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Research Article

Global Exponential Synchronization of Nonlinearly Coupled Complex Dynamical Networks with Time-Varying Coupling Delays

Yi-Ping Luo,[1,2] Li Shu,[1,2] and Bi-Feng Zhou[3]

[1] *Hunan Provincial Key Laboratory of Wind Generator and Its Control, Hunan Institute of Engineering, Xiangtan, Hunan 411104, China*
[2] *The Cooperative Innovation Center of Wind Power Equipment and Energy Conversion, Xiangtan, Hunan 411104, China*
[3] *Hunan Electrical College of Technology, Xiangtan, Hunan 411101, China*

Correspondence should be addressed to Li Shu; shuli@hnie.edu.cn

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This paper focuses on the global exponential synchronization problem of nonlinearly coupled complex dynamical networks with time-varying coupling delays. Several simple and generic global exponential synchronization criteria are derived based on the Lyapunov stability theory and the Dini derivatives using the Halanay and generalized Halanay inequalities. These criteria rely on system parameters alone and can be used conveniently in practical applications. In addition, the system parameters do not satisfy the conditions of the proposed criteria. That is, the system itself cannot synchronize. However, system synchronization can be achieved by adding the appropriate feedback controllers, thereby providing a practical and effective control method for complex dynamical networks. An estimation method of exponential convergence rate is also presented. Finally, the effectiveness of the proposed criteria is verified through numerical simulations.

1. Introduction

Complex dynamical networks have attracted considerable attention in the past several decades because of their potential applications in diverse fields, such as science, engineering, and societal systems [1–4]. In fact, a variety of real-world systems, such as the Internet, food webs, and ecological, neural, and social networks, can be described by complex dynamical networks. Consequently, synchronization, which is a typical collective behavior of complex dynamical networks, has become active popular research topic. Synchronization in dynamical networks has been extensively investigated by researchers from various fields [5–13].

Research on the synchronization of complex networks has two main aspects. In some cases, complex dynamical networks can achieve synchronization through their topological structure, communication quality, and the interaction of the intrinsic dynamical behavior of their nodes [14–17]. Wang and Chen [14, 15] were the first to propose a method for measuring synchronization capability by calculating eigenvalues; that is, synchronization will be achieved if the coupling strength is adequate. A previous study [16] utilized a novel Lyapunov–Krasovskii function and the Kronecker product and discussed synchronization problems for an array of coupled complex discrete-time networks. Huang et al. [17] introduced the complex dynamical network model with partial information transmission, for which certain synchronization criteria were derived by using an efficient decomposition method. However, the synchronization criteria in most existing works contain unknown parameters, thereby making system synchronization difficult for workers to verify.

A control action is introduced in the dynamical node to drive a complex dynamical network into synchronization if the network cannot synchronize by itself. Researchers have proposed several effective control methods, and significant conclusions have been obtained [18–25]. Guaranteed cost synchronization for complex networks was addressed in [18] by designing a dynamic feedback controller with guaranteed cost synchronization. Zhou et al. [19] investigated the local and global adaptive synchronizations of uncertain complex dynamical networks. Their hypotheses and adaptive controller design for network synchronization are rather simple in form. The global exponential synchronization of complex-valued dynamical networks with multiple time-varying delays and stochastic perturbations were considered in [20] in the design of a time-delayed impulsive control scheme. Cheng et al. [21] found that complex networks with the Watts–Strogatz or scale-free BA random topological architecture can be synchronized more easily than regular systems by pin-controlling fewer nodes. Intermittent pinning controllers were applied in [22] to synchronize interacting clusters of linearly coupled heterogeneous linear systems and nonlinear oscillators under a general coupling topology. These studies use the coupling matrix between nodes to affect synchronization, thereby creating the advantage of fully utilizing the information between nodes. Deficiency increases the number of adjustment parameters that affect system synchronization. Thus, verifying system synchronization can be difficult. However, system synchronization can be verified easily using a method based on these findings. This method relies only on the parameters of the system itself and does not need any other adjustment parameter, thereby reducing delays and providing convenience for controller design. However, few results on this study have been reported.

The key to studying complex networks is building a network model that can exactly describe a real network system. Delays should be considered in establishing a network model because of the finite speed of information transmission and traffic congestion [26, 27]. Time delay commonly changes over time during dynamic change. Fixed delay is an ideal state of time-varying delay. Therefore, time-varying delay should be considered when studying complex networks [28–32].

