Han et al. *Advances in Difference Equations* (2015) 2015:360
DOI 10.1186/s13662-015-0657-6
Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Global stability of May cooperative system with feedback controls

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Abstract

In this paper, a May cooperative system with feedback controls is proposed and studied. The dynamic behaviors of the system are discussed by using the Lyapunov function method. If $b_i \neq 0$, $i = 1, 2$, we show that feedback control variables have no influence on the global stability of the unique positive equilibrium of the system, which means that feedback control variables only change the position of the positive equilibrium and retain its global stability property. If $b_i = 0$, $i = 1, 2$, we can make the system which has a unique globally stable equilibrium or has unboundedly large solutions become globally stable. Some examples are given to illustrate the feasibility of the main results.

MSC: 34C25; 92D25; 34D20; 34D40

Keywords: May cooperative system; feedback controls; Lyapunov function; global stability

1 Introduction

May [1] suggested the following set of equations to describe a pair of mutualists:

$$\begin{aligned} \frac{dN_1}{dt} &= rN_1\left[1 - \frac{N_1}{K_1 + \alpha N_2}\right], \\ \frac{dN_2}{dt} &= rN_2\left[1 - \frac{N_2}{K_2 + \beta N_1}\right], \end{aligned} \tag{1.1}$$

where N_1, N_2 are the densities of the species, respectively. r, K_i, α, β, $i = 1, 2$, are positive constants. He showed that if the coefficients of system (1.1) satisfy

$$\alpha\beta < 1, \tag{1.2}$$

then system (1.1) has a global stability equilibrium point (N_1^*, N_2^*), where

$$N_1^* = \frac{K_1 + \alpha K_2}{1 - \alpha\beta}, \qquad N_2^* = \frac{K_2 + \beta K_1}{1 - \alpha\beta}.$$

If the amount of mutualistic interaction satisfies

$$\alpha\beta \geq 1, \tag{1.3}$$

then system (1.1) will 'run away', with both populations growing unboundedly large. So May brought the density restriction to the system and proposed the following system [2]:

$$\begin{aligned} \dot{x} &= r_1 x\left[1 - \frac{x}{K_1 + \alpha_1 y} - \varepsilon_1 x\right], \\ \dot{y} &= r_2 y\left[1 - \frac{y}{K_2 + \alpha_2 x} - \varepsilon_2 y\right], \end{aligned} \tag{1.4}$$

where r_i, K_i, α_i, ε_i, $i = 1, 2$ are positive constants. He showed that system (1.4) has a global stability equilibrium point.

Since that many excellent results concerned with the dynamic behaviors of May cooperative system are obtained. Cui and Chen [3] addressed the nonautonomous system

$$\begin{aligned} \dot{u} &= r_1(t)u\left[1 - \frac{u}{a_1(t) + b_1(t)v} - c_1(t)u\right], \\ \dot{v} &= r_2(t)v\left[1 - \frac{v}{a_2(t) + b_2(t)u} - c_2(t)v\right], \end{aligned} \tag{1.5}$$

under the assumption that $r_i(t), a_i(t), b_i(t), c_i(t), i = 1, 2$, are all continuous T-periodic functions, and they obtained sufficient conditions which guarantee the existence of a unique globally asymptotically stable strictly positive periodic solution. In [4], Cui further incorporated continuous time delays and generalized it to n species. In [5, 6], the authors incorporated discrete time delays in interspecies interaction and investigated the positive periodic and positive almost periodic solution of system.

