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Review Article

Gravity/Fluid Correspondence and Its Application on Bulk Gravity with $U(1)$ Gauge Field

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As the long wavelength limit of the AdS/CFT correspondence, the gravity/fluid correspondence has been shown to be a useful tool for extracting properties of the fluid on the boundary dual to the gravity in the bulk. In this paper, after briefly reviewing the algorithm of gravity/fluid correspondence, we discuss the results of its application on bulk gravity with a $U(1)$ gauge field. In the presence of a $U(1)$ gauge field, the dual fluid possesses more interesting properties such as its charge current. Furthermore, an external field A_μ^{ext} could affect the charge current, and the $U(1)$ Chern-Simons term also induces extra structures to the dual current giving anomalous transport coefficients.

1. Introduction

The AdS/CFT correspondence [1–4] provides a remarkable connection between a gravitational theory and a quantum field theory. According to this correspondence, the gravitational theory in an asymptotically AdS spacetime can be reformulated in terms of a quantum field theory on its boundary. In particular, the dynamics of a classical gravitational theory in the bulk can be mapped into a strongly coupled quantum field theory on the boundary. Therefore, the AdS/CFT correspondence provides a useful tool of investigating the strongly coupled field theory from the dual classical gravitational theory [5, 6].

Since the discovery of the AdS/CFT correspondence, there has been much work studying the hydrodynamical behavior of the dual quantum field theory using this correspondence [7–10]. A particular example is the computation of shear viscosity [11–43]. The reason that the correspondence applies here is because hydrodynamics is an effective description of interacting quantum field theory in the long wavelength limit, that is, when the length scales under consideration are much larger than the correlation length of the quantum field theory. Later, the long wavelength limit of the AdS/CFT correspondence has been developed as the gravity/fluid correspondence [44, 45], which could provide a systematic mapping of bulk gravity and boundary fluid. Through the gravity/fluid correspondence, one can construct the stress-energy tensor of the fluid order by order from the derivative expansion of the gravity solution and then extract certain transport coefficients of the fluid. For example, the shear viscosity η, entropy density s, and thus their ratio η/s can be calculated from the first-order stress-energy tensor [46–55]. In addition, in the presence of extra gauge fields in the bulk, the correspondence also allows us to construct the conserved charge current in the dual fluid and thereby allows us to extract the thermal and electrical conductivities of the dual fluid [46–48]. It should be pointed out that the presence of an external field A_μ^{ext} is needed if one wants to obtain the electrical conductivity [46, 49, 53, 54]. Furthermore, it has also been shown that introducing the topological Chern-Simons term in the bulk gravity brings interesting features such as the chiral magnetic effect (CME) and the chiral vortical effect (CVE) into the dual fluid [49, 50, 53, 54, 56, 57]. The Chern-Simons term was first discussed in Maxwell theory in three dimensions where it renders the gauge theory massive [58]. Such terms also affect the phase transition of holographic superconductors in four dimensions [59] and stability of the Reissner-Nordstrom (RN) black holes in AdS space in five dimensions [60]. In this paper, we will discuss the results of transport coefficients of dual boundary fluid in the presence of a $U(1)$ gauge field in the bulk.

The rest of this paper is organized as follows. We start Section 2 by briefly reviewing the algorithm of gravity/fluid correspondence. In Section 3, we discuss the application of gravity/fluid correspondence in studying fluid dynamics dual to bulk gravity in the presence of a $U(1)$ gauge field. One can see that both the stress tensor and the conserved charge current of the dual fluid can be extracted. Moreover, adding the external field A_μ^{ext} and Chern-Simons term in the bulk also introduces extra structures to the charge current of the dual fluid. The conclusion and discussion are given in Section 4.

