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Research Article

Morphism of m-Polar Fuzzy Graph

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The main purpose of this paper is to introduce the notion of m-polar h-morphism on m-polar fuzzy graphs. The action of m-polar h-morphism on m-polar fuzzy graphs is studied. Some elegant theorems on weak and coweak isomorphism are obtained. Also, some properties of highly irregular, edge regular, and totally edge regular m-polar fuzzy graphs are studied.

1. Introduction

Akram [1] introduced the notion of bipolar fuzzy graphs describing various methods of their construction as well as investigating some of their important properties. Bhutani [2] discussed automorphism of fuzzy graphs. Chen et al. [3] generalized the concept of bipolar fuzzy set to obtain the notion of m-polar fuzzy set. The notion of m-polar fuzzy set is more advanced than fuzzy set and eliminates doubtfulness more absolutely. Ghorai and Pal [4–6] studied some operations and properties of m-polar fuzzy graphs. Rashmanlou et al. [7] discussed some properties of bipolar fuzzy graphs and some of its results are investigated. Ramprasad et al. [8] studied product m-polar fuzzy graph, product m-polar fuzzy intersection graph, and product m-polar fuzzy line graph. Values between 0 and 1 are used to develop a set theory based on fuzziness by Zadeh [9–11].

In the present work the authors introduce the concepts of m-polar h-morphism, edge regular m-polar fuzzy graph, totally edge regular m-polar fuzzy graph, and highly irregular m-polar fuzzy graph in order to strengthen the decision-making in critical situations.

2. Preliminaries

Definition 1. Throughout the paper, $[0,1]^m$ (m copies of the closed interval $[0,1]$) is considered to be a poset with pointwise order $\leq$, where m is a natural number, $\leq$ is given by $w \leq f \Leftrightarrow$ for all $i = 1,2,\ldots,m$, $p_i(w) \leq p_i(f)$, where $w, f \in [0,1]^m$ and $p_i : [0,1]^m \to [0,1]$ is the ith projection mapping. An m-polar fuzzy set (or a $[0,1]^m$-set) on X is a mapping $W : X \to [0,1]^m$.

Definition 2. Let W be an m-polar fuzzy set on X. An m-polar fuzzy relation on W is an m-polar fuzzy set F of $X \times X$ such that $F(uv) \leq \min\{W(u), W(v)\}$ for all $u, v \in V$, that is, for each $i = 1,2,\ldots,m$, for all $u, v \in X$, $p_i \circ F(uv) \leq \min\{p_i \circ W(u), p_i \circ W(v)\}$.

Definition 3. A generalized m-polar fuzzy graph of a graph $G^* = (V, E)$ is a pair $G = (V, W, F)$, where $W : V \to [0,1]^m$ is an m-polar fuzzy set in V and $F : V \tilde{\times} V \to [0,1]^m$ is an m-polar fuzzy set in $V \tilde{\times} V$ such that $F(uv) \leq \min\{W(u), W(v)\}$ for all $uv \in V \tilde{\times} V$ and $F(uv) = 0$ for all $uv \in ((\widetilde{V^2}) - E)$ ($0 = 0, 0 \cdots 0$) is the smallest element in $[0,1]^m$. W is called the m-polar fuzzy set of G and F is called m-polar fuzzy edge set of G.

Definition 4. An m-polar fuzzy graph $G = (V, W, F)$ of the graph $G^* = (V, E)$ is said to be strong if $p_i \circ F(uv) = \min\{p_i \circ W(u), p_i \circ W(v)\}$ for all $uv \in E$, $i = 1,2,\ldots,m$.

3. A New Theory of Regularity in m-Polar Fuzzy Graphs

Using the existing graph theories a new m-polar fuzzy graph theory is introduced in this section.

Definition 5. Let $G = (V, W, F)$ be an m-polar fuzzy graph. Then the degree of a vertex u is defined for $i = 1, 2, \ldots, m$ as

$$d_G(u) = \sum_{uv \in E, u \neq v} p_i \circ F(uv) = \left\langle \sum_{uv \in E, u \neq v} p_1 \circ F(uv), \sum_{uv \in E, u \neq v} p_2 \circ F(uv), \sum_{uv \in E, u \neq v} p_3 \circ F(uv), \ldots, \sum_{uv \in E, u \neq v} p_m \circ F(uv) \right\rangle. \quad (1)$$

Definition 6. The degree of an edge $xy \in E$ in an m-polar fuzzy graph $G = (V, W, F)$ is defined as

$$d_G(xy) = d_G(x) + d_G(y) - 2\langle p_1 \circ F(xy), p_2 \circ F(xy), p_3 \circ F(xy), \ldots, p_m \circ F(xy) \rangle. \quad (2)$$

Definition 7. The total degree of an edge $xy \in E$ in an m-polar fuzzy graph $G = (V, W, F)$ is defined as

$$td_G(xy) = d_G(x) + d_G(y) - \langle p_1 \circ F(xy), p_2 \circ F(xy), p_3 \circ F(xy), \ldots, p_m \circ F(xy) \rangle. \quad (3)$$

Definition 8. The degree of an edge $w_j w_k \in E$ in a crisp graph G^* is $d_{G^*}(w_j w_k) = d_{G^*}(w_j) + d_{G^*}(w_k) - 2$.

Example 9. Consider an m-polar fuzzy graph $G = (V, W, F)$ of $G^* = (V, E)$, where

$$V = \{K, L, M, N\},$$
$$E = \{KL, LM, MN, NK\},$$
$$W = \left\{ \frac{\langle 0.2, 0.5, 0.6 \rangle}{K}, \frac{\langle 0.3, 0.5, 0.7 \rangle}{L}, \frac{\langle 0.3, 0.6, 0.7 \rangle}{M}, \frac{\langle 0.4, 0.7, 0.8 \rangle}{N} \right\}, \quad (4)$$
$$F = \left\{ \frac{\langle 0.2, 0.4, 0.5 \rangle}{KL}, \frac{\langle 0.3, 0.4, 0.6 \rangle}{LM}, \frac{\langle 0.3, 0.5, 0.6 \rangle}{MN}, \frac{\langle 0.2, 0.3, 0.6 \rangle}{NK} \right\},$$

as in Figure 1.