On the other hand, in the real world, ecosystems are continuously disturbed by the unpredictable force. It is mostly humans' interference. Furthermore human production activities is an important reason of species extinction [7, 8]. In some situations, one may wish to retain its stability. This is the significance in the control procedure of ecology balance. Therefore, scholars introduced the feedback control variables to the system. The dynamic behaviors of the Lotka-Volterra cooperative system with feedback controls have been discussed extensively; see [9–13]. In [10], Chen *et al.* first proposed the nonautonomous n-species cooperation system with continuous delays and feedback controls as follows:

$$\begin{aligned} \frac{dx_i(t)}{dt} &= r_i(t)x_i(t)\left[1 - \frac{x_i(t)}{a_i(t) + \sum_{j=1,j\neq i}^{n} b_{ij}\int_{-T_{ij}}^{0} K_{ij}(s)x_j(t+s)\,ds} - c_i(t)x_i(t)\right] \\ &\quad - d_i(t)u_i(t)x_i(t) - e_i(t)x_i(t)\int_{-\tau_i}^{0} H_i(s)u_i(t+s)\,ds, \\ \frac{du_i(t)}{dt} &= -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t) + \gamma_i(t)\int_{-\eta_i}^{0} G_i(s)x_i(t+s)\,ds, \end{aligned} \tag{1.6}$$

where $x_i(t)$, $i = 1, \ldots, n$ are the densities of the cooperation species X_i. $u_i(t)$, $i = 1, \ldots, n$ are the feedback control variables. They obtained a set of complicated sufficient conditions to guarantee the permanence and global attractivity of the system. Recently, by applying a new integral inequality, Chen and Xie [14] showed that in the above system, the feedback control variables have no influence on the persistent property of system (1.6). With regard to an autonomous system, since 2003, Gopalsamy and Weng [15] introduced feedback control variables into a two species competition system as follows:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= x_1(t)\bigl(b_1 - a_{11}x_1(t) - a_{12}x_2(t) - \alpha_1 u_1(t)\bigr),\\
\frac{dx_2(t)}{dt} &= x_2(t)\bigl(b_2 - a_{21}x_1(t) - a_{22}x_2(t) - \alpha_2 u_2(t)\bigr),\\
\frac{du_1(t)}{dt} &= -\eta_1 u_1(t) + a_1 x_1(t),\\
\frac{du_2(t)}{dt} &= -\eta_2 u_2(t) + a_2 x_2(t).
\end{aligned} \tag{1.7}$$

They obtained sufficient conditions to guarantee the existence of a globally attracting positive equilibrium of the system with feedback controls. Chen and Chen [16] discussed the global stability of a unique interior equilibrium for a Leslie-Gower predator-prey model with feedback controls:

$$\begin{aligned}
\dot{N}_1(t) &= (r_1 - a_1 N_2 - b_1 N_1 - c_1 u_1)N_1,\\
\dot{N}_2(t) &= \left(r_2 - a_2\frac{N_2}{N_1} - c_2 u_2\right)N_2,\\
\dot{u}_1(t) &= -f_1 u_1 + g_1 N_1,\\
\dot{u}_2(t) &= -f_2 u_2 + g_2 N_2.
\end{aligned} \tag{1.8}$$

They showed that under the assumption $a_2 b_1 \geq a_1 r_2$, the unique interior equilibrium of system (1.8) is globally stable. However, the authors invoked numerical calculations to found that the unique interior equilibrium of system (1.8) is globally stable also under the condition that $a_2 b_1 < a_1 r_2$ holds. So the condition $a_2 b_1 \geq a_1 r_2$ is unnecessary for the globally stable property of system (1.8). This is a question to be studied in the future. Recently, Li *et al.* [8] proposed and studied the following competitive system with feedback controls:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\left(b_1 - a_{11}x_1(t) - a_{12}\int_0^{+\infty} K_1(s)x_2(t-s)\,ds - c_1 u_1(t)\right),\\
\dot{x}_2(t) &= x_2(t)\left(b_2 - a_{21}\int_0^{+\infty} K_2(s)x_1(t-s)\,ds - a_{22}x_2(t) - c_2 u_2(t)\right),\\
\dot{u}_1(t) &= -e_1 u_1(t) + d_1 x_1(t),\\
\dot{u}_2(t) &= -e_2 u_2(t) + d_2 x_2(t),
\end{aligned} \tag{1.9}$$