2. Algorithm of Gravity/Fluid Correspondence

After several years' development, there exists now a well-formulated algorithm of how to extract information on the boundary fluid from the dual gravity using the gravity/fluid correspondence. In this section, we just give a brief introduction and more details can be found in [44, 45]. For illustrative purposes, let us take the 5-dimensional Einstein gravity in the bulk as an example. Its action can be written as

$$I = \frac{1}{16\pi G}\int_{\mathcal{M}} d^5x\sqrt{-g^{(5)}}\,(R-2\Lambda)\,, \tag{1}$$

where R is the Ricci scalar and Λ is a negative cosmological constant $\Lambda = -6/\ell^2$. For later convenience, we set $\ell = 1$ and $16\pi G = 1$. The equation of motion then reads

$$R_{MN} - \frac{1}{2}Rg_{MN} + \Lambda g_{MN} = 0. \tag{2}$$

The starting point is the black brane solution written in the Eddington-Finkelstein coordinate

$$\begin{aligned} ds^2 &= g_{MN}dx^M dx^N \\ &= 2H(r,b)\,dv dr - r^2 f(r,b)\,dv^2 \\ &\quad + r^2\left(dx^2+dy^2+dz^2\right), \end{aligned} \tag{3}$$

where $x^M = (v, x_i, r)$ and b is a constant related to the mass of the black brane. This black brane solution is asymptotical to the AdS_5 solution, which means $H(r) \to 1$, $f(r) \to 1$ when r approaches infinity. The advantage of using the Eddington-Finkelstein coordinate system is that it avoids the coordinate singularity.

By boosting the black brane solution in (3), one obtains a solution with more parameters which can then be related to the degrees of freedom of the dual boundary fluid

$$\begin{aligned} ds^2 &= 2H(r,b)\,u_\mu dx^\mu dr - r^2 f(r,b)\left(u_\mu dx^\mu\right)^2 \\ &\quad + r^2 P_{\mu\nu}dx^\mu dx^\nu \end{aligned} \tag{4}$$

with

$$\begin{gathered} u^v = \frac{1}{\sqrt{1-\beta_i^2}}, \qquad u^i = \frac{\beta_i}{\sqrt{1-\beta_i^2}}, \\ P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu, \end{gathered} \tag{5}$$

where $x^\mu = (v, x_i)$, velocities β^i are constants, $P_{\mu\nu}$ is the projector onto spatial directions, and the indices in the boundary are raised and lowered with the Minkowski metric $\eta_{\mu\nu}$. The metric (4) describes the uniform boosted black brane moving at velocity β^i [44].

The transport coefficients of the dual fluid can be extracted by perturbing the system away from equilibrium. On the gravity side, this can be achieved by promoting the parameters in the boosted black brane solution (4) to functions of boundary coordinates x^μ. Since the parameters now depend on the boundary coordinates, the metric (4) is no longer a solution of the equation of motion (2); extra correction terms are needed to make the new metric a solution. It is useful to define the following tensor:

$$W_{IJ} = R_{IJ} + 4g_{IJ}, \tag{6}$$

which arises from the left hand side of (2). When the parameters become functions of boundary coordinates x^μ, $W_{\mu\nu}$ no longer vanishes and is proportional to the derivatives of the parameters. These terms are the source terms which will be canceled by extra correction terms introduced into the metric. If we expand the parameters around $x^\mu = 0$ to the first order

$$\beta_i = \partial_\mu \beta_i|_{x^\mu=0}x^\mu, \qquad b = b(0) + \partial_\mu b|_{x^\mu=0}x^\mu, \tag{7}$$

where we have assumed $\beta^i(0) = 0$, we find the first-order source terms by inserting the metric (4) with (7) into W_{IJ}. By choosing an appropriate gauge like the background field gauge in [44] (G represents the metric)

$$G_{rr} = 0, \qquad G_{r\mu} \propto u_\mu, \qquad \mathrm{Tr}\left(\left(\mathrm{G}^{(0)}\right)^{-1}\mathrm{G}^{(1)}\right) = 0, \tag{8}$$

and taking into account the spatial SO(3) symmetry preserved in the background metric (3), the first-order correction terms around $x^\mu = 0$ can be written as

$$\begin{aligned} ds^{(1)2} &= \frac{k(r)}{r^2}dv^2 + 2h(r)\,dvdr + 2\frac{j_i(r)}{r^2}dvdx^i \\ &\quad + r^2\left(\alpha_{ij}(r) - \frac{2}{3}h(r)\,\delta_{ij}\right)dx^i dx^j. \end{aligned} \tag{9}$$

The first-order perturbative solution can then be obtained by requiring a cancelation of the source terms and the correction terms. Note that, after obtaining the solution around $x^\mu = 0$, one could convert it into a covariant form so that it applies to other spacetime points [44].