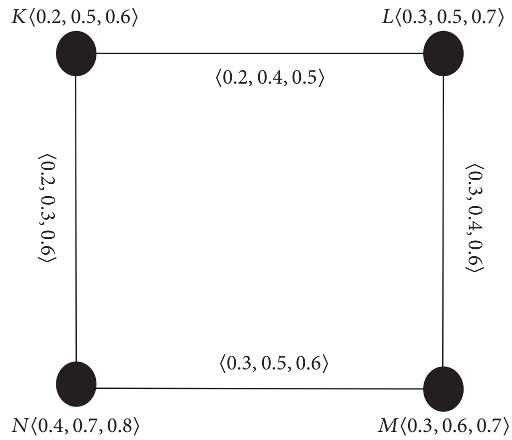

Figure 1: 3-polar fuzzy graph G.

Then, we have

$$d_G(K) = \langle 0.4, 0.7, 1.1 \rangle,$$
$$d_G(L) = \langle 0.5, 0.8, 1.1 \rangle,$$
$$d_G(M) = \langle 0.6, 0.9, 1.2 \rangle,$$
$$d_G(N) = \langle 0.5, 0.8, 1.2 \rangle,$$
$$d_G(KL) = \langle 0.5, 0.7, 1.2 \rangle,$$
$$d_G(LM) = \langle 0.5, 0.9, 1.1 \rangle,$$
$$d_G(MN) = \langle 0.5, 0.7, 1.2 \rangle,$$
$$d_G(NK) = \langle 0.5, 0.9, 1.1 \rangle,$$
$$td_G(KL) = \langle 0.7, 1.1, 1.7 \rangle,$$
$$td_G(LM) = \langle 0.8, 1.3, 1.7 \rangle,$$
$$td_G(MN) = \langle 0.8, 1.2, 1.8 \rangle,$$
$$td_G(NK) = \langle 0.7, 1.2, 1.7 \rangle. \quad (5)$$

Definition 10. If every vertex in an m-polar fuzzy graph $G = (V, W, F)$ has the same degree $\langle l_1, l_2, \ldots, l_m \rangle$, then $G = (V, W, F)$ is called regular m-polar fuzzy graph or m-polar fuzzy graph of degree $\langle l_1, l_2, \ldots, l_m \rangle$.

Definition 11. If every edge in an m-polar fuzzy graph $G = (V, W, F)$ has the same degree $\langle l_1, l_2, \ldots, l_m \rangle$, then $G = (V, W, F)$ is called an edge regular m-polar fuzzy graph.

Definition 12. If every edge in an m-polar fuzzy graph $G = (V, W, F)$ has the same total degree $\langle l_1, l_2, \ldots, l_m \rangle$, then $G = (V, W, F)$ is called totally edge regular m-polar fuzzy graph.

Example 13. Consider an m-polar fuzzy graph $G = (V, W, F)$ of $G^* = (V, E)$, where

$$V = \{K, L, M, N\},$$
$$E = \{KL, LM, MN, NK\},$$
$$W = \left\{ \frac{\langle 0.2, 0.6, 0.1 \rangle}{K}, \frac{\langle 0.3, 0.5, 0.1 \rangle}{L}, \frac{\langle 0.3, 0.5, 0.2 \rangle}{M}, \frac{\langle 0.4, 0.6, 0.3 \rangle}{N} \right\}, \quad (6)$$
$$F = \left\{ \frac{\langle 0.2, 0.2, 0.1 \rangle}{KL}, \frac{\langle 0.2, 0.2, 0.1 \rangle}{LM}, \frac{\langle 0.2, 0.5, 0.1 \rangle}{MN}, \frac{\langle 0.2, 0.5, 0.1 \rangle}{NK} \right\},$$

as in Figure 2.

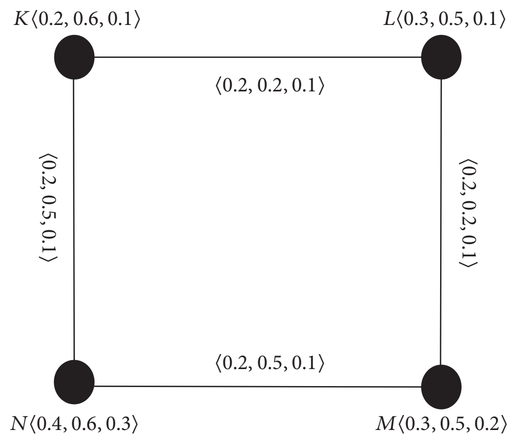

Figure 2: An edge regular m-polar fuzzy graph G.

Then, we have $d_G(KL) = d_G(LM) = d_G(MN) = d_G(NK) = \langle 0.4, 0.7, 0.2\rangle$.

Theorem 14. *Let $G = (V, W, F)$ be an m-polar fuzzy graph on a cycle $G^* = (V, E)$. Then*

$$\sum_{w_j \in V} d_G(w_j) = \sum_{w_j w_k \in E, j \neq k} d_G(w_j w_k). \tag{7}$$

Proof. Suppose that $G = (V, W, F)$ is an m-polar fuzzy graph and G^* is a cycle $w_1 w_2 w_3 \cdots w_n w_1$.

