^i) *characterized by:*

$$R_s < D^i \leq \tilde{D}^i(x^i) := R_r - \frac{1}{x^i}\tilde{x}^j(R_r - D^j) \tag{6}$$

where

$$\tilde{x}^j := \max\left\{\alpha^j\left(1 + \frac{(1-\beta)(p_r R_s - 1)}{\beta(m-1)}\right), \frac{(1-\beta)(m-1)}{\beta(m-1) + (1-\beta)(m - p_r R_s)}\right\}$$

then j's optimal reaction is to offer a contract $C^j = (\tilde{x}^j, R_s)$*, so that screening takes place with* i *serving the Risky borrowers and* j *serving the Safe ones.*

Proof. See Appendix A. □

Again, the intuition is standard: to obtain screening, Risky borrowers must be given better conditions via a reduction of the repayment D^j (the informational rent). At the same time some of the Safe borrowers must be rationed.

When the conditions in lemmas 1 and 2 are not satisfied, options (ii), (iii) and (iv) in the taxonomy above are relevant. Let $\Pi^j_{UR} := (1-\beta)[(m-1) - p_r x^j(R_r - R_s)]$ be the profit MFI j can earn by undercutting MFI i when $C^i = (1, D^i)$, with $D^i > R_s$. This is the profit that MFI i would earn in a screening equilibrium. Next lemma describes the best responses in these cases.

Lemma 3. *(i) If* $D^i \leq R_s$ *and* $1 > x^i > \hat{x}(D^i)$*, then j's optimal reaction is to set* $C^j = (1, R_s)$ *when* $\Pi_{ResB} > \Pi_{ResR}$ *and* $C^j = (1, R_r)$ *when* $\Pi_{ResR} > \Pi_{ResB}$*.*
(ii) If $D^i > \tilde{D}^i(x^i)$*, then j's optimal reaction is to set* $C^j = (1, R_s)$ *when* $\Pi_{ResB} > \Pi^j_{UR}$ *and* $C^j = (1, D^i)$ *when* $\Pi^j_{UR} > \Pi_{ResB}$*.*

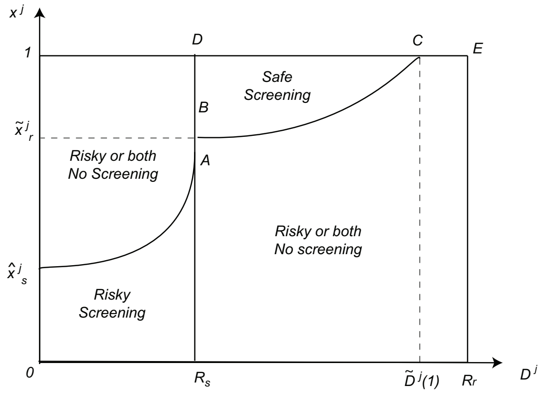

Figure 1: MFI i Strategies as a function of C^j

Proof. It follows immediately from the proofs of lemmas 1 and 2. □

An important implication of the lemmas above (whose results are represented in Figure 1) is that if specialization is an equilibrium in a microfinance market, then it is an equilibrium with *credit rationing.* This rationing is due to the combined effect of adverse selection and oligopolistic competition. Different than in Stiglitz and Weiss (1981), where rationing is a consequence of the presence of 'bad' types in the market, in our model the value of x is determined by the outside option of the competitor. In Lemma 1, i chooses the level of rationing in order to make the screening strategy optimal for j. In Lemma 2, i increases the information rent offered to the Risky borrowers in order to reduce rationing of the Safe ones and increase j's profit.

Let j be the MFI serving the Risky borrowers. Knowing the MFIs' reaction functions, we can now describe the conditions making screening equilibria possible.

Proposition 1. *Suppose that $\alpha^j \geq (1-\beta)$ for $j \in \{a,b\}$. Then, in the simultaneous model, when the following condition is satisfied:*

$$\tilde{x}^i < \hat{x}(R_s) \tag{7}$$

for $i \neq j$, $i \in \{a,b\}$, there exist a screening equilibrium in which MFI i serves the Safe types setting $C^i = (\hat{x}(R_s), R_s)$ and MFI j serves the Risky types setting $C^j = (1, \tilde{D}^j(1))$.