Given the first-order perturbative solution for the bulk gravity, we are able to extract information of the dual fluid using the gravity/fluid correspondence. According to the correspondence, the stress tensor of dual fluid $\tau_{\mu\nu}$ can be obtained from the following relation [61]:

$$\sqrt{-h}h^{\mu\nu}\left\langle\tau_{\nu\sigma}\right\rangle=\lim_{r\to\infty}\sqrt{-\gamma}\gamma^{\mu\nu}T_{\nu\sigma}, \tag{10}$$

where $h^{\mu\nu}$ is the background metric upon which the dual field theory resides, $\gamma^{\mu\nu}$ is the boundary metric obtained from the well-known ADM decomposition

$$ds^2=\gamma_{\mu\nu}\left(dx^\mu+V^\mu dr\right)\left(dx^\nu+V^\nu dr\right)+N^2dr^2, \tag{11}$$

and $T_{\mu\nu}$ is the boundary stress tensor defined as

$$T_{\mu\nu}=\frac{2}{\sqrt{-\gamma}}\frac{\delta}{\delta\gamma^{\mu\nu}}\left(I+I_{\text{sur}}+I_{\text{ct}}^0\right), \tag{12}$$

where

$$I_{\text{sur}}=-\frac{1}{8\pi G}\int_{\partial\mathcal{M}}d^4x\sqrt{-\gamma}K \tag{13}$$

is the Gibbons-Hawking surface term and K is the trace of the extrinsic curvature $K_{\mu\nu}$ of the boundary, which is given by $K_{\mu\nu}=-(1/2)(\nabla_\mu n_\nu+\nabla_\nu n_\mu)$ with n^μ being the normal vector of the constant hypersurface $r=r_c$ pointing toward increasing r direction. In addition,

$$I_{\text{ct}}^0=\frac{1}{8\pi G}\int_{\partial\mathcal{M}}d^4x\sqrt{-\gamma}\left[-3-\frac{\mathsf{R}}{4}\right] \tag{14}$$

is the boundary counterterm and R is the curvature scalar associated with the induced metric on the boundary γ_{ab} [62–65].

From (12), the boundary stress tensor is

$$T_{\mu\nu}=\frac{1}{8\pi G}\left[K_{\mu\nu}-\gamma_{\mu\nu}K-3\gamma_{\mu\nu}+\frac{1}{2}\left(\mathsf{R}_{\mu\nu}-\frac{1}{2}\gamma_{\mu\nu}\mathsf{R}\right)\right], \tag{15}$$

which can be used to compute the boundary stress tensor of the first-order perturbative black brane solution.

The background metric upon which the dual field theory resides usually is chosen as $h_{\mu\nu}=\lim_{r\to\infty}(\ell^2/r^2)\gamma_{\mu\nu}$ [61], and

$$ds^2=h_{\mu\nu}dx^\mu dx^\nu=-dv^2+dx^2+dy^2+dz^2 \tag{16}$$

is the Minkowski metric. From (10) and the result for the boundary stress tensor $T_{\mu\nu}$, the first-order stress tensor of the dual fluid $\tau_{\mu\nu}$ can be found to be

$$\tau_{\mu\nu}=P\left(\eta_{\mu\nu}+4u_\mu u_\nu\right)-2\eta\sigma_{\mu\nu}, \tag{17}$$

from which one immediately reads off the pressure and viscosity of the dual fluid.