Now, we get

$$\begin{aligned}\sum_{j=1}^{n} d_G(w_j w_{j+1}) &= d_G(w_1 w_2) + d_G(w_2 w_3) + \cdots \\ &\quad + d_G(w_n w_1), \quad \text{where } w_{n+1} = w_1, \\ &= d_G(w_1) + d_G(w_2) - 2\langle p_1 \circ F(w_1 w_2), p_2 \\ &\quad \circ F(w_1 w_2), \dots, p_m \circ F(w_1 w_2)\rangle + d_G(w_2) \\ &\quad + d_G(w_3) - 2\langle p_1 \circ F(w_2 w_3), p_2 \circ F(w_2 w_3), \dots, \\ &\quad p_m \circ F(w_2 w_3)\rangle + \cdots + d_G(w_n) + d_G(w_1) - 2\langle p_1 \\ &\quad \circ F(w_n w_1), p_2 \circ F(w_n w_1), \dots, p_m \circ F(w_n w_1)\rangle \\ &= 2\sum_{w_j \in V} d_G(w_j) - 2\sum_{j=1}^{n}\langle p_1 \circ F(w_j w_{j+1}), p_2 \\ &\quad \circ F(w_j w_{j+1}), \dots, p_m \circ F(w_j w_{j+1})\rangle \\ &= \sum_{w_j \in V} d_G(w_j) + \sum_{w_j \in V} d_G(w_j) - 2\sum_{j=1}^{n}\langle p_1 \\ &\quad \circ F(w_j w_{j+1}), p_2 \circ F(w_j w_{j+1}), \dots, p_m \\ &\quad \circ F(w_j w_{j+1})\rangle = \sum_{w_j \in V} d_G(w_j) + 2\sum_{j=1}^{n}\langle p_1 \\ &\quad \circ F(w_j w_{j+1}), p_2 \circ F(w_j w_{j+1}), \dots, p_m \\ &\quad \circ F(w_j w_{j+1})\rangle - 2\sum_{j=1}^{n}\langle p_1 \circ F(w_j w_{j+1}), p_2 \\ &\quad \circ F(w_j w_{j+1}), \dots, p_m \circ F(w_j w_{j+1})\rangle \\ &= \sum_{w_j \in V} d_G(w_j).\end{aligned} \tag{8}$$

Hence,

$$\sum_{w_j \in V} d_G(w_j) = \sum_{w_j w_k \in E, j \neq k} d_G(w_j w_k). \tag{9}$$

□

Remark 15. Let $G = (V, W, F)$ be an m-polar fuzzy graph on a crisp graph G^*. Then

$$\begin{aligned}\sum_{w_j w_k \in E} d_G(w_j w_k) &= \sum_{w_j w_k \in E} d_{G^*}(w_j w_k)\langle p_1 \\ &\quad \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k)\rangle,\end{aligned} \tag{10}$$

where $d_{G^*}(w_j w_k) = d_{G^*}(w_j) + d_{G^*}(w_k) - 2$, for all $w_j w_k \in E$.

Theorem 16. *Let $G = (V, W, F)$ be an m-polar fuzzy graph on a c-regular crisp graph G^*. Then*

$$\sum_{w_j w_k \in E} d_G(w_j w_k) = (c-1)\sum_{w_j \in V} d_G(w_j). \tag{11}$$

Proof. From Remark 15, we have

$$\begin{aligned}\sum_{w_j w_k \in E} d_G(w_j w_k) &= \sum_{w_j w_k \in E} d_{G^*}(w_j w_k)\langle p_1 \\ &\quad \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k)\rangle \\ &= \sum_{w_j w_k \in E}\left(d_{G^*}(w_j) + d_{G^*}(w_k) - 2\right)\langle p_1 \\ &\quad \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k)\rangle.\end{aligned} \tag{12}$$

Since G^* is a regular crisp graph, we have the degree of every vertex in G^* as c.

That is, $d_{G^*}(w_j) = c$, so

$$\begin{aligned}\sum_{w_j w_k \in E} d_G(w_j w_k) &= (c + c - 2)\sum_{w_j w_k \in E}\langle p_1 \\ &\quad \circ F(w_j w_k), p_2 \circ F(w_j w_k), \dots, p_m \circ F(w_j w_k)\rangle, \\ \sum_{w_j w_k \in E} d_G(w_j w_k) &= 2(c-1)\sum_{w_j w_k \in E}\langle p_1 \circ F(w_j w_k), p_2\end{aligned}$$

$$\circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle,$$
$$\sum_{w_jw_k \in E} d_G(w_jw_k) = (c-1) \sum_{w_jw_k \in E} (d_G(w_j)). \tag{13}$$

□

Theorem 17. *Let $G = (V, W, F)$ be an m-polar fuzzy graph on a crisp graph G^*. Then*

$$\sum_{w_jw_k \in E} td_G(w_jw_k) = \sum_{w_jw_k \in E} d_{G^*}(w_jw_k) \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle + \sum_{w_jw_k \in E} \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle. \tag{14}$$

Proof. From the definition of total edge degree of G, we get

$$\sum_{w_jw_k \in E} td_G(w_jw_k) = \sum_{w_jw_k \in E} (d_G(w_jw_k) + \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle) = \left(\sum_{w_jw_k \in E} d_G(w_jw_k) + \sum_{w_jw_k \in E} \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle\right). \tag{15}$$

From Remark 15, we have

$$\sum_{w_jw_k \in E} td_G(w_jw_k) = \sum_{w_jw_k \in E} d_{G^*}(w_jw_k) \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle + \sum_{w_jw_k \in E} \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle. \tag{16}$$

□

Theorem 18. *Let $G = (V, W, F)$ be an m-polar fuzzy graph. Then the function $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function if and only if the following conditions are equivalent.*

(i) *G is an edge regular m-polar fuzzy graph.*

(ii) *G is a totally edge regular m-polar fuzzy graph.*

Proof. Suppose that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function. Then

$$\langle p_1 \circ F(xy), p_2 \circ F(xy), \dots, p_m \circ F(xy)\rangle = \langle k_1, k_2, \dots, k_m\rangle \quad \forall xy \in E, \tag{17}$$

where $k_1, k_2, \dots, k_m$ are constants and $k_1, k_2, \dots, k_m \in [0, 1]$. Let G be an edge regular m-polar fuzzy graph. Then, for all $w_jw_k \in E$, $d_G(w_jw_k) = \langle r_1, r_2, \dots, r_m\rangle$.