Proof. See Appendix A □

Screening is only possible when the capacity of the MFI serving the Risky types is high enough to serve them all. Where it not the case, some of the Risky borrowers would apply for credit to the MFI targeting the Safe borrowers making the equilibrium unsustainable. Interestingly, screening equilibria are more likely to exist when the level of heterogeneity is high. In fact, $\tilde{x}_s^i$ is increasing in R_s, whereas the threshold defined in Proposition 1 is decreasing. For a given value of m, an increase of R_s can be interpreted as a reduction of the level of heterogeneity. The result is then quite intuitive: when heterogeneity is high, the opportunity cost of serving the 'wrong' type is larger.

Note that to prove this result we make no use of the assumption $\alpha^a + \alpha^b \leq 1$. Indeed the result is valid more generally. Nonetheless, the more α^a differs from α^b, the larger is the range of parameters for which screening equilibria exist. Moreover, the high capacity MFI is more likely to serve the Risky types in equilibrium. This is particularly true when $\alpha^a > \max\{\beta, 1-\beta\}$. It is easy to show that in this case α^b is smaller than $1-\beta$.[13] So if a targets the Risky, b's outside options to the screening strategy (in particular the option of undercutting a) are clearly less interesting.

What happens when the conditions in Proposition 1 are not satisfied? We can show that under our hypothesis, there always exists a pooling Nash equilibrium in which MFIs do not screen the borrowers. In order to prove it, define $D^*(i)$ as the repayment such that:

$$\alpha^i[\beta(p_s D^*(i) - 1) + (1-\beta)(p_r D^*(i) - 1)] = \max\{\Pi^i_{ResR}, \Pi^i_{ResB}\}$$

$D^*(i)$ is the repayment such that the profit from serving both types is equal to the profit from serving the residual demand. We can introduce the following proposition:

Proposition 2. *The pair of contracts $C^a = (1, D^*(b))$, $C^b = (1, D^*(b))$, is a Nash equilibrium of the simultaneous game with profit maximizing MFIs.*

Proof. See Appendix A □

This last result hinges on the hypothesis that $\alpha^a + \alpha^b \leq 1$. As showed in the proof, this implies that $D^*(a) = D^*(b)$ so that no MFI has incentives to deviate. The hypothesis on the capacity constraints is not unrealistic since despite the rapid increase of credit supply in development countries, many markets are still not saturated, and most MFIs are still struggling to

[13] Let $\beta > (1-\beta)$. Then $\alpha^a > \beta \Rightarrow 1-\alpha^a < 1-\beta \Rightarrow \alpha^b < (1-\beta)$.
If instead $\beta < (1-\beta)$, then $\alpha^a > (1-\beta) \Rightarrow 1-\alpha^a < \beta \Rightarrow \alpha^b < \beta \Rightarrow \alpha^b < (1-\beta)$

increase their outreach. Note that the equilibrium described above always exists. Thus, for some parameters, the model has multiple equilibria.

3.2 Multiple Borrowing Allowed

In Section 2.1 we showed how the incentive for borrowers to take multiple loans depends on the contracts available in the market. Clearly, the decision concerning which contract to offer depends on the competitive interaction between MFIs. In this section we reconsider the equilibria described above to see how and if the prediction we made in the previous section are influenced by the existence of agreements to share information. We show that for a large range of parameters the screening equilibria are robust to this assumption.

In the simultaneous model with two profit maximizing firms, we showed that there exist equilibria in which screening takes place. In these equilibria the MFI targeting the Safe types, say MFI i, sets $x^i < 1$ and D^i as high as possible, namely equal to R_s. The competing MFI j serves instead the Risky borrowers setting $x^j = 1$ and D^j low enough in order to leave them with the necessary informational rent.

