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*Research Article*

# Tracking Control for Switched Cascade Nonlinear Systems

**Xiaoxiao Dong, Linyan Chang, Fangfang Wu, and Na Hu**

*School of Science, Shenyang University of Technology, Shenyang 110870, China*

Correspondence should be addressed to Xiaoxiao Dong; dongxiaoxiao0331@sina.com

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The issue of $H_\infty$ output tracking for switched cascade nonlinear systems is discussed in this paper, where not all the linear parts of subsystems are stabilizable. The conditions of the solvability for the issue are given by virtue of the structural characteristics of the systems and the average dwell time method, in which the total activation time for stabilizable subsystems is longer than that for the unstabilizable subsystems. At last, a simulation example is used to demonstrate the validity and advantages of the proposed approach.

## 1. Introduction

With the development of society, the control theory also confronts lots of challenges. The famous control effect cannot be achieved, if the technique which simply relies on continuous-time or discrete-time system is employed. Therefore, hybrid systems have attracted much attention [1–4]. Switched system is a particular class of hybrid system, which includes a finite number of subsystems and a rule specifying the switching among these subsystems. Stability analysis and control synthesis of switched systems have been two important topics, and the average dwell time approach is an effective tool to handle the stability problem for switched systems [5–9].

On the other hand, the problem of output tracking for nonlinear systems has an important theoretical and practical significance, because it is a fundamental problem in control theory and has widespread application in engineering, such as aeronautics, robot control, and flight control [10–13]. However, the output tracking problem for nonlinear systems is harder to investigate than stability. This is because output tracking requires the output of systems to track the reference signal besides the internal stability. Furthermore, for switched systems, due to the interaction between the continuous and discrete dynamics, the problem of output tracking becomes more difficult.

As far as we know, there are only a few results about the output tracking problem of switched systems in the literature. Reference [14] studied the tracking control problem for linear switched systems based on a reference model. Reference [15] gave sufficient conditions for $H_\infty$ output tracking problem of linear switched systems to be solvable under asynchronous switching. For switched cascade nonlinear systems, References [16, 17] addressed the output tracking problem with external disturbance using a variable structure control method, Reference [18] investigated the observer-based model tracking problem, Reference [19] discussed the $H_\infty$ output tracking problem based on the method of multiple Lyapunov function, and [20] studied the $H_\infty$ output tracking problem based on the average dwell time method, in which all subsystems of switched systems are stabilizable.

Prompted by the above results, this paper studies the solvability of the $H_\infty$ output tracking control problem of switched cascade nonlinear systems. The main contribution of this paper is that the extended results of output tracking problem for switched cascade nonlinear systems are obtained by relaxing the conditions that all subsystems are stabilizable. The solvability conditions for the output tracking problem are proposed, if the total activation time of stabilizable and unstabilizable subsystems satisfies certain relations. A numerical example is demonstrated to verify the effectiveness of the main results.

## 2. Problem Formulation and Preliminaries

In this paper, we consider a switched cascade nonlinear system:

$$\dot{x}_1 = A_{1\sigma(t)}x_1(t) + A_{2\sigma(t)}x_2(t) + B_{\sigma(t)}u_{\sigma(t)}(t) + \omega,$$
$$\dot{x}_2 = f_{2\sigma(t)}(x_2(t)), \qquad (1)$$
$$y(t) = C_{\sigma(t)}x_1(t),$$

where $x_1 \in R^{n-d}$ and $x_2 \in R^d$, $u_\sigma \in R^m$, $y \in R^p$, and $\omega$ are the states, the control input, the measurable output, and the external disturbance, respectively. The switching law $\sigma : [0,\infty) \to I_N[1,\ldots,N]$ is a right continuous piecewise constant function of time, and the switching sequence $\Sigma = \{x_0; (i_0,t_0),(i_1,t_1),\ldots,(i_j,t_j)\ldots \mid i_j \in I_N\}$ means that the $i_j$th subsystem is active when $t \in [t_j, t_{j+1})$. $A_{1i}$, $A_{2i}$, $B_i$, and $C_i$ are known matrices, and $f_{2i}(x_2)$ are known smooth vector fields with appropriate dimensions.