3. Applications on the Bulk Gravity with $U(1)$ Gauge Field

In the previous section, we illustrate the algorithm of the gravity/fluid correspondence with a simple pure gravity model, which yields a viscosity for the dual fluid on the boundary. In general, the gravity/fluid correspondence could be applied to studying fluid dynamics dual to various bulk gravity configurations with matter fields in the bulk gravity, and it has been found that after adding the matter fields to the bulk gravity, more interesting properties of the dual fluid could be extracted. We discuss here the impact on the dual boundary fluid of adding a $U(1)$ gauge field to the bulk.

3.1. The Simplest Case. The action with a $U(1)$ gauge field can be written as [47, 48]

$$\begin{aligned}I=&\frac{1}{16\pi G}\int_{\mathcal{M}}d^5x\sqrt{-g^{(5)}}\left(R-2\Lambda\right)\\&-\frac{1}{4g^2}\int_{\mathcal{M}}d^5x\sqrt{-g^{(5)}}F^2,\end{aligned} \tag{18}$$

which gives the following equations of motion:

$$\begin{gathered}R_{AB}-\frac{1}{2}Rg_{AB}+\Lambda g_{AB}-\frac{1}{2g^2}\left(F_{AC}F_B^C-\frac{1}{4}g_{AB}F^2\right)=0,\\\nabla_BF_A^B=0.\end{gathered} \tag{19}$$

Following the general algorithm described in Section 2, we start with the five-dimensional charged RN-AdS black brane solution [66–68]

$$ds^2=\frac{dr^2}{r^2f(r)}+r^2\left(\sum_{i=1}^3dx_i^2\right)-r^2f(r)\,dt^2, \tag{20}$$

where

$$f(r)=1-\frac{2M}{r^4}+\frac{Q^2}{r^6},\qquad F=-g\frac{2\sqrt{3}Q}{r^3}dt\wedge dr. \tag{21}$$

The outer horizon of the black brane is located at $r=r_+$, where r_+ is the largest root of $f(r)=0$ and its Hawking temperature is

$$T_+=\left.\frac{\left(r^2f(r)\right)'}{4\pi}\right|_{r=r_+}=\frac{1}{2\pi r_+^3}\left(4M-\frac{3Q^2}{r_+^2}\right). \tag{22}$$

Writing the above black brane solution in the Eddington-Finkelstein coordinate system, one has

$$\begin{aligned}ds^2&=-r^2f(r)\,dv^2+2dvdr+r^2\left(dx^2+dy^2+dz^2\right),\\F&=-g\frac{2\sqrt{3}Q}{r^3}dv\wedge dr,\end{aligned} \tag{23}$$

where $v=t+r_*$ and r_* is the tortoise coordinate satisfying $dr_*=dr/(r^2f)$. The boosted solution can then be written as

$$ds^2=-r^2f(r)\left(u_\mu dx^\mu\right)^2-2u_\mu dx^\mu dr+r^2P_{\mu\nu}dx^\mu dx^\nu, \tag{24}$$

$$F=-g\frac{2\sqrt{3}Q}{r^3}u_\mu dx^\mu\wedge dr,\qquad A=-\frac{\sqrt{3}gQ}{r^2}u_\mu dx^\mu \tag{25}$$

with

$$u^v = \frac{1}{\sqrt{1-\beta_i^2}}, \qquad u^i = \frac{\beta_i}{\sqrt{1-\beta_i^2}}, \qquad P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu, \tag{26}$$

where velocities β^i, M, and Q are constants.

