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On generalized sequence spaces via modulus function

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Abstract

In this paper, we introduce and study the concept of lacunary strongly $(\mathbf{A},\varphi)$-convergence with respect to a modulus function and lacunary $(\mathbf{A},\varphi)$-statistical convergence and examine some properties of these sequence spaces. We establish some connections between lacunary strongly $(\mathbf{A},\varphi)$-convergence and lacunary $(\mathbf{A},\varphi)$-statistical convergence.

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1 Introduction

Let s denote the set of all real and complex sequences $x = (x_k)$. By l_∞ and c, we denote the Banach spaces of bounded and convergent sequences $x = (x_k)$ normed by $\|x\| = \sup_n |x_n|$, respectively. A linear functional L on l_∞ is said to be a Banach limit [1] if it has the following properties:

(1) $L(x) \geq 0$ if $n \geq 0$ (*i.e.* $x_n \geq 0$ for all n),
(2) $L(e) = 1$, where $e = (1, 1, \ldots)$,
(3) $L(Dx) = L(x)$, where the shift operator D is defined by $D(x_n) = \{x_{n+1}\}$.

Let B be the set of all Banach limits on l_∞. A sequence $x \in \ell_\infty$ is said to be almost convergent if all Banach limits of x coincide. Let $\hat{c}$ denote the space of almost convergent sequences. Lorentz [2] has shown that

$$\hat{c} = \left\{x \in l_\infty : \lim_m t_{m,n}(x) \text{ exists uniformly in } n\right\},$$

where

$$t_{m,n}(x) = \frac{x_n + x_{n+1} + x_{n+2} + \cdots + x_{n+m}}{m+1}.$$

By a lacunary $\theta = (k_r)$; $r = 0, 1, 2, \ldots$, where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and $h_r = k_r - k_{r-1}$. The ratio $\frac{k_r}{k_{r-1}}$ will be denoted by q_r. The space of lacunary strongly convergent sequences N_θ was defined by Freedman *et al.* [3] as follows:

$$N_\theta = \left\{x = (x_k) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} |x_k - l| = 0, \text{for some } l\right\}.$$

In the special case where $\theta = (2^r)$ (see [3]) we have $N_\theta = w$, which is defined by

$$w = \left\{ x = (x_k) : \lim_n \frac{1}{n} \sum_{k=0}^{n} |x_k - l| = 0, \text{for some } l \right\}.$$

Das and Mishra [4] have introduced the space AC_θ of lacunary almost convergent sequences and the space $|AC_\theta|$ of lacunary strongly almost convergent sequences as follows:

$$AC_\theta = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} (x_{k+n} - L) = 0, \text{for some } L \text{ uniformly in } n \right\}$$

and

$$|AC_\theta| = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} |x_{k+n} - L| = 0, \text{for some } L \text{ uniformly in } n \right\}.$$

Ruckle used the idea of a modulus function f to construct a class of FK spaces,

$$L(f) = \left\{ x = (x_k) : \sum_{k=1}^{\infty} f(|x_k|) < \infty \right\}.$$

The space $L(f)$ is closely related to the space l_1, which is an $L(f)$ space with $f(x) = x$ for all real $x \geq 0$.

In 1999, Savaş [5] generalized the concept of strong almost convergence by using a modulus f and $p = (p_k)$ is a sequence of strictly positive real numbers as follows:

$$\left[\hat{c}(f,p)\right] = \left\{ x : \lim_n \frac{1}{n} \sum_{k=1}^{n} f\left(|x_{k+m} - L|\right)^{p_k} = 0, \text{for some } L, \text{uniformly in } m \right\}$$

and

$$\left[\hat{c}(f,p)\right]_0 = \left\{ x : \lim_n \frac{1}{n} \sum_{k=1}^{n} f\left(|x_{k+m}|\right)^{p_k} = 0, \text{uniformly in } m \right\}.$$

More investigations in this direction and more applications of the modulus can be found in [6–12].

Following Ruckle [13], a modulus function f is a function from $[0,\infty)$ to $[0,\infty)$ such that

(i) $f(x) = 0$ if and only if $x = 0$,
(ii) $f(x + y) \leq f(x) + f(x)$ for all $x, y \geq 0$,
(iii) f increasing,
(iv) f is continuous from the right at zero.

By a φ-function we understand a continuous non-decreasing function $\varphi(u)$ defined for $u \geq 0$ and such that $\varphi(0) = 0$, $\varphi(u) > 0$, for $u > 0$ and $\varphi(u) \to \infty$ as $u \to \infty$.

