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Dynamics of a new delayed stage-structured predator-prey model with impulsive diffusion and releasing

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Abstract

In this work, we propose a new delayed stage-structured predator-prey model with impulsive diffusion and releasing. By the stroboscopic map of the discrete dynamical system, we obtain a prey-extinction boundary periodic solution. Furthermore, we prove that the prey-extinction boundary periodic solution is globally attractive. We also prove that the investigated system is permanent by the theory on the delay and impulsive differential equations. Our results indicate that time delay, impulsive diffusion, and impulsive releasing have influence to the dynamical behaviors of the investigated system. The results of this paper also provide a tactical basis for pest management.

Keywords: delayed stage structure; predator-prey model; impulsive diffusion; impulsive releasing; prey-extinction

1 Introduction

Many authors [1–8] and papers [9, 10] have studied the predator-prey, competitive, and cooperative models. Permanence and extinction are significant concepts of those models which also show many interesting results. However, the stage structure of a species has been considered very little. In the real world, almost all animals have the stage structure of being immature and mature. Recently, [11–17] studied the stage structure of species with or without time delays. Aiello *et al.* [12] considered a time delayed stage structure of being immature and mature of the population model

$$\begin{cases} \frac{dx_i(t)}{dt} = \alpha x_m(t) - rx_i(t) - \alpha e^{-r\tau} x_m(t-\tau), \\ \frac{dx_m(t)}{dt} = \alpha e^{-r\tau} x_m(t-\tau) - \beta x_m^2(t), \end{cases} \tag{1.1}$$

where $x_i(t)$ denotes the immature population density at time t, $x_m(t)$ denotes the mature population density at time t, $\alpha > 0$ represents the birth rate, $r > 0$ represents the immature death rate, $\beta > 0$ represents the mature death and the overcrowding rate, $\tau > 0$, represents the time to maturity rate, the term $\alpha e^{-r\tau} x_m(t-\tau)$ represents the immature who were born at time $t-\tau$ and survive at time t (with the immature death rate r) and therefore represents the transformation of the immature to the mature.

Dispersal is a ubiquitous phenomenon in the natural world. It is important for us to understand the ecological and evolutionary dynamics of populations mirrored by the large number of mathematical models devoted to it in the scientific literature [13–24]. If the population dynamics with the effects of spatial heterogeneity is modeled by a diffusion process, most previous papers focused on the population dynamical system modeled by the ordinary differential equations. But in practice, it is often the case that diffusion occurs in regular pulse. For example, when winter comes, birds will migrate between patches in search for a better environment, whereas they do not diffuse in other seasons, and the excursion of foliage seeds occurs at a fixed period of time every year. Thus impulsive diffusion provides a more natural description. Lately theories of impulsive differential equations [25, 26] have been introduced into population dynamics. Impulsive differential equations are found in most domains of applied science [16, 17, 20, 24, 27–29].

The organization of this paper is as follows. In the next section, we introduce the model and background concepts. In Section 3, some important lemmas are presented. In Section 4, we give the conditions of global attractivity and permanence for system (2.1). In Section 5, a brief discussion is given in the last section to conclude this work.

2 The model

Wang and Chen [17] considered a single population with impulsive diffusion. Jiao [24] considered a delayed predator-prey model with impulsive diffusion on predator and stage structure on prey. Inspired by [17, 24], we establish a new delayed stage-structured predator-prey model with impulsive diffusion and releasing

$$\begin{cases} \left.\begin{aligned} &\frac{dx_1(t)}{dt} = r_1y_1(t) - r_1e^{-w_{11}\tau_1}y_1(t-\tau_1) - w_{11}x_1(t),\\ &\frac{dy_1(t)}{dt} = r_1e^{-w_{11}\tau_1}y_1(t-\tau_1) - w_{12}y_1(t) - \beta_1y_1(t)z_1(t),\\ &\frac{dz_1(t)}{dt} = k_1\beta_1y_1(t)z_1(t) - w_{13}z_1(t),\\ &\frac{dx_2(t)}{dt} = r_2y_2(t) - r_2e^{-w_{21}\tau_2}y_2(t-\tau_2) - w_{21}x_1(t),\\ &\frac{dy_2(t)}{dt} = r_2e^{-w_{21}\tau_2}y_2(t-\tau_2) - w_{22}y_2(t) - \beta_2y_2(t)z_2(t),\\ &\frac{dz_2(t)}{dt} = k_2\beta_2y_2(t)z_2(t) - w_{23}z_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau,\\ \left.\begin{aligned} &\triangle x_1(t) = 0,\\ &\triangle y_1(t) = 0,\\ &\triangle z_1(t) = D(z_2(t) - z_1(t)),\\ &\triangle x_2(t) = 0,\\ &\triangle y_2(t) = 0,\\ &\triangle z_2(t) = D(z_1(t) - z_2(t)), \end{aligned}\right\} t = (n+l)\tau, n \in Z^+,\\ \left.\begin{aligned} &\triangle x_1(t) = 0,\\ &\triangle y_1(t) = 0,\\ &\triangle z_1(t) = \mu_1,\\ &\triangle x_2(t) = 0,\\ &\triangle y_2(t) = 0,\\ &\triangle z_2(t) = \mu_2, \end{aligned}\right\} t = (n+1)\tau, n \in Z^+, \end{cases} \tag{2.1}$$

with initial condition

$$\begin{aligned} &\big(\varphi_1(\zeta),\varphi_2(\zeta),\varphi_3(\zeta),\varphi_4(\zeta),\varphi_5(\zeta),\varphi_6(\zeta)\big)\in C_+ = C\big([-\tau_1,0],R_+^6\big),\\ &\varphi_i(0)>0,\quad i=1,2,3,4,5,6, \end{aligned}$$