where b_i, a_{ij}, c_i, e_i, d_i, $i,j = 1,2$ are positive constants. $x_i(t)$, $i = 1,2$, denote the densities of the populations $x_i(t)$. $u_i(t)$, $i = 1,2$, denote feedback control variables. They showed that if the Lotka-Volterra competitive system is globally stable, the feedback control variables had no influence on the global stability of the system (1.9). If the Lotka-Volterra competitive system is showing extinction, they can make the extinct species become globally stable or still keep the property of extinction. They showed that the feedback control variables play an important role on the dynamic behavior of the system (1.9).

It brings to our attention that in systems (1.4) and (1.1), we may ask: how do feedback control variables affect the global stability of the May cooperative system? To find an answer to this question, we consider a May cooperative system with feedback controls as follows:

$$\begin{aligned} \frac{dx_1}{dt} &= r_1x_1\left[1 - b_1x_1 - \frac{x_1}{\alpha x_2 + k_1} - c_1u_1\right], \\ \frac{dx_2}{dt} &= r_2x_2\left[1 - b_2x_2 - \frac{x_2}{\beta x_1 + k_2} - c_2u_2\right], \\ \frac{du_1}{dt} &= -\eta_1u_1 + e_1x_1, \\ \frac{du_2}{dt} &= -\eta_2u_2 + e_2x_2, \end{aligned} \tag{1.10}$$

where r_i, b_i, α, β, k_i, c_i, e_i, η_i, $i = 1, 2$, are positive constants. $x_i(t)$, $i = 1, 2$, are the densities of the species at time t. $u_i(t)$, $i = 1, 2$, denote feedback control variables.

System (1.10) satisfies the initial values

$$x_i(0) > 0, \qquad u_i(0) > 0, \quad i = 1, 2. \tag{1.11}$$

Obviously, the solutions of system (1.10) with initial values (1.11) are positive for all $t \geq 0$.

The rest of the paper is organized as follows. We will state and prove the main results in next section. In Section 3, numerical simulations are presented to illustrate our results. We end this work by a brief conclusion.

2 Main results

Lemma 2.1 *System* (1.10) *admits a unique positive equilibrium* $P(x_1^*, x_2^*, u_1^*, u_2^*)$.

Proof Obviously, $P(x_1^*, x_2^*, u_1^*, u_2^*)$ satisfies the following equations:

$$\begin{cases} 1 - b_1x_1 - \frac{x_1}{\alpha x_2 + k_1} - c_1u_1 = 0, \\ 1 - b_2x_2 - \frac{x_2}{\beta x_1 + k_2} - c_2u_2 = 0, \\ -\eta_1u_1 + e_1x_1 = 0, \\ -\eta_2u_2 + e_2x_2 = 0. \end{cases} \tag{2.1}$$

By the third and the fourth equations of (2.1), we have $u_i = \frac{e_ix_i}{\eta_i}$, $i = 1, 2$. Substituting them into the first and the second equations of (2.1), respectively, we have

$$x_2 = \frac{k_2 + \beta x_1}{1 + A_2k_2 + A_2\beta x_1} \tag{2.2}$$

and

$$(1 - A_1x_1)(k_1 + \alpha x_2) = x_1, \tag{2.3}$$

where $A_i = b_i + \frac{c_ie_i}{\eta_i}$, $i = 1, 2$. Substituting (2.2) into (2.3), we have

$$Dx_1^2 + Ex_1 + F = 0, \tag{2.4}$$

where

$$\begin{aligned} &D = \beta(A_2 + k_1A_1A_2 + A_1\alpha), \qquad F = -(k_1 + k_1k_2A_2 + k_2\alpha), \\ &E = k_1(A_1 - A_2\beta) + k_2(A_2 + A_1\alpha + k_1A_1A_2) + (1 - \alpha\beta). \end{aligned}$$

From $D > 0$, $F < 0$, we see that (2.4) admits a unique positive solution x_1^*. Substituting x_1^* to (2.2), we have a unique positive solution x_2^*. So system (1.10) admits a unique positive equilibrium $P(x_1^*, x_2^*, u_1^*, u_2^*)$, where $u_i^* = \frac{e_i x_i^*}{\eta_i}$, $i = 1, 2$, which completes the proof. □

Before we state and prove the global stability of this work, we need to state a definition and a useful lemma.