Inspired by the above discussion, this paper aims to investigate the global exponential synchronization of a class of complex dynamical networks with time-varying delay. We consider a complex network model composed of nonlinear coupling nodes and has unknown but bounded nonlinear vector functions and time-varying delayed and nondelayed couplings. Several simple global exponential synchronization criteria are derived from the Lyapunov stability theory and the Dini derivatives using the Halanay and generalized Halanay inequalities. These criteria rely on system parameters alone and can be used conveniently in practical applications. In addition, the system parameters do not satisfy the conditions of the proposed criteria, that is, the system itself cannot achieve synchronization. However, the system can be driven to synchronize by adding appropriate feedback controllers. These conditions provide a practical and effective control method for complex network systems. Finally, the validity of the proposed scheme is verified by numerical simulations.

The rest of this paper is organized as follows. Section 2 introduces the complex dynamical network model and provides certain preliminaries. Section 3 presents the criteria for ensuring the global exponential synchronization of complex dynamical networks with time-varying delay and an estimation method for exponential convergence rate. Section 4 presents the numerical simulations used to validate the proposed scheme. Finally, Section 5 concludes this paper.

2. Problem Formulation and Preliminaries

A complex network with N identical time-varying delayed dynamical nodes with nonlinear couplings is considered. Each node of the network has n dimensions. The state of the ith node can be described as follows:

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + c_1\sum_{j=1}^{N} c_{ij} g_j(x_j(t)) \\ &\quad + c_2\sum_{j=1}^{N} d_{ij} g_j(x_j(t-\tau(t))), \quad i = 1, 2, \ldots, N,\end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state vector of the ith node, $f(x) \in R^n$ is a continuously nonlinear vector-valued function that describes the dynamics of a node, and $g(x) \in R^n$ is a continuous nonlinear coupling function. The coupling time delay $\tau(t)$ is unknown but is bounded by a known constant, that is, $0 \le \tau(t) \le \tau$, where positive constants c_1 and c_2 are the coupling strengths. $C = (c_{ij})_{N\times N}$ and $D = (d_{ij})_{N\times N}$ are the coupling configuration matrices. If node i and j $(j \ne i)$ are connected, then $c_{ij} > 0$, $d_{ij} > 0$; otherwise, $c_{ij} = 0$, $d_{ij} = 0$. The diagonal elements of matrices C and D are defined as $c_{ii} = -\sum_{j=1,j\ne i}^{N} c_{ij}$, $d_{ii} = -\sum_{j=1,j\ne i}^{N} d_{ij}$.

Definition 1. Generally, when $t \to \infty$, $x_1(t), x_2(t), \ldots, x_N(t) \to s(t)$. $s(t) \in R^n$ is the solution of the following isolated node system (2). The complex dynamic system (1) synchronizes with the homogenous trajectory

$$\dot{s}(t) = f(s(t)). \tag{2}$$

Let $e_i(t) = x_i(t) - s(t)$ be the synchronization error. We obtain the error system on the basis of network (1) and (2) as follows:

$$\begin{aligned}\dot{e}_i(t) &= \dot{x}_i(t) - \dot{s}(t) \\ &= f_i(x_i(t)) - f(s(t)) + c_1\sum_{j=1}^{N} c_{ij}\phi_j(e_j(t)) \\ &\quad + c_2\sum_{j=1}^{N} d_{ij}\phi_j(e_j(t-\tau(t))),\end{aligned} \tag{3}$$

where $\phi_j(e_j(t)) = g_j(x_j(t)) - g(s(t))$, $\phi_j(e_j(t-\tau(t))) = g_j(x_j(t-\tau(t))) - g(s(t-\tau(t)))$.

Definition 2. $|x_i(t)| = (|x_{i1}(t)|, |x_{i2}(t)|, \ldots, |x_{in}(t)|)^T \in R^n$, $i = 1, 2, \ldots, N$, where $|x|$ denotes the absolute value of x.

Definition 3. A positive constant γ and vector $M = (m_1, m_2, \ldots, m_n)^T > 0$ exist if the vector function $e_i(t) = (e_{i1}(t), e_{i2}(t), \ldots, e_{in}(t))^T \in R^n$ satisfies

$$|e_i(t)| \le M\exp(-\gamma t). \tag{4}$$

The error system (3) is globally and exponentially stable, which implies that the complex dynamical network (1) achieves global exponential synchronization.