where system (2.1) is constructed of two patches. $x_i(t)$, $y_i(t)$ and $z_i(t)$ represent the immature prey population, mature prey population, predator population in patch i ($i=1,2$) at time t. It is assumed that birth into the immature prey population is proportional to the existing mature prey population with proportionality constant r_i in patch i ($i=1,2$). τ_i represents a constant time to maturity of prey population in patch i ($i=1,2$), that is, immature prey individuals and mature individuals are divided by age τ_i in patch i ($i=1,2$). The natural death rates w_{i1}, w_{i2} and w_{i3} ($i=1,2$) are assumed for the immature prey population, mature prey population, and predator population in patch i ($i=1,2$). β_i ($i=1,2$) is the mature prey population capture rate by the predator population in patch i ($i=1,2$). k_i ($i=1,2$) is the conversion rate of nutrients into the reproduction of the predator population in patch i ($i=1,2$). The pulse diffusion occurs every $\tau>0$ period. The system evolves from its initial state without being further affected by diffusion until the next pulse appears. $\triangle y_i((n+l)\tau)=y_i((n+l)\tau^+)-y_i((n+l)\tau)$ where $y_i((n+l)\tau^+)$ represents the density of population in the ith patch immediately after the nth diffusion pulse at time $t=(n+l)\tau$, while $y_i((n+l)\tau)$ represents the density of population in the ith patch before the nth diffusion pulse at time $t=(n+l)\tau$ ($n\in Z_+$, $0<l<1$). $0<D<1$ is the dispersal rate of the predator population between two patches. It is assumed here that the net exchange from the jth patch to the ith patch is proportional to the difference y_j-y_i of the predator population densities. The predator population is released with μ_i in patch i ($i=1,2$) at moment $t=(n+1)\tau$, $n\in Z_+$.

Because $x_i(t)$ ($i=1,2$) does not affect the other equations of (2.1), we can simplify system (2.1) and restrict our attention to the following system:

$$\begin{cases} \left.\begin{aligned} &\frac{dy_1(t)}{dt}=r_1e^{-w_{11}\tau_1}y_1(t-\tau_1)-w_{12}y_1(t)-\beta_1y_1(t)z_1(t),\\ &\frac{dz_1(t)}{dt}=k_1\beta_1y_1(t)z_1(t)-w_{13}z_1(t),\\ &\frac{dy_2(t)}{dt}=r_2e^{-w_{21}\tau_2}y_2(t-\tau_2)-w_{22}y_2(t)-\beta_2y_2(t)z_2(t),\\ &\frac{dz_2(t)}{dt}=k_2\beta_2y_2(t)z_2(t)-w_{23}z_2(t), \end{aligned}\right\} t\neq(n+l)\tau, t\neq(n+1)\tau,\\ \left.\begin{aligned} &\triangle y_1(t)=0,\\ &\triangle z_1(t)=D(z_2(t)-z_1(t)),\\ &\triangle y_2(t)=0,\\ &\triangle z_2(t)=D(z_1(t)-z_2(t)), \end{aligned}\right\} t=(n+l)\tau, n\in Z^+,\\ \left.\begin{aligned} &\triangle y_1(t)=0,\\ &\triangle z_1(t)=\mu_1,\\ &\triangle y_2(t)=0,\\ &\triangle z_2(t)=\mu_2, \end{aligned}\right\} t=(n+1)\tau, n\in Z^+, \end{cases} \tag{2.2}$$

with initial condition

$$\big(\varphi_2(\zeta),\varphi_3(\zeta),\varphi_5(\zeta),\varphi_6(\zeta)\big)\in C_+ = C\big([-\tau_1,0],R^4_+\big),$$
$$\varphi_i(0)>0,\quad i=2,3,5,6.$$

3 The lemmas

The solution of (2.1), denoted by $X(t)=(x_1(t),y_1(t),z_1(t),x_2(t),y_2(t),z_2(t))$, is a piecewise continuous function $X:R_+\to R^6_+$, $X(t)$ is continuous on $(n\tau,(n+l)\tau]$, $((n+l)\tau,(n+1)\tau]$, $n\in Z_+$ and $X(n\tau^+)=\lim_{t\to n\tau^+}X(t)$, $X((n+l)\tau^+)=\lim_{t\to(n+l)\tau^+}X(t)$ exist. Obviously the global existence and uniqueness of solutions of (2.1) are guaranteed by the smoothness properties of f, which denotes the mapping defined by the right side of system (2.1) (see Lakshmikantham, [25]). Before we have the the main results, we need to give some lemmas which will be used in the following.

According to the biological meaning, it is assumed that $x_i(t)\ge 0$, $y_i(t)\ge 0$, and $z_i(t)\ge 0$ $(i=1,2)$.

Let $V:R_+\times R^6_+\to R_+$, then V is said to belong to class V_0, if:

(i) V is continuous in $(n\tau,(n+l)\tau]\times R^6_+$ and $((n+l)\tau,(n+1)\tau]\times R^6_+$, for each $z\in R^6_+$, $n\in Z_+$, $V(n\tau^+,z)=\lim_{(t,y)\to(n\tau^+,z)}V(t,y)$, $V((n+l)\tau^+,z)=\lim_{(t,y)\to((n+l)\tau^+,y)}V(t,y)$ exist.

(ii) V is locally Lipschitzian in z.

Definition 3.1 $V\in V_0$, then, for $(t,z)\in(n\tau,(n+l)\tau]\times R^6_+$ and $((n+l)\tau,(n+1)\tau]\times R^6_+$, the upper right derivative of $V(t,z)$ with respect to the impulsive differential system (2.1) is defined as

$$D^+V(t,z)=\limsup_{h\to 0}\frac{1}{h}\big[V\big(t+h,z+hf(t,z)\big)-V(t,z)\big].$$

Lemma 3.2 ([26]) *Let the function $m\in PC'[R^+,R]$ satisfy the inequalities*

$$\begin{cases} m'(t)\le p(t)m(t)+q(t), & \\ t\ge t_0, t\ne t_k, & k=1,2,\dots, \\ m(t_k^+)\le d_k m(t_k)+b_k, & t=t_k, \end{cases} \tag{3.1}$$

where $p,q\in PC[R^+,R]$ and $d_k\ge 0$, b_k are constants, then

$$\begin{aligned} m(t)\le{}& m(t_0)\prod_{t_0<t_k<t}d_k\exp\left(\int_{t_0}^t p(s)\,ds\right)\\ &+\sum_{t_0<t_k<t}\left(\prod_{t_k<t_j<t}d_j\exp\left(\int_{t_0}^t p(s)\,ds\right)\right)b_k\\ &+\int_{t_0}^t\prod_{s<t_k<t}d_k\exp\left(\int_s^t p(\sigma)\,d\sigma\right)q(s)\,ds,\quad t\ge t_0. \end{aligned}$$

Now, we show that all solutions of (2.1) are uniformly ultimately bounded.

