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Optically simulated universal quantum computation

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Abstract. Recently, classical optics based systems to emulate quantum information processing have been proposed. The analogy is based on the possibility of encoding a quantum state of a system with a 2^N-dimensional Hilbert space as an image in the input of an optical system. The probability amplitude of each state of a certain basis is associated with the complex amplitude of the electromagnetic field in a given slice of the laser wavefront. Temporal evolution is represented as the change of the complex amplitude of the field when the wavefront pass through a certain optical arrangement. Different modules that represent universal gates for quantum computation have been implemented. For instance, unitary operations acting on the qbits space (or U(2) gates) are represented by means of two phase plates, two spherical lenses and a phase grating in a typical image processing set up. In this work, we present CNOT gates which are emulated by means of a cube prism that splits a pair of adjacent rays incoming from the input image. As an example of application, we present an optical module that can be used to simulate the quantum teleportation process. We also show experimental results that illustrate the validity of the analogy. Although the experimental results obtained are promising and show the capability of the system for simulate the real quantum process, we must take into account that any classical simulation of quantum phenomena, has as fundamental limitation the impossibility of representing non local entanglement. In this classical context, quantum teleportation has only an illustrative interpretation.

Keywords: Quantum Information, Imaging and optical processing, Quantum teleportation
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INTRODUCTION

Quantum information processing has received special attention not only for the variety of the problems of practical interest that it raises (quantum computation, quantum key distribution, etc) but also for many others problems that show the most counterintuitive aspects of quantum mechanics. Otherwise, the analogy between quantum mechanics and classical optics has been recently explored [1–6]. The key idea is to exploit the wave nature of the electromagnetic field in order to represent the quantum state of one or more particles. In this representation, the probability amplitude of each state of a basis is associated to the amplitude of the electromagnetic field and temporal evolutions are simulated by means of the propagation of the electromagnetic field through an optical system. The possibility of performing simulations of quantum phenomena by means of classical analogies is interesting from many points of view. Quantum algorithms can be understood as a consequence of the wave nature of the evolution of quantum states. In this sense, the wave character of the electromagnetic field allows us to emulate in an elegant way how the quantum algorithms work.

In this work, we present results that show how universal cuantum computation can be optically simulated. We show how quantum states can be represented as images and universal quantum gates can be represented as coherent optical processors. As an example, we show how all these elements can be combined in order to obtain an optical setup for simulating the quantum teleportation process. We present experimental results where the classical representation of the operations of one qbit is done. Finally, the illustrative character of this classical simulation is discussed.

BACKGROUND

Quantum computation and quantum information is the study of the information processing that can be accomplished using quantum mechanical systems. The bit is the fundamental concept of classical computation. Quantum computation and quantum information are built upon an analogous concept, the *quantum bit* or *qbit* for short. A qbit is simply an state of the two-dimensional Hilbert space H_2 and it can be denoted as a complex linear combination of the two states of the computational basis $\{|0\rangle, |1\rangle\}$. An state of the 2^N-dimensional space of a system composed by N qbits $H_2^{\otimes N}$ can be denoted as the product $|\Psi(1)\rangle_1 |\Psi(2)\rangle_2 \ldots |\Psi(3)\rangle_N$, where $|\Psi(j)\rangle = \alpha_j |0\rangle_j + \beta_j |1\rangle_j$ is the qbit

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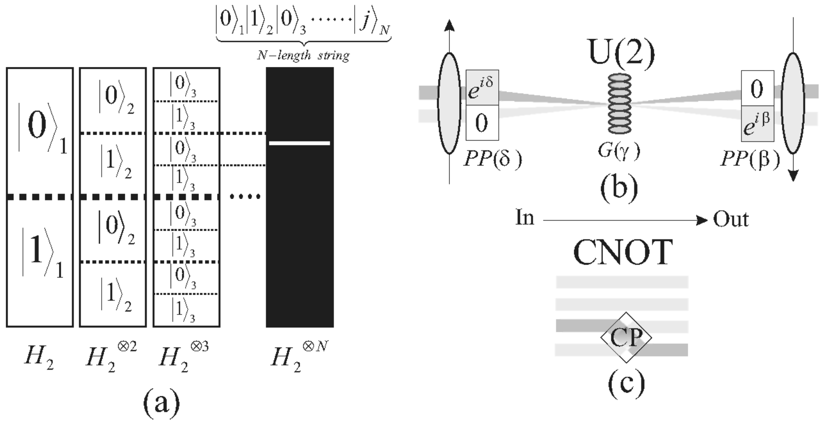