Remark 4. If the state vector of the error system (3) satisfies the conditions of Definition 3, then $\|e_i(t)\| = \sqrt{e_i^T(t)e_i(t)}$, where $\|\cdot\|$ indicates the Euclidean norm, is globally and exponentially stable.

Definition 5 (see [33]). The Dini derivatives of f at $t_0 \in (\alpha, \beta)$ for the vector-valued function $f : (\alpha, \beta) \to R$, $t \to f(t)$ are defined as

$$D^+ f(t_0) = \lim_{t \to t_0^+} \sup \frac{f(t) - f(t_0)}{t - t_0} \tag{5}$$

Definition 6 (see [33]). A matrix $A = (a_{ij})_{N\times N}$ is called an M-matrix if it satisfies $a_{ii} > 0$, $a_{ij} \le 0$, $i \ne j$, $A^{-1} \ge 0$. According to the M-matrix property, a constant $\theta_j > 0$ exists such that $\sum_{j=1}^{N} \theta_j a_{ij} > 0$, $i = 1, 2, \ldots, N$.

Assumption 7. The nonlinear function $f(x_i)$, $g(x_i)$ is assumed to satisfy the uniform semi-Lipschitzian condition, where constants σ_i and l_i satisfy

$$\begin{aligned} 0 &\le \frac{f(\xi_1) - f(\xi_2)}{\xi_1 - \xi_2} \le \sigma_i, \quad i = 1, 2, \ldots, N, \\ 0 &\le \frac{g(\xi_1) - g(\xi_2)}{\xi_1 - \xi_2} \le l_i, \quad i = 1, 2, \ldots, N. \end{aligned} \tag{6}$$

3. Main Results

3.1. Global Exponential Synchronization of System (1) Using the Halanay Inequality

Lemma 8 (Halanay [34]). *Let $w(t) : [t_0 - \tau, \infty) \to [0, \infty)$ be a continuous function, where constants $a > b > 0$ exist such that*

$$\dot{w}(t) \le -a w(t) + b \overline{w}(t) \tag{7}$$

holds for $t \ge t_0$, in which $\overline{w}(t) = \sup_{t-\tau \le s \le t} w(s)$, $\tau \ge 0$, and then

$$w(t) \le \overline{w}(t_0) \exp\{-\gamma (t - t_0)\}, \quad t \ge t_0, \tag{8}$$

where $\gamma > 0$ is the unique positive solution of the following equation:

$$\gamma = a - b \exp\{\gamma \tau\}. \tag{9}$$

Theorem 9. *Supposing that Assumption 7 holds and satisfies the condition*

$$\min_{1\le i\le N} (\varepsilon c_1 l_i) > \max_{1\le i\le N} \left(\sigma_i + \sum_{j=1, j\ne i}^{N} c_1 l_i c_{ij} \right) + 2 \max_{1\le i\le N} c_2 \lambda l_i, \tag{10}$$

where λ is the maximum eigenvalue of the matrix $D = (d_{ij})_{N\times N}$, then the complex dynamical network (1) can achieve global exponential synchronization. $\mu/2$ is the convergence rate, where μ is the unique positive solution of the equation, $\mu = a - b\exp(\mu t)$, $a = 2(\min_{1\le i\le N}(\varepsilon c_1 l_i) - \max_{1\le i\le N}(\sigma_i + \sum_{j=1, j\ne i}^{N} c_1 l_i c_{ij} + c_2 \lambda l_i/2))$, $b = \max_{1\le i\le N} c_2 \lambda l_i$.

Proof. The Lyapunov function is constructed as follows:

$$V(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^T(t) e_i(t). \tag{11}$$

Taking the derivative of $V(t)$ with respect to time t along the solutions of error system (3) yields

$$\begin{aligned} \dot{V}(t) = {} & \sum_{i=1}^{N} e_i^T(t) \left(f(x_i(t)) - f(s(t)) \right) \\ & + c_1 \sum_{i=1}^{N} \sum_{j=1, j\ne i}^{N} c_{ij} e_i^T(t) \phi_i(e_i(t)) \\ & + c_1 \sum_{i=1}^{N} c_{ii} e_i^T(t) \phi_i(e_i(t)) \\ & + c_2 \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij} e_i^T(t) \phi_j\left(e_j(t - \tau(t))\right). \end{aligned} \tag{12}$$