Lemma 3.3 *There exists a constant $M>0$ such that $x_i(t)\le M$, $y_i(t)\le M$, $z_i(t)\le M$ $(i=1,2)$ for each solution $(x_1(t),y_1(t),z_1(t),x_2(t),y_2(t),z_2(t))$ of (2.1) with all t large enough.*

Proof Define

$$V(t)=\sum_{i=1}^{2}\bigl[k_ix_i(t)+k_iy_i(t)+z_i(t)\bigr],$$

and $d=\min\{w_{11},w_{12},w_{13},w_{21},w_{22},w_{23}\}$, then $t\neq(n+l)\tau$, $t\neq(n+1)\tau$, we have

$$\begin{aligned}D^+V(t)+dV(t)&=\sum_{i=1}^{2}k_ir_ix_i(t)\\&\quad-\sum_{i=1}^{2}\bigl[k_i(w_{i1}-d)x_i(t)+k_i(w_{i2}-d)y_i(t)+(w_{i3}-d)z_i(t)\bigr]\\&\leq\sum_{i=1}^{2}k_ir_ix_i(t)<\zeta.\end{aligned}$$

When $t=(n+l)\tau$,

$$\begin{aligned}V\bigl((n+l)\tau^+\bigr)&=\sum_{i=1}^{2}\bigl[x_i\bigl((n+l)\tau^+\bigr)+y_i\bigl((n+l)\tau^+\bigr)+z_i\bigl((n+l)\tau^+\bigr)\bigr]\\&=\sum_{i=1}^{2}\bigl[x_i\bigl((n+l)\tau\bigr)+y_i\bigl((n+l)\tau\bigr)+z_i\bigl((n+l)\tau\bigr)\bigr]=V(n\tau).\end{aligned}$$

When $t=(n+1)\tau$,

$$\begin{aligned}V\bigl((n+1)\tau^+\bigr)&=\sum_{i=1}^{2}\bigl[x_i\bigl((n+1)\tau^+\bigr)+y_i\bigl((n+1)\tau^+\bigr)+z_i\bigl((n+1)\tau^+\bigr)\bigr]\\&=\sum_{i=1}^{2}\bigl[x_i\bigl((n+1)\tau\bigr)+y_i\bigl((n+1)\tau\bigr)+z_i\bigl((n+1)\tau\bigr)\bigr]+\mu_1+\mu_2\\&=V\bigl((n+1)\tau\bigr)+\mu_1+\mu_2.\end{aligned}$$

By Lemma 3.2, for $t\in(n\tau,(n+1)\tau]$, we have

$$\begin{aligned}V(t)&\leq V\bigl(0^+\bigr)e^{-dt}+\frac{\zeta}{d}\bigl(1-e^{-dt}\bigr)+(\mu_1+\mu_2)\frac{e^{-d(t-\tau)}}{1-e^{-d\tau}}+(\mu_1+\mu_2)\frac{e^{d\tau}}{e^{d\tau}-1}\\&\to\frac{\zeta}{d}+(\mu_1+\mu_2)\frac{e^{d\tau}}{e^{d\tau}-1},\quad\text{as }t\to\infty.\end{aligned}$$

So $V(t)$ is uniformly ultimately bounded. Hence, by the definition of $V(t)$, there exists a constant $M>0$ such that $x_i(t)\leq M/k_i$, $y_i(t)\leq M/k_i$, $z_i(t)\leq M$ $(i=1,2)$ for t large enough. The proof is complete. □

If $y_i(t) = 0$ $(i = 1, 2)$, we have the following subsystem of (2.2):

$$\begin{cases} \left.\begin{aligned} \frac{dz_1(t)}{dt} &= -w_{13}z_1(t), \\ \frac{dz_2(t)}{dt} &= -d_{23}z_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle z_1(t) &= D(z_2(t) - z_1(t)), \\ \triangle z_2(t) &= D(z_1(t) - z_2(t)), \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle z_1(t) &= \mu_1, \\ \triangle z_2(t) &= \mu_2, \end{aligned}\right\} t = (n+1)\tau, n \in Z^+. \end{cases} \tag{3.2}$$

We can easily obtain the analytic solution of (3.2) between pulses as follows:

$$\begin{cases} z_1(t) = \begin{cases} z_1(n\tau^+)e^{-w_{13}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_1((n+l)\tau^+)e^{-w_{13}(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ z_2(t) = \begin{cases} z_2(n\tau^+)e^{-w_{23}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_2((n+l)\tau^+)e^{-w_{23}(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau). \end{cases} \end{cases} \tag{3.3}$$

Considering the third and fourth equations of (3.2), we have

$$\begin{cases} z_1((n+l)\tau^+) = (1-D)e^{-w_{13}l\tau}z_1(n\tau^+) + De^{-w_{23}l\tau}z_2(n\tau^+), \\ z_2((n+l)\tau^+) = De^{-w_{13}l\tau}z_1(n\tau^+) + (1-D)e^{-w_{23}l\tau}z_2(n\tau^+). \end{cases} \tag{3.4}$$

Considering the fifth and sixth equations of (3.2), we also have

$$\begin{cases} z_1((n+1)\tau^+) = z_1((n+l)\tau^+)e^{-w_{13}(1-l)\tau} + \mu_1, \\ z_2((n+1)\tau^+) = z_2((n+l)\tau^+)e^{-w_{23}(1-l)\tau} + \mu_2. \end{cases} \tag{3.5}$$

Substituting (3.4) into (3.5), we have the stroboscopic map of (3.2)

$$\begin{cases} z_1((n+1)\tau^+) = (1-D)e^{-w_{13}\tau}z_1(n\tau^+) + De^{-[w_{13}(1-l)+w_{23}l]\tau}z_2(n\tau^+) + \mu_1, \\ z_2((n+1)\tau^+) = De^{-[w_{13}l+w_{23}(1-l)]\tau}z_1(n\tau^+) + (1-D)e^{-w_{23}\tau}z_2(n\tau^+) + \mu_2. \end{cases} \tag{3.6}$$

Equation (3.6) has one fixed point:

$$\begin{cases} z_1^* = \frac{\mu_2(1-A_1)+\mu_1 A_2}{(1-A_1)(1-B_2)-A_2B_1} > 0, \\ z_2^* = \frac{\mu_2 B_1+\mu_1(1-B_2)}{(1-A_1)(1-B_2)-A_2B_1} > 0, \end{cases} \tag{3.7}$$

where

$$A_1 = (1-D)e^{-w_{13}\tau} < 1,$$

$$B_1 = De^{-[w_{13}(1-l)+w_{23}l]\tau} < 1,$$

$$A_2 = De^{-[w_{13}l+w_{23}(1-l)]\tau} < 1,$$

$$B_2 = (1-D)e^{-w_{23}\tau} < 1.$$

Lemma 3.4 *The fixed point* (z_1^*, z_2^*) *of* (3.6) *is globally asymptotically stable.*