FIGURE 1. (a) Spatial organization of the input plane in order to perform the optical representation of the N qbits state.(b) Coherent optical processor with two phase plates and a phase grating in the Fourier plane as the optical single qbit gate.(c) A cube prism as the optically simulated CNOT gate.

state associated to the jth qbit (we have omitted the tensor product symbol "$\otimes$" between each factor for simplicity). Usually, quantum computation process can be thought as a *circuit* whose input and output states are generally multiple qbits states. According to this circuital model of quantum computation, the input state is mapping onto the output state by a unitary operator. It can be demonstrated the following *universality* result: *Any multiple qbit logic gate may be composed from* CNOT *and single qbit gates* [7]. Single qbit gates are the unitary operators acting on one qbit states. The CNOT gate has two input qbits, known as the control qbit and the target qbit, respectively. The action of the gate can be described as follows. If the first qbit (that is the control qbit) is set to 0, then the second qbit (that is the target qbit) is unchanged. If the control qbit is set to 1, then the target qbit is flipped. So, the action of the CNOT gate on each element of the two qbits computational basis is $|0\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|0\rangle_2; |0\rangle_1|1\rangle_2 \rightarrow |0\rangle_1|1\rangle_2; |1\rangle_1|0\rangle_2 \rightarrow |1\rangle_1|1\rangle_2$ and $|1\rangle_1|1\rangle_2 \rightarrow |1\rangle_1|0\rangle_2$.

OPTICAL SIMULATION

Let us briefly introduce how the quantum states can be emulated as spatial distributions of light [4]. If we divide the wavefront of a laser beam in 2^N portions, the amplitude of the electromagnetic field in each portion can be associated with the probability amplitude of a certain computational state. For example, let us consider that the input plane is limited to a square region, which is splitted in two halves. We use the convention that the up and the down regions correspond to the two computational states of a single qbit: up is associated with the state $|0\rangle$ and down is associated with the state $|1\rangle$. Once the up and down regions (in the entire plane) are defined, within each one of these regions, we can define another up and down regions with the same convention, in order to represent the computational states of a second qbit. Following the same idea we can represent a state of N qbits. In fact at the ending of the previous process we will have a one to one mapping between each region of the entire plane and the set of N-length binary strings. It should be noted that the mapping is exponentially inefficient since in order to represent the state of a quantum computer with N qbits it is necessary to divide the plane in 2^N regions [8]. In Fig.(1a) we show an scheme of the spatial organization of the input scene. A single slice into each region of the plane will be associated with a certain N-length binary string. The state corresponding to a well defined value for the binary string is one where the electromagnetic field amplitude is zero everywhere in the plane except for a single slice. More general states (complex linear combinations of the previous states) can be generated using a mask generated by using a medium where the complex amplitude of the electromagnetic field can be controlled. This can be done by means, for example, of a programmable LCD displays as we will discuss later.

As we have mentioned above unitary temporal evolutions can be simulated by means of optical processors. In general any unitary evolution of N qbits may be implemented exactly by composing unitary single qbits gates and CNOT gates [7]. We show in this section how to implement these two kind of operators by means of classical optics.

Let us begin with the single qbit $\mathbf{U}(2)$ operators. We have recently proposed a combination of optical elements to obtain the Hadamard operator acting on a single qbit [5]. The Hadamard operator is a particular case of a more general $\mathbf{U}(2)$ operators that we describe here. We will exploit the fact that the complete set of $\mathbf{U}(2)$ operators can be decomposed as a product of rotations generated by the Pauli $\mathbf{Z}$ operator (phase shifts) and rotations generated by the Pauli $\mathbf{Y}$ operator (ordinary $\mathbf{SO}(2)$ rotations) [7]. The arbitrary 2×2 unitary matrix could be decomposed as:

$$\mathbf{U} = e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\beta} \end{pmatrix} \begin{pmatrix} \cos\frac{\gamma}{2} & -\sin\frac{\gamma}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\gamma}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \tag{1}$$

where α, β, γ and δ are real-valued. The idea is very simple: Once the qbit is spatially distributed in the input, a phase plate PP(δ) that introduces a retard in the phase of a certain amount δ over the down half of the wavefront is used. Then, a convergent lens is used in order to obtain the optical Fourier transform of the image. In the Fourier plane a phase grating is placed. The frequency of the grating is selected to produce diffracted orders whose separation in the final plane is equal to the distance between the two qbit images. The phase grating is constructed in such a way that the three principal orders (-1, 0 and 1) have relative intensities $\cos(\gamma/2)$ for the zero order and $\sin(\gamma/2)$ for the one and the minus one orders (the real parameter γ is controlled by modifying the level of the phase modulation of the grating). We denote the grating G(γ) as it is shown in Fig.(1b). Following the grating, a second phase plate PP(β) is placed. A second lens produces the inverse Fourier transform and the transformed qbit is obtained in the output plane. In Fig.(1b) the set-up for the optically simulated U(2) operator is shown. In the case of CNOT gate acting on two qbits state, the classical optics simulation can be made by using a cube prism (CP) that splits the pair of rays coming from the down half of the entire input plane as we shown in Fig.(1c). Since each one of these four incoming beams represents the complex amplitude of two qbits basis state in the same order and with the same convention that in Fig(1a) the cube prism only splits the parity label of the complex amplitude (the second or target qbit) if the pair of rays come from the down half of the plane (the first or control bit in the logical $|1\rangle$) .

AN EXAMPLE: QUANTUM TELEPORTATION

Quantum teleportation is usually explained as follows: Two persons (Alice and Bob) were together for some time but now they live far apart. While together they generated an entangled quantum state of two qbits, then each one takes one qbit of the entangled EPR pair [9] and finally they separate themselves. It has been shown that Alice can send a certain qbit $|\Psi\rangle$ to Bob, and she can do this only sending him classical information [10–14]. Let us suppose that the state to be teleported is $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are unknown complex amplitudes. The process begins with the three qbits state $|\Psi\rangle_1 |0\rangle_2 |0\rangle_3$. For simplicity, we have divided the full process in four stages or modules. Alice and Bob (that are still together at this time) begin the process by interacting qbits 2 and 3 in order to create an entangled pair. Two operations are necessary for doing this: The Hadamard gate (a single qbit unitary operation) that maps the basis states $|0\rangle$ and $|1\rangle$ into the superposition $(|0\rangle + |1\rangle)/\sqrt{2}$ and $(|0\rangle - |1\rangle)/\sqrt{2}$ respectively; and the CNOT gate. After a Hadamard operator on the qbit 2 and the CNOT operator on qbits 2 and 3 (with qbit 2 as control), the prepared state at $t = 0$ results $|\Phi(t=0)\rangle = |\Psi\rangle_1 (|0\rangle_2 |0\rangle_3 + |1\rangle_2 |1\rangle_3)/\sqrt{2}$. This is the end of the first stage.

At this time, Bob moves apart from Alice. The first two qbits (1 and 2) belong to Alice, while the third qbit goes far away with Bob. Once Alice has the qbit to be teleported together with her part of the entangled pair, she begins the second stage of the process that is usually called *Bell analysis* on qbits 1 and 2 [9, 15]. First, she sends her qbits (1 and 2) through a CNOT gate using the unknown qbit as control and her part of the entangled pair as target. Then, she applies the Hadamard operator to her first qbit. This transformations put all three qbits into the state $|\Phi(t=1)\rangle = \mathbf{H}_1 \mathbf{CNOT}_{12} |\Phi(t=0)\rangle$ whose expression naturally breaks down into the following four terms:

$$\begin{aligned} |\Phi(t=1)\rangle \; = \; & \frac{1}{2}|0\rangle_1|0\rangle_2(\alpha|0\rangle + \beta|1\rangle)_3 + \frac{1}{2}|0\rangle_1|1\rangle_2(\alpha|1\rangle + \beta|0\rangle)_3 + \\ & + \frac{1}{2}|1\rangle_1|0\rangle_2(\alpha|0\rangle - \beta|1\rangle)_3 + \frac{1}{2}|1\rangle_1|1\rangle_2(\alpha|1\rangle - \beta|0\rangle)_3 \end{aligned} \tag{2}$$