Assuming $c_{11} = c_{22} = \cdots = c_{NN} = -\varepsilon < 0$, the assumption on the coupling matrix C is not conservative. This because if $K = (k_{ij})_{N\times N}$ is any square matrix that satisfies the conditions of the defined coupling configuration matrices, that is, $k_{ij} \ge 0$, $i \ne j$, $k_{ii} = -\sum_{j=1, j\ne i}^{N} k_{ij}$ and $k_{ii} \ne 0$ ($i = 1, 2, \ldots, N$), then all diagonal entries of the matrix $\widetilde{K} = (\widetilde{k}_{ij})_{N\times N} = (k_{ij}/\sum_{j=1, j\ne i}^{N} k_{ij})_{N\times N}$ are easily verified as -1, and thus we can design the coupling matrix as $C = \varepsilon \widetilde{K}$.

Using Assumption 7 ($2x^T y \le x^T x + y^T y, \forall x, y \in R^n$), the following inequalities can be estimated:

$$\begin{aligned} \dot{V}(t) \le {} & \sum_{i=1}^{N} \sigma_i e_i^T(t) e_i(t) + c_1 \sum_{i=1}^{N} \sum_{j=1, j\ne i}^{N} l_i c_{ij} e_i^T(t) e_j(t) \\ & - \varepsilon c_1 \sum_{i=1}^{N} l_i e_i^T(t) e_i(t) \\ & + c_2 \sum_{i=1}^{N} \sum_{j=1}^{N} l_i d_{ij} e_i^T(t) e_j(t - \tau(t)) \le -\Bigg(\min_{1\le i\le N} (\varepsilon c_1 l_i) \\ & - \max_{1\le i\le N} \left(\sigma_i + \sum_{j=1, j\ne i}^{N} c_1 l_i c_{ij} \right) \Bigg) \sum_{j=1}^{N} e_j^T(t) e_j(t) \\ & + \max_{1\le i\le N} \frac{c_2 \lambda l_i}{2} \sum_{j=1}^{N} e_j^T(t) e_j(t) + \max_{1\le i\le N} \frac{c_2 \lambda l_i}{2} \end{aligned}$$

$$\cdot \sum_{j=1}^{N} e_j^T (t-\tau(t)) e_j (t-\tau(t)) \le -2 \left(\min_{1\le i\le N} (\varepsilon c_1 l_i) - \max_{1\le i\le N} \left(\sigma_i + \sum_{j=1, j\ne i}^{N} c_1 l_i c_{ij} \right) + \max_{1\le i\le N} \frac{c_2 \lambda l_i}{2} \right) V(t) + \max_{1\le i\le N} c_2 \lambda l_i \overline{V}(t) = -aV(t) + b\overline{V}(t), \tag{13}$$

where $\overline{V}(t) = (1/2)\sum_{i=1}^{N} \overline{e}_i^T(t)\overline{e}_i(t)$, $\overline{e}_i(t) = \sup_{t-\tau\le s\le t} e_i(s)$.

According to the conditions of Theorem 9 and Lemma 8, we obtain

$$V(t) \le \overline{V}(t_0) \exp\{-\mu(t-t_0)\}, \tag{14}$$

where μ is the unique positive solution of the equation $\mu = a - b\exp(\mu t)$

Thus, we obtain

$$\sqrt{\sum_{i=1}^{N} e_i^T(t) e_i(t)} \le \sqrt{\sum_{i=1}^{N} \overline{e}_i^T(t_0) \overline{e}_i(t_0) \exp\left\{-\frac{\mu}{2}(t-t_0)\right\}}. \tag{15}$$

Hence, the zero of the error system (3) is globally and exponentially stable. This completes the proof of Theorem 9. □

Remark 10. Recently, significant effort has been devoted to the study of the synchronization of complex dynamical networks [11–25]. However, the synchronization criteria in most of the existing studies contain unknown parameters, thereby making system synchronization difficult for workers to verify. In this study, a method for verifying system synchronization is obtained in Theorem 9 by constructing a simple Lyapunov function. The method relies only on the parameters of the system itself and does need any other adjustment parameter, thereby reducing delays and providing convenience for controller design.