In order to solve the output tracking problem, motivated by [20], we apply the controller as follows:

$$\dot{z}(t) = y(t) - y_d(t), \qquad (2)$$

$$u = K_{1i}x_1 + K_{2i}x_2 + K_{3i}z, \qquad (3)$$

where $e(t) \triangleq y(t) - y_d(t)$ is the tracking error, $y_d(t)$ is the reference input, and $K_{1i}$, $K_{2i}$, and $K_{3i}$ are the gain matrices, which will be determined later.

Combining (1) and (3), we get the following augmented system [20]:

$$\dot{\overline{x}}_1(t) = \overline{A}_{1\sigma}\overline{x}_1(t) + \overline{A}_{2\sigma}x_2(t) + \varpi,$$
$$\dot{x}_2 = f_{2\sigma(t)}(x_2(t)), \qquad (4)$$

where $\overline{x}_1(t) = [x_1^T(t), z^T]^T$, $\overline{A}_{1i} = \begin{bmatrix} A_{1i}+B_iK_{1i} & B_iK_{3i} \\ C_i & 0 \end{bmatrix}$, $\overline{A}_{2i} = \begin{bmatrix} A_{2i}+B_iK_{2i} \\ 0 \end{bmatrix}$, and $\varpi = \begin{pmatrix} \omega \\ -y_d(t) \end{pmatrix}$.

In the development to follow, we introduce a definition first.

*Definition 1* (see [14]). System (1) is said to satisfy weighted $H_\infty$ tracking performance, if the following conditions are satisfied:

(i) Internal stability: the system

$$\dot{\overline{x}}_1(t) = \overline{A}_{1\sigma}\overline{x}_1(t) + \overline{A}_{2\sigma}x_2(t),$$
$$\dot{x}_2 = f_{2\sigma(t)}(x_2(t)), \qquad (5)$$

is asymptotically (or exponentially) stable.

(ii) Tracking performance: given the performance index as

$$J_L = \int_0^\infty e^{-\lambda t}\Bigg[x^T(t)Q_{11}x(t) + \left(\int_0^t e(t)\,dt\right)^T Q_{22}\left(\int_0^t e(t)\,dt\right) + u^T(t)Ru(t)\Bigg]dt, \qquad (6)$$

the tracking performance index $J_L$ can meet certain upper bound, where $Q_{11} \in R^{n\times n}$ and $Q_{22} \in R^{p\times p}$ are positive semidefinite matrices and $R \in R^{m\times m}$ is a positive definite matrix.

*Definition 2* (see [21]). For any switching signal $\sigma(t)$ and any $t > \tau > 0$, let $N_\sigma(\tau,t)$ denote the number of switching of $\sigma(t)$ over $(\tau,t)$. If $N_\sigma(\tau,t) \le N_0 + (t-\tau)/\tau_a$ holds for $N_0 \ge 0$ and $\tau_a > 0$, then $\tau_a$ is called the average dwell time. In general, we assume $N_0 = 0$.

For nonswitched cascade nonlinear system (7), we give a lemma as follows:

$$\dot{x}_1 = A_1x_1(t) + A_2x_2(t) + \varpi,$$
$$\dot{x}_2 = f_2(x_2(t)), \qquad (7)$$
$$y(t) = Cx_1(t).$$

**Lemma 3** (see [20]). *For a given constant $\gamma > 0$, if there exist constants $\lambda^- > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, and $\beta > 0$, positive definite matrices $Q_{11}$, $R$, and $P$, a matrix $K$, and a function $W_2(x_2)$, such that the following conditions are satisfied for system (7)*

$$A_1^TP + PA_1 + Q_{11} + K^TRK + \gamma^{-2}PP + I + \lambda^- P < 0,$$
$$\alpha_1\|x_2\|^2 \le W(x_2) \le \alpha_2\|x_2\|^2, \qquad (8)$$
$$\frac{\partial W(x_2)}{\partial x}f(x_2) \le -\beta\|x_2\|^2,$$