Now we have to show that G is a totally edge regular m-polar fuzzy graph.

Now

$$td_G(w_jw_k) = d_G(w_jw_k) + \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle = \langle r_1, r_2, \dots, r_m\rangle + \langle k_1, k_2, \dots, k_m\rangle = \langle r_1 + k_1, r_2 + k_2, \dots, r_m + k_m\rangle \quad \forall w_jw_k \in E. \tag{18}$$

Thus G is a totally edge regular graph.

Now, let G be a $\langle h_1, h_2, \dots, h_m\rangle$-totally edge regular m-polar fuzzy graph. Then

$$td_G(w_jw_k) = \langle h_1, h_2, \dots, h_m\rangle \quad \forall w_jw_k \in E. \tag{19}$$

So, we have

$$td_G(w_jw_k) = d_G(w_jw_k) + \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle = \langle h_1, h_2, \dots, h_m\rangle. \tag{20}$$

Hence

$$d_G(w_jw_k) = \langle h_1, h_2, \dots, h_m\rangle - \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle = \langle h_1 - p_1 \circ F(w_jw_k), h_2 - p_2 \circ F(w_jw_k), \dots, h_m - p_m \circ F(w_jw_k)\rangle = \langle h_1 - k_1, h_2 - k_2, \dots, h_m - k_m\rangle. \tag{21}$$

Then G is an $\langle h_1 - k_1, h_2 - k_2, \dots, h_m - k_m\rangle$-edge regular m-polar fuzzy graph.

Conversely, suppose that G is an edge regular m-polar fuzzy graph and G is a totally edge regular m-polar fuzzy graph which are equivalent. We have to prove that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function. In a contrary way, we suppose that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is not a constant function. Then

$$\langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle \neq \langle p_1 \circ F(w_rw_s), p_2 \circ F(w_rw_s), \dots, p_m \circ F(w_rw_s)\rangle \tag{22}$$

for at least one pair of edges $w_jw_k, w_rw_s \in E$. Let G be an $\langle r_1, r_2, \dots, r_m\rangle$-edge regular m-polar fuzzy graph. Then

$d_G(w_jw_k) = d_G(w_rw_s) = \langle r_1, r_2, \dots, r_m\rangle$. Hence, for every $w_jw_k \in E$ and for every $w_rw_s \in E$,

$$\begin{aligned} td_G(w_jw_k) &= d_G(w_jw_k) + \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle \\ &= \langle r_1, r_2, \dots, r_m\rangle + \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle = \langle r_1 + p_1 \circ F(w_jw_k), r_2 + p_2 \circ F(w_jw_k), \dots, r_m + p_m \circ F(w_jw_k)\rangle, \\ td_G(w_rw_s) &= d_G(w_rw_s) + \langle p_1 \circ F(w_rw_s), p_2 \circ F(w_rw_s), \dots, p_m \circ F(w_rw_s)\rangle = \langle r_1, r_2, \dots, r_m\rangle \\ &+ \langle p_1 \circ F(w_rw_s), p_2 \circ F(w_rw_s), \dots, p_m \circ F(w_rw_s)\rangle = \langle r_1 + p_1 \circ F(w_rw_s), r_2 + p_2 \circ F(w_rw_s), \dots, r_m + p_m \circ F(w_rw_s)\rangle. \end{aligned} \tag{23}$$

Since

$$\begin{aligned} &\langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle \\ &\neq \langle p_1 \circ F(w_rw_s), p_2 \circ F(w_rw_s), \dots, p_m \circ F(w_rw_s)\rangle, \end{aligned} \tag{24}$$

we have $td_G(w_jw_k) \neq td_G(w_rw_s)$. Hence, G is not a totally edge regular m-polar fuzzy graph. This is a contradiction to our assumption. Hence, $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function. In the same way, we can prove that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function, when G is a totally edge regular m-polar fuzzy graph. □

Theorem 19. *Let G^* be a h-regular crisp graph and $G = (V, W, F)$ be an m-polar fuzzy graph on G^*. Then, $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function if and only if G is both regular m-polar fuzzy graph and totally edge regular m-polar fuzzy graph.*

Proof. Let $G = (V, W, F)$ be an m-polar fuzzy graph on G^* and let G^* be a h-regular crisp graph. Assume that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function. Then

$$\begin{aligned} &\langle p_1 \circ F(xy), p_2 \circ F(xy), \dots, p_m \circ F(xy)\rangle \\ &= \langle k_1, k_2, \dots, k_m\rangle \quad \forall xy \in E, \end{aligned} \tag{25}$$

where $k_1, k_2, \dots, k_m$ are constants and $k_1, k_2, \dots, k_m \in [0, 1]$. From the definition of degree of a vertex, we get

$$\begin{aligned} d_G(w_j) &= \sum_{w_jw_k \in E} \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle \\ &= \sum_{w_iw_j \in E} \langle k_1, k_2, \dots, k_m\rangle = \langle hk_1, hk_2, \dots, hk_m\rangle \quad \text{for every } w_j \in V. \end{aligned} \tag{26}$$

So $d_G(w_j) = \langle hk_1, hk_2, \dots, hk_m\rangle$. Therefore, G is regular m-polar fuzzy graph.