A φ-function φ is called non-weaker than a φ-function ψ if there are constants $c, b, k, l > 0$ such that $c\psi(lu) \leq b\varphi(ku)$ (for all large u) and we write $\psi \prec \varphi$.

A φ-function φ and ψ are called equivalent if there are positive constants b_1, b_2, c, k_1, k_2, l such that $b_1\varphi(k_1 u) \leq c\psi(lu) \leq b_2\varphi(k_2 u)$ (for all large u) and we write $\varphi \sim \psi$.

A φ-function φ is said to satisfy the condition (Δ_2) (for all large u) if for some constant $k > 1$ there is satisfied the inequality $\varphi(2u) \leq k\varphi(u)$ (see [12, 14]).

In this paper, we introduce and study some properties of the following sequence space which is generalization of Savaş [14].

2 Main results

Let φ and f be a given φ-function and modulus function, respectively, and let $p = (p_n)$ be a sequence of positive real numbers. Moreover, let $\mathbf{A} = (\mathbf{A}_i)$ be the generalized three parametric real matrix with $A_i = (a_{n,k}(i))$ and a lacunary sequence θ be given. Then we define the following sequence spaces:

$$N_\theta^0(\mathbf{A}, \varphi, f, p) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p_n} = 0, \text{uniformly in } i \right\}.$$

If $x \in N_\theta^0(\mathbf{A}, \varphi, f)$, the sequence x is said to be lacunary strong $(\mathbf{A}, \varphi)$-convergent to zero with respect to a modulus f. When $\varphi(x) = x$ for all x, we obtain

$$N_\theta^0(\mathbf{A}, f, p) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i) x_k \right| \right)^{p_n} = 0, \text{uniformly in } i \right\}.$$

If we take $f(x) = x$, we write

$$N_\theta^0(\mathbf{A}, \varphi, p) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} \left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right|^{p_n} = 0, \text{uniformly in } i \right\}.$$

If we take $p_k = p$, for all k, we have

$$N_\theta^0(\mathbf{A}, \varphi, f) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p} = 0, \text{uniformly in } i \right\}.$$

If we take $\mathbf{A} = I$ and $\varphi(x) = x$, respectively, then we have

$$N_\theta^0 = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} f(|x_k|)^{p_n} = 0 \right\}.$$

If we define the matrix $A = (a_{nk}(i))$ as follows:

$$a_{nk}(i) := \begin{cases} \frac{1}{n}, & \text{if } n \geq k, \\ 0, & \text{otherwise}, \end{cases}$$

then we have

$$N_\theta^0(\mathbf{C}, \varphi, f, p) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} f\left(\left| \frac{1}{n} \sum_{k=1}^{n} \varphi(|x_k|) \right| \right)^{p_n} = 0, \text{uniformly in } i \right\},$$

$$a_{nk}(i) := \begin{cases} \frac{1}{n}, & \text{if } i \leq k \leq i + n - 1, \\ 0, & \text{otherwise}, \end{cases}$$

then we have

$$N_\theta^0(\hat{c},\varphi,f,p) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n\in I_r} f\left(\left| \frac{1}{n} \sum_{k=i}^{i+n} \varphi(|x_k|) \right| \right)^{p_n} = 0, \text{uniformly in } i \right\}.$$

If $x \in N_\theta^0(\hat{c},\varphi,f)$, the sequence x is said to be almost lacunary strong φ-convergent to zero with respect to a modulus f. In the next theorem we establish inclusion relations between $w(A,\varphi,f,p)$ and $N_\theta^0(\mathbf{A},\varphi,f,p)$. We now have the following.

Theorem 2.1 *Let f be any modulus function and let there be a φ-function φ and a generalized three parametric real matrix* $\mathbf{A}$; *let $p = (p_n)$ be a sequence of positive real numbers and the sequence θ be given. If*

$$w(\mathbf{A},\varphi,f,p) = \left\{ x = (x_k) : \lim_m \frac{1}{m} \sum_{n=1}^{m} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p_n} = 0, \textit{uniformly in } i \right\},$$

then the following relations are true:

(a) *If* $\liminf_r q_r > 1$ *then we have* $w(A,\varphi,f,p) \subseteq N_\theta^0(\mathbf{A},\varphi,f,p)$.

(b) *If* $\sup_r q_r < \infty$, *then we have* $N_\theta^0(\mathbf{A},\varphi,f,p) \subseteq w(A,\varphi,f,p)$.

(c) $1 < \liminf_r q_r \le \limsup_r q_r < \infty$, *then we have* $N_\theta^0(\mathbf{A},\varphi,f,p) = w(A,\varphi,f,p)$.

