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Research Article

Global Positive Periodic Solutions of Generalized n-Species Gilpin-Ayala Delayed Competition Systems with Impulses

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We consider the following generalized n-species Lotka-Volterra type and Gilpin-Ayala type competition systems with multiple delays and impulses: $x_i'(t) = x_i(t)[a_i(t) - b_i(t)x_i(t) - \sum_{j=1}^n c_{ij}(t)x_j^{\alpha_{ij}}(t-\rho_{ij}(t)) - \sum_{j=1}^n d_{ij}(t)x_j^{\beta_{ij}}(t-\tau_{ij}(t)) - \sum_{j=1}^n e_{ij}(t)\int_{-\eta_{ij}}^0 k_{ij}(s)x_j^{\gamma_{ij}}(t+s)ds - \sum_{j=1}^n f_{ij}(t)\int_{-\theta_{ij}}^0 K_{ij}(\xi)x_i^{\delta_{ij}}(t+\xi)x_j^{\sigma_{ij}}(t+\xi)d\xi]$, a.e, $t > 0, t \neq t_k$; $x_i(t_k^+) - x_i(t_k^-) = h_{ik}x_i(t_k), i = 1, 2, \ldots, n, k \in Z_+$. By applying the Krasnoselskii fixed-point theorem in a cone of Banach space, we derive some verifiable necessary and sufficient conditions for the existence of positive periodic solutions of the previously mentioned. As applications, some special cases of the previous system are examined and some earlier results are extended and improved.

1. Introduction

In the recent decades, the traditional Lotka-Volterra competition systems have been studied extensively. One of the models is the following competition system:

$$\frac{dx_i(t)}{dt} = x_i(t)\left[b_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t)\right], \quad i = 1, 2, \ldots, n. \tag{1}$$

Many results concerned with the permanence, global asymptotic stability, and the existence of positive periodic solutions of system (1) are obtained; we refer to [1–10] and the reference therein. However, the Lotka-Volterra type models have often been severely criticized. One of the criticisms is that, in such a model, the per capita rate of change of the density of each species is a linear function of densities of the interacting species. In 1973, Ayala et al. [11] conducted experiments on fruit fly dynamics to test the validity of ten models of competitions. One of the models accounting best for the experimental results is given by

$$\begin{aligned} x_1'(t) &= r_1x_1(t)\left[1 - \left(\frac{x_1(t)}{K_1}\right)^{\theta_1} - \alpha_{12}\frac{x_2(t)}{K_2}\right], \\ x_2'(t) &= r_2x_2(t)\left[1 - \left(\frac{x_2(t)}{K_2}\right)^{\theta_2} - \alpha_{21}\frac{x_1(t)}{K_1}\right]. \end{aligned} \tag{2}$$

In order to fit data in their experiments and to yield significantly more accurate results, Gilpin and Ayala [12] claimed that a slightly more complicated model was needed and proposed the following competition model:

$$\frac{dx_i(t)}{dt} = r_ix_i(t)\left[1 - \left(\frac{x_i(t)}{K_i}\right)^{\theta_i} - \sum_{j=1, j\neq i}^n a_{ij}\frac{x_j(t)}{K_j}\right], \quad i = 1, 2, \ldots, n, \tag{3}$$

where x_i is the population density of the ith species, r_i is the intrinsic exponential growth rate of the ith species, K_i is the environmental carrying capacity of species i in the absence of competition, θ_i provides a nonlinear measure of interspecific interference, and a_{ij} provides a measure of interspecific interference. [13–15] obtained sufficient conditions which guarantee the global asymptotic stability of system (3). Chen [16] investigated the following n-species nonautonomous Gilpin-Ayala competitive model:

$$\frac{dx_i(t)}{dt} = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)\, x_j^{\theta_{ij}}(t)\right], \quad i = 1, 2, \ldots, n. \tag{4}$$

For each $r \le n$, they established a series of criteria under which r of the species of system (4) were permanent while the remaining $n-r$ species were driven to extinction. In [17], Fan and Wang further studied the following delay Gilpin-Ayala type competition model:

$$\frac{dx_i(t)}{dt} = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)\, x_j^{\theta_{ij}}\left(t - \tau_{ij}(t)\right)\right], \quad i = 1, 2, \ldots, n. \tag{5}$$

They obtained a set of easily verifiable sufficient conditions for the existence of at least one positive periodic solution of the system (5) by applying the coincidence degree theory. Recently, in [18], Chen investigated the following n-species Gilpin-Ayala type competition systems:

$$\begin{aligned} y_i'(t) = y_i(t)\Bigg[r_i(t) &- \sum_{j=1}^{n} a_{ij}(t)\, x_j^{\alpha_{ij}}(t) \\ &- \sum_{j=1}^{n} b_{ij}(t)\, x_j^{\beta_{ij}}\left(t - \tau_{ij}(t)\right) \\ &- \sum_{j=1}^{n} \int_{-\sigma_{ij}}^{0} c_{ij}(t,s)\, y_j^{\gamma_{ij}}(t+s)\, ds \Bigg], \\ & i = 1, 2, \ldots, n. \end{aligned} \tag{6}$$

He established a series of criteria under which r of the species in the system (6) were permanent while the remaining $n-r$ species were driven to extinction. In [19], Xia et al. considered the following almost periodic nonlinear n-species competitive Lotka-Volterra model:

$$\begin{aligned} y_i'(t) = y_i(t)\Bigg[r_i(t) &- \sum_{j=1}^{N} a_{ij}(t)\, x_j^{\alpha_{ij}}(t) \\ &- \sum_{j=1}^{N} b_{ij}(t)\, x_j^{\alpha_{ij}}\left(t - \tau_{ij}(t)\right) \\ &- \sum_{j=1}^{N} c_{ij}(t)\, y_i^{\alpha_{ii}}(t)\, y_j^{\alpha_{ij}}(t) \Bigg], \\ & i = 1, 2, \ldots, n. \end{aligned} \tag{7}$$

By using comparison theorem and constructing suitable Lyapunov functional, they derived a set of sufficient conditions for the existence and global attractivity of a unique positive almost periodic solution of the previously mentioned model. Motivated by the previous ideas, in [20], Yan considered the following generalized periodic n-species Gilpin-Ayala type competition models in periodic environments with deviating arguments of the form:

$$\begin{aligned} y_i'(t) = y_i(t)\Bigg[r_i(t) &- \sum_{j=1}^{n} a_{ij}(t)\, x_j^{\alpha_{ij}}(t) \\ &- \sum_{j=1}^{n} b_{ij}(t)\, x_j^{\beta_{ij}}\left(t - \tau_{ij}(t)\right) \\ &- \sum_{j=1}^{n} c_{ij}(t) \int_{-\sigma_{ij}}^{0} K_{ij}(\xi)\, y_i^{\gamma_{ij}} \\ &\times (t+\xi)\, y_j^{\delta_{ij}}(t+\xi)\, d\xi \Bigg], \\ & i = 1, 2, \ldots, n. \end{aligned} \tag{8}$$

By using a fixed point theorem in cone and the proof by contradiction, he obtained a necessary and sufficient condition for the existence of positive periodic solutions (with strictly positive components) of the system (8).