Now, for $i = 1, 2, \dots, m$,

$$\begin{aligned} td_G(w_jw_k) &= \sum (p_i \circ F(w_jw_k)) + \sum (p_i \circ F(w_kw_l)) + (p_i \circ F(w_jw_l)) \\ &= \sum_{\substack{w_jw_k \in E \\ j \neq k}} \langle k_1, k_2, \dots, k_m\rangle + \sum_{\substack{w_kw_l \in E \\ k \neq l}} \langle k_1, k_2, \dots, k_m\rangle + \langle k_1, k_2, \dots, k_m\rangle \\ &= (h-1)\langle k_1, k_2, \dots, k_m\rangle + (h-1)\langle k_1, k_2, \dots, k_m\rangle + \langle k_1, k_2, \dots, k_m\rangle \\ &= (2h-1)\langle k_1, k_2, \dots, k_m\rangle \quad \forall w_jw_k \in E. \end{aligned} \tag{27}$$

Hence, G is also a totally edge regular m-polar fuzzy graph.

Conversely, assume that G is both regular and totally edge regular m-polar fuzzy graph. Now we have to prove that $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function. Since G is regular, $d_G(w_j) = \langle l_1, l_2, \dots, l_m\rangle$ for all $w_j \in V$. Also G is totally edge regular. Hence, $td_G(w_jw_k) = \langle h_1, h_2, \dots, h_m\rangle$ for all $w_jw_k \in E$. From the definition of total edge degree, we get

$$\begin{aligned} td_G(w_jw_k) &= d_G(w_j) + d_G(w_k) - \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle \quad \forall w_jw_k \in E, \\ \langle h_1, h_2, \dots, h_m\rangle &= \langle l_1, l_2, \dots, l_m\rangle + \langle l_1, l_2, \dots, l_m\rangle - \langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle, \end{aligned} \tag{28}$$

so

$$\begin{aligned} &\langle p_1 \circ F(w_jw_k), p_2 \circ F(w_jw_k), \dots, p_m \circ F(w_jw_k)\rangle \\ &= 2\langle l_1, l_2, \dots, l_m\rangle - \langle h_1, h_2, \dots, h_m\rangle \\ &= \langle 2l_1 - h_1, 2l_2 - h_2, \dots, 2l_m - h_m\rangle \quad \forall w_jw_k \in E. \end{aligned} \tag{29}$$

Hence $\langle p_1 \circ F, p_2 \circ F, \dots, p_m \circ F\rangle$ is a constant function. □

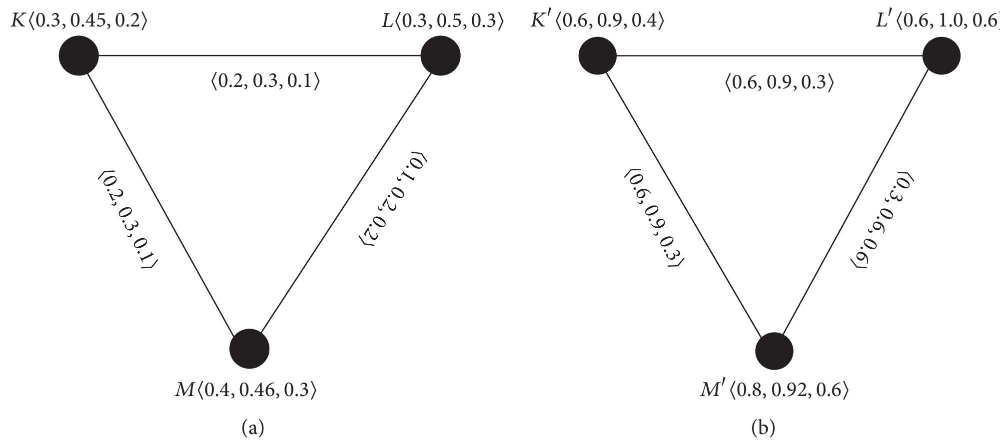

Figure 3: h-morphism of m-polar fuzzy graphs G_1 and G_2.

4. h-Morphism on m-Polar Fuzzy Graphs

Definition 20. Let $G_1 = (V_1, W_1, F_1)$ and $G_2 = (V_2, W_2, F_2)$ be two m-polar fuzzy graphs of the graphs $G_1{}^* = (V_1, E_1)$ and $G_2{}^* = (V_2, E_2)$, respectively.

A homomorphism g from G_1 to G_2 is a mapping $g : V_1 \to V_2$ such that, for each $i = 1, 2, \ldots, m$,

$$\begin{aligned} p_i \circ W_1(u) &\le p_i \circ W_2(g(u)), \quad \forall u \in V_1, \\ p_i \circ F_1(uv) &\le p_i \circ F_2(g(u)g(v)), \quad \forall uv \in E_1. \end{aligned} \tag{30}$$

An isomorphism g from G_1 to G_2 is a bijective mapping $g : V_1 \to V_2$ which satisfies the following conditions:

$$\begin{aligned} p_i \circ W_1(u) &= p_i \circ W_2(g(u)), \quad \forall u \in V_1, \\ p_i \circ F_1(uv) &= p_i \circ F_2(g(u)g(v)), \quad \forall uv \in E_1. \end{aligned} \tag{31}$$

A weak isomorphism g from G_1 to G_2 is a bijective mapping $g : V_1 \to V_2$ which satisfies the following conditions:

g is homomorphism and $p_i \circ W_1(u) = p_i \circ W_2(g(u))$, $\forall u \in V_1$.

A coweak isomorphism g from G_1 to G_2 is a bijective mapping $g : V_1 \to V_2$ which satisfies the following conditions:

g is homomorphism and $p_i \circ F_1(uv) = p_i \circ F_2(g(u)g(v))$, $\forall uv \in E_1$.