Proof For convenience, we use the notation $(z_1^n, z_2^n) = (z_1(n\tau^+), z_2(n\tau^+))$. The linear form of (3.6) can be written as

$$\begin{pmatrix} z_1^{n+1} \\ z_2^{n+1} \end{pmatrix} = M \begin{pmatrix} z_1^n \\ z_2^n \end{pmatrix}. \tag{3.8}$$

Obviously, the near dynamics of (z_1^*, z_2^*) is determined by linear system (3.6). The stability of (z_1^*, z_2^*) is determined by the eigenvalue of M less than 1. If M satisfies the *Jury* criterion [30], we can know the eigenvalue of M is less than 1,

$$1 - \operatorname{tr} M + \det M > 0. \tag{3.9}$$

We can easily know that (z_1^*, z_2^*) is unique fixed point of (3.6), and

$$M = \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \end{pmatrix}. \tag{3.10}$$

For

$$\begin{aligned}
&1 - \operatorname{tr} M + \det M \\
&\quad = 1 - (A_1 + B_2) + (A_1 B_2 - A_2 B_1) \\
&\quad = (1 - A_1)(1 - B_2) - A_2 B_1 \\
&\quad = \left[1 - (1-D)e^{-w_{13}\tau}\right] \times \left[1 - (1-D)e^{-w_{23}\tau}\right] - D^2 e^{-(w_{13}+d_2)\tau} \\
&\quad = \left[\left(1 - e^{-w_{13}\tau}\right) + De^{-w_{13}\tau}\right]\left[\left(1 - e^{-w_{23}\tau}\right) + De^{-w_{23}\tau}\right] - D^2 e^{-(w_{13}+w_{23})\tau} \\
&\quad = \left(1 - e^{-w_{13}\tau}\right) \times \left(1 - e^{-w_{13}\tau}\right) + De^{-w_{23}\tau}\left(1 - e^{-w_{13}\tau}\right) + De^{-w_{13}\tau}\left(1 - e^{-w_{23}\tau}\right) \\
&\quad > 0.
\end{aligned}$$

From the *Jury* criterion, (z_1^*, z_2^*) is locally stable, then it is globally asymptotically stable. This completes the proof. □

Lemma 3.5 *The periodic solution* $(\widetilde{z_1(t)}, \widetilde{z_2(t)})$ *of system* (3.2) *is globally asymptotically stable, where*

$$\begin{cases}
\widetilde{z_1(t)} = \begin{cases} z_1^* e^{-w_{13}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_1^{**} e^{-w_{13}(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\
\widetilde{z_2(t)} = \begin{cases} z_2^* e^{-w_{23}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_2^{**} e^{-w_{23}(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases}
\end{cases} \tag{3.11}$$

where z_1^* *and* z_2^* *are determined as* (3.7), z_1^{**} *and* z_2^{**} *are defined as*

$$\begin{cases}
z_1^{**} = (1-D)e^{-w_{13}l\tau} z_1^* + De^{-w_{23}l\tau} z_2^*, \\
z_2^{**} = De^{-w_{13}l\tau} z_1^* + (1-D)e^{-w_{23}l\tau} z_2^*.
\end{cases} \tag{3.12}$$

Lemma 3.6 ([31]) *Consider the following equation*:

$$\frac{dx(t)}{dt} = a_1 x(t-\omega) - a_2 x(t),$$

where $a_1, a_2, \omega > 0$; $x(t) > 0$ *for* $-\omega \le t \le 0$, *we have*:

(i) *if* $a_1 < a_2$, *then*, $\lim_{t\to\infty} x(t) = 0$,

(ii) *if* $a_1 > a_2$, *then*, $\lim_{t\to\infty} x(t) = +\infty$.

4 The dynamics

From the above discussion, we know there exists a prey-extinction boundary periodic solution $(0, \widetilde{z_1(t)}, 0, \widetilde{z_2(t)})$ of system (2.2). In this section, we will prove that the prey-extinction boundary periodic solution $(0, \widetilde{z_1(t)}, 0, \widetilde{z_2(t)})$ of system (2.2) is globally attractive.

Theorem 4.1 *If*

$$\max_{i=1,2}\left\{r_i e^{-w_{i1}\tau_i} - \left[w_{i2} + \beta_i\left(z_i^* + z_i^{**}\right)\right]\right\} < 0 \quad (i = 1,2) \tag{4.1}$$

holds, the prey-extinction boundary periodic solution $(0, \widetilde{z_1(t)}, 0, \widetilde{z_2(t)})$ *of* (2.2) *is globally attractive, where* z_i^* $(i = 1,2)$ *is determined by* (3.7), z_i^{**} $(i = 1,2)$ *is defined by* (3.12).

Proof From (4.1), we can obtain

$$r_i e^{-w_{i1}\tau_i} < \left[w_{i2} + \beta_i\left(z_i^* + z_i^{**}\right)\right]. \tag{4.2}$$

Then, we can choose ε_0 sufficiently small such that

$$r_i e^{-w_{i1}\tau_i} < w_{i2} + \beta_i\left[\left(z_i^* + z_i^{**}\right) - \varepsilon_0\right]. \tag{4.3}$$

From the second and fourth equations of system (2.2), we obtain $\frac{dz_i(t)}{dt} \ge -w_{i3} z_i(t)$ $(i = 1,2)$. So we consider the following comparison impulsive differential system:

$$\begin{cases} \left.\begin{aligned} \frac{dz_{11}(t)}{dt} &= -w_{13} z_{11}(t), \\ \frac{dz_{21}(t)}{dt} &= -w_{23} z_{21}(t), \end{aligned}\right\} t \ne (n+l)\tau, t \ne (n+1)\tau, \\ \left.\begin{aligned} \triangle z_{11}(t) &= D(z_{21}(t) - z_{11}(t)), \\ \triangle z_{21}(t) &= D(z_{11}(t) - z_{21}(t)), \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle z_{11}(t) &= \mu_1, \\ \triangle z_{21}(t) &= \mu_2, \end{aligned}\right\} t = (n+1)\tau, n \in Z^+. \end{cases} \tag{4.4}$$

In view of Lemma 3.4 and (3.5), we find that the boundary periodic solution of system (4.1)

$$\begin{cases} \widetilde{z_{11}(t)} = \begin{cases} z_1^* e^{-w_{13}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_1^{**} e^{-w_{13}(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ \widetilde{z_{21}(t)} = \begin{cases} z_2^* e^{-w_{23}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_2^{**} e^{-w_{23}(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \end{cases} \tag{4.5}$$

is globally asymptotically stable, where z_1^* and z_2^* are determined as (3.7), z_1^{**} and z_2^{**} are defined as (3.12).