We characterized these equilibria in a model in which we assumed that MFIs have perfect information about the borrowers' level of indebtedness. We now want to check whether, and under which conditions, these equilibria are robust to changes in the informational structures. In other words, we want to understand whether the screening contracts described above create *ex-ante* incentives for multiple-borrowing. We know from Section 2.1 that in order to avoid multiple borrowing the following conditions must be simultaneously satisfied (these are simply conditions (2) and (3) rearranged):

$$p_r p_r' x^a x^b (2R_r - D^b - D^a) + x^b(1 - x^a) p_r (R_r - D^b) + x^a(1 - x^b) p_r (R_r - D^a)$$
$$> x^a p_r (R_r - D^a) \qquad \text{(Risky borrowers)}$$

$$p_s p_s' x^a x^b (2R_s - D^b - D^a) + x^b(1 - x^a) p_s (R_s - D^b) + x^a(1 - x^b) p_s (R_s - D^a)$$
$$> x^b p_s (R_s - D^b) \qquad \text{(Safe borrowers)}$$

Therefore, we consider the equilibrium contracts described in the previous section and we check whether they satisfy the conditions above. We show that when p_i' is relatively low, the constraints are both satisfied. Intuitively, this is due to the fact that borrowers recognize that in case of failure, they will have to repay two loans rather than one and enjoy no rent even in case of partial failure. In that respect, taking a second loan actually decreases the chances to enjoy some rent since the income from one project might be lost to repay the other. In the following propositions we state formally the conditions under which multiple borrowing does not take place. We start by considering the screening equilibria.

Proposition 3. *When two profit maximizing MFIs compete, in the screening equilibria there are no* ex-ante *incentives for multiple-borrowing if* $p_r' < \hat{p}_r'$*, where*

$$\hat{p}_r' := \frac{(1-\beta)(m-1) - \Pi^a_{ResB}}{(1-\beta)(2m - p_r R_s - 1) - \Pi^a_{ResB}} < 1.$$

Proof. See Appendix A □

The proposition above shows that screening equilibria are robust to the specific type of incomplete information we are considering when the probability of succeeding in the second project is low enough. Note that no conditions are required on p_s' since the Safe borrowers incentive constraint is not binding. The implication of this result is that, if the contracts are properly defined, multiple lending is *ex-ante* not a problem whenever the market is risky enough.

The assumption that borrowers repay as much as they can plays a crucial role. This highlight the fact that the very frequent installments characterizing micro-loans' repayment schedules are one of the fundamental ingredients that allowed for the success of microfinance. As other researchers pointed out, this seems to be more relevant than group lending to explain the impressively low default rates of poor borrowers. Our result identify a different reason why frequent installments can be of fundamental importance for an MFI operating in a competitive market.

As noticed in the previous section, screening is not the only possible outcome of the competitive interaction between MFIs. We showed that, for some values of the parameters, pooling equilibria can prevail. The result of Proposition 3 extends to these cases in a very similar way: when MFIs offer identical contracts, there are no incentives for ex-ante multiple borrowing as long as the market is risky enough. The result is formalized in the next proposition.

Proposition 4. *When two profit maximizing MFIs compete, in the pooling equilibria there are no ex-ante incentives for multiple borrowing if* $p_s' < 1/2$.

Proof. See Appendix A □

It is interesting to compare the results of propositions 3 and 4 to understand the circumstances in which multiple borrowing is more likely to take place. The relative performance of screening equilibria versus no-screening ones is unfortunately ambiguous in some cases. Clearly, when $\hat{p}'_r > 1/2$ then screening makes multiple lending unambiguously less likely. By using the definition of $\hat{p}'_r$ we can note that

$$\hat{p}'_r > 1/2 \quad \Leftrightarrow \quad (1-\alpha^a)(1-\beta)(p_r R_s - 1) > \alpha^a \beta (m - p_r R_s).$$

If $\alpha^a > 1/2$, then the condition is satisfied when the fraction $(1-\beta)$ of risky borrowers is high and/or when the difference between the safe and the risky borrowers is relatively small in terms of return and probability of success (so that $(p_r R_s - 1)$ is close to $(m-1)$). If $\alpha^a < 1/2$, then the condition is more easily satisfied. Since by assumption $\alpha^a \geq \alpha^b$, this means that screening is particularly useful and likely to take place when the market is still largely unserved and the fraction of Risky borrowers is high. Both hypothesis fit very well the typical microcredit market. When $\hat{p}'_r < 1/2$, no clear comparison is possible.

A different way to put it is to say that if $\hat{p}'_r > 1/2$ and $p'_r \in [1/2, \hat{p}'_r]$, than screening can become a way to solve the problem of ex-ante multiple borrowing. In this case, in fact, with a non-screening strategy multiple borrowing is unavoidable. Screening, instead allows to eliminate the incentives for all the borrowers to take more than one loan. Note, moreover, that screening is in this case much easier to sustain in equilibrium since the profit from all the outside options, for both MFI a and b is importantly reduced.[14]

In any case, since $p'_s > p'_r$, whenever the Safe types have incentives to multiple borrow also the Risky ones have, the results above have an important implication summarized in the next corollary.