The third stage begins with the measurement of qbits 1 and 2 in the computational basis. The first term of Eq.(3) has Alice's qbits in the state $|0\rangle_1 |0\rangle_2$, and Bob's qbits in the state $(\alpha|0\rangle + \beta|1\rangle)_3$, which is the original state. Therefore, if Alice performs a measurement and obtains the result 00, then Bob's qbit will be in the state $|\Psi\rangle$. Similarly, from Eq.(3), if the Alice's measurement is 01, 10 or 11 then the state of Bob's qbit becomes $(\alpha|1\rangle + \beta|0\rangle)_3$, $(\alpha|0\rangle - \beta|1\rangle)_3$ or $(\alpha|1\rangle - \beta|0\rangle)_3$ respectively. Note that in each of these three cases there is an unitary transformation that restores the

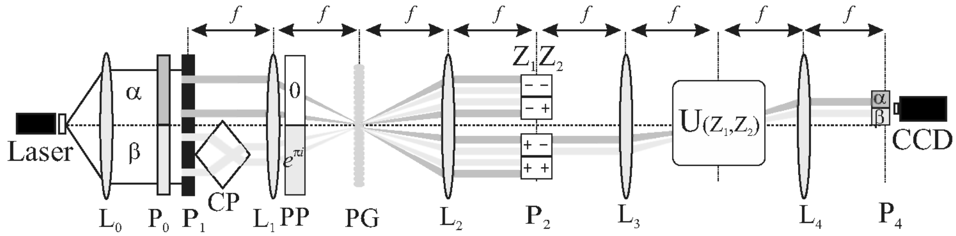

FIGURE 2. Experimental set-up for performing the optically simulated quantum teleportation

state of Bob's qbit to Alice's original (unknown) state. In the first case he can apply the Pauli **X** (which interchanges $|0\rangle$ and $|1\rangle$), in the second case, he can apply the Pauli **Z** operator (which leaves $|0\rangle$ alone but changes the sign of $|1\rangle$), and in the third case, he can apply the **ZX** operator. After performing the measurement, Alice must send to Bob the result of her measurement outcome (two bits of classical information) through some classical channel. The fourth stage can be described as a conditional correction of the third qbit state (in Bob's lab) after Alice's measurement. Once Bob has learnt the measurement outcome, in order to restore the state $|\Psi\rangle$ he has to apply the correcting unitary operation depending on the result obtained for Alice.

In the experimental set-up shown in Fig.2 , we can see the classical optics representation of each one of the four stages mentioned above. A laser is expanded, filtered and then collimated with lens L_0. The collimated beam impinges onto the mask P_0 that modulates the complex amplitude of the electromagnetic field in order to encode the information contained in the first qbit. Then, a binary mask P_1 is placed to generate the three qbit state $|\Phi(t=0)\rangle$. The second stage consists in the Bell analysis of qbits 1 and 2. CNOT gate on the first and second qbits is simulated by a cube prism CP that splits the rays coming from the down half of the entire input plane. Hadamard gate on the first qbit is simulated by using a phase plate PP($\delta=\pi$) and a phase grating G($\gamma=\pi/2$) between a pair of spherical lenses (L_1 and L_2) [5]. In this case the plate was constructed by deposition of a transparent film over a plane glass plate. The input plane lies on the same plane that the binary mask P_1. Lens L_1 (focal length 70 cm) allows to obtain the Fourier transform of the input plane over the phase grating G which is used to perform the Hadamard transform. The grating is constructed in such a way that the three principal orders (-1,0 and 1) have identical intensities. In our case we synthesized an holographic bleached grating with a period of 50 1/mm. A third lens L_2 (focal length 70 cm) is placed in order to perform the inverse Fourier transformation.