Proof (a) Let us suppose that $x \in w(A,\varphi,f,p)$. There exists $\delta > 0$ such that $q_r > 1 + \delta$ for all $r \ge 1$ and we have $h_r/k_r \ge \delta/(1+\delta)$ for sufficiently large r. Then, for all i,

$$\begin{aligned} &\frac{1}{k_r} \sum_{n=1}^{k_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p_n} \\ &\quad \ge \frac{1}{k_r} \sum_{n\in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p_n} \\ &\quad = \frac{h_r}{k_r} \frac{1}{h_r} \sum_{n\in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p_n} \\ &\quad \ge \frac{\delta}{1+\delta} \frac{1}{h_r} \sum_{n\in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}\varphi(|x_k|) \right| \right)^{p_n}. \end{aligned}$$

Hence, $x \in N_\theta^0(\mathbf{A},\varphi,f,p)$.

(b) If $\limsup_r q_r < \infty$ then there exists $M > 0$ such that $q_r < M$ for all $r \ge 1$. Let $x \in N_\theta^0(\mathbf{A},\varphi,f,p)$ and ε is an arbitrary positive number, then there exists an index j_0 such that for every $j \ge j_0$ and all i,

$$R_j = \frac{1}{h_j} \sum_{n\in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p_n} < \varepsilon.$$

Thus, we can also find $K > 0$ such that $R_j \le K$ for all $j = 1,2,\ldots$. Now let m be any integer with $k_{r-1} \le m \le k_r$, then we obtain, for all i,

$$I = \frac{1}{m} \sum_{n=1}^{m} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p_n} \le \frac{1}{k_{r-1}} \sum_{n=1}^{k_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \right| \right)^{p_n} = I_1 + I_2,$$

where

$$I_1 = \frac{1}{k_{r-1}} \sum_{j=1}^{j_0} \sum_{n \in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)^{p_n},$$

$$I_2 = \frac{1}{k_{r-1}} \sum_{j=j_0+1}^{m} \sum_{n \in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)^{p_n}.$$

It is easy to see that

$$\begin{aligned}
I_1 &= \frac{1}{k_{r-1}} \sum_{j=1}^{j_0} \sum_{n \in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)^{p_n} \\
&= \frac{1}{k_{r-1}} \left(\sum_{n \in I_1} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)^{p_n} + \cdots + \sum_{n \in I_{j_0}} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)^{p_n} \right) \\
&\le \frac{1}{k_{r-1}} (h_1 R_1 + \cdots + h_{j_0} R_{j_0}) \\
&\le \frac{1}{k_{r-1}} j_0 k_{j_0} \sup_{1 \le i \le j_0} R_i \\
&\le \frac{j_0 k_{j_0}}{k_{r-1}} K.
\end{aligned}$$

Moreover, we have for all i

$$\begin{aligned}
I_2 &= \frac{1}{k_{r-1}} \sum_{j=j_0+1}^{m} \sum_{n \in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}\varphi(|x_k|)\right|\right)^{p_n} \\
&= \frac{1}{k_{r-1}} \sum_{j=j_0+1}^{m} \frac{1}{h_j} \sum_{n \in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}\varphi(|x_k|)\right|\right)^{p_n} h_j \\
&\le \varepsilon \frac{1}{k_{r-1}} \sum_{j=j_0+1}^{m} h_j \\
&\le \varepsilon \frac{k_r}{k_{r-1}} \\
&= \varepsilon q_r < \varepsilon \cdot M.
\end{aligned}$$

Thus $I \le \frac{j_0 k_{j_0}}{k_{r-1}} K + \varepsilon \cdot M$. Finally, $x \in w(A, \psi, f, p)$.

The proof of (c) follows from (a) and (b). This completes the proof. □

Theorem 2.2 *Let f, f_1, be modulus functions. Then we have*

$$N_\theta^0(A, f_1, \varphi, p) \subset (A, \varphi, fof_1, p).$$

Proof This can be proved by using techniques similar to those used in the theorem of Savaş [14]. □

Recently Savaş [14] defined $(\mathbf{A}, \varphi)$-statistical convergence as follows.