Corollary 1. *In both screening and pooling equilibria, MFIs do not want to multi-lend.*

Intuitively, MFIs might have incentives to multi-lend to safe borrowers

[14] If a non-screening strategy leads to multiple borrowing, whereas a screening one avoids it, the type of equilibria described in Proposition 1 are easier to attain. A more formal characterization would require to re-calculate the thresholds described above. But in order to do it is necessary to model the behavior of borrowers when only partial reimbursement is possible. As discussed above, they could give priority to one MFI rather than the other (because of different enforcement power, dynamic incentives etc.). Modelling all this explicitly is interesting for other purposes, but qualitatively would not add anything to our discussion. We believe our results are able to describe the mechanics and the forces leading to multiple borrowing without making arbitrary assumptions.

when $p'_s > p_r > p'_r$. Our results show that even in this case, MFIs prefer to avoid it.

4 Conclusions

Microfinance has attracted an important variety of actors, pursuing different objectives and competing with each other to attract clients. Our model describes the interaction between these actors in a tractable framework capturing the special features of microcredit markets.

Our contribution is to show that even if increasing competition can make informational asymmetries harsher, proper contract design can help mitigating some of the consequent negative effects. We concentrate on the *ex-ante* incentive to multiple-borrow in order to evaluate the effects of the absence of a credit bureau. Understanding the mechanism driving our results is very important for those who are working to enlarge the outreach and promote the development of microfinance.

Our model does not tackle all the issues created by insufficient information sharing between MFIs. In particular, using a static model, we concentrate only on the *ex-ante* incentives to multiple borrow. A dynamic set-up would allow to address the *ex-post* incentives arising from unpredicted negative shocks. Thus, our result should not be read as aiming at understating the importance of a credit bureau. Our emphasis is rather on how MFIs can minimize their risk when information sharing is impossible. We believe this is an interesting approach since in many developing countries the conditions making the creation of a credit bureau possible are still far from being fulfilled.

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A Appendix

Proof of Lemma 1: Suppose that i is willing to serve the Safe borrowers only, and that she offers the contract described in Lemma 1. We show that j's optimal reaction is to offer a screening contract. We start by computing the profits j would get serving the Risky borrowers only, that is when $B^j(C^i, C^j) = (1-\beta)$. In this case, her maximization problem is given by:

$$\max_{x^j, D^j} \Pi^j_{rs} = (1-\beta)x^j(p_r D^j - 1)$$

In order to have $B^j(C^i, C^j) = 1 - \beta$, we need the following conditions to hold.

$$\begin{aligned} D^j &\leq R_r & PC1 \\ D^i &\leq R_s & PC2 \\ x^j p_r(R_r - D^j) &\geq x^i p_r(R_r - D^i) & IC1 \\ x^i p_s(R_s - D^i) &\geq x^j p_s(R_s - D^j) & IC2 \end{aligned}$$

Consider first the constraints $PC1$ and $IC1$. The IC1 is always binding since the left hand side is decreasing in D^j. Solving it for D^j we get:

$$D^j = R_r - \frac{x^i}{x^j}(R_r - D^i)$$

What about x^j? Substituting D^j in the profit function we get:

$$\Pi^j_{rs} = (1-\beta)x^j[p_r R_r - p_r \frac{x^i}{x^j}(R_r - D^i) - 1] = (1-\beta)(x_r p_r R_r - x^j - p_r x^i(R_r - D^i))$$

that is clearly maximized for $x^j = 1$ given that $p_r R_r = m > 1$. So j can set:

$$\begin{cases} x^j = 1 \\ D^j = R_r - \frac{x^i}{x^j}(R_r - D^i) \end{cases} \tag{8}$$

that gives her the expected profit:

$$\Pi^j_{rs} = (1-\beta)[(m-1) - p_r x^i(R_r - D^i)] \tag{9}$$

This profit must be compared with j's outside options. She can:

1. Target the Risky sector, but serve only the residual demand of the Risky. It is then optimal to set $D^j = R_r$ and $x^j = 1$, that gives profit $\Pi_{ResR} = \alpha^j(1-\beta)(m-1)$.
2. Target the residual demand of Both types. This leads to profit $\Pi_{ResB} = \alpha^j[\beta(m-1) + (1-\beta)(p_r R_s - 1)]$.
3. Target both types undercutting the Incumbent's contract. This can be done by setting $x^j = 1$ and $D^j = D^i$. The profit is then the same as in point 2: $\Pi_{Both} = \alpha^j[\beta(m-1) + (1-\beta)(p_r R_s - 1)]$.