*then the following inequality is satisfied*

$$\dot{V}(t) + \lambda_0^- V(t) + \Gamma(t) \le 0, \qquad (9)$$

*where $\lambda_0^- > 0$ is a constant, $V(t)$ is a Lyapunov function for system (7), and $\Gamma(t) = x^T(t)Qx(t) + u^T(t)Ru(t) - \gamma^2\varpi^T(t)\varpi(t)$ and $(u(t) = \overline{K}x(t), \overline{K} = [K \;\; K_2])$.*

## 3. Main Results

The objective of this paper is to design switching scheme such that system (1) has $H_\infty$ output tracking performance. We first consider the nonswitched nonlinear system (7).

**Lemma 4.** *For a given constant $\gamma > 0$, if there exist positive constants $\lambda^+$, $\alpha_1$, $\alpha_2$, and $\beta$, matrices $Q_{11} > 0$, $R > 0$, $P > 0$, and $K$, and a function $W_2(x_2)$, such that the following conditions are satisfied for system (7)*

$$A_1^TP + PA_1 + Q_{11} + K^TRK + \gamma^{-2}PP + I - \lambda^+ P < 0,$$
$$\alpha_1\|x_2\|^2 \le W(x_2) \le \alpha_2\|x_2\|^2, \qquad (10)$$
$$\frac{\partial W(x_2)}{\partial x}f(x_2) \le -\beta\|x_2\|^2,$$

*then following inequality is satisfied*

$$\dot{V}(t) - \lambda_0^+ V(t) + \Gamma(t) \le 0, \qquad (11)$$

*where $\lambda_0^+ > 0$ is a constant and $V(t)$ is a Lyapunov function for system (7).*

*Proof.* According to the proof of Lemma 3 and the condition of Lemma 4, we obtain

$$\begin{aligned}\dot{V}+\Gamma(t) &\leq x_1^T\lambda^+Px_1+\frac{\beta}{2\alpha_1}lW(x_2)-\frac{\beta}{2\alpha_1}lW(x_2)\\ &\quad -x_1^Tx_1+6p\|x_1\|\|x_2\|-l\beta x_2^Tx_2\\ &\quad +2qx_2^Tx_2\\ &\leq \lambda_0^+V-x_1^Tx_1+6p\|x_1\|\|x_2\|-\frac{3\beta}{2}lx_2^Tx_2\\ &\quad +2qx_2^Tx_2,\end{aligned}\tag{12}$$

where $\lambda_0^+=\max\{\lambda^+,\beta/2\alpha_1\}$ and $l>2(9p^2+2q)/3\beta$. Thus,

$$\dot{V}(t)+\Gamma(t)\leq\lambda_0^+V(t).\tag{13}$$

Consider the switched cascade nonlinear system (1). For the problem of $H_\infty$ output tracking, suppose that not all the linear parts of the subsystems of system (4) are stabilizable. Without loss of generality, we suppose that the first $r$ subsystems are stabilizable (the positive integer $r$ satisfies $1\leq r<N$) and the other subsystems are unstabilizable. For any switching law $\sigma(t)$ and any $0\leq\tau<t$, we let $T^-(\tau,t)$ (resp., $T^+(\tau,t)$) denote the total activation time of stabilizable (resp., unstabilizable) subsystems during $[\tau,t)$. Let $t_0<t_1<t_2<\cdots<t_k<\cdots$ be a specified sequence of time instants. □

A sufficient condition for $H_\infty$ output tracking performance of system (1) is proposed as follows.