Definition 2.2 [16] A matrix $A = ((a_{ij})_{n\times n})$ is said to be an M matrix if $a_{ij} \le 0$, $i \ne j$, $i, j = 1, 2, \dots, n$, and any one of the following conditions holds:

(1) all of the eigenvalues of the matrix A have positive real parts;
(2) the order principal minor of matrix A is positive;
(3) matrix A is nonsingular and $A^{-1} \ge 0$;
(4) there exists a vector $x > 0$ such that $Ax > 0$;
(5) there exists a vector $y > 0$ such that $A^T y > 0$.

Lemma 2.3 [17] *If A is an M matrix, then there exists a positive diagonal matrix* $D = \operatorname{diag}(d_1, d_2, \dots, d_n)$, $d_i > 0$, $i = 1, \dots, n$, *such that matrix* $B = \frac{1}{2}(DA + A^T D)$ *is positive definite.*

Theorem 2.4 *The unique positive equilibrium $P(x_1^*, x_2^*, u_1^*, u_2^*)$ of system* (1.10) *is globally stable.*

Proof Now let us construct a Lyapunov function

$$V(t) = \beta_1 \int_{x_1^*}^{x_1} \frac{\theta - x_1^*}{\theta}\, d\theta + \beta_2 \int_{x_2^*}^{x_2} \frac{\theta - x_2^*}{\theta}\, d\theta + \beta_3 \int_{u_1^*}^{u_1} (\theta - u_1^*)\, d\theta + \beta_4 \int_{u_2^*}^{u_2} (\theta - u_2^*)\, d\theta,$$

where β_i, $i = 1, \dots, 4$ are positively undetermined coefficients. Calculating the upper right derivative of $V(t)$ along the solution of system (1.10), we have

$$\begin{aligned} D^+V(t) = {} & \beta_1 r_1 (x_1 - x_1^*)\left[-b_1(x_1 - x_1^*) - \frac{x_1}{\alpha x_2 + k_1} + \frac{x_1^*}{\alpha x_2^* + k_1} - c_1(u_1 - u_1^*)\right] \\ & + \beta_2 r_2 (x_2 - x_2^*)\left[-b_2(x_2 - x_2^*) - \frac{x_2}{\beta x_1 + k_2} + \frac{x_2^*}{\beta x_1^* + k_2} - c_2(u_2 - u_2^*)\right] \\ & + \beta_3 (u_1 - u_1^*)\left[-\eta_1(u_1 - u_1^*) + e_1(x_1 - x_1^*)\right] \\ & + \beta_4 (u_2 - u_2^*)\left[-\eta_2(u_2 - u_2^*) + e_2(x_2 - x_2^*)\right], \end{aligned}$$

we take $\beta_{i+2} = \frac{\beta_i r_i c_i}{e_i}$, $i = 1, 2$, then

$$\begin{aligned} D^+V(t) = {} & -\beta_1 r_1 \left[b_1 + \frac{1}{\alpha x_2 + k_1}\right](x_1 - x_1^*)^2 - \beta_2 r_2 \left[b_2 + \frac{1}{\beta x_1 + k_2}\right] \\ & \times (x_2 - x_2^*)^2 + \left[\frac{\beta_1 r_1 \alpha x_1^*}{(\alpha x_2^* + k_1)(\alpha x_2 + k_1)} + \frac{\beta_2 r_2 \beta x_2^*}{(\beta x_1^* + k_2)(\beta x_1 + k_2)}\right] \\ & \times (x_1 - x_1^*)(x_2 - x_2^*) - \beta_3 \eta_1 (u_1 - u_1^*)^2 - \beta_4 \eta_2 (u_2 - u_2^*)^2 \\ \le {} & -\frac{1}{2} Y^T (DG + G^T D) Y - \beta_3 \eta_1 (u_1 - u_1^*)^2 - \beta_4 \eta_2 (u_2 - u_2^*)^2, \end{aligned}$$