The equality between Π_{ResB} and Π_{ResR} is due to the assumption $\alpha^i + \alpha^j < 1$. Depending on the parameters and on the assumptions about MFIs' behavior, one of the remaining options dominates the other. When $\Pi^j_{ResR} \geq \Pi^j_{ResB}$ we need this condition to hold for j to engage in screening:

$$(1-\beta)[(m-1) - p_r x^i(R_r - D^i)] > \alpha^j(1-\beta)(m-1) \tag{10}$$

Solving the inequality for x^i we find the threshold:

$$\hat{x}(D^i) := \frac{(1-\alpha^j)(m-1)}{m - p_r D^i} \tag{11}$$

When $\Pi^j_{ResB} \geq \Pi^j_{ResR}$, the following condition is needed:

$$(1-\beta)[(m-1) - p_r x^i (R_r - D^i)] > \alpha^j [\beta(m-1) + (1-\beta)(p_r R_s - 1)] \tag{12}$$

and solving for x^i we get:

$$\hat{x}(D^i) := \frac{(1-\beta)(m-1) - \alpha^j[\beta(m-1) + (1-\beta)(p_r R_s - 1)]}{(1-\beta)p_r(R_r - D^i)} \tag{13}$$

Note that in all these cases $\hat{x}(D^i)$ is not necessarily in $[0,1)$. If $\hat{x}(D^i)$ is greater than one, then screening is clearly possible for any $x^i < 1$.
We still have to show that these values of $\hat{x}(D^i)$ make screening possible. We have to verify that given j's optimal reaction, the value $\hat{x}(D^i)$ satisfies also condition (IC2). Replacing $x^j = 1$ and $D^j = R_r - \frac{x^i}{x^j}(R_r - D^i)$ in the IC2 we get:

$$x^i(R_s - D^i) \geq [R_s - R_r + x^i(R_r - D^i)] \Rightarrow x^i(R_s - R_r) \geq R_s - R_r$$

that is satisfied for any $x^i \in [0,1)$. □

Proof of Lemma 2: Suppose that i wants to specialize in the Risky sector inducing j to serve the Safe sector and offer an incentive compatible contract. In this case j solves this maximization problem:

$$\max_{x^j, D^j} \Pi^j_{sr} = \beta x^j (p_s D^j - 1)$$

To have $B^j(C^i, C^j) = \beta$, the following conditions must be fulfilled:

$$\begin{aligned} D^j &\leq R_s & PC1 \\ D^i &\leq R_r & PC2 \\ x^i p_r (R_r - D^i) &\geq x^j p_r (R_r - D^j) & IC1 \\ x^j p_s (R_s - D^j) &\geq x^i p_s (R_s - D^i) & IC2 \end{aligned}$$

Note first that i sets $D^i \geq R_s$. We show that, as long as $D^i > R_s$, i can raise j's profit from screening by setting a lower D^i. Consider first the IC2. When $D^i \geq R_s$ the RHS is negative, and the PC binds. Thus j can set $D^j = R_s$. In order to attain screening, IC1 must be satisfied. Solving it for x^j we find the condition:

$$x^j \leq \frac{x^i(R_r - D^i)}{R_r - D^j} \tag{14}$$

that is binding at the optimum. Notice that if $D^i = R_r$, (14) is true only for $x^j = 0$. So i must offer a contract with $D^i < R_r$. j's expected profit is then:

$$\Pi^j_{sr} = \beta \hat{x}(m-1) \tag{15}$$

This must be compared with j's outside options. She can:

1. Target both types offering a non incentive compatible contract characterized by $D^j = R_s$ and $x^j = 1$. This strategy gives profit $\Pi^j_{br} = \alpha^j(\beta(m-1)+(1-\beta)(p_rR_s-1))$. In this case, for j to prefer serving the Safe types, we need $\Pi^j_{sr} \geq \Pi^j_{br}$. In formulas:

$$\beta x^j(m-1) \geq \alpha^j(\beta(m-1)+(1-\beta)(p_rR_s-1)) \Longrightarrow x^j \geq \alpha^j(1+\frac{(1-\beta)(p_rR_s-1)}{\beta(m-1)})$$

Replacing x^j with (14) we get:

$$D^i \leq R_r - \frac{\alpha^j}{x^i}\Big[1+\frac{(1-\beta)(p_rR_s-1)}{\beta(m-1)}\Big](R_r-R_s) := \tilde{D}^i$$

2. Target the Risky sector, undercutting i: also in this case, as showed above, to induce screening i must set $D^i = R_r - x^j/x^i(R_r - R_s)$. We can determine the relevant value of x^j by solving the inequality :

$$\beta x^j(m-1) \geq (1-\beta)[(m-1) - p_rx^j(R_r-R_s)] \Longrightarrow$$
$$x^j \geq \frac{(1-\beta)(m-1)}{\beta(m-1)+(1-\beta)(m-p_rR_s)}.$$

Now replacing again x^j with (14) we get:

$$D^i \leq R_r - \frac{1}{x^i}\Big[\frac{(1-\beta)(m-1)}{\beta(m-1)+(1-\beta)(m-p_rR_s)}\Big](R_r-R_s) := \tilde{D}^i$$

If we define

$$\tilde{x}^j := \max\left\{\alpha^j(1+\frac{(1-\beta)(p_rR_s-1)}{\beta(m-1)}), \frac{(1-\beta)(m-1)}{\beta(m-1)+(1-\beta)(m-p_rR_s)}\right\}$$

then $\tilde{D}^i(\tilde{x}^j)$ gives the upper bound for D^i.

□

Proof of Proposition 1: The proof hinges on Lemma 1 and Lemma 2. Suppose that MFI i, with $i \in \{a, b\}$ has offered an incentive compatible contract targeting the Safe borrowers, that is a contract such that $D^i \leq R_s$ and $x^i < 1$. Assume also that $\alpha^j \geq 1-\beta$, with $j \neq i$, $j \in \{a, b\}$. Then MFI j's reaction is to offer an incentive compatible contract, too (that is a contract characterised by $D^j = R_r - x^i(R_r - D^i)$ and $x^j = 1$) if the profit from screening is higher than the best possible outside option. MFI j's profit from serving the Risky types in a screening set-up is given by

$$\Pi^j_r(C^i) = (1-\beta)[(m-1) - x^i(m - p_rD^i)]$$

The best outside option for j, given i's contract, is to undercut it offering $D^j = D^i$ and $x^j = 1$. That would give her $\Pi^j_{Both}(C^i) = \alpha^j(\beta(p_sD^i - 1) + (1-\beta)(p_rD^i - 1))$. Thus the condition for MFI j to prefer screening is: $\Pi^j_r(C^i) > \Pi^j_{Both}(C^i)$. As showed in Lemma 1 and 2, MFI i optimally sets $D^i = R_s$. Thus, the condition above can be rewritten as:

$$x^i \leq \frac{(1-\beta)(m-1) - \alpha^j[\beta(m-1) + (1-\beta)(p_r R_s - 1)]}{(1-\beta)(m - p_r R_s)} \quad (16)$$

In order for the strategies defined above to be an equilibrium, we need MFI i to prefer setting x^i smaller than the upper bound above rather than playing her outside options. Several alternatives are available to i. Assume first that $\alpha^i > \beta$. There are then two cases:
(i) The best outside option is to serve both types setting $D^i = R_s$ and $x^i = 1$. In this case for i to prefer a screening strategy we need this condition to hold:

$$x^i \geq \frac{\alpha^i[\beta(m-1) + (1-\beta)(p_r R_s - 1)]}{\beta(m-1)} := \tilde{x}^i \quad (17)$$

(ii) The best outside option is to undercut MFI j's contract. We have to distinguish two sub-cases. If $\alpha^i \geq (1-\beta)$ the screening condition is:

$$x^i \geq \frac{(1-\beta)(m-1)}{\beta(m-1) + (1-\beta)(m - p_r R_s)} := \tilde{x}^i \quad (18)$$