From Lemma 3.5 and comparison theorem of impulsive equation [2], we have $z_i(t) \geq z_{i1}(t)$ $(i = 1, 2)$ and $z_{i1}(t) \to \widetilde{z_i(t)}$ as $t \to \infty$. Then there exists an integer $k_2 > k_1$, $t > k_2$ such that

$$z_i(t) \geq z_{i1}(t) \geq \widetilde{z_i(t)} - \varepsilon_0 \quad (i = 1, 2), n\tau < t \leq (n + 1)\tau, n > k_2,$$

that is,

$$z_i(t) > \widetilde{z_i(t)} - \varepsilon_0 \geq \left(z_i^* + z_i^{**}\right) - \varepsilon_0 \stackrel{\Delta}{=} \varrho_i \quad (i = 1, 2), n\tau < t \leq (n + 1)\tau, n > k_2.$$

From (2.2), we get

$$\frac{dy_i(t)}{dt} \leq r_i e^{-w_{i1}\tau_i} y_i(t - \tau_1) - (w_{i2} + \beta_i \varrho_i) y_i(t) \quad (i = 1, 2), t > n\tau + \tau_1, n > k_2. \tag{4.6}$$

Consider the following comparison differential system referring to (4.6):

$$\frac{dR_i(t)}{dt} = r_i e^{-w_{i1}\tau_i} R_i(t - \tau_1) - (w_{i2} + \beta_i \varrho_i) R_i(t) \quad (i = 1, 2), t > n\tau + \tau_1, n > k_2. \tag{4.7}$$

From (4.3) and Lemma 3.6, we have $\lim_{t\to\infty} R_i(t) = 0$.

Let $(y_1(t), z_1(t), y_2(t), z_2(t))$ be the solution of system (2.2) with initial conditions and $y_1(\zeta) = \varphi_2(\zeta)$ $(\zeta \in [-\tau_1, 0])$, $y_2(\zeta) = \varphi_5(\zeta)$ $(\zeta \in [-\tau_1, 0])$. $R_i(t)$ $(i = 1, 2)$ is the solution of system (4.7) with initial conditions $R_1(\zeta) = \varphi_2(\zeta)$ $(\zeta \in [-\tau_1, 0])$, $R_2(\zeta) = \varphi_5(\zeta)$ $(\zeta \in [-\tau_1, 0])$. By the comparison theorem, we have

$$\lim_{t\to\infty} y_i(t) < \lim_{t\to\infty} R_i(t) = 0.$$

Incorporating the positivity of $y_i(t)$, we know that $\lim_{t\to\infty} y_i(t) = 0$. Therefore, for any $\varepsilon_1 > 0$ (sufficiently small) and $\varepsilon_1 < \min\{\frac{w_{i3}}{k_i\beta_i}\}$, there exists an integer k_3 $(k_3\tau > k_2\tau + \tau_1)$ such that $y_i(t) < \varepsilon_1$ $(i = 1, 2)$ for all $t > k_3\tau$.

From the second and fourth equations of system (2.2), we have

$$-w_{i3} z_i(t) \leq \frac{dz_i(t)}{dt} \leq -(w_{i3} - k_i\beta_i\varepsilon_1) z_i(t) \quad (i = 1, 2). \tag{4.8}$$

Then we have $z_{i2}(t) \leq z_i(t) \leq z_{i3}(t)$ and $z_{i2}(t) \to \widetilde{z_i(t)}, z_{i3}(t) \to \widetilde{z_{i3}(t)}$ $(i = 1, 2)$ as $t \to \infty$. While $(z_{12}(t), z_{22}(t))$ and $(z_{13}(t), z_{23}(t))$ are the solutions of

$$\begin{cases} \left.\begin{aligned} \frac{dz_{12}(t)}{dt} &= -w_{13} z_{12}(t), \\ \frac{dz_{22}(t)}{dt} &= -w_{23} z_{22}(t), \end{aligned}\right\} t \neq (n + l)\tau, t \neq (n + 1)\tau, \\ \left.\begin{aligned} \triangle z_{12}(t) &= D(z_{22}(t) - z_{12}(t)), \\ \triangle z_{22}(t) &= D(z_1(t) - z_{22}(t)), \end{aligned}\right\} t = (n + l)\tau, \\ \left.\begin{aligned} \triangle z_{12}(t) &= \mu_1, \\ \triangle z_{22}(t) &= \mu_2, \end{aligned}\right\} t = (n + 1)\tau, n \in Z^+, \end{cases} \tag{4.9}$$

and

$$\begin{cases} \left.\begin{aligned} \frac{dz_{13}(t)}{dt} = -(w_{13} - k_1\beta_1\varepsilon_1)z_{13}(t), \\ \frac{dz_{23}(t)}{dt} = -(w_{23} - k_2\beta_2\varepsilon_1)z_{23}(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle z_{13}(t) = D(z_{23}(t) - z_{13}(t)), \\ \triangle z_{23}(t) = D(z_{13}(t) - z_{23}(t)), \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle z_{13}(t) = \mu_1, \\ \triangle z_{23}(t) = \mu_2, \end{aligned}\right\} t = (n+1)\tau, n \in Z^+, \end{cases} \tag{4.10}$$

respectively, we have

$$\begin{cases} \widetilde{z_{13}(t)} = \begin{cases} z_{13}^* e^{-(w_{13}-k_1\beta_1\varepsilon_1)(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_{13}^{**} e^{-(w_{13}-k_1\beta_1\varepsilon_1)(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ \widetilde{z_{23}(t)} = \begin{cases} z_{23}^* e^{-(w_{23}-k_2\beta_2\varepsilon_1)(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_{23}^{**} e^{-(w_{23}-k_2\beta_2\varepsilon_1)(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \end{cases} \tag{4.11}$$

