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Beam Distributions Beyond RMS

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Abstract

The beam is often represented only by its position (mean) and the width (rms = root mean squared) of its distribution. To achieve these beam parameters in a noisy condition with high backgrounds, a Gaussian distribution with offset (4 parameters) is fitted to the measured beam distribution. This gives a very robust answer and is not very sensitive to background subtraction techniques. To get higher moments of the distribution, like skew or kurtosis, a fitting function with one or two more parameters is desired which would model the higher moments. In this paper we will concentrate on an Asymmetric Gaussian and a Super Gaussian function that will give something like the skew and the kurtosis of the distribution. – This information is used to quantify special beam distribution. Some are unwanted like beam tails (skew) from transverse wakefields, higher order dispersive aberrations or potential well distortion in a damping ring. A negative kurtosis of a beam distribution describes a more rectangular, compact shape like with an over-compressed beam in z or a closed to double-horned energy distribution, while a positive kurtosis looks more like a "Christmas tree" and can quantify a beam mismatch after filamentation. Besides the advantages of the quantification, there are some distributions which need a further investigation like long flat tails which create background particles in a detector. In particle simulations on the other hand a simple rms number might grossly overestimate the effective size (e.g. for producing luminosity) due to a few particles which are far away from the core. This can reduce the practical gain of a big theoretical improvement in the beam size.

1. INTRODUCTION

Beam distributions are measured by different techniques. Transverse distributions are generated simply by screens or projections directly by wire scanners. In the longitudinal phase space, the z-distribution is measured by Streak cameras and the energy distribution is measured by the distribution at a dispersive region. These one-dimensional distributions (or the one-dimensional projections) have a Gaussian shape, if the mechanism for generating the shape is purely statistical. An example is the transverse distribution after the radiative damping in a damping ring. Different effects can disturb this shape and can therefore be an indication for the origin of the disturbance. Transverse wakefields kick the tail of the bunch and create an asymmetric distribution. Quantifying this effect with an Asymmetric Gaussian fit function has help to improve the SLC performance. The next chapter discusses this function, the relation to the skew and the causes of different other asymmetric distributions.

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Then the next higher moment is studied, which can be fitted by a Super Gaussian fit function and gives a value for the kurtosis. These higher moments give hints of how much a size can be reduced by which effect. A simple increase in the Gaussian beam size (or emittance) from one point to another is more difficult to attack.

At the end there are some examples given how an rms number generated by a simulation can lead to wrong estimates and answers. The right effect can be more easily implemented, but might need more CPU time.

2. ASYMMETRIC GAUSSIAN

An asymmetric Gaussian fit function was developed to get a quantitative answer for the asymmetry of a beam spot especially induced by transverse wakefield. Different approaches are discussed elsewhere [1,2], which include a more detailed understanding of the wakefields. A simple additional parameter to a Gaussian fit function can give most of the desired information.

2.1 Fit Function

A Gaussian function is represented by 4 parameters which cover an offset, the peak height, the centering, and the size:

$$A + B \cdot g(x - x_0)\ , \quad \text{with} \qquad g(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-x^2}{2\sigma^2}\right).$$

With an additional parameter E the skewness of the distribution can be estimated:

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-x^2}{2(\sigma \cdot (1 + \operatorname{sign}(x) \cdot E))^2}\right).$$

This is like fitting a left and right half of a Gaussian to the distribution with

$$\sigma = \frac{\sigma_r + \sigma_l}{2}, \text{ and } E = \frac{\sigma_r - \sigma_l}{\sigma_r + \sigma_l}.$$

The precise values for the rms and the skew of this distribution can be calculated and are:

$$rms = \sigma \cdot \sqrt{1 + \left(3 - \frac{8}{\pi}\right) \cdot E^2}, \text{ and}$$

$$skew = E \cdot \sqrt{\frac{8}{\pi}} \cdot \left(1 + \left[\frac{16}{\pi} - 5\right] E^2\right)$$

giving a small correction to just σ and E.

E gives roughly the amount of improvement possible, the exact value for the small σ is:

$$\sigma_{min} = \sigma \cdot (1 - |E|).$$

Figure 1 shows a distribution of a beam profile with a Gaussian and an asymmetric Gaussian distribution.