The procedure of solving for the first-order perturbative solution is essentially the same as in the previous section. Now the tensors inducing the source terms become

$$\begin{aligned} W_{IJ} &= R_{IJ} + 4g_{IJ} + \frac{1}{2g^2}\left(F_{IK}F_J^K + \frac{1}{6}g_{IJ}F^2\right), \\ W_A &= \nabla_B F_A^B. \end{aligned} \tag{27}$$

Letting the parameters be x^μ-dependent and expanding them around $x^\mu = 0$ to the first-order, one has

$$\begin{aligned} \beta_i = \partial_\mu \beta_i|_{x^\mu=0} x^\mu, \qquad M = M(0) + \partial_\mu M|_{x^\mu=0} x^\mu, \\ Q = Q(0) + \partial_\mu Q|_{x^\mu=0} x^\mu, \end{aligned} \tag{28}$$

and the corresponding required first-order correction terms around $x^\mu = 0$ are then given by

$$\begin{aligned} ds^{(1)^2} &= \frac{k(r)}{r^2}dv^2 + 2h(r)\,dv\,dr + 2\frac{j_i(r)}{r^2}dv\,dx^i \\ &\quad + r^2\left(\alpha_{ij}(r) - \frac{2}{3}h(r)\,\delta_{ij}\right)dx^i dx^j, \\ A^{(1)} &= a_v(r)\,dv + a_i(r)\,dx^i. \end{aligned} \tag{29}$$

Note that the gauge $a_r(r) = 0$ has been chosen. Therefore, the first-order perturbative gravitational and Maxwell solution could be achieved by requiring a cancelation of the source terms and the correction terms.

The solutions then allow one to compute the boundary stress tensor. According to the dictionary of fluid-gravity correspondence, the first-order stress tensor of the dual fluid $\tau_{\mu\nu}$ then reads

$$\begin{aligned} \tau_{\mu\nu} &= \frac{1}{16\pi G}\left[\frac{2M}{\ell^3}\left(\eta_{\mu\nu} + 4u_\mu u_\nu\right) - \frac{2r_+^3}{\ell^3}\sigma_{\mu\nu}\right] \\ &= P\left(\eta_{\mu\nu} + 4u_\mu u_\nu\right) - 2\eta\sigma_{\mu\nu} \end{aligned} \tag{30}$$

with the pressure and viscosity given by

$$P = \frac{M}{8\pi G\ell^3}, \qquad \eta = \frac{r_+^3}{16\pi G\ell^3}, \tag{31}$$

where the coupling $16\pi G$ and ℓ are recovered for comparison purposes.

Following the AdS/CFT correspondence, the dual operator of $\widetilde{A}_\mu$ (here $\widetilde{A}_\mu$ is the boundary value of A_μ projected on the boundary) is the charge current of the dual fluid. One can thus extract the charge current of the dual fluid as

$$J^\mu = \lim_{r\to\infty}\frac{r^4}{\sqrt{-\gamma}}\frac{\delta S_{\text{cl}}}{\delta\widetilde{A}_\mu} = \lim_{r\to\infty}\frac{r^4 N}{g^2}F^{r\mu}, \tag{32}$$

where the factor r^4 comes from the conformal transformation. From this equation and the result for the first-order perturbative solution, one obtains the charge current

$$J^\mu = J^\mu_{(0)} + J^\mu_{(1)}, \tag{33}$$

with the zeroth-order particle number current

$$J^\mu_{(0)} = \frac{2\sqrt{3}Q}{g}u^\mu := nu^\mu \tag{34}$$

and the first-order charge current

$$J^\mu_{(1)} = -\kappa P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right), \tag{35}$$

where $\mu = \sqrt{3}gQ/r_+^2$ is the chemical potential and $\kappa = \pi^2T^3r_+^7/4g^2M^2$ is the thermal conductivity. From this simplest case, it is obvious that after adding the $U(1)$ gauge field in the bulk, the charge current of the dual fluid could also be extracted.

3.2. In the Presence of an External Field A_μ^{ext}. It has been found that in the presence of an external field A_μ^{ext}, the charge current of dual fluid has extra structures, whereas this external field does not change the stress tensor of dual fluid [46, 49, 53, 54]. The key point is that a constant external field A_μ^{ext} could be added into (25) like [46]:

$$A = \left(A_\mu^{\text{ext}} - \frac{\sqrt{3}gQ}{r^2}u_\mu\right)dx^\mu. \tag{36}$$

It is obvious (36) is still the solution since the equations of motion in the previous subsection involve the field strength only.