3.2. Global Exponential Synchronization of System (1) Using Generalized Halanay Inequalities

Lemma 11 (see [35]). *For any vector function $x(t), y(t) \in R^n$, $\overline{x}(t) = \sup_{t-\tau\le s\le t} x(s)$ and $\overline{y}(t) = \sup_{t-\tau\le s\le t} y(s)$ satisfy the following condition:*

(1) $x(t) < y(t)$, $t_0 - \tau \le t \le t_0$.

(2) $D^+ y(t) > F(t, y(t), \overline{y}(t))$, $t \ge t_0 \ge 0$; $D^+ x(t) \le F(t, x(t), \overline{x}(t))$, $t \ge t_0 \ge 0$, *where* $F(t, x(t), \overline{x}(t)) = (A+B)x(t) + C\overline{x}(t)$, $A < 0$, *is a diagonal matrix. Then,* $x(t) \le y(t)$, $t \ge t_0$.

Theorem 12. *If Assumption 7 holds and $M = -(p_{ii} + q_{ij} + r_{ij})_{N\times N}$ is an M-matrix, in which*

$$p_{ii} = -\varepsilon c_1 l_i + \sigma_i; \quad q_{ij} = \begin{cases} c_1 l_j c_{ij}, & i \ne j \\ 0, & i = j; \end{cases} \quad r_{ij} = c_2 l_j |d_{ij}|, \quad i, j = 1, 2, \ldots, N, \tag{16}$$

then the complex dynamical network (1) can achieve global exponential synchronization.

Proof. Taking the Dini derivative of $|e_i(t)|$, we obtain

$$\begin{aligned} D^+ |e_i(t)| &= \dot{e}_i \operatorname{sgn} e_i \\ &\le |f_i(x_i(t)) - f(s(t))| + c_1 \sum_{j=1, j\ne i}^{N} c_{ij} |\phi_j(e_j(t))| + c_1 c_{ii} |\phi_i(e_i(t))| + c_2 \sum_{j=1}^{N} |d_{ij}| |\phi_j(e_j(t-\tau(t)))| \\ &\le (-\varepsilon c_1 l_i + \sigma_i)|e_i(t)| + \sum_{j=1, j\ne i}^{N} c_1 l_j c_{ij} |e_j(t)| + \sum_{j=1}^{N} c_2 l_j |d_{ij}| |\overline{e}_j(t)| \\ &= p_{ii} |e_i(t)| + \sum_{j=1}^{N} q_{ij} |e_j(t)| + \sum_{j=1}^{N} r_{ij} |\overline{e}_j(t)| \\ &= F(t, e(t), \overline{e}(t)), \quad i = 1, 2, \ldots, N. \end{aligned} \tag{17}$$

From the property of M-matrix, constants $\delta > 0$, $\theta_j > 0$, $(j = 1, 2, \ldots, N)$ exist such that

$$p_{ii}\theta_i + \sum_{j=1}^{N} (q_{ij} + r_{ij})\theta_j < -\delta, \quad i = 1, 2, \ldots, N. \tag{18}$$

Let $0 < \alpha \ll 1$, we can obtain

$$\alpha\theta_i + p_{ii}\theta_i + \sum_{j=1}^{N} (q_{ij}\theta_j + r_{ij}\theta_j \exp(\alpha\tau)) < 0. \tag{19}$$

When $t \in [t_0 - \tau, t_0]$, let $R \gg 1$, such that

$$R\theta_i \exp(-\alpha t) > 1. \tag{20}$$

For any vector $\Upsilon = (\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_N)^T > 0$, we construct a vector function

$$q_i(t) = R\theta_i \left[\sum_{j=1}^{N} \overline{e}_j(t_0) + \Upsilon \right] \exp\{-\alpha(t-t_0)\}. \tag{21}$$

Taking the Dini derivative of $|q_i(t)|$, we obtain

$$\begin{aligned} D^{+}\left|q_i(t)\right| &= -\alpha R\theta_i\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|+\Upsilon\right]\exp\left\{-\alpha(t-t_0)\right\}. \end{aligned} \tag{22}$$