Let θ be a lacunary sequence, and let $\mathbf{A} = (a_{nk}(i))$ be the generalized three parametric real matrix, the sequence $x = (x_k)$, the φ-function $\varphi(u)$ and a positive number $\varepsilon > 0$ be given. We write, for all i,

$$K_\theta^r(A,\varphi,\varepsilon) = \left\{ n \in I_r : \sum_{k=1}^{\infty} a_{nk}(i)\varphi\bigl(|x_k|\bigr) \geq \varepsilon \right\}.$$

The sequence x is said to be $(\mathbf{A},\varphi)$-statistically convergent to a number zero if for every $\varepsilon > 0$

$$\lim_r \frac{1}{h_r}\mu\bigl(K_\theta^r(A,\varphi,\varepsilon)\bigr) = 0, \quad \text{uniformly in } i,$$

where $\mu(K_\theta^r(A,\varphi,\varepsilon))$ denotes the number of elements belonging to $K_\theta^r(\mathbf{A},\varphi,\varepsilon)$. We denote by $S_\theta^0(\mathbf{A},\varphi)$, the set of sequences $x = (x_k)$ which are lacunary $(\mathbf{A},\varphi)$-statistical convergent to zero and we write

$$S_\theta^0(\mathbf{A},\varphi) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r}\mu\bigl(K_\theta^r(A,\varphi,\varepsilon)\bigr) = 0, \text{uniformly in } i \right\}.$$

More investigations in this direction can be found in [15–20].

We now establish inclusion relations between $N_\theta^0(\mathbf{A},\varphi,f,p)$ and $S_\theta^0(A,\varphi)$.

In the following theorem we assume that $0 < h = \inf p_n \leq p_n \leq \sup p_k \leq H \leq \infty$.

Theorem 2.3 (a) *If the matrix A and the sequence θ and functions f and φ are given, then*

$$N_\theta^0(A,\varphi,f,p) \subset S_\theta^0(A,\varphi).$$

(b) *If the φ-function $\varphi(u)$ and the matrix A are given, and if the modulus function f is bounded, then*

$$S_\theta^0(A,\varphi) \subset N_\theta^0(A,\varphi,f,p).$$

Proof (a) Let f be a modulus function and let ε be a positive numbers. We write the following inequalities, for all i,

$$\begin{aligned}
&\frac{1}{h_r}\sum_{n\in I_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi\bigl(|x_k|\bigr)\right|\right)^{p_n} \\
&\quad = \frac{1}{h_r}\sum_{n\in I_r^1} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi\bigl(|x_k|\bigr)\right|\right)^{p_n} \\
&\quad \geq \frac{1}{h_r}\sum_{n\in I_r^1}\bigl[f(\varepsilon)\bigr]^{p_n} \\
&\quad \geq \frac{1}{h_r}\sum_{n\in I_r^1}\min\bigl(\bigl[f(\varepsilon)\bigr]^{\inf p_n},\bigl[f(\varepsilon)\bigr]^H\bigr) \\
&\quad \geq \frac{1}{h_r}\mu\bigl(K_\theta^r(A,\varphi,\varepsilon)\bigr)\min\bigl(\bigl[f(\varepsilon)\bigr]^{\inf p_n},\bigl[f(\varepsilon)\bigr]^H\bigr),
\end{aligned}$$

where

$$I_r^1 = \left\{ n \in I_r : \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) \geq \varepsilon \right\}.$$

Finally, if $x \in N_\theta^0(A, \varphi, f, p)$ then $x \in S_\theta^0(A, \varphi, f)$.

(b) Let us suppose that $x \in S_\theta^0(A, \varphi)$. If the modulus function f is a bounded function, then there exists an integer K such that $f(x) < K$ for $x \geq 0$. Let us take

$$I_r^2 = \left\{ n \in I_r : \sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|) < \varepsilon \right\}.$$

Thus we have, for all i,

$$\begin{aligned}
&\frac{1}{h_r}\sum_{n\in I_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)^{p_n} \\
&\quad\leq \frac{1}{h_r}\sum_{n\in I_r^1} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)^{p_n} \\
&\qquad + \frac{1}{h_r}\sum_{n\in I_r^2} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)^{p_n} \\
&\quad\leq \frac{1}{h_r}\sum_{n\in I_r^1} \max\left(K^h, K^H\right) + \frac{1}{h_r}\sum_{n\in I_r^2} \left[f(\varepsilon)\right]^{p_n} \\
&\quad\leq \max\left(K^h, K^H\right)\frac{1}{h_r}\mu\left(K_\theta^r(A, \varphi, \varepsilon)\right) + \max\left(\left[f(\varepsilon)\right]^h, \left[f(\varepsilon)\right]^H\right).
\end{aligned}$$

Taking the limit as $\varepsilon \to 0$, we observe that $x \in N_\theta^0(A, \varphi, f, p)$.

This completes the proof. □

Competing interests

The author declares that they have no competing interests.

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