After requiring the external field A_μ^{ext} to be a function of x^μ, we can expand it around $x^\mu = 0$ to the first order as in (28),

$$A_\mu^{\text{ext}} = A_\mu^{\text{ext}}(0) + \partial_\nu A_\mu^{\text{ext}}|_{x^\mu=0}x^\nu. \tag{37}$$

The corresponding correction terms around $x^\mu = 0$ could be the same as (29). One finds that the external field A_μ^{ext} changes the coefficients $a_i(r)$, $j_i(r)$ in (29) and hence change the charge current of dual fluid [46, 53, 54]. Finally, the result of the first-order charge current now becomes

$$J^\mu_{(1)} = -\kappa P^{\mu\nu}\partial_\nu\frac{\mu}{T} + \sigma_E u^\lambda F^{\text{ext}}_{\lambda\mu}, \tag{38}$$

where

$$\kappa = \frac{\pi^2T^3r_+^7}{4g^2M^2}, \qquad \sigma_E = \frac{\pi^2T^2r_+^7}{4g^2M^2} \tag{39}$$

are the thermal and electrical conductivity, respectively. Obviously, a new structure related to the electrical conductivity appears in the first-order charge current.

3.3. Adding $U(1)$ Chern-Simons Term in the Bulk. Recently Chern-Simons terms have attracted much attention, as introducing such terms in the bulk leads to anomalous transport coefficients on the dual fluid side. The bulk gravity action with a $U(1)$ Chern-Simons term is [47–49, 53, 54]

$$I = \frac{1}{16\pi G}\int_{\mathscr{M}} d^5x\sqrt{-g^{(5)}}\,(R-2\Lambda) - \frac{1}{4g^2}\int_{\mathscr{M}} d^5x\sqrt{-g^{(5)}}\left(F^2 + \frac{4\kappa_{cs}}{3}\epsilon^{LABCD}A_L F_{AB}F_{CD}\right), \tag{40}$$

and the equations of motion become

$$R_{AB} - \frac{1}{2}Rg_{AB} + \Lambda g_{AB} - \frac{1}{2g^2}\left(F_{AC}F_B{}^C - \frac{1}{4}g_{AB}F^2\right) = 0,$$
$$\nabla_B F_A^B - \kappa_{cs}\epsilon_{ABCDE}F^{BC}F^{DE} = 0. \tag{41}$$

Although the Maxwell equation is different from (19), it turns out that the five-dimensional charged RN-AdS black brane solution (24) and (25) still solves (41). In the following we also insert an external field A_μ^{ext} into the bulk solution. Now the tensors inducing the source terms become

$$W_{IJ} = R_{IJ} + 4g_{IJ} + \frac{1}{2g^2}\left(F_{IK}F_J^K + \frac{1}{6}g_{IJ}F^2\right), \tag{42}$$

$$W_A = \nabla_B F_A^B - \kappa_{cs}\epsilon_{ABCDE}F^{BC}F^{DE}. \tag{43}$$

One can solve the first-order perturbative gravitational and Maxwell solution as before. Note that the Chern-Simons term appears in the second term of W_A in (43), while it does not change W_{IJ} in (42). As a consequence, $h(r)$, $k(r)$, α_{ij} are the same as in the case without the Chern-Simons term, while $a_i(r)$ and $j_i(r)$ are different [53, 54]. Therefore, the Chern-Simons term will not change the first-order stress tensor of the dual fluid $\tau_{\mu\nu}$. However, in the presence of the Chern-Simons term, the charge current of the dual fluid can be computed as

$$J^\mu = \lim_{r\to\infty}\frac{r^4}{\sqrt{-\gamma}}\frac{\delta S_{\text{cl}}}{\delta\widetilde{A}_\mu} = \lim_{r\to\infty}\frac{r^4 N}{g^2}\left(F^{r\mu} + \frac{4\kappa_{cs}}{3}\epsilon^{r\mu\rho\sigma\tau}A_\rho F_{\sigma\tau}\right). \tag{44}$$