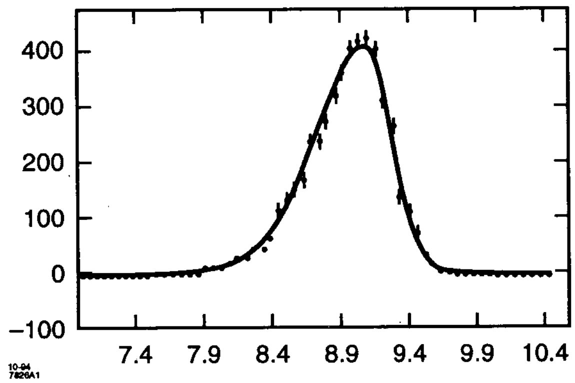

Fig. 1: Asymmetric Gaussian Fit Function

A Gaussian fit to an asymmetric distribution would only indicate the asymmetry, while the asymmetric fit gives an estimate of the beam blow up due to the skewness, (E = −0.35 in this case).

2.2 Reasons for Beam Asymmetry

Besides wakefields, higher order dispersion T_{166} and potential well distortion can lead to asymmetries in the bunch shape.

2.2.1 Wakefields

A beam offset in a cavity will generate transverse wakefields, which will kick the tail of the bunch. A betatron oscillation will drive the tail further and further out. Fig. 2 shows a simulation for two betatron oscillations and the resulting distribution for $3 \cdot 10^{10}$ particles with an asymmetric fit. The asymmetry parameter of $T = 18\%$ indicates most of the possible improvement of 24%.

2.2.2 Higher Order Dispersion

Besides the normal linear dispersion η there can be higher order terms like the quadratic T_{166} term which will generate beam tail.

2.2.3 Potential Well Distortion

The longitudinal beam shape in the SLC damping ring is strongly influenced by longitudinal wakefields which distort the focusing potential well, giving an asymmetric distribution which was calculated [3] and measured with an E-parameter of 0.5.

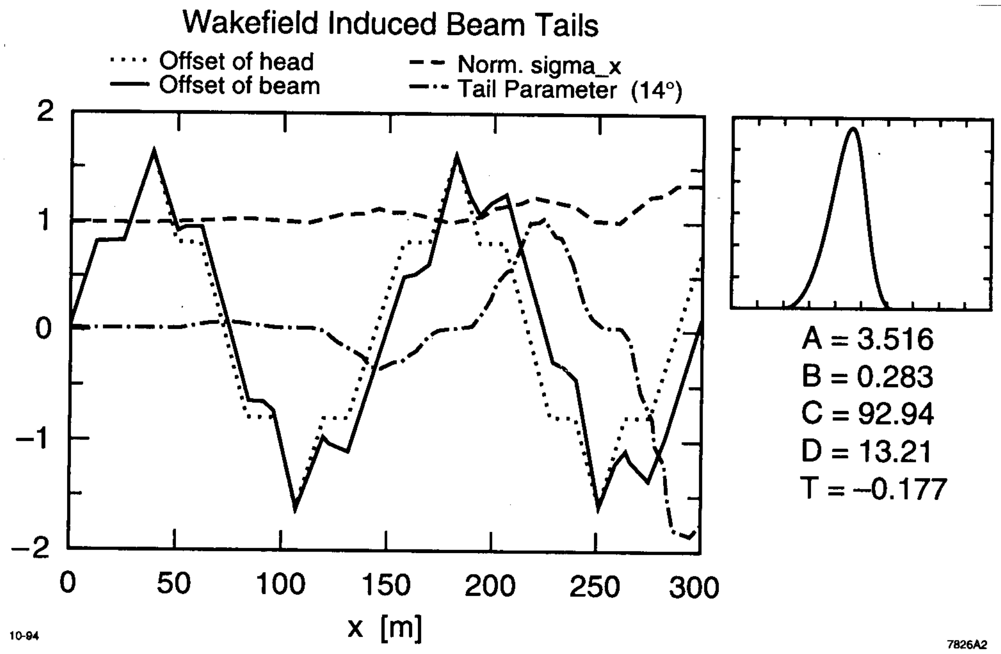

Fig. 2: Simulation of a Betatron Oscillation with Wakefields

A betatron oscillation (left) creates a wakefield tail which blows up the beam size sigma_x. The asymmetry (tail) is well parametrized by an asymmetric Gaussian (right) and gives quantitative values for the possible improvement.

3. SUPER GAUSSIAN DISTRIBUTION

Super Gaussian distributions are used in laser physics to describe higher order beam modes and therefore more rectangular distributions. The steepness of the rectangular shape gives hints for a possible reduction in size and can be quantified by an additional parameter in the exponent. First the mathematical function, then some beam distributions like a longitudinal bunch distribution with over-compression, energy distributions, and special transverse distribution from mismatched beam after filementation are discussed.