However, the ecological system is often deeply perturbed by human exploitation activities such as planting and harvesting, which makes it unsuitable to be considered continually. For having a more accurate description of such a system, we need to consider the impulsive differential equations. The theory of impulsive differential equations is not only richer than the corresponding theory of differential equations without impulses, but also represents a more natural framework for mathematical modeling of many real-world phenomena (see [21–23]). In recent years, some impulsive equations have been recently introduced in population dynamics in relation to population ecology; we refer the reader to [24–36] and the reference therein. However, to this day, only a little work has been done on the existence of positive periodic solutions to the generalized periodic n-species Gilpin-Ayala type competition models in periodic environments with deviating arguments of the form and impulses. Motivated by this, in this paper, we mainly consider the following n-species Gilpin-Ayala type competition models in periodic environments with deviating arguments of the form and impulses:

$$\begin{aligned} x_i'(t) = x_i(t)\Bigg[a_i(t) &- b_i(t)\, x_i(t) \\ &- \sum_{j=1}^{n} c_{ij}(t)\, x_j^{\alpha_{ij}}\left(t - \rho_{ij}(t)\right) \\ &- \sum_{j=1}^{n} d_{ij}(t)\, x_j^{\beta_{ij}}\left(t - \tau_{ij}(t)\right) - \sum_{j=1}^{n} e_{ij}(t) \end{aligned}$$

$$\times \int_{-\eta_{ij}}^{0} k_{ij}(s)\, x_j^{\gamma_{ij}}(t+s)\, ds - \sum_{j=1}^{n} f_{ij}(t) \times \int_{-\theta_{ij}}^{0} K_{ij}(\xi)\, x_i^{\delta_{ij}}(t+\xi)\, x_j^{\sigma_{ij}}(t+\xi)\, d\xi \Bigg],$$

$$\text{a.e, } t > 0,\ t \neq t_k;$$

$$x_i(t_k^+) - x_i(t_k^-) = h_{ik} x_i(t_k), \quad i = 1, 2, \ldots, n,\ k \in Z_+, \tag{9}$$

with initial conditions

$$x_i(\vartheta) = \phi_i(\vartheta), \quad \vartheta \in [-\tau, 0], \quad \phi_i(0) > 0,$$
$$\phi_i \in C([-\tau, 0), [0, +\infty)), \quad i = 1, 2, \ldots, n, \tag{10}$$

where x_i represents the density of the ith species X_i at time t, $a_i(t)$ is the intrinsic growth rate of the ith species X_i at this time, $a_i(t)$ may be negative while $\overline{a}_i = (1/\omega)\int_0^\omega a_i(t)dt > 0$; $c_{ij}(t)$, $d_{ij}(t)$, $e_{ij}(t)$ $(i \neq j)$ denote the competitive coefficient between the ith species X_i and jth species X_j, and $a_i(t)$, $b_i(t)$, $c_{ij}(t)$, $d_{ij}(t)$, $e_{ij}(t)$ are continuous ω-periodic functions with $b_i(t) \geq 0$, $c_{ij}(t) \geq 0$, $d_{ij}(t) \geq 0$, $e_{ij}(t) \geq 0$, $f_{ij}(t) \geq 0$, $\rho_{ij}(t) \geq 0$, $\tau_{ij}(t) \geq 0$, $\eta_{ij} \geq 0$, $\theta_{ij} \geq 0$ corresponding to the time delays with $\tau = \max_{i,j\in[1,n]}\{\rho_{ij}(t), \tau_{ij}(t), \eta_{ij}, \theta_{ij}\} \geq 0$; $\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij}, \sigma_{ij}$ are positive constants. Assume that h_{ik}, $i = 1, 2, \ldots, n$, $k \in Z_+ = \{1, 2, 3, \ldots\}$ are constants and there exists an integer $q > 0$ such that $t_{k+q} = t_k + \omega$, $h_{i(k+q)} = h_{ik}$, $i = 1, 2, \ldots, n$, where $0 < t_1 < t_2 < \cdots < t_q < \omega$.

Throughout this paper, we make the following notation and assumptions:

$C_\omega = \{x \mid x \in C(R,R), x(t+\omega) = x(t)\}$, with the norm defined by $|x|_0 = \max_{t\in[0,\omega]}|x(t)|$;

$C_\omega^1 = \{x \mid x \in C^1(R,R), x(t+\omega) = x(t)\}$, with the norm defined by $\|x\| = \max_{t\in[0,\omega]}\{|x|_0, |x'|_0\}$;

$PC = \{x \mid x : R \to R^+, \lim_{s\to t} x(s) = x(t), \text{if } t \neq t_k, \lim_{t\to t_k^-} x(t) = x(t_k), \lim_{t\to t_k^+} x(t) \text{ exists}, k \in Z^+\}$;

$PC^1 = \{x \mid x : R \to R^+, x' \in PC\}$;

$PC_\omega = \{x \mid x \in PC, x(t+\omega) = x(t)\}$, with the norm defined by $|x|_0 = \max_{t\in[0,\omega]}|x(t)|$;

$PC_\omega^1 = \{x \mid x \in PC^1, x(t+\omega) = x(t)\}$, with the norm defined by $\|x\| = \max_{t\in[0,\omega]}\{|x|_0, |x'|_0\}$.

The previously mentioned spaces are all Banach spaces. We also denote

$$\overline{g} = \frac{1}{\omega}\int_0^\omega g(t)\, dt, \quad \text{for any } g \in PC_\omega,$$
$$\Delta_{ik} = 1 + h_{ik}, \quad i = 1, 2, \ldots, n, k \in Z_+, \tag{11}$$

and make the following assumptions:

(H_1) $a_i(t), b_i(t), c_{ij}(t), d_{ij}(t), e_{ij}(t), f_{ij}(t), \tau_{ij}(t), \rho_{ij}(t) \in PC_\omega$, $i, j = 1, 2, \ldots, n$, and $\eta_{ij} \geq 0, \theta_{ij} \geq 0$.

(H_2) $\{t_k\}, k \in Z_+$ satisfies $0 < t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to+\infty} t_k = +\infty$.

(H_3) $\{h_{ik}\}$ is a real sequence with $\Delta_{ik} = h_{ik} + 1 > 0$, and $\prod_{0<t_k<t}\Delta_{ik}$ is an ω-periodic function.

(H_4) $k_{ij} \in C([-\eta_{ij}, 0], R_+)$, $K_{ij} \in C([-\theta_{ij}, 0], R_+)$ are constants such that $\int_{-\eta_{ij}}^0 k_{ij}(t)dt = 1$, $\int_{-\theta_{ij}}^0 K_{ij}(t)dt = 1$.

To conclude this section, we summarize in the following definition and lemma that will be needed in our arguments.

Definition 1. A function $x_i : R \to (0, +\infty)$ is said to be a positive solution of the system (9) and (10), if the following conditions are satisfied:

(a) $x_i(t)$ is absolutely continuous on each (t_k, t_{k+1}).

(b) for each $k \in Z_+$, $x_i(t_k^+)$ and $x_i(t_k^-)$ exist, and $x_i(t_k^-) = x_i(t_k)$.

(c) $x_i(t)$ satisfies the first equation of the system (9) and (10) for almost everywhere (for short a.e.) in $[0, \infty] \setminus \{t_k\}$ and satisfies $x_i(t_k^+) = \Delta_{ik} x_i(t_k)$ for $t = t_k$, $k \in Z_+$.

Under the previously mentioned hypotheses (H_1)–(H_4), we consider the following nonimpulsive Lotka-Volterra competitive systems:

$$\frac{dy_i(t)}{dt} = y_i(t)\Bigg[a_i(t) - B_i(t)\, y_i(t) - \sum_{j=1}^{n} C_{ij}(t)\, y_i^{\alpha_{ij}}\left(t - \rho_{ij}(t)\right) - \sum_{j=1}^{n} D_{ij}(t)\, y_j^{\beta_{ij}}\left(t - \tau_{ij}(t)\right) - \sum_{j=1}^{n} E_{ij}(t) \times \int_{-\eta_{ij}}^{0} k_{ij}(s)\, y_j^{\gamma_{ij}}(t+s)\, ds - \sum_{j=1}^{n} F_{ij}(t) \times \int_{-\theta_{ij}}^{0} K_{ij}(\xi)\, y_i^{\delta_{ij}}(t+\xi)\, y_j^{\sigma_{ij}}(t+\xi)\, d\xi\Bigg],$$
$$i = 1, 2, \ldots, n, \tag{12}$$

with initial conditions

$$y_i(\zeta) = \varphi_i(\zeta), \quad \zeta \in [-\tau, 0], \quad \varphi_i(0) > 0,$$
$$\varphi_i \in C([-\tau, 0), [0, +\infty)), \quad i = 1, 2, \ldots, n, \tag{13}$$

where

$$B_i(t) = b_i(t)\prod_{0<t_k<t}(1 + \theta_{ik}),$$

$$C_{ij}(t) = c_{ij}(t)\prod_{0<t_k<t-\rho_{ij}(t)}(1 + \theta_{ik})^{\alpha_{ij}},$$

$$D_{ij}(t) = d_{ij}(t) \prod_{0<t_k<t-\tau_{ij}(t)} (1+\theta_{ik})^{\beta_{ij}},$$

$$E_{ij}(t) = e_{ij}(t) \prod_{0<t_k<t} (1+\theta_{ik})^{\gamma_{ij}},$$

$$F_{ij}(t) = f_{ij}(t) \prod_{0<t_k<t} (1+\theta_{ik})^{\delta_{ij}+\sigma_{ij}}, \quad i=j=1,2,\ldots,n. \tag{14}$$

By a solution $y(t) = (y_1(t),\ldots,y_n(t))^T$ of the system (12) and (13), it means an absolutely continuous function $y(t) = (y_1(t),\ldots,y_n(t))^T$ defined on $[-\tau,0]$ that satisfies (12) and (13).