where

$$\begin{cases} z_{13}^* = \frac{\mu_2(1-A_{13})+\mu_1 A_{23}}{(1-A_{13})(1-B_{23})-A_{23}B_{13}} > 0, \\ z_{23}^* = \frac{\mu_2 B_{13}+\mu_1(1-B_{23})}{(1-A_{13})(1-B_{23})-A_{23}B_{13}} > 0, \end{cases} \tag{4.12}$$

and

$$\begin{cases} z_{13}^{**} = (1-D)e^{-(w_{13}-k_1\beta_1\varepsilon_1)l\tau} z_{13}^* + De^{-(w_{23}-k_2\beta_2\varepsilon_1)l\tau} z_{23}^*, \\ z_{23}^{**} = De^{-(w_{13}-k_1\beta_1\varepsilon_1)l\tau} z_{13}^* + (1-D)e^{-(w_{23}-k_2\beta_2\varepsilon_1)l\tau} z_{23}^*, \end{cases} \tag{4.13}$$

and

$$\begin{aligned} A_{13} &= (1-D)e^{-(w_{13}-k_1\beta_1\varepsilon_1)\tau} < 1, \\ B_{13} &= De^{-[(w_{13}-k_1\beta_1\varepsilon_1)(1-l)+(w_{23}-k_2\beta_2\varepsilon_1)l]\tau} < 1, \\ A_{23} &= De^{-[(w_{13}-k_1\beta_1\varepsilon_1)l+(w_{23}-k_2\beta_2\varepsilon_1)(1-l)]\tau} < 1, \\ B_{23} &= (1-D)e^{-(w_{23}-k_2\beta_2\varepsilon_1)\tau} < 1. \end{aligned}$$

Therefore, for any $\varepsilon_2 > 0$ (ε_2 is small enough), there exists an integer k_4, $n > k_4$ such that $\widetilde{z_{i2}(t)} - \varepsilon_2 < z_i(t) < \widetilde{z_{i3}(t)} + \varepsilon_2$ $(i = 1, 2)$. Let $\varepsilon_1 \to 0$, so we have $\widetilde{z_i(t)} - \varepsilon_2 < z_i(t) < \widetilde{z_i(t)} + \varepsilon_2$ $(i = 1, 2)$, for t large enough. This implies $z_i(t) \to \widetilde{z_i(t)}$ $(i = 1, 2)$ as $t \to \infty$. This completes the proof. □

The next work is to investigate the permanence of system (2.2). Before starting our theorem, we give the following definition.

Definition 4.2 System (2.2) is said to be permanent if there are constants $m, M > 0$ (independent of initial value) and a finite time T_0 such that, for all solutions $(y_1(t), z_1(t),$

$y_2(t), z_2(t))$ with all initial values $y_i(0^+) > 0$, $z_i(0^+) > 0$ $(i = 1,2)$, $m \leq y_i(t) \leq M$, $m \leq z_i(t) \leq M$ $(i = 1,2)$ hold for all $t \geq T_0$. Here T_0 may depend on the initial values $(y_1(0^+), z_1(0^+), y_2(0^+), z_2(0^+))$.

Theorem 4.3 *If*

$$\min_{i=1,2}\Big[r_i e^{-w_{i1}\tau_i} - w_{i2} - \beta_i\big(z_{i4}^{*} e^{-(w_{i3}-k_i\beta_i y_i^*)l\tau} + z_{i4}^{**} e^{-(w_{i3}+k_i\beta_i y_i^*)(1-l)\tau}\big)\Big] > 0,$$

there is a positive constant q such that each positive solution $(y_1(t), z_1(t), y_2(t), z_2(t))$ *of* (2.2) *satisfies* $y_i(t) \geq q$, *for t large enough, where* y_i^* $(i = 1,2)$ *is determined by*

$$r_i e^{-w_{i1}\tau_i} = w_{i2} + \beta_i\big(z_{i4}^{*} e^{-(w_{i3}-k_i\beta_i y_i^*)l\tau} + z_{i4}^{**} e^{-(w_{i3}+k_i\beta_i y_i^*)(1-l)\tau}\big) \quad (i = 1,2),$$

where z_{i4}^* $(i = 1,2)$ *and* z_{i4}^{**} $(i = 1,2)$ *are defined as* (4.19) *and* (4.20), *respectively.*

Proof The second and fourth equations of (2.2) can be rewritten as

$$\begin{aligned}\frac{dy_i(t)}{dt} &= \big[r_i e^{-w_{i1}\tau_i} - \big(w_{i2} + \beta_i z_i(t)\big)\big]y_i(t) \\ &\quad - r_i e^{-w_{i1}\tau_i}\frac{d}{dt}\int_{t-\tau_i}^{t} y_i(u)\,du \quad (i = 1,2).\end{aligned} \tag{4.14}$$

According to (4.14), $Q_i(t)$ $(i = 1,2)$ is defined as

$$Q_i(t) = y_i(t) + r_i e^{-w_{i1}\tau_i}\int_{t-\tau_i}^{t} y_i(u)\,du \quad (i = 1,2).$$

We calculate the derivative of $Q_i(t)$ $(i = 1,2)$ along the solution of (2.2):

$$\frac{dQ_i(t)}{dt} = \big[r_i e^{-w_{i1}\tau_i} - \big(w_{i2} + \beta_i z_i(t)\big)\big]y_i(t) \quad (i = 1,2). \tag{4.15}$$

Since

$$r_i e^{-w_{i1}\tau_i} > w_{i2} + \beta_i\big[z_{i4}^{*} e^{-(w_{i3}-k_i\beta_i y_i^*)l\tau} + z_{i4}^{**} e^{-(w_{i3}+k_i\beta_i y_i^*)(1-l)\tau}\big] \quad (i = 1,2),$$

we can easily see that there exists a sufficiently small $\varepsilon > 0$ such that

$$r_i e^{-w_{i1}\tau_i} > w_{i2} + \beta_i\big\{\big[z_{i4}^{*} e^{-(w_{i3}-k_i\beta_i y_i^*)l\tau} + z_{i4}^{**} e^{-(w_{i3}+k_i\beta_i y_i^*)(1-l)\tau}\big] + \varepsilon\big\} \quad (i = 1,2).$$