Substituting (19) into (22) yields

$$\begin{aligned} D^{+}\left|q_i(t)\right| &> \left[p_{ii}\theta_i+\sum_{j=1}^{N}\left(q_{ij}\theta_j+r_{ij}\theta_j\exp(\alpha\tau)\right)\right] \\ &\cdot R\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|+\Upsilon\right]\exp\left\{-\alpha(t-t_0)\right\} \\ &= p_{ii}\theta_i R\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|+\Upsilon\right]\exp\left\{-\alpha(t-t_0)\right\} \\ &+\sum_{j=1}^{N}q_{ij}\theta_j R\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|+\Upsilon\right]\exp\left\{-\alpha(t-t_0)\right\} \\ &\cdot\sum_{j=1}^{N}r_{ij}\theta_j R\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|+\Upsilon\right]\exp\left\{-\alpha(t-t_0)\right\} \\ &\cdot\exp(\alpha\tau)\geq p_{ii}\left|q_i(t)\right|+\sum_{j=1}^{N}q_{ij}\left|q_j(t)\right| \\ &+\sum_{j=1}^{N}r_{ij}\left|\bar{q}_j(t)\right| = F\left(t,q(t),\bar{q}(t)\right). \end{aligned} \tag{23}$$

According to (20), when $t\in[t_0-\tau,t_0]$, we can derive

$$\begin{aligned} \left|q_i(t)\right| &= R\theta_i\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|+\Upsilon\right]\exp\left\{-\alpha(t-t_0)\right\} \\ &> \sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|+\Upsilon. \end{aligned} \tag{24}$$

It is obvious that $|e_i(t)|\leq\sum_{j=1}^{N}|\bar{e}_j(t_0)|+\Upsilon, t\in[t_0-\tau,t_0]$; thus

$$\left|e_i(t)\right|<\left|q_i(t)\right|,\quad t\in\left[t_0-\tau,t_0\right]. \tag{25}$$

In view of (17), (23), and (25) and by using Lemma 11, we can derive

$$\begin{aligned} \left|e_i(t)\right| &< \left|q_i(t)\right| \\ &= R\theta_i\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|+\Upsilon\right]\exp\left\{-\alpha(t-t_0)\right\}, \\ &\qquad t\geq t_0. \end{aligned} \tag{26}$$

Let $\Upsilon\to 0^{+}$, $\sum_{i=1}^{N}R\theta_i=\xi_i$, and we obtain

$$\left|e_i(t)\right|\leq\xi_i\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|\right]\exp\left\{-\alpha(t-t_0)\right\},\quad t\geq t_0, \tag{27}$$

where ξ_i, α are positive constants. According to Definition 3, the error system (3) is globally and exponentially stable, which implies that global exponential synchronization is achieved. This completes the proof of Theorem 12. □

Remark 13. Several synchronization criteria for the system are derived by constructing Lyapunov functions and using the Halanay inequality [36, 37]. However, these synchronization criteria contain unknown parameters. In this paper, we avoid such issues by building the Lyapunov function. The Dini derivative loosen the requirements of the system state function; that is, it does not require the state function to be derivable. Moreover, certain points of the system are not derivable if the system is under discontinuous control. A direct solution of the error vector function for the Dini derivative can reduce the conservativeness of conditions. Moreover, a synchronization criterion is obtained in Theorem 12 by using generalized Halanay inequalities. The synchronization criterion relies only on the parameters of the system itself and does not contain any unknown adjustment parameter. Thus, our model provides convenience for practical applications.

Remark 14. In this paper, we provide conservative estimates of the size of α as it cannot be confirmed (Theorem 12); however, exact values are preferred.

Theorem 15. *Supposing that the condition of Theorem 12 holds and constants $\varsigma>0$ exist such that $\widetilde{M}=-(\widetilde{p}_{ii}+q_{ij}+r_{ij})_{N\times N}$ is still an M-matrix, one can derive*

$$\begin{aligned} \left|e_i(t)\right| &\leq \tilde{\xi}_i\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|\right]\exp\left\{-\varsigma(t-t_0)\right\}, \\ &\qquad i=1,2,\ldots,N,\ t\geq t_0, \end{aligned} \tag{28}$$

where

$$\begin{aligned} \widetilde{p}_{ii} &= -\varepsilon c_1 l_i+\sigma_i+\varsigma; \\ q_{ij} &= \begin{cases} c_1 l_j c_{ij}, & i\neq j \\ 0, & i=j; \end{cases} \\ r_{ij} &= c_2 l_j\left|d_{ij}\right|, \\ &\qquad i,j=1,2,\ldots,N. \end{aligned} \tag{29}$$

and then the complex dynamical network (1) can achieve global exponential synchronization with ς as the convergence rate.