If instead $\alpha^i < (1-\beta)$ the condition is:

$$\beta x^i(m-1) \geq \alpha^i(1-\beta)[(m-1) - x^i(m - p_r R_s))$$

that can be rewritten as:

$$x^i \geq \frac{\alpha^i(1-\beta)(m-1)}{\beta(m-1) + \alpha^i(1-\beta)(m - p_r R_s)} := \tilde{x}^i \quad (19)$$

To have an equilibrium, equation (16) and one of the three equations defining $\tilde{x}^i$ ((17), (18), (19)) must be satisfied simultaneously.
Consider now the case in which $\alpha^i < \beta$. It easy to see that in this case equilibria similar to the one described above are still possible. If $\alpha^i > \tilde{x}^i$ than the results showed above hold true. If the capacity is instead very small, then the level of screening is implicitly defined by α^i. To see that, just observe that when $\alpha^i < \tilde{x}^i_s$ and $D^i \leq R_s$ the outside option examined at point (i) can be ruled out. In fact, j can impose a screening strategy just by giving the Risky borrowers the adequate informational rent. □

Proof of Proposition 2: Suppose first that $\Pi_{ResR} > \Pi_{ResB}$, and that MFI a offers a contract with $x^a = 1$ and $D^a = D^*(b)$. We describe the optimal reaction of b. Given a's capacity constraint, the residual demand is given by $1 - \alpha^a$, but by assumption $\alpha^b \leq (1 - \alpha^a)$. So b cannot do better than offering $D^*(b)$. In fact, by definition $D^*(b)$ satisfies this condition:

$$\alpha^b[\beta(p_s D^*(b) - 1) + (1-\beta)(p_r D^*(b) - 1)] = \alpha^b(1-\beta)(m-1)$$

that can be rewritten as $\beta(p_s D^*(b) - 1) + (1-\beta)(p_r D^*(b) - 1) = (1-\beta)(m-1)$. We now show that offering $x^a = 1$ and $D^a = D^*(b)$ is a best reaction for a given b's contract. For a not to be willing to undercut b's contract, $D^*(b)$ must satisfy this condition:

$$\alpha^a[\beta(p_s D^*(b) - 1) + (1-\beta)(p_r D^*(b) - 1)] = \alpha^a(1-\beta)(m-1),$$

since $\alpha^a \leq (1-\alpha^b)$. The condition is clearly satisfied. So a's best reply, given our tie-breaking rule, is also to offer $x^a = 1$ and $D^a = D^*(b)$. Analogous reasoning can be used for the case in which $\Pi_{ResR}(\alpha^b) < \Pi_{ResB}(\alpha^b)$. □

Proof of Proposition 3: Assume that MFI b serves the Safe borrowers and MFI a serves the Risky ones. In the equilibria with screening of the simultaneous model b sets $x^b = \hat{x}_s < 1$ and $D^b = R_s$, whereas a sets $x^a = 1$ and $D^a = \hat{D}_r$, where $\hat{x}_s$ and $\hat{D}_r$ are defined as in Lemma 1.

When we substitute these values in equations (2) and (3), we get the following conditions:

$$(R_r - R_s)p'_r < (R_r - \hat{D}_r)(1 - p'_r)$$

for the Risky not to multiple-borrow, and

$$(R_s - \hat{D}_r)(p'_s x^b + 1 - x^b) < x^b(R_s - R_s)(1 - p'_s) = 0$$

for the Safe not to multiple borrow. The second condition is always satisfied since $\hat{D}_r > R_s$. The first condition is satisfied for

$$p'_r < \frac{R_r - \hat{D}_r}{2R_r - R_s - \hat{D}_r} = \frac{(1-\beta)(m-1) - \alpha_a[p_r R_s - 1 + \beta(m - p_r R_s)]}{(1-\beta)(2m - p_r R_s - 1) - \alpha_a[p_r R_s - 1 + \beta(m - p_r R_s)]}$$

Note that the threshold is well defined since it always belong to the interval $[0, 1]$. □

Proof of Proposition 4. The result follows immediately from Proposition 2 and equations (2) and (3) by replacing $D^a = D^b = D^*$ and $x^a = x^b = 1$. □