3.1 Super Gaussian Function

A distribution which has a symmetric higher moment can be approximated with a Super Gaussian function where the exponent of the Gaussian is a variable N and will give a Gaussian for $N = 2$. For big N the function will describe a more rectangular distribution, while for small N it fits to a distribution with long tails on both sides

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left(\frac{-(\mathrm{abs}(x))^N}{2\sigma_0^N} \right) \quad \text{with} \quad \sigma = \sigma_0 \cdot \left(\frac{\pi}{2} \right)^{2/N-1} .$$

The difference between σ and σ_0 helps to keep σ close to the right number of a normal Gaussian fit.

3.2 Reasons for Big Beam Kurtosis

The beam can get a rectangular-like shape by folding it on top of itself in the other dimension of the phase space. By smearing it out like filamentation, the beam distribution gets wide symmetric tails.

3.2.1 Rectangular Distribution

For N bigger than 2 the value $N/2$ gives a factor of how much smaller the beam spot would be if it were a simple Gaussian with the rise and fall slopes of the more rectangular distribution. Fig. 3 shows an example of an over-compressed beam [4] indicating the 2.5 times smaller possible bunch length. (The bunch is purposely formed in that way to compensate longitudinal wakefields giving a small energy spread at the end of the SLC linac.) A double-horned energy distribution and a filamented beam offset are other examples giving an S-shape or respectively donut in phase space.

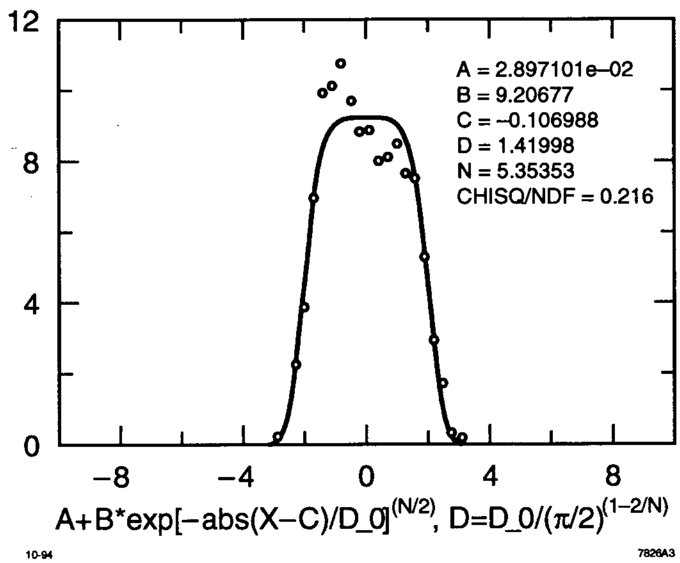

Fig. 3: Simulated Longitudinal Bunch Shape and Fit

The non-linearity of the rf in a compression scheme can be used to form the bunch in such a way that it will give a small energy spread at the end. The fit can quantify this distribution in an analytical way to use it for other studies and comparison with experiments.

3.1.2 Christmas Tree Distribution

If the form parameter N is less than 2 it will describe distributions with a small peak and wide tails (like a Christmas tree). Such distributions were observed earlier with no direct tool to quantify and fix. In simulations such a distribution is easily achieved by a betatron mismatch and filamentation (smearing out the elliptic concentrically in phase space). The distributions and fits for different mismatches are shown in Fig. 4. A measured distribution can now be quantified and compared with simulations to give a prediction for the size of the mismatch.