Proof. Let $|y_i(t)| = |e_i(t)|\exp(\varsigma t)$, $i = 1, 2, \ldots, N$. Taking the Dini derivative of $|y_i(t)|$, we obtain

$$\begin{aligned} D^+ \left|y_i(t)\right| &\leq \varsigma \exp(\varsigma t)\left|e_i(t)\right| + \exp(\varsigma t) D^+\left|e_i(t)\right| \\ &\leq \exp(\varsigma t)\left(\varsigma\left|e_i(t)\right| + p_{ii}\left|e_i(t)\right| + \sum_{j=1}^{N} q_{ij}\left|e_j(t)\right| \right. \\ &\quad \left. + \sum_{j=1}^{N} r_{ij}\left|\bar{e}_j(t)\right|\right) \leq \widetilde{p}_{ii}\left|y_i(t)\right| + \sum_{j=1}^{N} q_{ij}\left|y_j(t)\right| \\ &\quad + \sum_{j=1}^{N} r_{ij}\left|\bar{y}_j(t)\right|. \end{aligned} \tag{30}$$

Using Theorem 12, we obtain

$$\left|y_i(t)\right| \leq \xi_i\left[\sum_{j=1}^{N}\left|\bar{y}_j(t_0)\right|\right]\exp\left\{-\alpha(t-t_0)\right\}, \quad t \geq t_0. \tag{31}$$

Thus

$$\begin{aligned} \left|e_i(t)\right| &\leq \widetilde{\xi}_i\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|\right]\exp\left\{(-\alpha-\varsigma)(t-t_0)\right\} \\ &\leq \widetilde{\xi}_i\left[\sum_{j=1}^{N}\left|\bar{e}_j(t_0)\right|\right]\exp\left\{-\varsigma(t-t_0)\right\}. \end{aligned} \tag{32}$$

This completes the proof of Theorem 15. □

If system (1) does not satisfy the conditions of the above theorem, that is, the system cannot achieve synchronization by itself, we can apply the appropriate controller to achieve synchronization.

Remark 16. On the basis of Theorem 12, we provide a simple and feasible method for calculating the exponential convergence rate in Theorem 15.

3.3. Global Exponential Synchronization of the Controlled System (1). The controlled complex dynamical network can be described as

$$\begin{aligned} \dot{x}_i(t) &= f\left(x_i(t)\right) + c_1\sum_{j=1}^{N} c_{ij} g_j\left(x_j(t)\right) \\ &\quad + c_2\sum_{j=1}^{N} d_{ij} g_j\left(x_j(t-\tau(t))\right) + u_i(t), \\ &\qquad i = 1, 2, \ldots, N; \end{aligned} \tag{33}$$

the controller $u_i(t)$ ($i = 1, 2, \ldots, N$) is designed as follows:

$$u_i(t) = -k_i x_i(t), \tag{34}$$

where $k_i > 0$ is the adaptive feedback gain to be designed.

We obtain the error system on the basis of network (33) and (2) as follows:

$$\begin{aligned} \dot{e}_i(t) &= \dot{x}_i(t) - \dot{s}(t) \\ &= f\left(x_i(t)\right) - f\left(s(t)\right) + c_1\sum_{j=1}^{N} c_{ij}\phi_j\left(e_j(t)\right) \\ &\quad + c_2\sum_{j=1}^{N} d_{ij}\phi_j\left(e_j(t-\tau(t))\right) - k_i x_i(t), \end{aligned} \tag{35}$$

where $\phi_i(e_i(t)) = g_i(x_i(t)) - g_i(s(t))$, $\phi_i(e_i(t-\tau(t))) = g_i(x_i(t-\tau(t))) - g_i(s(t-\tau(t)))$.

Theorem 17. *Supposing that Assumption 7 holds and satisfies the following condition:*

$$\begin{aligned} \min_{1\leq i\leq N} k_i + \min_{1\leq i\leq N}\left(\varepsilon c_1 l_i\right) &> \max_{1\leq i\leq N}\left(\sigma_i + \sum_{j=1, j\neq i}^{N} c_1 l_i c_{ij}\right) \\ &\quad + 2\max_{1\leq i\leq N}\frac{c_2\lambda l_i}{2}, \end{aligned} \tag{36}$$

where λ is the maximum eigenvalue of the matrix $D = (d_{ij})_{N\times N}$, then the complex dynamical network (33) can achieve global exponential synchronization. $\mu/2$ is the convergence rate, where μ is the same as in Theorem 9.