**Theorem 5.** *Consider the switched cascade nonlinear system (1). For a given positive constant $\gamma$, if there exist positive constants $\lambda^-$, $\lambda^+$, $\alpha_1$, $\alpha_2$, and $\beta$, matrices $\overline{Q}_{11}>0$, $R>0$, $P_i>0$, and $K_i$, and functions $W_i(x_2)$, such that the following conditions are satisfied for the system (4)*

(1)

$$\overline{A}_{1i}^TP_i+P_i\overline{A}_{1i}+\overline{Q}_{11}+K_i^TRK_i+\gamma^{-2}P_iP_i+I+\lambda^-P_i<0,\quad 1\leq i\leq r,\tag{14}$$

$$\overline{A}_{1i}^TP_i+P_i\overline{A}_{1i}+\overline{Q}_{11}+K_i^TRK_i+\gamma^{-2}P_iP_i+I-\lambda^+P_i<0,\quad r<i\leq N,\tag{15}$$

$$P_i\leq\mu P_j,\quad i,j=1,\ldots,N\tag{16}$$

(2)

$$\alpha_1\|x_2\|^2\leq W_i(x_2)\leq\alpha_2\|x_2\|^2,\tag{17}$$

$$\frac{\partial W_i(x_2)}{\partial x}f_i(x_2)\leq-\beta\|x_2\|^2,\tag{18}$$

*then, under the average dwell time scheme*

$$\tau_a\geq\tau_a^*=\frac{\ln\widehat{\mu}}{\lambda}\tag{19}$$

*and the condition*

$$\frac{T^-}{T^+}\geq\frac{\lambda_0^++\lambda^*}{\lambda_0^--\lambda^*},\tag{20}$$

*the $H_\infty$ output tracking control problem is solved, where $\lambda_0^-=\min\{\lambda^-,\beta/2\alpha_2\}$, $\lambda_0^+=\max\{\lambda^+,\beta/2\alpha_1\}$, $\widehat{\mu}=\max\{\mu,\alpha_2/\alpha_1\}$, $\lambda\in(0,\lambda^*)$, and $\lambda^*\in(0,\lambda_0^-)$.*

*Proof.* Consider the following Lyapunov function for the switched system (4):

$$V(x)=V_{\sigma(t)}(x)=x_1^TP_{\sigma(t)}x_1+lW_{\sigma(t)}(x_2).\tag{21}$$

When $i$th subsystem is active, according to Lemmas 3 and 4 and the conditions of Theorem 5, there exist constants $\lambda_0^+$ and $\lambda_0^-$, such that the Lyapunov function (21) satisfies

$$\dot{V}(t)+\Gamma(t)\leq\begin{cases}-\lambda_0^-V(t) & \text{if } i\leq r,\\ \lambda_0^+V(t) & \text{if } i>r.\end{cases}\tag{22}$$

When $\varpi=0$, according to the above inequality, we have

$$\dot{V}(t)\leq\begin{cases}-\lambda_0^-V(t) & \text{if } i\leq r,\\ \lambda_0^+V(t) & \text{if } i>r.\end{cases}\tag{23}$$

From (16), (17), and (21), we get

$$V_i(t)\leq\widehat{\mu}V_j(t),\tag{24}$$

$$\widehat{\mu}=\max\left\{\mu,\frac{\alpha_2}{\alpha_1}\right\}.\tag{25}$$

Therefore, on every switching instant $t_j$, we get that

$$\begin{aligned}V(t)&\leq e^{\lambda_0^+T^+(t_j,t)-\lambda_0^-T^-(t_j,t)}V(t_j)\\ &\leq\widehat{\mu}e^{\lambda_0^+T^+(t_j,t)-\lambda_0^-T^-(t_j,t)}V(t_j^-)\\ &\leq\widehat{\mu}e^{\lambda_0^+T^+(t_{j-1},t)-\lambda_0^-T^-(t_{j-1},t)}V(t_{j-1})\leq\cdots\\ &\leq\widehat{\mu}^{N_\sigma(0,t)}e^{\lambda_0^+T^+(0,t)-\lambda_0^-T^-(0,t)}V(0).\end{aligned}\tag{26}$$

According to (20), we have

$$\lambda_0^+T^+(0,t)-\lambda_0^-T^-(0,t)\leq-\lambda^*t.\tag{27}$$

Because $N_\sigma(0,t)\leq t/\tau_a^*$, according to (19), it holds that

$$N_\sigma(0,t)\ln\widehat{\mu}\leq\lambda t.\tag{28}$$

Thus, $e^{\lambda_0^+T^+(0,t)-\lambda_0^-T^-(0,t)+N_\sigma(0,t)\ln\widehat{\mu}}\leq e^{-(\lambda^*-\lambda)t}$.