where $Y = (|x_1 - x_1^*|, |x_2 - x_2^*|)^T$, $D = \operatorname{diag}(\beta_1, \beta_2)$, $\beta_i > 0$, $i = 1, 2$, and

$$G = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \tag{2.5}$$

where

$$\begin{aligned}
a_{11} &= r_1\left(b_1 + \frac{1}{\alpha x_2 + k_1}\right), \\
a_{12} &= -\frac{r_1 \alpha x_1^*}{(\alpha x_2^* + k_1)(\alpha x_2 + k_1)}, \\
a_{21} &= -\frac{r_2 \beta x_2^*}{(\beta x_1^* + k_2)(\beta x_1 + k_2)}, \\
a_{22} &= r_2\left(b_2 + \frac{1}{\beta x_1 + k_2}\right).
\end{aligned}$$

Note first that the off-diagonal elements of the matrix G are a_{21} and a_{21}, which are negative. Furthermore, by simple algebraic computation, the two order principal minors of the matrix G are

$$\begin{aligned}
a_{11} &= r_1\left(b_1 + \frac{1}{\alpha x_2 + k_1}\right) > 0, \\
\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} &= r_1 r_2 b_1 b_2 + \frac{r_1 r_2 b_1}{\beta x_1 + k_2} + \frac{r_1 r_2 b_2}{\alpha x_2 + k_1} + \frac{r_1 r_2}{(\beta x_1 + k_2)(\alpha x_2 + k_1)} \\
&\quad - \frac{r_1 r_2 \beta x_1^* \alpha x_2^*}{(\beta x_1 + k_2)(\alpha x_2 + k_1)(\beta x_1^* + k_2)(\alpha x_2^* + k_1)} > 0.
\end{aligned}$$

From Definition 2.2, it follows that G is an M matrix; according to Lemma 2.3, there exists a positive diagonal matrix $D = \operatorname{diag}(\beta_1, \beta_2)$, $\beta_i > 0$, $i = 1, 2$, such that the matrix $\frac{1}{2}(DG + G^T D)$ is positive definite. So $\frac{dV}{dt} < 0$ strictly for all $x_1 > 0$, $x_2 > 0$, $u_1 > 0$, $u_2 > 0$, except the positive equilibrium $P(x_1^*, x_2^*, u_1^*, u_2^*)$, where $\frac{dV}{dt} = 0$. So $V(t)$ satisfies Lyapunov's asymptotic stability theorem [18] and the unique interior equilibrium $P(x_1^*, x_2^*, u_1^*, u_2^*)$ is globally stable, which completes the proof. □

Next we consider the dynamic behaviors of system (1.1) incorporate feedback control variables, in system (1.10), set $b_i = 0$, $i = 1, 2$, and we have

$$\begin{aligned}
\frac{dx_1}{dt} &= r_1 x_1\left[1 - \frac{x_1}{\alpha x_2 + k_1} - c_1 u_1\right], \\
\frac{dx_2}{dt} &= r_2 x_2\left[1 - \frac{x_2}{\beta x_1 + k_2} - c_2 u_2\right], \\
\frac{du_1}{dt} &= -\eta_1 u_1 + e_1 x_1, \\
\frac{du_2}{dt} &= -\eta_2 u_2 + e_2 x_2.
\end{aligned} \tag{2.6}$$

Lemma 2.5 *System* (2.6) *admits a unique equilibrium* $P_1(x_{10}, x_{20}, u_{10}, u_{20})$.