The following lemma will be used in the proofs of our results. The proof of Lemma 2 is similar to that of Theorem 1 in [24].

Lemma 2. *Suppose that* (H_1)–(H_4) *hold. Then*

(i) *if* $y_i(t)$ $(i=1,2,\ldots,n)$ *is a solution of* (12) *and* (13) *on* $[-\tau,+\infty)$, *then* $x_i(t) = \prod_{0<t_k<t}\Delta_{ik} y_i(t)$ $(i=1,2,\ldots,n)$ *is a solution of* (9) *and* (10) *on* $[-\tau,+\infty)$;

(ii) *if* $x_i(t)$ $(i=1,2,\ldots,n)$ *is a solution of* (9) *and* (10) *on* $[-\tau,+\infty)$, *then* $y_i(t) = \prod_{0<t_k<t}\Delta_{ik}^{-1} x_i(t)$ $(i=1,2,\ldots,n)$ *is a solution of* (12) *and* (13) *on* $[-\tau,+\infty)$.

Proof. (i) It is easy to see that $x_i(t) = \prod_{0<t_k<t}\Delta_{ik} y_i(t)$ $(i=1,2,\ldots,n)$ is absolutely continuous on every interval $(t_k,t_{k+1}]$, $t\neq t_k$, $k=1,2,\ldots$,

$$\begin{aligned}
x_i'(t) &- x_i(t)\Bigg[a_i(t) - b_i(t)x_i(t) - \sum_{j=1}^n c_{ij}(t) x_j^{\alpha_{ij}}(t-\rho_{ij}(t)) \\
&- \sum_{j=1}^n d_{ij}(t) x_j^{\beta_{ij}}(t-\tau_{ij}(t)) - \sum_{j=1}^n e_{ij}(t) \\
&\times \int_{-\eta_{ij}}^0 k_{ij}(v) x_j^{\gamma_{ij}}(t+v)\,dv - \sum_{j=1}^n f_{ij}(t) \\
&\times \int_{-\xi_{ij}}^0 K_{ij}(v) x_i^{\delta_{ij}}(t+v) x_j^{\sigma_{ij}} \\
&\times (t+v)\,dv\Bigg] \\
&= \prod_{0<t_k<t}\Delta_{ik} y_i'(t) - \prod_{0<t_k<t}\Delta_{ik} y_i(t) \\
&\times \Bigg[a_i(t) - b_i(t)\prod_{0<t_k<t}\Delta_{ik} y_i(t) \\
&- \sum_{j=1}^n c_{ij}(t) \prod_{0<t_k<t-\rho_{ij}(t)} \Delta_{ik}^{\alpha_{ij}} y_j^{\alpha_{ij}}(t-\rho_{ij}(t)) \\
&- \sum_{j=1}^n d_{ij}(t) \prod_{0<t_k<t-\tau_{ij}(t)} \Delta_{ik}^{\beta_{ij}} y_j^{\beta_{ij}}(t-\tau_{ij}(t)) \\
&- \sum_{j=1}^n e_{ij}(t) \prod_{0<t_k<t} \Delta_{ik}^{\gamma_{ij}} \int_{-\eta_{ij}}^0 k_{ij}(v) y_j^{\gamma_{ij}}(t+v)\,dv \\
&- \sum_{j=1}^n f_{ij}(t) \prod_{0<t_k<t} \Delta_{ik}^{\delta_{ij}+\sigma_{ij}} \\
&\times \int_{-\xi_{ij}}^0 K_{ij}(v) y_i^{\delta_{ij}}(t+v) y_j^{\sigma_{ij}}(t+v)\,dv\Bigg] \\
&= \prod_{0<t_k<t}\Delta_{ik}\Bigg\{y_i'(t) - y_i(t) \\
&\times \Bigg[a_i(t) - B_i(t) y_i(t) \\
&- \sum_{j=1}^n C_{ij}(t) y_i^{\alpha_{ij}}(t-\rho_{ij}(t)) \\
&- \sum_{j=1}^n D_{ij}(t) y_j^{\beta_{ij}}(t-\tau_{ij}(t)) - \sum_{j=1}^n E_{ij}(t) \\
&\times \int_{-\eta_{ij}}^0 k_{ij}(u) y_j^{\gamma_{ij}}(t+u)\,du - \sum_{j=1}^n F_{ij}(t) \\
&\times \int_{-\xi_{ij}}^0 K_{ij}(v) y_i^{\delta_{ij}}(t+v) y_j^{\sigma_{ij}} \\
&\times (t+v)\,dv\Bigg]\Bigg\} = 0.
\end{aligned} \tag{15}$$

On the other hand, for any $t=t_k$, $k=1,2,\ldots$,

$$\begin{aligned}
x_i(t_k^+) &= \lim_{t\to t_k^+} \prod_{0<t_j<t} \Delta_{ik} y_i(t) = \prod_{0<t_j\le t_k} \Delta_{ik} y_i(t_k), \\
x_i(t_k) &= \prod_{0<t_j<t_k} \Delta_{ik} y_i(t_k), \quad i=1,2,\ldots,n;
\end{aligned} \tag{16}$$

thus,

$$x_i(t_k^+) = \Delta_{ik} x_i(t_k), \quad i=1,2,\ldots,n,\ k=1,2,\ldots. \tag{17}$$

It follows from (15)–(17) that $x_i(t)$ $(i=1,2,\ldots,n)$ is a solution of the system (9) and (10). Similarly, if $y_i(t)$ $(i=1,2,\ldots,n)$ is a solution of the system (12) and (13), we can prove that $x_i(t)$ $(i=1,2,\ldots,n)$ is a solution of the system (9) and (10).

(ii) Since $x_i(t) = \prod_{0<t_k<t}\Delta_{ik} y_i(t)$ is absolutely continuous on every interval $(t_k,t_{k+1}]$, $t\neq t_k$, $k=1,2,\ldots$, and in view of (16), it follows that for any $k=1,2,\ldots$,

$$\begin{aligned}
y_i(t_k^+) &= \prod_{0<t_j\le t_k} \Delta_{ik}^{-1} x_i(t_k^+) = \prod_{0<t_j<t_k} \Delta_{ik}^{-1} x_i(t_k) = y_i(t_k), \\
y_i(t_k^-) &= \prod_{0<t_j<t_k} \Delta_{ik}^{-1} x_i(t_k^-) = \prod_{0<t_j\le t_k^-} \Delta_{ik}^{-1} x_i(t_k^-) = y_i(t_k),
\end{aligned} \tag{18}$$

which implies that $y_i(t)$ is continuous on $[-\tau, +\infty)$. It is easy to prove that $y_i(t)$ is absolutely continuous on $[-\tau, +\infty)$. Similar to the proof of (i), we can check that $y_i(t) = \prod_{0<t_k<t} \Delta_{ik}^{-1} x_i(t)$ $(i = 1, 2, \ldots, n)$ are solutions of (8) on $[-\tau, +\infty)$. Similarly, if $x_i(t)$ $(i = 1, 2, \ldots, n)$ is a solution of the system (9) and (10), we can prove that $y_i(t)$ $(i = 1, 2, \ldots, n)$ is a solution of the system (12) and (13). The proof of Lemma 2 is completed. □

In the following section, we only discuss the existence of a periodic solution for the system (12) and (13).

The paper is organized as follows. In the next section, we give some definitions and lemmas. In Section 3, we derive a necessary and sufficient condition ensuring at least one positive periodic solution of the system, by using the Krasnoselskii fixed-point theorem in the cone of Banach space. In Section 4, as applications, we consider some particular cases of the system which have been investigated extensively in the references mentioned previously.