We claim that for any $t_0 > 0$, it is impossible that $y_i(t) < y_i^*$ $(i = 1,2)$ for all $t > t_0$. Suppose that the claim is not valid. Then, there is a $t_0 > 0$ such that $y_i(t) < y_i^*$ $(i = 1,2)$ for all $t > t_0$. It follows from the first and third equations of (2.2) that for all $t > t_0$

$$\frac{dz_i(t)}{dt} < -\big(w_{i3} - k_i\beta_i y_i^*\big)z_i(t) \quad (i = 1,2). \tag{4.16}$$

Consider the following comparison impulsive system for all $t > t_0$:

$$\begin{cases} \left.\begin{aligned} \frac{dz_{14}(t)}{dt} = -(w_{13} - k_1\beta_1 y_1^*)z_{14}(t), \\ \frac{dz_{24}(t)}{dt} = -(w_{23} - k_2\beta_2 y_2^*)z_{24}(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle z_{14}(t) = D(z_{24}(t) - z_{14}(t)), \\ \triangle z_{24}(t) = D(z_{14}(t) - z_{24}(t)), \end{aligned}\right\} t = (n+l)\tau, \\ \left.\begin{aligned} \triangle z_{14}(t) = \mu_1, \\ \triangle z_{24}(t) = \mu_2, \end{aligned}\right\} t = (n+1)\tau, n \in Z^+, \end{cases} \tag{4.17}$$

where

$$\begin{cases} \widetilde{z_{14}(t)} = \begin{cases} z_{14}^* e^{-(w_{13}-k_1\beta_1 y_1^*)(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_{14}^{**} e^{-(w_{13}-k_1\beta_1 y_1^*)(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ \widetilde{z_{24}(t)} = \begin{cases} z_{24}^* e^{-(w_{23}-k_2\beta_2 y_2^*)(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_{24}^{**} e^{-(w_{23}-k_2\beta_2 y_2^*)(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \end{cases} \tag{4.18}$$

here

$$\begin{cases} z_{14}^* = \frac{\mu_2(1-A_{14})+\mu_1 A_{24}}{(1-A_{14})(1-B_{24})-A_{24}B_{14}} > 0, \\ z_{24}^* = \frac{\mu_2 B_{14}+\mu_1(1-B_{24})}{(1-A_{14})(1-B_{24})-A_{24}B_{14}} > 0, \end{cases} \tag{4.19}$$

and

$$\begin{cases} z_{14}^{**} = (1-D)e^{-(w_{13}-k_1\beta_1 y_1^*)l\tau} z_{14}^* + De^{-(w_{23}-k_2\beta_2 y_2^*)l\tau} z_{24}^*, \\ z_{24}^{**} = De^{-(w_{13}-k_1\beta_1 y_2^*)l\tau} z_{14}^* + (1-D)e^{-(w_{23}-k_2\beta_2 y_2^*)l\tau} z_{24}^*, \end{cases} \tag{4.20}$$

and

$$\begin{aligned} A_{14} &= (1-D)e^{-(w_{13}-k_1\beta_1 y_1^*)\tau} < 1, \\ B_{14} &= De^{-[(w_{13}-k_1\beta_1 y_1^*)(1-l)+(w_{23}-k_2\beta_2 y_2^*)l]\tau} < 1, \\ A_{24} &= De^{-[(w_{13}-k_1\beta_1 y_1^*)l+(w_{23}-k_2\beta_2 y_2^*)(1-l)]\tau} < 1, \\ B_{24} &= (1-D)e^{-(w_{23}-k_2\beta_2 y_2^*)\tau} < 1. \end{aligned}$$

By the comparison theorem for impulsive differential equations [28], we know that there exists a sufficient small $\varepsilon > 0$ and t_1 ($> t_0 + \tau_1$) such that the inequality $z_i(t) \leq \widetilde{z_{i4}(t)} + \varepsilon$ ($i = 1, 2$) holds for $t \geq t_1$, thus $z_i(t) \leq [z_{i4}^* e^{-(w_{i3}-k_i\beta_i y_i^*)l\tau} + z_{i4}^{**} e^{-(w_{i3}+k_i\beta_i y_i^*)(1-l)\tau}] + \varepsilon$ for all $t \geq t_1$. We use the notation $\sigma_i \overset{\Delta}{=} [z_{i4}^* e^{-(w_{i3}-k_i\beta_i y_i^*)l\tau} + z_{i4}^{**} e^{-(w_{i3}+k_i\beta_i y_i^*)(1-l)\tau}] + \varepsilon$ ($i = 1, 2$) for convenience. So we have

$$r_i e^{-w_{i1}\tau_i} > w_{i2} + \beta_i\sigma_i \quad (i = 1, 2),$$

then we have

$$Q_i'(t) > y_2(t)\left[r_i e^{-w_{i1}\tau_i} - (w_{i2} + \beta_i\sigma_i)\right] \quad (i = 1, 2),$$

for all $t > t_1$. Set $y_i^m = \min_{t\in[t_1,t_1+\tau_1]} y_i(t)$, we will show that $y_i(t) \geq y_i^m$ for all $t \geq t_1$. Suppose the contrary, then there is a $T_0 > 0$ such that $y_i(t) \geq y_i^m$ for $t_1 \leq t \leq t_1 + \tau_1 + T_0$, $y_i(t_1 + \tau_1 + T_0) = y_i^m$ and $y_i'(t_1 + \tau_1 + T_0) < 0$. Hence, the second and fourth equations of system (2.2) imply that

$$\begin{aligned} y_i'(t_1 + \tau_1 + T_0) &= r_i e^{-w_{i1}\tau_i} y_i(t_1 + \tau_1 + T_0) - \big[w_{i1} + \beta_i z_i(t_1 + \tau_1 + T_0)\big] y_i(t_1 + \tau_1 + T_0) \\ &\geq \big[r_i e^{-w_{i1}\tau_i} - (w_{i2} + \beta_i\sigma_i)\big] I_i^m > 0 \quad (i = 1, 2). \end{aligned}$$

This is a contradiction. Thus, $y_i(t) \geq y_i^m$ for all $t > t_1$. As a consequence, then $Q_i'(t) > y_i^m[r_i e^{-w_{i1}\tau_i} - (w_{i2} + \beta_i\sigma_i)] > 0$ $(i = 1, 2)$ for all $t > t_1$. This implies that as $t \to \infty$, $Q_i(t) \to \infty$. It is a contradiction to $Q_i(t) \leq M(1 + \tau_i r_i e^{-w_{i1}\tau_i})$. Hence, the claim is complete.