Definition 21. The order of an m-polar fuzzy graph G is defined as

$$O(G) = \sum_{u\in V} p_i \circ W(u) = \left\langle \sum_{u\in V} p_1 \circ W(u), \sum_{u\in V} p_2 \circ W(u), \ldots, \sum_{u\in V} p_m \circ W(u) \right\rangle. \tag{32}$$

Definition 22. The size of an m-polar fuzzy graph G is defined as

$$S(G) = \sum_{\substack{u\neq v\\ uv\in E}} p_i \circ F(uv) = \left\langle \sum_{\substack{u\neq v\\ uv\in E}} p_1 \circ F(uv), \sum_{\substack{u\neq v\\ uv\in E}} p_2 \circ F(uv), \ldots, \sum_{\substack{u\neq v\\ uv\in E}} p_m \circ F(uv) \right\rangle. \tag{33}$$

Definition 23. Let G_1 and G_2 be two m-polar graphs on (V_1, E_1) and (V_2, E_2), respectively.

A bijective function $h : V_1 \to V_2$ is called an m-polar morphism or m-polar h-morphism if there exists two numbers $l_1 > 0$ and $l_2 > 0$ such that $p_i \circ W_2(h(u)) = l_1 p_i \circ W_1(u)$, $\forall u \in V_1$, $p_i \circ F_2(h(u)h(v)) = l_2 p_i \circ F_1(uv)$, $\forall uv \in E_1$, $i = 1, 2, \ldots, m$. In such a case, h will be called an (l_1, l_2) m-polar h-morphism from G_1 to G_2. If $l_1 = l_2 = l$, we call h, an m-polar l-morphism.

Example 24. Consider two m-polar fuzzy graphs $G_1 = (V_1, W_1, F_1)$ and $G_2 = (V_2, W_2, F_2)$ as shown in Figure 3.

An m-polar fuzzy graph $G_1 = (V_1, W_1, F_1)$ is shown in Figure 3(a) where

$$\begin{aligned} V_1 &= \{K, L, M\}, \\ E_1 &= \{KL, LM, MK\}, \\ W_1 &= \left\{ \frac{\langle 0.3, 0.45, 0.2\rangle}{K}, \frac{\langle 0.3, 0.5, 0.3\rangle}{L}, \frac{\langle 0.4, 0.46, 0.3\rangle}{M} \right\}, \\ F_1 &= \left\{ \frac{\langle 0.2, 0.3, 0.1\rangle}{KL}, \frac{\langle 0.1, 0.2, 0.2\rangle}{LM}, \frac{\langle 0.2, 0.3, 0.1\rangle}{MK} \right\}. \end{aligned} \tag{34}$$

Another m-polar fuzzy graph $G_2 = (V_2, W_2, F_2)$ is shown in Figure 3(b) where

$$V_2 = \{K', L', M'\},$$

$$E_2 = \{K'L', L'M', M'K'\},$$

$$W_2 = \left\{\frac{\langle 0.6, 0.9, 0.4\rangle}{K'}, \frac{\langle 0.6, 1.0, 0.6\rangle}{L'}, \frac{\langle 0.8, 0.92, 0.6\rangle}{M'}\right\},$$
$$F_2 = \left\{\frac{\langle 0.6, 0.9, 0.3\rangle}{K'L'}, \frac{\langle 0.3, 0.6, 0.6\rangle}{L'M'}, \frac{\langle 0.6, 0.9, 0.3\rangle}{M'K'}\right\}. \quad (35)$$

Here, there is an m-polar h-morphism such that $h(K) = K'$, $h(L) = L'$, $h(M) = M'$, $l_1 = 2$, and $l_2 = 3$.

Theorem 25. *The relation h-morphism is an equivalence relation in the collection of m-polar fuzzy graphs.*

Proof. Consider the collection of m-polar fuzzy graphs. Define the relation $G_1 \approx G_2$ if there exists a (l_1, l_2) h-morphism from G_1 to G_2 where both $l_1 \neq 0$ and $l_2 \neq 0$. Consider the identity morphism G_1 to G_1. It is a $(1,1)$-morphism from G_1 to G_1 and hence $\approx$ is reflexive. Let $G_1 \approx G_2$. Then there exists a (l_1, l_2) morphism from G_1 to G_2 for some $l_1 \neq 0$ and $l_2 \neq 0$. Therefore,

$$p_i \circ W_2(h(u)) = l_1 p_i \circ W_1(u), \quad \forall u \in V_1,$$
$$p_i \circ F_2(h(u)h(v)) = l_2 p_i \circ F_1(uv), \quad \forall uv \in E_1. \quad (36)$$

Consider $h^{-1} : G_2 \to G_1$. Let $m, n \in V_2$. Since h^{-1} is bijective, $m = h(u)$, $n = h(v)$, for some $u, v \in V_2$. Now,

$$p_i \circ W_1\left(h^{-1}(m)\right) = p_i \circ W_1\left(h^{-1}(h(u))\right) = p_i \circ W_1(u) = \frac{1}{l_1} p_i \circ W_2(h(u)) = \frac{1}{l_1} p_i \circ W_2(m),$$
$$p_i \circ F_1\left(h^{-1}(m)h^{-1}(n)\right) = p_i \circ F_1\left(h^{-1}(h(u))h^{-1}(h(v))\right) = p_i \circ F_1(uv) = \frac{1}{l_2} p_i \circ F_2(h(u)h(v)) = \frac{1}{l_2} p_i \circ F_2(mn). \quad (37)$$

Thus there exists $(1/l_1, 1/l_2)$ morphism from G_2 to G_1. Therefore, $G_2 \approx G_1$ and hence $\approx$ is symmetric.

Let $G_1 \approx G_2$ and $G_2 \approx G_3$. Then there exists a (l_1, l_2) morphism from G_1 to G_2, say h for some $l_1 \neq 0$ and $l_2 \neq 0$, and there exists (l_3, l_4) morphism from G_2 to G_3, say q for some $l_3 \neq 0$ and $l_4 \neq 0$. So, for $i = 1, 2, \ldots, m$,

$$p_i \circ W_3(q(x)) = l_3 p_i \circ W_2(x), \quad x \in V_2,$$
$$p_i \circ F_3(q(x)q(y)) = l_4 p_i \circ F_2(xy), \quad \forall xy \in E_2. \quad (38)$$

Let $r : q \circ p : G_1 \to G_3$.