2. Preliminaries

We will first make some preparations and list a few preliminary results. Let $X = \{y = (y_1(t), y_2(t), \ldots, y_n(t))^T \in C(R, R^n) \mid y_i(t + \omega) = y_i(t)\}$ with the norm $\|y\| = \sum_{i=1}^n |y_i|_0$, $|y_i|_0 = \sup_{t\in[0,\omega]} |y_i(t)|$. It is easy to verify that $(X, \|\cdot\|)$ is a Banach space.

We define an operator $A : X \to X$ as follows:

$$(Ay)(t) = ((Ay)_1(t), (Ay)_2(t), \ldots, (Ay)_n(t))^T, \tag{19}$$

where

$$\begin{aligned}(Ay)_i(t) &= \int_t^{t+\omega} \Bigg\{ G_i(t,v)\, y_i(v) \\ &\quad \times \Bigg[B_i(v)\, y_i(v) \\ &\quad + \sum_{j=1}^n C_{ij}(v)\, y_i^{\alpha_{ij}}\left(v - \rho_{ij}(v)\right) \\ &\quad + \sum_{j=1}^n D_{ij}(v)\, y_j^{\beta_{ij}}\left(v - \tau_{ij}(v)\right) + \sum_{j=1}^n E_{ij}(v) \\ &\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s)\, y_j^{\gamma_{ij}}(v+s)\, ds + \sum_{j=1}^n F_{ij}(v) \\ &\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi)\, y_i^{\delta_{ij}}(v+\xi)\, y_j^{\sigma_{ij}} \\ &\quad \times (v+\xi)\, d\xi \Bigg] \Bigg\} dv, \quad i = 1, 2, \ldots, n,\end{aligned}$$

$$\begin{aligned} G_i(t,v) &= \frac{e^{-\int_t^v a_i(u) du}}{1 - e^{-\int_0^\omega a_i(u) du}} = \frac{e^{\int_v^{t+\omega} a_i(u) du}}{e^{\int_0^\omega a_i(u) du} - 1}, \\ G(t,v) &= \operatorname{diag}\left[G_1(t,v), G_2(t,v), \ldots, G_n(t,v)\right], \\ &\qquad t \le v \le t+\omega. \end{aligned} \tag{20}$$

It is clear that $G_i(t+\omega, v+\omega) = G_i(t,v)$, $\partial G_i(t,v)/\partial t = a_i(t) G_i(t,v)$, $G_i(t,t) - G_i(t, t+\omega) = 1$.

In view of (H_1), for $1 \le i \le n$, we define

$$\begin{aligned} \alpha_i &:= \min_{0\le t\le s\le\omega} \left|G_i(t,v)\right| = \frac{1}{e^{\int_0^\omega a_i(u)du} - 1}, \\ \beta_i &:= \max_{0\le t\le s\le\omega} \left|G_i(t,v)\right| = \frac{e^{\int_0^\omega a_i(u)du}}{e^{\int_0^\omega a_i(u)du} - 1}, \\ \alpha &= \min_{1\le i\le n} \alpha_i, \qquad \beta = \max_{1\le i\le n} \beta_i, \qquad \sigma = \frac{\alpha}{\beta} \in (0,1). \end{aligned} \tag{21}$$

Define P as a cone in X by

$$\begin{aligned} P = \Big\{ y = (y_1(t), y_2(t), \ldots, y_n(t))^T \in X : y_i(t) \\ \ge \sigma |y_i|_0, t \in [0,\omega] \Big\}. \end{aligned} \tag{22}$$

We easily verify that P is a cone in X. For convenience of expressions, we define an operator $T : P \to X$ by

$$\begin{aligned}(Ty)_i(t) &= y_i(t) \Bigg[B_i(t)\, y_i(t) + \sum_{j=1}^n C_{ij}(t)\, y_i^{\alpha_{ij}}\left(t - \rho_{ij}(t)\right) \\ &\quad + \sum_{j=1}^n D_{ij}(t)\, y_j^{\beta_{ij}}\left(t - \tau_{ij}(t)\right) + \sum_{j=1}^n E_{ij}(t) \\ &\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s)\, y_j^{\gamma_{ij}}(t+s)\, ds + \sum_{j=1}^n F_{ij}(t) \\ &\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi)\, y_i^{\delta_{ij}}(t+\xi)\, y_j^{\sigma_{ij}}(t+\xi)\, d\xi \Bigg], \\ &\qquad i = 1, 2, \ldots, n. \end{aligned} \tag{23}$$

The proof of the main result in this paper is based on an application of the Krasnoselskii fixed-point theorem in cones. Firstly, we need to introduce some definitions and lemmas.

Definition 3. Let X be a real Banach space and P be a closed, nonempty subset of X. P is said to be a cone if

(1) $\alpha x + \beta y \in P$ for all $x, y \in P$, and $\alpha, \beta > 0$,

(2) $x, -x \in P$ imply $x = 0$.

Lemma 4 (see [37–39]). *Let P be a cone in a real Banach space X. Assume that Ω_{r_1} and Ω_{r_2} are open subsets of X with $0 \in \Omega_{r_1} \subset \overline{\Omega_{r_1}} \subset \Omega_{r_2}$, where $\Omega_{r_i} = \{x \in X : \|x\| < r_i\}$, $i = 1, 2$.*

Let $T : P \cap (\Omega_{r_2} \setminus \overline{\Omega_{r_1}}) \to P$ be a continuous and completely continuous operator satisfying

(1) $\|Tx\| \le \|x\|$, *for any* $x \in P \cap \partial\Omega_{r_1}$;

(2) *The fact that there exists* $\phi \in P \setminus \{0\}$ *such that* $y \ne Ty + \lambda y$, *for any* $y \in P \cap \partial\Omega_{r_2}$ *and* $\lambda > 0$.

Then, T has a fixed point in $P \cap (\overline{\Omega_{r_2}} \setminus \Omega_{r_1})$. The same conclusion remains valid if (1) *holds for for any $x \in P \cap \partial\Omega_{r_2}$ and* (2) *holds for any $y \in P \cap \partial\Omega_{r_1}$ and $\lambda > 0$.*

Lemma 5. *Assume that* (H_1)–(H_4) *hold. Then the solutions of the system* (12) *and* (13) *are defined on* $[-\tau, \infty)$ *and are positive.*

Proof. By Lemma 2, we only need to prove that the solutions $y_i(t)$ $(i = 1, 2, \dots, n)$ of (12) and (13) are defined on $[-\tau, \infty)$ and are positive on $[0, \infty)$. From (12), we have that for any $\varphi_i \in C([-\tau, 0), R^+)$ $(i = 1, 2, 3, \dots, n)$ and $t > 0$

$$\begin{aligned} y_i(t) &= \varphi_i(0) \exp\Bigg\{\int_0^t \Bigg[a_i(v) - B_i(v) y_i(v) \\ &\quad - \sum_{j=1}^n C_{ij}(v) y_i^{\alpha_{ij}}\left(v - \rho_{ij}(v)\right) \\ &\quad - \sum_{j=1}^n D_{ij}(v) y_j^{\beta_{ij}}\left(v - \tau_{ij}(v)\right) - \sum_{j=1}^n E_{ij}(v) \\ &\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s) y_j^{\gamma_{ij}}(v+s)\,ds - \sum_{j=1}^n F_{ij}(v) \\ &\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi) y_i^{\delta_{ij}}(v+\xi) y_j^{\sigma_{ij}} \\ &\quad \times (v+\xi)\,d\xi\Bigg]dv\Bigg\}, \\ &\qquad i = 1, 2, \dots, n. \end{aligned} \tag{24}$$

Therefore, $y_i(t)$ $(i = 1, 2, \dots, n)$ are defined on $[-\tau, \infty)$ and are positive on $[0, \infty)$. The proof of Lemma 5 is complete. □

Lemma 6. *Assume that* (H_1)–(H_4) *hold. Then* $A : P \to P$ *is well defined.*

Proof. In view of the definitions of P and T, for any $y \in P$, we have

$$\begin{aligned} (Ay)_i(t) &= \int_t^{t+\omega} G_i(t, v)(Ty)_i(v)\,dv, \\ (Ay)_i(t+\omega) &= \int_{t+\omega}^{t+2\omega} G_i(t, v+\omega)(Ty)_i(v+\omega)\,dv \\ &= \int_t^{t+\omega} G_i(t, v)(Ty)_i(v)\,dv = (Ay)_i(t). \end{aligned} \tag{25}$$