By the claim, we are left to consider two cases. First, $y_i(t) \geq y_i^*$ $(i = 1, 2)$ for all t large enough. Second, $y_i(t)$ $(i = 1, 2)$ oscillates about y_i^* $(i = 1, 2)$ for t large enough.

Define

$$q = \min\left\{\frac{y_1^*}{2}, \frac{y_2^*}{2}, q_1, q_2\right\}, \tag{4.21}$$

where $q_i = y_i^* e^{-(w_{i1}+\beta_i M)\tau_i}$ $(i = 1, 2)$. We hope to show that $y_i(t) \geq q_i$ $(i = 1, 2)$ for all t large enough. The conclusion is evident in the first case. For the second case, let $t^* > 0$ and $\xi > 0$ satisfy $y_i(t^*) = y_i(t^* + \xi) = y_i^*$ $(i = 1, 2)$ and $y_i(t) < y_i^*$ $(i = 1, 2)$ for all $t^* < t < t^* + \xi$ where t^* is sufficiently large such that $y_i(t) > \sigma_i$ $(i = 1, 2)$ for $t^* < t < t^* + \xi$, $y_i(t)$ $(i = 1, 2)$ is uniformly continuous. The positive solutions of (2.2) are ultimately bounded and $y_i(t)$ $(i = 1, 2)$ is not affected by impulses. Hence, there is a T $(0 < t < \tau_1)$ and T is dependent on the choice of t^* such that $y_i(t^*) > \frac{y_i^*}{2}$ $(i = 1, 2)$ for $t^* < t < t^* + T$. If $\xi < T$, there is nothing to prove. Let us consider the case $T < \xi < \tau_1$. Since $y_i'(t) > -(w_{i1} + \beta_i M) y_i(t)$ $(i = 1, 2)$ and $y_i(t^*) = y_i^*$ $(i = 1, 2)$, it is clear that $y_i(t) \geq q_i$ $(i = 1, 2)$ for $t \in [t^*, t^* + \tau_1]$. Then, proceeding exactly as the proof for the above claim, we see that $y_i(t) \geq q_i$ for $t \in [t^* + \tau_1, t^* + \xi]$. Because the kind of interval $t \in [t^*, t^* + \xi]$ is chosen in an arbitrary way (we only need t^* to be large). We conclude that $y_i(t) \geq q$ for all large t. In the second case, in view of the above discussion, the choice of q is independent of the positive solution, and we prove that any positive solution of (2.2) satisfies $y_i(t) \geq q$ for all sufficiently large t. This completes the proof of the theorem. □

Theorem 4.4 *If*

$$\min_{i=1,2}\big[r_i e^{-w_{i1}\tau_i} - w_{i2} - \beta_i\big(z_{i4}^* e^{-(w_{i3}-k_i\beta_i y_i^*)l\tau} + z_{i4}^{**} e^{-(w_{i3}+k_i\beta_i y_i^*)(1-l)\tau}\big)\big] > 0,$$

system (2.2) *is permanent.*

Proof Denote $(y_1(t), z_1(t), y_2(t), z_2(t))$ for any solution of system (2.2). From system (2.2) and Lemma 3.3, we can easily obtain

$$\frac{dz_i(t)}{dt} > -w_{i3} z_i(t) \quad (i = 1, 2). \tag{4.22}$$

Consider the comparison impulsive system (4.9) for all $t > t_0$. By Lemma 3.5, we obtain

$$\begin{cases} \widetilde{z_{12}(t)} = \begin{cases} z_1^* e^{-w_{13}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_1^{**} e^{-w_{13}(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ \widetilde{z_{22}(t)} = \begin{cases} z_2^* e^{-w_{23}(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ z_2^{**} e^{-w_{23}(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \end{cases} \tag{4.23}$$

here z_1^* and z_2^* are defined as (3.7), z_1^{**} and z_2^{**} are defined as (3.12). By the comparison theorem for impulsive differential equation [28], we know that there exists a sufficient small $\varepsilon > 0$ and t_1 ($> t_0 + \tau_1$) such that the inequality $z_i(t) \geq \widetilde{z_i(t)} - \varepsilon$ ($i = 1, 2$) holds for $t \geq t_1$, thus $z_i(t) \geq [z_i^* e^{-w_{i3} l\tau} + z_i^{**} e^{-w_{i3}(1-l)\tau}] - \varepsilon \stackrel{\Delta}{=} p_i$ for all $t \geq t_1$. By Theorem 4.3, Lemma 3.3, and the above discussion, system (2.2) is permanent. The proof of Theorem 4.4 is complete. □

5 Discussion

In this paper, we investigate a new delayed stage-structured predator-prey model with impulsive diffusion and releasing. We analyze that the prey-extinction boundary periodic solution of system (2.2) is globally attractive, and we also obtain the permanent condition of system (2.2). From Theorem 4.1 and Theorem 4.4, we can easily guess that there must exist a threshold μ^* ($\mu^* = \max_{i=1,2}\{\mu_i^*\}$ and μ_i^* ($i = 1, 2$)) is determined by the condition of Theorem 4.1), if $\mu > \mu^*$, the prey-extinction boundary periodic solution $(0, \widetilde{z_1(t)}, 0, \widetilde{z_2(t)})$ of (2.2) is globally attractive. If $\mu < \mu^{**}$ ($\mu^{**} = \min_{i=1,2}\{\mu_i^{**}\}$ and μ_i^{**} ($i = 1, 2$) is determined by the condition of Theorem 4.4), system (2.2) is permanent. From Theorem 4.1 and Theorem 4.4, we can also easily guess that there must exist a threshold D^* ($0 < D^* < 1$). If $D < D^*$, the prey-extinction boundary periodic solution $(0, \widetilde{z_1(t)}, 0, \widetilde{z_2(t)})$ of (2.2) is globally attractive. If $D > D^*$, system (2.2) is permanent. This indicates that impulsive diffusion and impulsive releasing can affect the dynamical behaviors of the investigated system (2.2). That is to say, impulsive diffusion and impulsive releasing of the predator population play important roles for the prey-extinction of system (2.2). The parameters as τ_i ($i = 1, 2$) and τ can also be discussed, its change also affects the dynamical system of (2.2). The results of this paper provide a tactical basis for pest management.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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