Now,

$$p_i \circ W_3(r(u)) = p_i \circ W_3((q \circ h)(u)) = p_i \circ W_3(q(h(u))) = l_3 p_i \circ W_2(h(u)) = l_3 l_1 p_i \circ W_1(u),$$
$$p_i \circ F_3(r(u)r(v)) = p_i \circ F_3((q \circ h)(u)(q \circ h)(v)) = p_i \circ F_3(q(h(u)))q(h(v)) = l_4 p_i \circ F_2(h(u)h(v)) = l_4 l_2 p_i \circ F_1(uv). \quad (39)$$

Thus there exists $(l_3 l_1, l_4 l_2)$ morphism r from G_1 to G_3. Therefore, $G_1 \approx G_3$ and hence $\approx$ is transitive. So, the relation h-morphism is an equivalence relation in the collection of m-polar fuzzy graphs. □

Theorem 26. *Let G_1 and G_2 be two m-polar fuzzy graphs such that G_1 is (l_1, l_2) m-polar morphism to G_2 for some $l_1 \neq 0$ and $l_2 \neq 0$. The image of a strong edge in G_1 is also a strong edge in G_2 if and only if $l_1 = l_2$.*

Proof. Let xy be a strong edge in G_1 such that $h(x)h(y)$ is also a strong edge in G_2.

Now as $G_1 \approx G_2$ for $i = 1, 2, \ldots, m$, we have

$$l_2 p_i \circ F_1(xy) = p_i \circ F_2(h(x)h(y)) = p_i \circ W_2(h(x) \wedge h(y)) = l_1 p_i \circ W_1(x) \wedge l_1 p_i \circ W_1(y) = l_1(p_i \circ W_1(x) \wedge p_i \circ W_1(y)) = l_1 p_i \circ F_1(xy) \quad \forall xy \in E_1. \quad (40)$$

Hence,

$$l_2 p_i \circ F_1(xy) = l_1 p_i \circ F_1(xy), \quad \forall xy \in E_1. \quad (41)$$

The equation holds if and only if $l_1 = l_2$. □

Theorem 27. *If an m-polar fuzzy graph G_1 is coweak isomorphic to G_2 and if G_1 is regular then G_2 is also regular.*

Proof. As an m-polar fuzzy graph G_1 is coweak isomorphic to G_2, there exists a coweak isomorphism $h : G_1 \to G_2$ which is bijective for $i = 1, 2, \ldots, m$ that satisfies

$$p_i \circ W_1(u) \leq p_i \circ W_2(h(u)), \quad \forall u \in V_1,$$
$$p_i \circ F_1(uv) = p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \quad (42)$$

As G_1 is regular, for $u \in V$, $\sum_{u \neq v, v \in V_1} p_i \circ F_1(uv) = \text{constant}$. Now $\sum_{h(u) \neq h(v)} p_i \circ F_2(h(u)h(v)) = \sum_{u \neq v, v \in V_1} p_i \circ F_1(uv) = \text{constant}$. Therefore, G_2 is regular. □

Theorem 28. *Let G_1 and G_2 be two m-polar fuzzy graphs. If G_1 is weak isomorphic to G_2 and if G_1 is strong then G_2 is also strong.*

Proof. As G_1 is an m-polar fuzzy graph which is weak isomorphic with G_2, then there exists a weak isomorphism $h : G_1 \to G_2$ which is bijective for $i = 1, 2, \ldots, m$ that satisfies

$$p_i \circ W_1(u) = p_i \circ W_2(h(u)), \quad \forall u \in V_1,$$
$$p_i \circ F_1(uv) \le p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \quad (43)$$

As G_1 is strong, $p_i \circ F_1(uv) = \min(p_i \circ W_1(u), p_i \circ W_1(v))$. Now, we get

$$p_i \circ F_2(h(u)h(v)) \ge p_i \circ F_1(uv) = \min(p_i \circ W_1(u), p_i \circ W_1(v)) = \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v))). \quad (44)$$

By the definition, $p_i \circ F_2(h(u)h(v)) \le \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v)))$. Therefore, $p_i \circ F_2(h(u)h(v)) = \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v)))$. So G_2 is strong. □

Theorem 29. *If an m-polar fuzzy graph G_1 is coweak isomorphic with a strong regular m-polar fuzzy graph G_2, then G_1 is strong regular m-polar fuzzy graph.*

Proof. As an m-polar fuzzy graph G_1 is coweak isomorphic to G_2. Then there exists a coweak isomorphism $h : G_1 \to G_2$ which is bijective for $i = 1, 2, \ldots, m$ that satisfies

$$p_i \circ W_1(u) \le p_i \circ W_2(h(u)), \quad \forall u \in V_1,$$
$$p_i \circ F_1(uv) = p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \quad (45)$$

Now, we get

$$p_i \circ F_1(uv) = p_i \circ F_2(h(u)h(v)) = \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v))) \ge \min(p_i \circ W_1(u), p_i \circ W_1(v)). \quad (46)$$

But, by the definition, we have

$$p_i \circ F_1(uv) \le \min(p_i \circ W_1(u), p_i \circ W_1(v)). \quad (47)$$

So, $p_i \circ F_1(uv) = \min(p_i \circ W_1(u), p_i \circ W_1(v))$.