Therefore, $(Ay) \in X$. Furthermore, for any $y \in P$, it follows from (20) that

$$\left|(Ay)_i\right|_0 \le \beta_i \int_0^\omega (Ty)_i(v)\,dv. \tag{26}$$

On the other hand, for any $y \in P$, we obtain

$$\begin{aligned} (Ay)_i(t) &\ge \alpha_i \int_0^\omega (Ty)_i(v)\,dv \\ &\ge \frac{\alpha_i}{\beta_i}\left|(Ay)_i\right|_0 \ge \sigma\left|(Ay)_i\right|_0. \end{aligned} \tag{27}$$

Therefore, $Ay \in P$. The proof of Lemma 6 is complete. □

Lemma 7. *The operator $A : P \to P$ is continuous and completely continuous.*

Proof. By using a standard argument one can show that ψ is continuous on P. Now, we show that A is completely continuous. Let r be any positive constant and $S_r = \{y \in X : |y_i|_0 \le r\}$ a bounded set. For any $y \in S_r$, by (20), we have

$$\begin{aligned} &\left|(Ay)_i\right|_0 \\ &\quad \le \beta_i \int_0^\omega (Ty)_i(v)\,dv \\ &\quad = \beta_i \int_0^\omega y_i(v) \Bigg[B_i(v) y_i(v) \\ &\qquad + \sum_{j=1}^n C_{ij}(v) y_i^{\alpha_{ij}}\left(v - \rho_{ij}(v)\right) \\ &\qquad + \sum_{j=1}^n D_{ij}(v) y_j^{\beta_{ij}}\left(v - \tau_{ij}(v)\right) + \sum_{j=1}^n E_{ij}(v) \\ &\qquad \times \int_{-\eta_{ij}}^0 k_{ij}(s) y_j^{\gamma_{ij}}(v+s)\,ds + \sum_{j=1}^n F_{ij}(v) \\ &\qquad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi) y_i^{\delta_{ij}}(v+\xi) y_j^{\sigma_{ij}} \\ &\qquad \times (v+\xi)\,d\xi\Bigg]dv \\ &\quad \le \omega\beta_i r\Bigg[\overline{B}_i r + \sum_{j=1}^n \Big(\overline{C}_{ij} r^{\alpha_{ij}} + \overline{D}_{ij} r^{\beta_{ij}} + \overline{E}_{ij} r^{\gamma_{ij}} \\ &\qquad + \overline{F}_{ij} r^{\delta_{ij}+\sigma_{ij}}\Big)\Bigg] := R_i. \end{aligned} \tag{28}$$

Therefore, for any $y \in S_r$, we obtain

$$\|Ay\| = \sum_{i=1}^n \left|(Ay)_i\right|_0 \le \sum_{i=1}^n R_i := R, \tag{29}$$

which implies that $A(S_r)$ is a uniformly bounded set. On the other hand, in view of the definitions of A and T, we have

$$\begin{aligned}\frac{d\left[(Ay)_i(t)\right]}{dt} &= G_i(t,t+\omega)(Ty)_i(t+\omega) - G_i(t,t)(Ty)_i(t) \\ &\quad + \int_t^{t+\omega} \frac{dG_i(t,v)}{dt}(Ty)_i(v)\,dv \\ &= -(Ty)_i(t) + a_i(t)\int_t^{t+\omega} G_i(t,v)(Ty)_i(v)\,dv \\ &= a_i(t)(Ay)_i(t) - (Ty)_i(t).\end{aligned} \tag{30}$$

Again, from (20), we obtain

$$\begin{aligned}\left|\frac{d\left[(Ay)_i(t)\right]}{dt}\right| &\le a_i^M R_i + r \\ &\quad \times \left[\overline{B}_i r + \sum_{j=1}^n \left(\overline{C}_{ij} r^{\alpha_{ij}} + \overline{D}_{ij} r^{\beta_{ij}} \right.\right. \\ &\quad \left.\left. + \overline{E}_{ij} r^{\gamma_{ij}} + \overline{F}_{ij} r^{\delta_{ij}+\sigma_{ij}}\right)\right] \\ &:= \overline{R_i} \le M := \max_{i\in[1,n]}\left\{\overline{R_i}\right\},\end{aligned} \tag{31}$$

which implies that $d[(Ay)_i(t)]/dt$, for any $y \in S_r$, is also uniformly bounded. Hence, $A(S_r) \subset X$ is a family of uniformly bounded and equi-continuous functions. By the well-known Ascoli-Arzela theorem, we know that the operator ψ is completely continuous. The proof of Lemma 7 is complete. □

Lemma 8. *Assume that (H_1)–(H_4) hold. The existence of positive ω-periodic solution of the system* (12) *and* (13) *is equivalent to that of nonzero fixed point of A in P.*

Proof. Assume that $y = (y_1(t), y_2(t), \dots, y_n(t))^T \in X$ is a periodic solution of (12) and (13). Then, we have

$$\begin{aligned}&\left[y_i(t)\,e^{-\int_0^t a_i(u)du}\right] \\ &= -e^{-\int_0^t a_i(u)du} y_i(t) \\ &\quad \times \left[B_i(t)\,y_i(t) + \sum_{j=1}^n C_{ij}(t)\,y_i^{\alpha_{ij}}\left(t-\rho_{ij}(t)\right)\right. \\ &\quad + \sum_{j=1}^n D_{ij}(t)\,y_j^{\beta_{ij}}\left(t-\tau_{ij}(t)\right) + \sum_{j=1}^n E_{ij}(t) \\ &\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s)\,y_j^{\gamma_{ij}}(t+s)\,ds + \sum_{j=1}^n F_{ij}(t) \\ &\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi)\,y_i^{\delta_{ij}}(t+\xi)\,y_j^{\sigma_{ij}} \\ &\quad \left.\times (t+\xi)\,d\xi\right], \quad i = 1,2,\dots,n.\end{aligned} \tag{32}$$

Integrating the previous equation over $[t, t+\omega]$, we can have

$$\begin{aligned}&\left[y_i(v)\,e^{-\int_0^v a_i(u)du}\right]\Big|_t^{t+\omega} \\ &= -\int_t^{t+\omega}\left\{e^{-\int_0^v a_i(u)du} y_i(v)\right. \\ &\quad \times \left[B_i(v)\,y_i(v) + \sum_{j=1}^n C_{ij}(v)\,y_i^{\alpha_{ij}}\left(v-\rho_{ij}(v)\right)\right. \\ &\quad + \sum_{j=1}^n D_{ij}(v)\,y_j^{\beta_{ij}}\left(v-\tau_{ij}(v)\right) + \sum_{j=1}^n E_{ij}(v) \\ &\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s)\,y_j^{\gamma_{ij}}(v+s)\,ds + \sum_{j=1}^n F_{ij}(v) \\ &\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi)\,y_i^{\delta_{ij}}(v+\xi)\,y_j^{\sigma_{ij}} \\ &\quad \left.\left.\times (v+\xi)\,d\xi\right]\right\} dv, \\ &\quad i = 1,2,\dots,n.\end{aligned} \tag{33}$$