Although the zeroth-order particle number current is the same as (34), the first-order charge current becomes [47–49, 53, 54]

$$J_{(1)}^\mu = -\kappa P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma_E E^\mu + \sigma_B B^\mu + \xi\omega^\mu + \ell\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}^{\text{ext}}A_\nu^{\text{ext}}, \tag{45}$$

where

$$\kappa = \frac{\pi^2T^3r_+^7}{4g^2M^2}, \qquad \sigma_E = \frac{\pi^2T^2r_+^7}{4g^2M^2},$$
$$\sigma_B = -\frac{\sqrt{3}\kappa_{cs}Q\left(3r_+^4 + 2M\right)}{gMr_+^2}, \qquad \xi = \frac{6\kappa_{cs}Q^2}{M}, \tag{46}$$
$$\ell = \frac{4\kappa_{cs}}{3g^2}, \qquad E^\mu = u^\lambda F_{\lambda^\mu}^{\text{ext}},$$
$$B^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}^{\text{ext}}, \qquad \omega^\mu = \epsilon^{\mu\nu\rho\sigma}u_\nu\partial_\rho u_\sigma.$$

Obviously, the Chern-Simons term changes the charge current of dual fluid, as can be seen from the last three terms in (45), whose coefficients are proportional to κ_{cs}. In addition, it should be emphasized that the external field A_μ^{ext} and its boundary value are also important, as there are three terms related to the external field A_μ^{ext}, and if one chooses $A_\mu^{\text{ext}}(0) = 0$ in (37), the first-order charge current becomes

$$J_{(1)}^\mu = -\kappa P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma_E E^\mu + \sigma_B B^\mu + \xi\omega^\mu, \tag{47}$$

which is the well-known result related to triangle anomalies [49]. Moreover, if one chooses $A_\mu^{\text{ext}}(0) = (C, 0, 0, 0)$, although the first-order current (45) becomes the same as (47), the chiral magnetic conductivity is given as $\sigma_B = -(\sqrt{3}\kappa_{cs}Q(3r_+^4 + 2M)/gMr_+^2) + 2\ell C$, where the second term is the extra contribution from the last term in (45) [54, 57].

4. Conclusion and Discussion

In this paper, we illustrated the algorithm of gravity/fluid correspondence to extracting transport coefficients of dual fluid from the bulk gravity with a simple pure gravity configuration and discussed its application on various bulk gravity configurations in the presence of a $U(1)$ gauge field. On the fluid side, the transport coefficients can be obtained by perturbing the system away from the equilibrium. On the gravity side, this can be achieved by promoting the parameters in the boosted black brane solution to functions of boundary coordinates. Extra correction terms are then needed to render the boundary-coordinate-dependent metric a solution of Einstein equations. This determines the extra correction terms and thus the perturbative solution of the bulk gravity, from which the transport coefficients of dual fluid can be determined using the dictionary of gravity/fluid correspondence.

As expected, adding a $U(1)$ gauge field to the bulk gravity induces charge current for the dual fluid. One also finds that the external field A_μ^{ext} and $U(1)$ Chern-Simons term in the bulk could affect the charge current. Some recent work investigated the dual fluid on a finite cutoff surface and showed that the electric and magnetic conductivity could still show up even without the external field A_μ^{ext}. A simple interpretation is that the projection of A_μ onto the cutoff surface naturally introduces its boundary value $\widetilde{A}_\mu$ as the external field A_μ^{ext} [55]. In addition, it should be emphasized that the introduction of Chern-Simons term in the bulk usually can lead to extra structures which generate anomalous transport coefficients [69]. For example, after adding the Chern-Simons term of the Maxwell field in the bulk, the conserved current J^μ contains additional terms related to the anomalous magnetic and vortical effects [47–49, 53, 54]. Moreover, it has been shown that if the gravitational Chern-Simons term is added in the bulk, a new term related to the Hall viscosity will appear in the stress tensor of the dual fluid [70–72]. Therefore, it will be interesting to further investigate the effects of the Chern-Simons terms in other modified gravity configurations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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