Therefore, $V(t)\leq e^{-(\lambda^*-\lambda)t}V(0)$. From (17), it easily holds that there exist constants $a_1$ and $a_2$ such that $a_1(\|\overline{x}_1\|^2+\|x_2\|^2)\leq V(t)\leq a_2(\|\overline{x}_1\|^2+\|x_2\|^2)$; furthermore, $\|\widetilde{x}(t)\|^2\leq(1/a_1)e^{-(\lambda^*-\lambda)t}V(0)\leq(a_2/a_1)e^{-(\lambda^*-\lambda)t}\|\widetilde{x}(0)\|^2$. Thus, $\|\widetilde{x}(t)\|\leq\sqrt{a_2/a_1}e^{-(\lambda^*-\lambda)t/2}\|\widetilde{x}(0)\|$, which implies asymptotic stability of the closed-loop system (4) when $\varpi=0$.

When $\varpi \neq 0$, $t \in [t_j, t_{j+1})$ $(0 \leq j \leq N_\sigma(0,t))$, by virtue of (22), we get

$$V(t) \leq \begin{cases} e^{-\lambda_0^-(t-t_j)} V(t_j) - \int_{t_j}^{t} e^{-\lambda_0^-(t-\tau)} \Gamma(\tau)\, d\tau & \text{if } i \leq r, \\ e^{\lambda_0^+(t-t_j)} V(t_j) - \int_{t_j}^{t} e^{\lambda_0^+(t-\tau)} \Gamma(\tau)\, d\tau & \text{if } i > r. \end{cases} \quad (29)$$

According to (20) and (24), on every switching instant $t_j$, we have

$$\begin{aligned} V(t) &\leq e^{\lambda_0^+ T^+(t_j,t) - \lambda_0^- T^-(t_j,t)} V(t_j) - \int_{t_j}^{t} e^{\lambda_0^+ T^+(\tau,t) - \lambda_0^- T^-(\tau,t)} \Gamma(\tau)\, d\tau \\ &\leq \hat{\mu} e^{\lambda_0^+ T^+(t_j,t) - \lambda_0^- T^-(t_j,t)} V(t_j^-) - \int_{t_j}^{t} e^{\lambda_0^+ T^+(\tau,t) - \lambda_0^- T^-(\tau,t)} \Gamma(\tau)\, d\tau \\ &\leq \hat{\mu} e^{\lambda_0^+ T^+(t_{j-1},t) - \lambda_0^- T^-(t_{j-1},t)} V(t_{j-1}) - \int_{t_j}^{t} e^{\lambda_0^+ T^+(\tau,t) - \lambda_0^- T^-(\tau,t)} \Gamma(\tau)\, d\tau \\ &\quad - \hat{\mu} e^{\lambda_0^+ T^+(t_j,t) - \lambda_0^- T^-(t_j,t)} - \int_{t_{j-1}}^{t_j} e^{\lambda_0^+ T^+(\tau,t) - \lambda_0^- T^-(\tau,t)} \Gamma(\tau)\, d\tau \leq \cdots \\ &\leq \hat{\mu}^{N_\sigma(0,t)} e^{\lambda_0^+ T^+(0,t) - \lambda_0^- T^-(0,t)} V(0) - \int_{0}^{t} \hat{\mu}^{N_\sigma(\tau,t)} e^{\lambda_0^+ T^+(\tau,t) - \lambda_0^- T^-(\tau,t)} \Gamma(\tau)\, d\tau \\ &= e^{\lambda_0^+ T^+(0,t) - \lambda_0^- T^-(0,t) + N_\sigma(0,t)\ln\hat{\mu}} V(0) - \int_{0}^{t} e^{\lambda_0^+ T^+(\tau,t) - \lambda_0^- T^-(\tau,t) + N_\sigma(\tau,t)\ln\hat{\mu}} \Gamma(\tau)\, d\tau. \end{aligned} \quad (30)$$