Therefore, we have

$$\begin{aligned}&y_i(t)\,e^{-\int_0^t a_i(u)du}\left[1 - e^{-\int_0^\omega a_i(u)du}\right] \\ &= \int_t^{t+\omega}\left\{e^{-\int_0^v a_i(u)du} y_i(v)\right. \\ &\quad \times \left[B_i(v)\,y_i(v) + \sum_{j=1}^n C_{ij}(v)\,y_i^{\alpha_{ij}}\left(v-\rho_{ij}(v)\right)\right. \\ &\quad + \sum_{j=1}^n D_{ij}(v)\,y_j^{\beta_{ij}}\left(v-\tau_{ij}(v)\right) + \sum_{j=1}^n E_{ij}(v) \\ &\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s)\,y_j^{\gamma_{ij}}(v+s)\,ds + \sum_{j=1}^n F_{ij}(v) \\ &\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi)\,y_i^{\delta_{ij}}(v+\xi)\,y_j^{\sigma_{ij}}\end{aligned}$$

$$\times (v+\xi)\, d\xi \Bigg]\Bigg\}\, dv,$$

$$i = 1, 2, \ldots, n, \tag{34}$$

which can be transformed into

$$\begin{aligned}
y_i(t) &= \int_t^{t+\omega} \Bigg\{ \frac{e^{-\int_0^v a_i(u)du}}{e^{-\int_0^t a_i(u)du}\left[1-e^{-\int_0^\omega a_i(u)du}\right]} y_i(v) \\
&\quad \times \Bigg[B_i(v)\, y_i(v) + \sum_{j=1}^n C_{ij}(v)\, y_i^{\alpha_{ij}}\left(v-\rho_{ij}(v)\right) \\
&\quad + \sum_{j=1}^n D_{ij}(v)\, y_j^{\beta_{ij}}\left(v-\tau_{ij}(v)\right) + \sum_{j=1}^n E_{ij}(v) \\
&\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s)\, y_j^{\gamma_{ij}}(v+s)\, ds + \sum_{j=1}^n F_{ij}(v) \\
&\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi)\, y_i^{\delta_{ij}}(v+\xi)\, y_j^{\sigma_{ij}} \\
&\quad \times (v+\xi)\, d\xi \Bigg]\Bigg\}\, dv \\
&= \int_t^{t+\omega} \Bigg\{ G_i(t,v)\, y_i(v) \\
&\quad \times \Bigg[B_i(v)\, y_i(v) + \sum_{j=1}^n C_{ij}(v)\, y_i^{\alpha_{ij}}\left(v-\rho_{ij}(v)\right) \\
&\quad + \sum_{j=1}^n D_{ij}(v)\, y_j^{\beta_{ij}}\left(v-\tau_{ij}(v)\right) + \sum_{j=1}^n E_{ij}(v) \\
&\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s)\, y_j^{\gamma_{ij}}(v+s)\, ds + \sum_{j=1}^n F_{ij}(v) \\
&\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi)\, y_i^{\delta_{ij}}(v+\xi)\, y_j^{\sigma_{ij}} \\
&\quad \times (v+\xi)\, d\xi \Bigg]\Bigg\}\, dv \\
&= \int_t^{t+\omega} G_i(t,v)\left(Ty\right)_i(v)\, dv, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{35}$$

Thus, $y_i(t)$ $(i = 1, 2, \ldots, n)$ is a periodic solution for system (12) and (13).

If $y = (y_1(t), y_2(t), \ldots, y_n(t))^T \in X$, and $Ay = ((Ay)_1, (Ay)_2, \ldots, (Ay)_n)^T = y$ with $y \neq 0$, then for any $t = t_k$ derivative the two sides of (20) about t,

$$\begin{aligned}
&\frac{d\left[(Ay)_i(t)\right]}{dt} \\
&\quad = G_i(t, t+\omega)\left(Ty\right)_i(t+\omega) - G_i(t,t)\left(Ty\right)_i(t) \\
&\qquad + \int_t^{t+\omega} \frac{dG_i(t,v)}{dt}\left(Ty\right)_i(v)\, dv \\
&\quad = -\left(Ty\right)_i(t) + a_i(t)\int_t^{t+\omega} G_i(t,v)\left(Ty\right)_i(v)\, dv \\
&\quad = a_i(t)\, y_i(t) - \left(Ty\right)_i(t) \\
&\quad = y_i(t)\Bigg[a_i(t) - B_i(t)\, y_i(t) \\
&\qquad - \sum_{j=1}^n C_{ij}(t)\, y_i^{\alpha_{ij}}\left(t-\rho_{ij}(t)\right) \\
&\qquad - \sum_{j=1}^n D_{ij}(t)\, y_j^{\beta_{ij}}\left(t-\tau_{ij}(t)\right) - \sum_{j=1}^n E_{ij}(t) \\
&\qquad \times \int_{-\eta_{ij}}^0 k_{ij}(s)\, y_j^{\gamma_{ij}}(t+s)\, ds - \sum_{j=1}^n F_{ij}(t) \\
&\qquad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi)\, y_i^{\delta_{ij}}(t+\xi)\, y_j^{\sigma_{ij}}(t+\xi)\, d\xi \Bigg] \\
&\quad = \frac{dy_i(t)}{dt}.
\end{aligned} \tag{36}$$

Hence, $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in X$ is a positive ω-periodic solution of (12) and (13). Thus we complete the proof of Lemma 8. □

3. Existence of Periodic Solution of the System

Now, we are at the position to study the existence of positive periodic solutions of system (9) and (10). We mainly apply the Krasnoselskii fixed-point theorem in the cone of Banach space under some conditions to prove the main Theorem 9.

Theorem 9. *Assume* (H_1)*–*(H_4)*. System* (9) *and* (10) *has at least one positive* ω*-periodic solution if and only if the condition*

$$l_0 = \min_{1\le i\le n}\left\{ \overline{B}_i + \sum_{j=1}^n\left(\overline{C}_{ij} + \overline{D}_{ij} + \overline{E}_{ij} + \overline{F}_{ij}\right)\right\} > 0 \tag{37}$$

holds.

Proof (sufficiency). Let

$$L_0 = \max_{i\in[1,n]}\left\{ \overline{b}_i + \sum_{j=1}^n\left(\overline{C}_{ij} + \overline{D}_{ij} + \overline{E}_{ij} + \overline{F}_{ij}\right)\right\}, \tag{38}$$

by condition (37), we know that $L_0 \geq l_0 > 0$. Choose a constant $L \geq L_0$ such that $1/\omega\beta_i L < 1$. Let $r = 1/\omega\beta_i L$ and

$$\Omega_r = \left\{ y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in X : |y_i|_0 < r, i = 1, 2, \ldots, n \right\}. \tag{39}$$

For any $y = y(t) \in P \cap \partial\Omega_r$, $\sigma|y_i|_0 \leq y_i(t) \leq |y_i|_0$, from (20), we obtain

$$\begin{aligned}
|(Ay)_i|_0 &\leq \beta_i \int_0^\omega (Ty)_i(t)\,dt \\
&= \beta_i \int_0^\omega y_i(t) \Bigg[B_i(t) y_i(t) + \sum_{j=1}^n C_{ij}(t) y_i^{\alpha_{ij}}(t - \rho_{ij}(t)) \\
&\quad + \sum_{j=1}^n D_{ij}(t) y_j^{\beta_{ij}}(t - \tau_{ij}(t)) + \sum_{j=1}^n E_{ij}(t) \\
&\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s) y_j^{\gamma_{ij}}(t+s)\,ds + \sum_{j=1}^n F_{ij}(t) \\
&\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi) y_i^{\delta_{ij}}(t+\xi) y_j^{\sigma_{ij}}(t+\xi)\,d\xi \Bigg] dt \\
&\leq \omega\beta_i |y_i|_0 \Bigg[\overline{B}_i r + \sum_{j=1}^n \Big(\overline{C}_{ij} r^{\alpha_{ij}} + \overline{D}_{ij} r^{\beta_{ij}} \\
&\quad + \overline{E}_{ij} r^{\gamma_{ij}} + \overline{F}_{ij} r^{\delta_{ij}+\sigma_{ij}} \Big) \Bigg] \\
&\leq \omega\beta_i |y_i|_0 \left[\overline{B}_i + \sum_{j=1}^n \left(\overline{C}_{ij} + \overline{D}_{ij} + \overline{E}_{ij} + \overline{F}_{ij} \right) \right] r \\
&\leq \omega\beta_i |y_i|_0 L_0 r \leq |y_i|_0.
\end{aligned} \tag{40}$$

Hence, for any $y = y(t) \in P \cap \partial\Omega_r$, $\sigma|y_i|_0 \leq y_i(t) \leq |y_i|_0$, we have

$$\|Ay\| = \sum_{j=1}^n |(Ay)_i|_0 \leq \sum_{j=1}^n |y_i|_0 = \|y\|, \tag{41}$$

which implies that condition (1) in Lemma 4 is satisfied.