The next stage consists in the measurement of the qbits 1 and 2 in the computational basis. As result of this measurement process we have the collapse of the quantum state into one of the four possibilities labeled with the eigenvalues of the $\mathbf{Z}_1$ and $\mathbf{Z}_2$ measure operators. The classical analogy of this process is to select randomly one of the four states of the qbits 1 and 2 with the same probability. This random selection is represented by the two rays unobstructed to the right of the plane P_2 as it is shown in Fig.2. Each of these two rays represents the amplitudes associated with the two logical states of the third qbit.

The fourth stage consists in a conditional unitary operator $\mathbf{U}(Z_1,Z_2)$ acting on qbit 3 (In Bob's lab) in order to recover the original information. This last operation is controlled by the result of the measurement performed in the previous stage (in Alice's Lab). Depending on the result of this measurement (the pair of rays selected in the previous stage) we have to perform a certain unitary operation on qbit 3. In Fig.3 we show how each unitary correcting operation $\mathbf{U}(Z_1,Z_2)$ is associated with a certain reduced optical set-up. In practice we have used a pair of spherical lenses with 50 cm of focal length. The phase grating $G(\gamma=\pi)$ was programmed in a spatial light modulator (SLM) working in a phase mostly mode. This device consists in a Sony liquid crystal display TV (LCTV) that combined with two polarizers and two wave plates, acts as a pure phase modulator [16]. The final image is captured by a videocamera (CCD). In the final image, after application of correcting $\mathbf{U}(Z_1,Z_2)$ operator, we must take into account the local inversion of the coordinates system whose senses are indicated with arrows on the L_3 and L_4 lenses in Fig.3.

Experimental results

In Fig.4 we show the experimental results obtained with the set-up described in the previous section. In the left we show the obtained images and in the right we show the corresponding intensity profiles (performed by averaging rows) in arbitrary units. In Fig.4(a) we show the input scene that represents the three qbit state at $t=0$. Fig.4(b) and

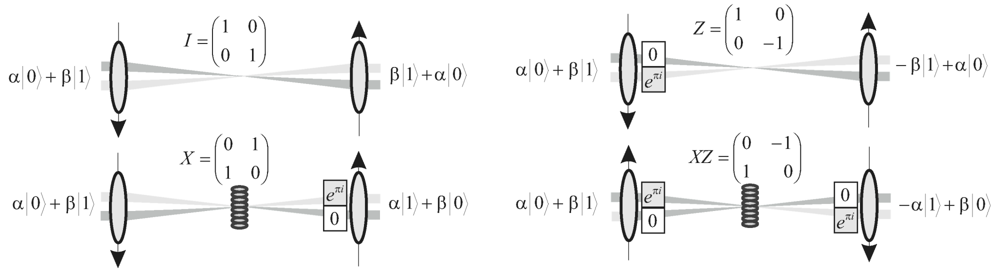

FIGURE 3. Reduced optical set-up for performing the optically simulated correcting operations for the four possible results of Alice measurement.