The above inequality and conditions (19)-(20) give that

$$\begin{aligned} &\int_0^t e^{-\lambda_0^-(t-\tau) - \lambda\tau} \left[\tilde{x}^T(\tau) Q \tilde{x}(\tau) + u^T(\tau) R u(\tau)\right] d\tau \\ &\quad \leq e^{-\lambda^* t} V(0) + \gamma^2 \int_0^t e^{-\lambda^*(t-\tau)} \varpi^T(\tau) \varpi(\tau)\, d\tau. \end{aligned} \quad (31)$$

Integrating both sides of the above inequality from $t = 0$ to $\infty$, we obtain

$$\begin{aligned} &\int_0^\infty e^{-\lambda\tau} \left[\tilde{x}^T(\tau) Q \tilde{x}(\tau) + u^T(\tau) R u(\tau)\right] d\tau \\ &\quad \leq \frac{\lambda_0^-}{\lambda^*} V(0) + \frac{\lambda_0^-}{\lambda^*} \gamma^2 \int_0^\infty \varpi^T(\tau) \varpi(\tau)\, d\tau, \end{aligned} \quad (32)$$

where $\tilde{x}^T(t) Q \tilde{x}(t) = x^T(t) Q_{11} x(t) + (\int_0^t e(t)dt) Q_{22} (\int_0^t e(t)dt)$, $Q = \operatorname{diag}\{Q_{11}^1 \ \ Q_{22} \ \ Q_2\}$, $\overline{Q}_{11} = \operatorname{diag}\{Q_{11}^1 \ \ Q_{22}\}$, and $Q_{11} = \operatorname{diag}\{Q_{11}^1 \ \ Q_2\}$.

Therefore,

$$J_L \leq \frac{\lambda_0^-}{\lambda^*} V(0) + \frac{\lambda_0^-}{\lambda^*} \gamma^2 \int_0^\infty \varpi^T(\tau) \varpi(\tau)\, d\tau. \quad (33)$$

To conclude, the output tracking problem is solved. □

## 4. Example

In this section, we will illustrate the effectiveness of the proposed results by an example.

Consider the cascade switched nonlinear system (4) with $A_{11} = \left[\begin{smallmatrix} -1.05 & 135.05 \\ -18.75 & -14.75 \end{smallmatrix}\right]$, $A_{21} = \left[\begin{smallmatrix} 0.5 \\ 1 \end{smallmatrix}\right]$, $B_1 = \left[\begin{smallmatrix} -100 \\ -5 \end{smallmatrix}\right]$, $C_1 = [25 \ \ -2]$, $A_{12} = \left[\begin{smallmatrix} -5.32 & -0.7 \\ -21.8 & -4.77 \end{smallmatrix}\right]$, $A_{22} = \left[\begin{smallmatrix} -0.5 \\ 1 \end{smallmatrix}\right]$, $B_2 = \left[\begin{smallmatrix} 7 \\ 0.8 \end{smallmatrix}\right]$, $C_2 = [25 \ \ -2]$, $f_{21} = -3x_2 - x_2^3$, $f_{22} = -2.5x_2 - x_2^3$, and $\gamma = 1$, $R = 0.1$, $y_d = e^{-2t}$, $Q = 0.1 \times I_{N\times N}$, and $\omega(t) = [-e^{-2t} \ \ e^{-t}]^T$.

Let $\lambda^+ = 3$, let $\lambda^- = 2$, and let $\mu = 1.2$; solving inequalities (14)–(16) yields

$$\begin{aligned} P_1 &= \begin{bmatrix} 3.1771 & -0.0228 & 1.2524 \\ -0.0228 & 3.1885 & 0.5618 \\ 1.2524 & 0.5618 & 1.7399 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 1.8751 & 0.6126 & 0.3827 \\ 0.6126 & 3.2298 & 1.5883 \\ 0.3827 & 1.5883 & 1.4901 \end{bmatrix}, \\ K_1 &= [0.24 \ \ 1.15 \ \ 0.05], \\ K_2 &= [1.02 \ \ 2.15 \ \ -0.45]. \end{aligned} \quad (34)$$

Therefore, $\overline{K}_1 = [0.24 \ \ 1.15 \ \ 0.05 \ \ 0.65]$ and $\overline{K}_2 = [1.02 \ \ 2.15 \ \ -0.45 \ \ 0.35]$.