On the other hand, we choose $0 < l \leq l_0$ such that $1/\omega\sigma\alpha_i l > 1$. Let $R = 1/\omega\sigma\alpha_i l > 1$ and suppose $u = (u_1, u_2, \ldots, u_n)^T \in P/\{0\}$. We show that, for any $y = y(t) \in P \cap \partial\Omega_R$ and $\lambda > 0$, $y \neq \psi y + \lambda u$. Otherwise, there exist $y_0 = y_0(t) \in P \cap \partial\Omega_R$ and $\lambda_0 > 0$, such that $y_0 = \psi y_0 + \lambda_0 u$. Let $u_{i_0} \neq 0$ $(1 \leq i_0 \leq n)$; since $y_{i_0}(t) \geq \sigma|y_{i_0}|_0$, it follows that

$$\begin{aligned}
y_{i_0} &= (Ay)_{i_0}(t) + \lambda_0 u_{i_0} \\
&= \int_t^{t+\omega} G_{i_0}(t,v) (Ty)_{i_0}(v)\,dv + \lambda_0 u_{i_0} \\
&\geq \sigma\alpha_i |y_{i_0}|_0 \int_0^\omega \Bigg[B_i(t) y_i(t) \\
&\quad + \sum_{j=1}^n C_{ij}(t) y_i^{\alpha_{ij}}(t - \rho_{ij}(t)) \\
&\quad + \sum_{j=1}^n D_{ij}(t) y_j^{\beta_{ij}}(t - \tau_{ij}(t)) + \sum_{j=1}^n E_{ij}(t) \\
&\quad \times \int_{-\eta_{ij}}^0 k_{ij}(s) y_j^{\gamma_{ij}}(t+s)\,ds + \sum_{j=1}^n F_{ij}(t) \\
&\quad \times \int_{-\theta_{ij}}^0 K_{ij}(\xi) y_i^{\delta_{ij}}(t+\xi) y_j^{\sigma_{ij}} \\
&\quad \times (t+\xi)\,d\xi \Bigg] dt + \lambda_0 u_{i_0} \\
&\geq \omega\sigma\alpha_i |y_{i_0}|_0 \Bigg[\overline{B}_i R + \sum_{j=1}^n \Big(\overline{C}_{ij} R^{\alpha_{ij}} + \overline{D}_{ij} R^{\beta_{ij}} \\
&\quad + \overline{E}_{ij} R^{\gamma_{ij}} + \overline{F}_{ij} R^{\delta_{ij}+\sigma_{ij}} \Big) \Bigg] \\
&\geq \omega\sigma\alpha_i |y_i|_0 \left[\overline{B}_i + \sum_{j=1}^n \left(\overline{C}_{ij} + \overline{D}_{ij} + \overline{E}_{ij} + \overline{F}_{ij} \right) \right] R + \lambda_0 u_{i_0} \\
&\geq \omega\sigma\alpha_i l_0 R |y_i|_0 + \lambda_0 u_{i_0} \geq |y_{i_0}|_0 + \lambda_0 u_{i_0} > |y_{i_0}|_0,
\end{aligned} \tag{42}$$

which is a contradiction. This proves that condition (2) in Lemma 4 is also satisfied. By Lemmas 4 and 8, system (12) and (13) has at least one positive omega-periodic solution. From Lemma 2, system (9) and (10) has at least one positive ω-periodic solution.

(Necessity). Suppose that (37) does not hold. Then, there exists at least an i_0 $(1 \leq i_0 \leq n)$ such that

$$\overline{B}_{i_0} = 0, \qquad \overline{C}_{i_0 j} = \overline{D}_{i_0 j} = \overline{E}_{i_0 j} = \overline{F}_{i_0 j} = 0, \quad j \in [1, n]. \tag{43}$$

If system (12) and (13) has a positive ω-periodic solution $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T$, then we have

$$\frac{dy_{i_0}(t)}{dt} = a_{i_0}(t) y_{i_0}(t). \tag{44}$$

Integrating the previous equation over $[t, t+\omega]$, we can have

$$0 = \ln \frac{y_{i_0}(t+\omega)}{y_{i_0}(t)} = \int_0^\omega a_{i_0}(t)\,dt > 0, \tag{45}$$

which is a contradiction. The proof of Theorem 9 is complete. □

4. Applications

In this section, we apply the results obtained in the previous section to some n-species competition systems which are mentioned in Section 1. If we consider the environmental or biological factors, the assumption of the periodic oscillation of the parameters and impulse functions seems realistic and reasonable in view of any seasonal phenomena which they might be subject to, for example, mating habits, availability of food, weather conditions, and so forth.

Application 1. We consider the following three classes of periodic n-species competition systems with impulses:

$$\frac{dx_i(t)}{dt} = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)\,x_j(t)\right], \quad t \in R, \quad t \neq t_k,\ i = 1,2,\ldots,n, \qquad (46)$$
$$\Delta x_i(t_k) = h_{ik}x_i(t_k), \quad k \in Z_+,\ i = 1,2,\ldots,n;$$

$$\frac{dx_i(t)}{dt} = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)\,x_j^{\theta_{ij}}(t)\right], \quad t \in R, \quad t \neq t_k,\ i = 1,2,\ldots,n, \qquad (47)$$
$$\Delta x_i(t_k) = h_{ik}x_i(t_k), \quad k \in Z_+,\ i = 1,2,\ldots,n;$$

$$\frac{dx_i(t)}{dt} = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)\,x_j^{\theta_{ij}}\left(t-\tau_{ij}(t)\right)\right], \quad t \in R, \quad t \neq t_k,\ i = 1,2,\ldots,n,$$
$$\Delta x_i(t_k) = h_{ik}x_i(t_k), \quad k \in Z_+,\ i = 1,2,\ldots,n, \qquad (48)$$

which are special cases of system (9), where $b_i(t), a_{ij}(t), \tau_{ij}(t)$, θ_{ij}, h_{ik} are the same as in (H_1)–(H_4). We denote

$$A_{ij}(t) = a_{ij}(t)\prod_{0<t_k<t}(1+h_{ik}),$$
$$A*_{ij}(t) = a_{ij}(t)\prod_{0<t_k<t-\tau_{ij}(t)}(1+h_{ik}), \qquad (49)$$

from Theorem 9, we have the following results.

Corollary 10. *Assume that (H_1)–(H_4) hold. The systems* (46) *and* (47) *have at least one positive ω-periodic solution if and only if the following condition*

$$\min_{1\le i\le n}\left\{\overline{A}_{ij}\right\} > 0 \qquad (50)$$

holds.

Corollary 11. *Assume that (H_1)–(H_4) hold. The system* (48) *has at least one positive ω-periodic solution if and only if the following condition*

$$\min_{1\le i\le n}\left\{\overline{A*}_{ij}\right\} > 0 \qquad (51)$$

holds.

Application 2. We consider the following n-species Gilpin-Ayala type competition systems with impulses:

$$y_i'(t) = y_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)\,y_j^{\alpha_{ij}}(t) - \sum_{j=1}^{n} b_{ij}(t)\,y_j^{\beta_{ij}}\left(t-\tau_{ij}(t)\right) - \sum_{j=1}^{n} c_{ij}(t) \times \int_{-\sigma_{ij}}^{0} k_{ij}(s)\,y_j^{\gamma_{ij}}(t+s)\,ds\right], \qquad (52)$$
$$\text{a.e, } t>0,\ t\neq t_k,\ i=1,2,\ldots,n,$$
$$y_i(t_k^+) - y_i(t_k^-) = h_{ik}y_i(t_k), \quad i=1,2,\ldots,n,\ k\in Z_+,$$

which is a special case of system (9), where $r_i(t), a_{ij}(t), b_{ij}(t)$, $c_{ij}(t), \tau_{ij}(t) \in C(R, R_+)$ are ω-periodic, $k_{ij} \in C([-\eta_{ij}, 0], R_+)$ are constants such that $\int_{-\eta_{ij}}^{0} k_{ij}(t)dt = 1$, and $\alpha_{ij} \ge 0$, $\beta_{ij} \ge 0$, $\gamma_{ij} \ge 0$, $i = 1,2,\ldots,n$. We denote

$$A_{ij}(t) = a_{ij}(t)\prod_{0<t_k<t}(1+h_{ik}),$$
$$B_{ij}(t) = b_{ij}(t)\prod_{0<t_k<t-\tau_{ij}(t)}(1+h_{ik}), \qquad (53)$$
$$C_{ij}(t) = c_{ij}(t)\prod_{0<t_k<t-\sigma_{ij}}(1+h_{ik}),$$

from Theorem 9, we have the following result.