Proof Obviously, $P_1(x_{10}, x_{20}, u_{10}, u_{20})$ satisfies the following equations:

$$\begin{cases} 1 - \frac{x_1}{\alpha x_2 + k_1} - c_1 u_1 = 0, \\ 1 - \frac{x_2}{\beta x_1 + k_2} - c_2 u_2 = 0, \\ -\eta_1 u_1 + e_1 x_1 = 0, \\ -\eta_2 u_2 + e_2 x_2 = 0. \end{cases} \tag{2.7}$$

After simple algebraic computations, we have

$$D_1 x_1^2 + E_1 x_1 + F_1 = 0, \tag{2.8}$$

where

$$D_1 = \beta(B_2 + k_1 B_1 B_2 + B_1 \alpha), \qquad F_1 = -(k_1 + k_1 k_2 B_2 + k_2 \alpha),$$
$$E_1 = k_1(B_1 - B_2\beta) + k_2(B_2 + B_1\alpha + k_1 B_1 B_2) + (1 - \alpha\beta),$$

and $B_i = \frac{c_i e_i}{\eta_i}$, $i = 1, 2$. From $D_1 > 0$, $F_1 < 0$, we see that (2.8) admits a unique positive solution x_{10}. Similar to the analysis of Lemma 2.1, system (2.6) admits a unique positive equilibrium $P_1(x_{10}, x_{20}, u_{10}, u_{20})$. □

Theorem 2.6 *The unique positive equilibrium* $P_1(x_{10}, x_{20}, u_{10}, u_{20})$ *of system* (2.6) *is globally stable.*

Proof The proof of Theorem 2.6 is similar to that of Theorem 2.4, and we omit the details here. □

3 Examples

The following three examples show the feasibility of our main results.

Example 3.1 Consider the following equations:

$$\dot{x_1} = x_1\left(1 - 2x_1 - \frac{2x_1}{x_2 + 4}\right), \qquad \dot{x_2} = x_2\left(2 - x_2 - \frac{3x_2}{x_1 + 1}\right). \tag{3.1}$$

By calculation, there exists a unique positive equilibrium of system (3.1), which is globally stable. Figure 1 shows the dynamic behavior of system (3.1).

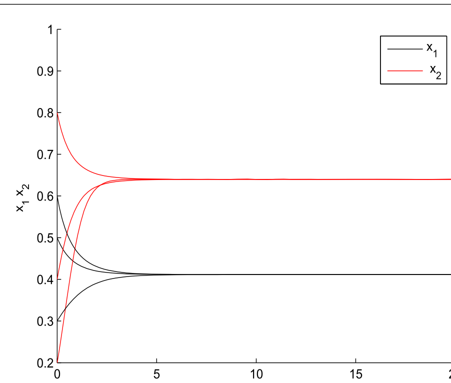

Figure 1 Dynamic behavior of the solution ($x_1(t), x_2(t)$) to system (3.1) with the initial conditions ($x_1(0), x_2(0)$) = (0.5, 0.2), (0.3, 0.4) and (0.6, 0.8), respectively.

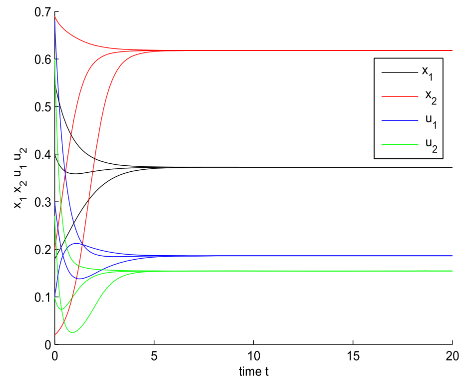

Figure 2 Dynamic behavior of the solution $(x_1(t), x_2(t), u_1(t), u_2(t))$ to system (3.2) with the initial conditions $(x_1(0), x_2(0), u_1(0), u_2(0)) = (0.4, 0.2, 0.68, 0.1)$, $(0.18, 0.02, 0.3, 0.27)$ and $(0.55, 0.69, 0.1, 0.6)$, respectively.