Obviously, the first subsystem is stabilizable and the second subsystem is unstabilizable.

We choose $W_1(x_2) = 0.5x_2^2$ and $W_2(x_2) = 0.8x_2^2$ and define $V(x) = \overline{x}_1^T P_i \overline{x}_1 + l W_i(x_2)$. Selecting $\alpha_1 = 0.5$, $\alpha_1 = 0.8$, and $\beta = 3$, we can obtain $\hat{\mu} = \max\{\mu, \alpha_2/\alpha_1\} = 1.6$, $\lambda_0^- = \min\{\lambda^-, \beta/2\alpha_2\} = 1.875$, and $\lambda_0^+ = \max\{\lambda^+, \beta/2\alpha_1\} = 3$. $l = 50$. Let $\lambda^* = 0.8$ and let $\lambda = 0.5$; then, $\tau_a{}^* = 0.56$ and $T^-/T^+ \geq 1.92$.

By virtue of Theorem 5, the output tracking problem is solvable, and the simulation results are depicted in Figures 1–3.

## 5. Conclusion

The $H_\infty$ output tracking problem for switched cascade nonlinear systems with the stabilizable linear parts and unstabilizable linear parts has been studied. Sufficient conditions for the solvability of the $H_\infty$ output tracking problem are demonstrated. The average dwell time technique is utilized

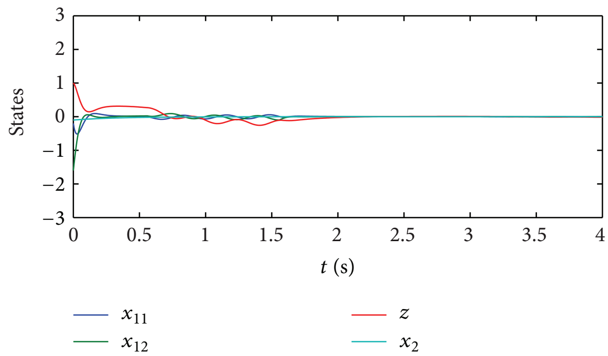


Figure 1: State response of the switched system.

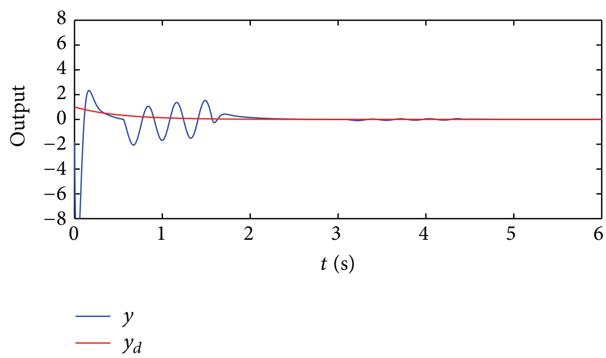


Figure 2: Output $y$ and the reference input $y_d$.

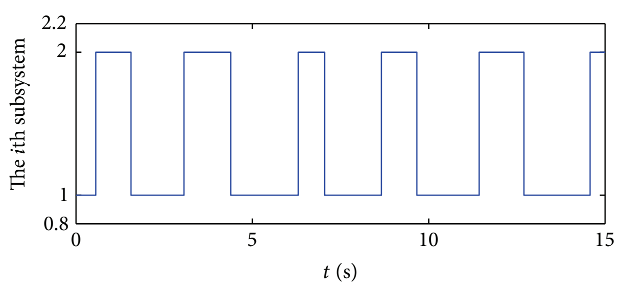


Figure 3: The switching signal.

to obtain the main results. A numerical example shows the effectiveness of the proposed switching schemes.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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