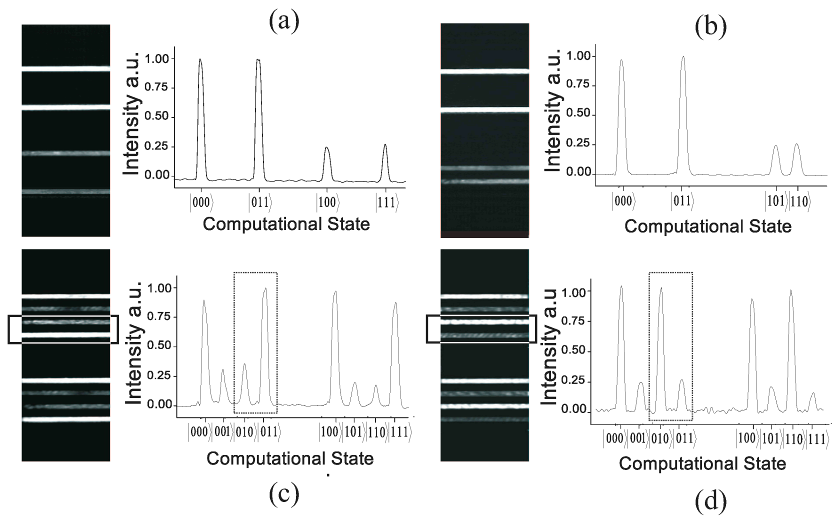

FIGURE 4. Experimental results: (a) Image representation of the three qbits state $|\Phi(t=0)\rangle$. (b) Image representation of the resulting state after CNOT operation on qbits 1 (control) and 2 (target). (c) Image representation after Hadamard operation on the first qbit of the state representd in (b). (d) Image representation of the four possible results after the correcting operations. The rectangles in (c) and (d) suggests the random selection performed by Alice measurement

Fig.4(c) are the otput images after the operations of the second stage. Fig.4(b) is the output image after CNOT gate acting on qbits 1 and 2 and Fig.4(c) shows the superposition performed by the optically simulated Hadamard gate acting on the first qbit. The next step is the quantum measurement of qbits 1 and 2 in the computational basis. As we have commented above, in our classical representation this is completely equivalent to select randomly one of the four adjacent pair of outcoming rays after Hadamard operation. This random selection is suggested by the rectangles in Figs.4(c) and (d). Finally, in Fig.4(d) we show the images after application of correcting $\mathbf{U}(Z_1,Z_2)$ operations. Each one of the correcting gates have been simulated separately but, in Fig.4(d), we have presented the final images and profiles obtained from these corrections jointly for simplicity. A more realistic situation is that one where all pairs of adjacent outcoming beams from the second stage are obstructed except one of them as we have shown in Fig.4. In our experiment we have selected the complex amplitudes α y β in such a way that $|\beta|^2/|\alpha|^2 \approx 1/4$. As it can be observed in Fig.4. a) to d) this rate is preserved (within the experimental error) in the output image of each operation, in a good agreement with the predicted temporal evolution. It must be considered that, the high coherence of the light source introduces speckle noise and the aberrations of the optical elements can introduce undesired phases. These effects can be observed, for instance, in Fig.4. a) to d) where the intensity profiles are partially corrupted by noise and irregularities. However, the proposed set-up can transfer with high fidelity the information encoded in the input state to the output state. This can be observed in the final profile, where the initial rate between intensities is transferred from the initial to the final state with good accuracy by the system.

CONCLUSIONS

We have shown how to perform optical simulations of quantum information processing by using imaging architectures. Quantum states are emulated by means of images and spherical lenses, phase plates, phase gratings and cube prisms, can be used to optically simulate any $\mathbf{U}(2)$ gate and the CNOT gate in an optical processing architecture. As an example, we have implemented an optical setup to classically simulate the quantum teleportation algorithm. The process begins with the representation of the state to be processed as an image organized in two halves. Then, we process the input image with an optical set up composed by several modules. Each one of these modules emulates one step of the real quantum process. The obtained images observed in Fig. 4. show the capability of the system for "teleporting" the information from the full input plane to a portion of the output plane with very high fidelity. It should be emphasized that the classical interpretation of the quantum process is very simple. First the entire image was divided in two halves. After a sequence of optical operations, the information of amplitude and phase encoded in this two halves, is transferred to other portion of the final image. In our case, this is shown in the squared region in the image of Fig.4(d). From a conceptual point of view we can say that meanwhile the realistic *nonlocal* quantum process has as amazing consequence the possibility of transfer one qbit of quantum information to an arbitrarily far place by only sending two bits of classical information; this (or any other) classical simulation will have necessarily a more innocent interpretation since the *nonlocality* nature of quantum mechanics is absent. For this particular simulation, the classical counterpart of quantum teleportation is a consequence of the spatial representation of the states and can be thought as a simple change of scale. The fact of that the information encoded in the first qbit is *locally* transferred to the third qbit, means that the complex amplitudes encoded in the up or the down half of the entire input plane, appears in the up or the down half of a little piece of the entire plane once ending the teleportation process. However, in spite of this limitation that is common to all classical systems, we have obtained experimental results that simulate the teleportation process as an imaging process by means of an inexpensive equipment. In fact, our optical set up reproduces with good agreement the expected probability amplitudes in each stage of the whole teleportation process.

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