Example 3.2 Now let us further incorporate the feedback control variables to system (3.1) and consider the following system:

$$\begin{aligned} \dot{x}_1 &= x_1\left(1 - 2x_1 - \frac{2x_1}{x_2 + 4} - 0.5u_1\right), \\ \dot{x}_2 &= x_2\left(2 - x_2 - \frac{3x_2}{x_1 + 1} - 0.2u_2\right), \\ \dot{u}_1 &= -2u_1 + x_1, \\ \dot{u}_2 &= -4u_2 + x_2. \end{aligned} \tag{3.2}$$

From Theorem 2.4, the positive equilibrium of system (3.2) is globally stable. Figure 2 shows the dynamic behavior of system (3.2).

Example 3.3 Consider the following equations:

$$\begin{aligned} \dot{x}_1 &= x_1\left(2 - \frac{x_1}{x_2 + 2} - 0.2u_1\right), \\ \dot{x}_2 &= x_2\left(3 - \frac{2x_2}{x_1 + 1} - 0.5u_2\right), \\ \dot{u}_1 &= -3u_1 + 2x_1, \\ \dot{u}_2 &= -2u_2 + x_2. \end{aligned} \tag{3.3}$$

From Theorem 2.6, there exists a unique positive equilibrium of system (3.3), which is globally stable. Figure 3 shows the dynamic behavior of system (3.3).

4 Conclusion

In this paper, we propose and study May cooperative system with feedback controls. In Theorem 2.4, by constructing a suitable Lyapunov function, we show that feedback control variables have no influence on the global stability of the system. Our result improve the corresponding result of Chen *et al.* [10]. In [16], Chen and Chen have a conjecture that the condition $a_2b_1 \geq a_1r_2$ is not needed to ensure the global stability of the unique interior equilibrium. In this paper, corresponding to a May cooperative system, we give a strict proof of an affirmative answer which is without any conditions. Compared with Chen *et*

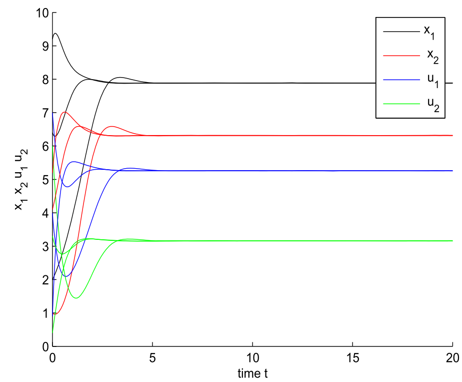

Figure 3 Dynamic behavior of the solution $(x_1(t), x_2(t), u_1(t), u_2(t))$ to system (3.3) with the initial conditions $(x_1(0), x_2(0), u_1(0), u_2(0)) = (2, 1, 4, 6), (6.4, 5.2, 7, 0.4)$ and $(9.2, 4.1, 0.9, 3.3)$, respectively.

al. [13, 14], the authors showed that the feedback control variables have no influence on the permanence of the cooperation system. We have a further insight.

System (3.3) shows that

$$1 \leq \alpha\beta = 3,$$

which implies condition (1.3) holds, that is, both populations are growing unboundedly large. However, Figure 3 and Theorem 2.6 show that the unique positive equilibrium is globally stable, which implies the feedback control variables which make both populations growing unboundedly large now become globally stable.

Competing interests
The authors declare that there is no conflict of interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Acknowledgements
The authors are grateful to anonymous referees for their excellent suggestions, which greatly improve the presentation of the paper. Also, the research was supported by the Natural Science Foundation of Fujian Province (2015J010121, 2015J01019).

Received: 12 June 2015 Accepted: 30 September 2015 Published online: 24 November 2015

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