Corollary 12. *Assume that (H_1)–(H_4) and the condition*

$$R_2 = \min_{1\le i\le n}\left\{\sum_{j=1}^{n}\left(\overline{A}_{ij} + \overline{B}_{ij} + \overline{C}_{ij}\right)\right\} > 0 \qquad (54)$$

hold, the system (52) *has at least one positive ω-periodic solution.*

Application 3. We study the following periodic nonlinear n-species competitive Lotka-Volterra models with impulses:

$$y_i'(t) = y_i(t)\left[r_i(t) - \sum_{j=1}^{N} a_{ij}(t)\,y_j^{\alpha_{ij}}(t) - \sum_{j=1}^{N} b_{ij}(t)\,y_j^{\beta_{ij}}\left(t-\tau_{ij}(t)\right) - \sum_{j=1}^{N} c_{ij}(t)\,y_i^{\gamma_{ij}}(t)\,y_j^{\delta_{ij}}(t)\right],$$
$$\text{a.e, } t>0,\ t\neq t_k,\ i=1,2,\ldots,N,$$
$$y_i(t_k^+) - y_i(t_k^-) = h_{ik}y_i(t_k), \quad i=1,2,\ldots,N,\ k\in Z_+, \qquad (55)$$

which is a special case of system (9), where $r_i(t), a_{ij}(t), b_{ij}(t), c_{ij}(t), \tau_{ij}(t) \in C(R, R_+)$ are ω-periodic, and $\alpha_{ij} \geq 0$, $\beta_{ij} \geq 0$, $\gamma_{ij} \geq 0$, $\delta_{ij} \geq 0$. We denote

$$\begin{aligned} A_{ij}(t) &= a_{ij}(t) \prod_{0<t_k<t} (1+h_{ik})^{\alpha_{ij}}, \\ B_{ij}(t) &= b_{ij}(t) \prod_{0<t_k<t-\tau_{ij}(t)} (1+h_{ik})^{\beta_{ij}}, \\ C_{ij}(t) &= c_{ij}(t) \prod_{0<t_k<t} (1+h_{ik})^{\gamma_{ij}+\delta_{ij}}, \end{aligned} \tag{56}$$

from Theorem 9, we have the following result.

Corollary 13. *Assume that* (H_1)*–*(H_4) *and the condition*

$$R_3 = \min_{1\leq i\leq N} \left\{ \sum_{j=1}^{N} \left(\overline{A}_{ij} + \overline{B}_{ij} + \overline{C}_{ij} \right) \right\} > 0 \tag{57}$$

hold, the system (55) *has at least one positive* ω*-periodic solution.*

Application 4. We study the following generalized periodic n-species Gilpin-Ayala type competition models in periodic environments with deviating arguments of the form and impulses:

$$\begin{aligned} y_i'(t) = y_i(t) \Bigg[& r_i(t) - \sum_{j=1}^{n} a_{ij}(t) y_j^{\alpha_{ij}}(t) \\ & - \sum_{j=1}^{n} b_{ij}(t) y_j^{\beta_{ij}} \left(t - \tau_{ij}(t)\right) - \sum_{j=1}^{n} c_{ij}(t) \\ & \times \int_{-\sigma_{ij}}^{0} K_{ij}(\xi) y_i^{\gamma_{ij}}(t+\xi) y_j^{\delta_{ij}} \\ & \times (t+\xi)\, d\xi \Bigg], \quad t \neq t_k, \\ \Delta y_i(t_k) = \theta_k y(t_k), & \quad i = 1, 2, \ldots, n, \ k \in Z_+, \ t = t_k, \end{aligned} \tag{58}$$

which is a special case of system (9), where $r_i(t), a_{ij}(t), b_{ij}(t), c_{ij}(t), \tau_{ij}(t) \in C(R, R_+)$ are ω-periodic,
$K_{ij} \in C([-\sigma_{ij}, 0], R_+)$ are constants such that $\int_{-\sigma_{ij}}^{0} K_{ij}(t)dt = 1$, and $\alpha_{ij} \geq 0$, $\beta_{ij} \geq 0$, $\gamma_{ij} \geq 0$, $\delta_{ij} \geq 0$, $i = 1, 2, \ldots, n$, $1 + H_{ik} > 0$. We denote

$$\begin{aligned} A_{ij}(t) &= a_{ij}(t) \prod_{0<t_k<t} (1+h_{ik})^{\alpha_{ij}}, \\ B_{ij}(t) &= b_{ij}(t) \prod_{0<t_k<t-\tau_{ij}(t)} (1+h_{ik})^{\beta_{ij}}, \\ C_{ij}(t) &= c_{ij}(t) \prod_{0<t_k<t-\sigma_{ij}(t)} (1+h_{ik})^{\gamma_{ij}+\delta_{ij}}, \end{aligned} \tag{59}$$

from Theorem 9, we have the following result.

Corollary 14. *Assume that* (H_1)*–*(H_4) *and the condition*

$$R_4 = \min_{1\leq i\leq 2} \left\{ \overline{A}_i + \sum_{i=1}^{n} \left(\overline{B}_{ij} + \overline{C}_{ij} \right) \right\} > 0 \tag{60}$$

hold, the system (58) *has at least one positive* ω*-periodic solution.*

Application 5. We study the following two-species competitive systems with impulses:

$$\begin{aligned} y_1'(t) = y_1(t) \Bigg[& r_1(t) - a_1(t) y_1(t) \\ & - \sum_{j=1}^{n} b_{1j}(t) y_1^{\alpha_{1j}} \left(t - \tau_j(t)\right) \\ & - \sum_{j=1}^{n} c_{1j}(t) y_2^{\beta_{2j}} \left(t - \rho_j(t)\right) \Bigg], \\ y_2'(t) = y_2(t) \Bigg[& r_2(t) - a_2(t) y_2(t) \\ & - \sum_{j=1}^{n} b_{2j}(t) y_2^{\alpha_{2j}} \left(t - \eta_j(t)\right) \\ & - \sum_{j=1}^{n} c_{2j}(t) y_1^{\beta_{1j}} \left(t - \sigma_j(t)\right) \Bigg], \quad t \neq t_k, \\ \Delta y_i(t_k) = h_{ik} y_i(t_k), & \quad i = 1, 2, \ k \in Z_+, \ t = t_k, \end{aligned} \tag{61}$$

which is a special case of system (9), where $r_i(t), a_{ij}(t), b_{ij}(t), c_{ij}(t), \tau_j(t), \rho_j(t), \eta_j(t), \sigma_j(t) \in C(R, R_+)$ $(i = 1, 2)$ are ω-periodic. We denote

$$\begin{aligned} A_i(t) &= a_i(t) \prod_{0<t_k<t} (1+\theta_{ik}), \\ B_{ij}(t) &= b_{ij}(t) \prod_{0<t_k<t-\tau_j(t)} (1+\theta_{ik})^{\alpha_{ij}} \quad (i = 1, 2), \\ C_{1j}(t) &= C_{1j}(t) \prod_{0<t_k<t-\rho_j(t)} (1+\theta_{ik})^{\beta_{2j}} C_{2j}(t) \\ &= c_{2j}(t) \prod_{0<t_k<t-\sigma_j(t)} (1+\theta_{ik})^{\beta_{1j}}, \end{aligned} \tag{62}$$

from Theorem 9, we have the following result.

Corollary 15. *Assume that* (H_1)*–*(H_4) *and the condition*

$$R_4 = \min_{1\leq i\leq 2} \left\{ \overline{A}_i + \sum_{i=1}^{n} \left(\overline{B}_{ij} + \overline{C}_{ij} \right) \right\} > 0 \tag{63}$$

hold, the system (61) *has at least one positive* ω*-periodic solution.*

Remark 16. We apply the main results obtained in the previous section to study some examples which have some biological implications; from the previous corollaries, we see that, under the appropriate conditions, the impulsive perturbations do not affect the existence of periodic solution of systems. However, if the impulsive perturbations are unbounded, some properties of the solution of systems could be changed significantly, which will be our further work.

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