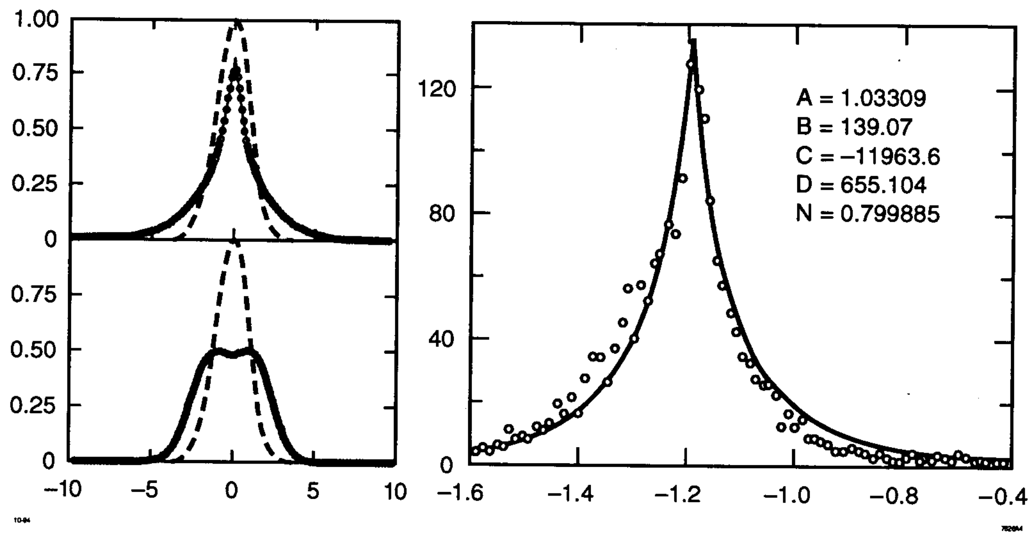

Fig. 4: Filamented Beam Shape after a Betatron Mismatch

A peaked distribution with wide tails give a hint for a mismatch which is already filamented. By fitting a Super Gaussian function to that shape the amount of the mismatch can be measured (Bmag more than 5, left). An offset will filament to a donut shape in phase space which will give a more rectangular shape.

4. RMS IN SIMULATIONS

To get a quick result of a beam size in simulations the rms (root mean square) is used. This number might have not much in common with the effective size of the distribution which will be show in an example and some simulation results.

4.1 RMS Example

Let's assume a Gaussian beam distribution in y. The luminosity is proportional to 1/size. Now we take 2% of that distribution and put it to a big halo around the beam, so that the rms number goes up by a factor of two indicating half the luminosity. On the other hand the real luminosity is only reduced by 4% since 2% of each bunch are more or less not contributing to the luminosity.

4.2 Simulation of Effective Size

The effective size of a distribution depends on the subject you are studying. If the concern is background in the detector more interest is spend on the behavior of tail and halo particles, while the core is relevant for luminosity. The right effective size for luminosity can be calculated by convoluting the distribution of the two colliding beams, which is essentially a simulation of the collision.

Figure 5 shows the effect of a large higher order chromatic term (U_{3466}) [5] in the final focus optics for different angular divergences. The simple rms value would predict a large degradation in spot size, while the effective size enlargement is much more moderate and closer to the linear optics results.

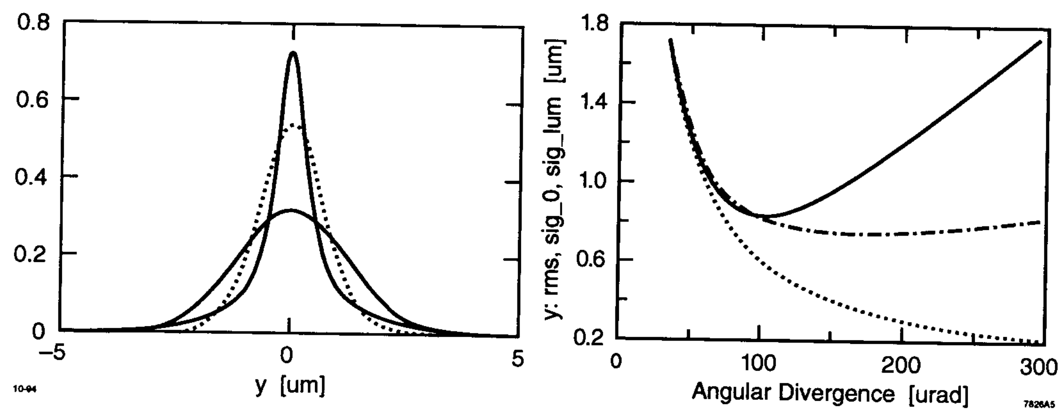

Fig. 5: RMS and Effective Sizes of a Beam Distribution

At the interaction point the beam distribution gets influenced by higher order chromatic terms giving wide tails and therefore a bigger rms value for higher angular divergences. The effective beam size is still going down since it depends on the core where the peak height is still rising.

CONCLUSION

Higher moments in a distribution can be fitted with special functions, Asymmetric Gaussian for the 3rd and Super Gaussian for the 4th moment. They give the advantage of a special form that is robust against varying pedestrial offsets below the measured distribution. Different beam conditions are discussed, which are quite remarkably fitted by these functions which quantify the measured effect. A simple rms value in simulations can lead to wrong conclusions if wide tails are present.

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