Therefore, G_1 is strong. Also for $u \in V_1$, $\sum_{u \ne v, v \in V_1} p_i \circ F_1(uv) = \sum p_i \circ F_2(h(u)h(v)) =$ constant as G_2 is regular. Therefore, G_1 is regular. □

Theorem 30. *Let G_1 and G_2 be two isomorphic m-polar fuzzy graphs; then G_1 is strong regular if and only if G_2 is strong regular.*

Proof. As an m-polar fuzzy graph G_1 is isomorphic with an m-polar fuzzy graph G_2, there exists an isomorphism $h : G_1 \to G_2$ which is bijective for $i = 1, 2, \ldots, m$ that satisfies

$$p_i \circ W_1(u) = p_i \circ W_2(h(u)), \quad \forall u \in V_1,$$
$$p_i \circ F_1(uv) = p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \quad (48)$$

Now, G_1 is strong if and only if $p_i \circ F_1(uv) = \min(p_i \circ W_1(u), p_i \circ W_1(v))$, if and only if $p_i \circ F_2(h(u)h(v)) = \min(p_i \circ W_2(h(u)), p_i \circ W_2(h(v)))$, and if and only if G_2 is strong.

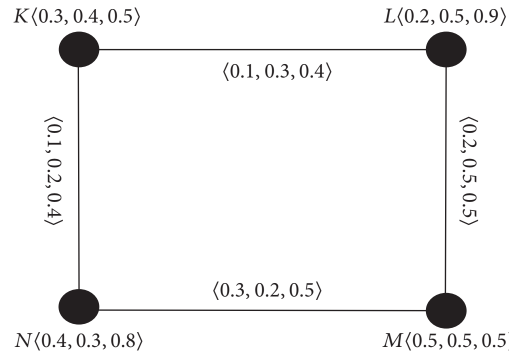

Figure 4: A highly irregular m-polar fuzzy graph.

G_1 is regular if and only if, for $u \in V_1$, $\sum_{u \ne v, v \in V_1} p_i \circ F_1(uv) =$ constant, if and only if $\sum_{p(u) \ne p(v)} p_i \circ F_2(h(u)h(v)) =$ Constant, for all $h(u) \in V_2$, and if and only if G_2 is regular. □

Definition 31. Let $G = (V, W, F)$ be a connected m-polar fuzzy graph. Then G is said to be a highly irregular m-polar fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees.

Example 32. Consider an m-polar fuzzy graph $G = (V, W, F)$ of $G^* = (V, E)$, where

$$V = \{K, L, M, N\},$$
$$E = \{KL, LM, MN, NK\},$$
$$W = \left\{\frac{\langle 0.3, 0.4, 0.5\rangle}{K}, \frac{\langle 0.2, 0.5, 0.9\rangle}{L}, \frac{\langle 0.5, 0.5, 0.5\rangle}{M}, \frac{\langle 0.4, 0.3, 0.8\rangle}{N}\right\},$$
$$F = \left\{\frac{\langle 0.1, 0.3, 0.4\rangle}{KL}, \frac{\langle 0.2, 0.5, 0.5\rangle}{LM}, \frac{\langle 0.3, 0.2, 0.5\rangle}{MN}, \frac{\langle 0.1, 0.2, 0.4\rangle}{NK}\right\}, \quad (49)$$

as in Figure 4.

By usual calculations, we get

$$d_G(K) = \langle 0.2, 0.5, 0.8\rangle,$$
$$d_G(L) = \langle 0.3, 0.8, 0.9\rangle,$$
$$d_G(M) = \langle 0.5, 0.7, 1.0\rangle,$$
$$d_G(N) = \langle 0.4, 0.4, 0.9\rangle. \quad (50)$$

We see that every vertex of G is adjacent to vertices with distinct degrees.

Theorem 33. *For any two isomorphic highly irregular m-polar fuzzy graphs, their order and size are the same.*

Proof. If $h : G_1 \to G_2$ is an isomorphism between the two highly irregular m-polar fuzzy graphs G_1 and G_2 with the underlying sets V_1 and V_2, respectively, then, for $i = 1, 2, \ldots, m$,

$$p_i \circ W_1(u) = p_i \circ W_2(h(u)), \quad \forall u \in V_1,$$
$$p_i \circ F_1(uv) = p_i \circ F_2(h(u)h(v)), \quad \forall uv \in E_1. \tag{51}$$

So, we get

$$\begin{aligned} O(G_1) &= \sum_{x_1 \in V_1} p_i \circ W_1(x_1) = \sum_{x_1 \in V_1} p_i \circ W_2(h(x_1)) \\ &= \sum_{x_2 \in V_2} p_i \circ W_2(x_2) = O(G_2), \\ S(G_1) &= \sum_{x_1 y_1 \in E_1} p_i \circ F_1(x_1 y_1) \\ &= \sum_{x_1 y_1 \in E_1} p_i \circ F_2(h(x_1)h(y_1)) \\ &= \sum_{x_2 y_2 \in E_2} p_i \circ F_2(x_2 y_2) = S(G_2). \end{aligned} \tag{52}$$

□

Theorem 34. *If G_1 and G_2 are isomorphic highly irregular m-polar fuzzy graphs, then, the degrees of the corresponding vertices u and $h(u)$ are preserved.*

Proof. If $h : G_1 \to G_2$ is an isomorphism between the highly irregular m-polar fuzzy graphs G_1 and G_2 with the underlying sets V_1 and V_2, respectively, then, for $i = 1, 2, \ldots, m$,

$$p_i \circ F_1(uv) = p_i \circ F_2(h(u)h(v)) \quad \forall u, v \in V_1. \tag{53}$$

Therefore,

$$\begin{aligned} d_{G_1}(u) &= \sum_{u,v \in V_1} p_i \circ F_1(uv) = \sum_{u,v \in V_1} p_i \circ F_2(h(u)h(v)) \\ &= d_{G_2}(h(u)). \end{aligned} \tag{54}$$

That is, the degrees of the corresponding vertices of G_1 and G_2 are the same. □

5. Conclusion

Any dissimilar fuzzy graph hypothesis needs large data for training to be able to help in decision-making which is crucial to utilitarian research in science and technology. The new method developed in this paper based on the pattern of unique cases helps us to make a better choice in contrast to the established fuzzy graph solutions. The concept of h-morphism, highly irregular m- polar fuzzy graphs is discussed in this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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