A similar approach in Theorem 9 can be used for its proof.

Theorem 18. *Supposing that Assumption 7 holds and $\overline{M} = -(\overline{p}_{ii} + q_{ij} + r_{ij})_{N\times N}$ is an M-matrix, where*

$$\begin{aligned} \overline{p}_{ii} &= -k_i - \varepsilon c_1 l_i + \sigma_i; \\ q_{ij} &= \begin{cases} c_1 l_j c_{ij}, & i \neq j \\ 0, & i = j; \end{cases} \\ r_{ij} &= c_2 l_j\left|d_{ij}\right|, \\ &\qquad i, j = 1, 2, \ldots, N, \end{aligned} \tag{37}$$

then the complex dynamical network (33) can achieve global exponential synchronization.

A similar approach in Theorem 12 can be used for its proof.

Theorem 19. *Supposing that the condition of Theorem 18 holds and constants $\varsigma > 0$ exist such that $\overline{\overline{M}} = -(\tilde{\overline{p}}_{ii} + q_{ij} + r_{ij})_{N\times N}$ is still an M-matrix, one can derive*

$$\left|e_i(t)\right| \leq \tilde{\xi}_i \left[\sum_{j=1}^{N} \left|\overline{e}_j(t_0)\right|\right] \exp\{-\varsigma(t-t_0)\}, \quad i = 1,2,\ldots,N,\ t \geq t_0, \tag{38}$$

where

$$\begin{gathered}\overline{p}_{ii} = -k_i - \varepsilon c_1 l_i + \sigma_i + \varsigma;\\ q_{ij} = \begin{cases} c_1 l_j c_{ij}, & i \neq j \\ 0, & i = j; \end{cases}\\ r_{ij} = c_2 l_j \left|d_{ij}\right|, \quad i,j = 1,2,\ldots,N.\end{gathered} \tag{39}$$

The complex dynamical network (33) can then achieve global exponential synchronization with ς as the convergence rate.

A similar approach in Theorem 15 can be used for its proof.

Remark 20. We provide several synchronization criteria for adding the controller in Theorems 17, 18, and 19. The feedback gain k_i of the controllers is related only to the inherent parameters of the system.

4. Numerical Simulation

A dynamical network model with 10 dynamical nodes is considered. The state of the ith node is as presented in (1). Without loss of generality, we choose the system parameters that satisfy Theorem 15, that is, $c_1 = c_2 = 1$, $\sigma = 2$, $l = 2$ after considering the coupling matrices:

$$C = \begin{pmatrix} -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -4 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -4 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & -4 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & -4 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & -4 \end{pmatrix},$$

$$D = \begin{pmatrix} -3 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -3 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -3 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -2 \end{pmatrix}. \tag{40}$$

Figure 1 illustrates the trajectories of a system error. The error states are exponentially stable at zero equilibrium points, thereby implying that global exponential synchronization was obtained by the dynamic nodes of the complex dynamical network with time-varying delays.

The parameters of the system are adjusted such that the conditions of Theorem 15 are not established. Figures 2, 4, and 6 show the trajectories of an error system with unstable error states. According to Theorem 19, we added the feedback controller (34) to the time-varying delay complex network (33). Figures 3, 5, and 7 show the obtained trajectories of the error system. When the control gain $k_i > 5$, the complex dynamical network can achieve global exponential synchronization.

5. Conclusion

In this study, several simple and generic synchronization criteria are derived to guarantee that a complex dynamical network achieves global exponential synchronization. These criteria do not contain unknown parameters related to the intrinsic parameters of the system. The specific form of time delay is unknown, thereby making practical application convenient. If the system cannot establish synchronization by itself, adding appropriate feedback controllers would ensure that sufficient conditions for system synchronization are obtained. The gain of the controllers is related only to the intrinsic parameters of the system itself. The estimation of exponential convergence rate has also been studied. Hence, a method for estimating the exponential convergence rate is also presented in this paper. For instance, solving transcendental equation (9) to obtain the exponential convergence rate is easier using related mathematical software. Theorems 15 and 19 also illustrate a simple and feasible method for calculating the exponential convergence rate.

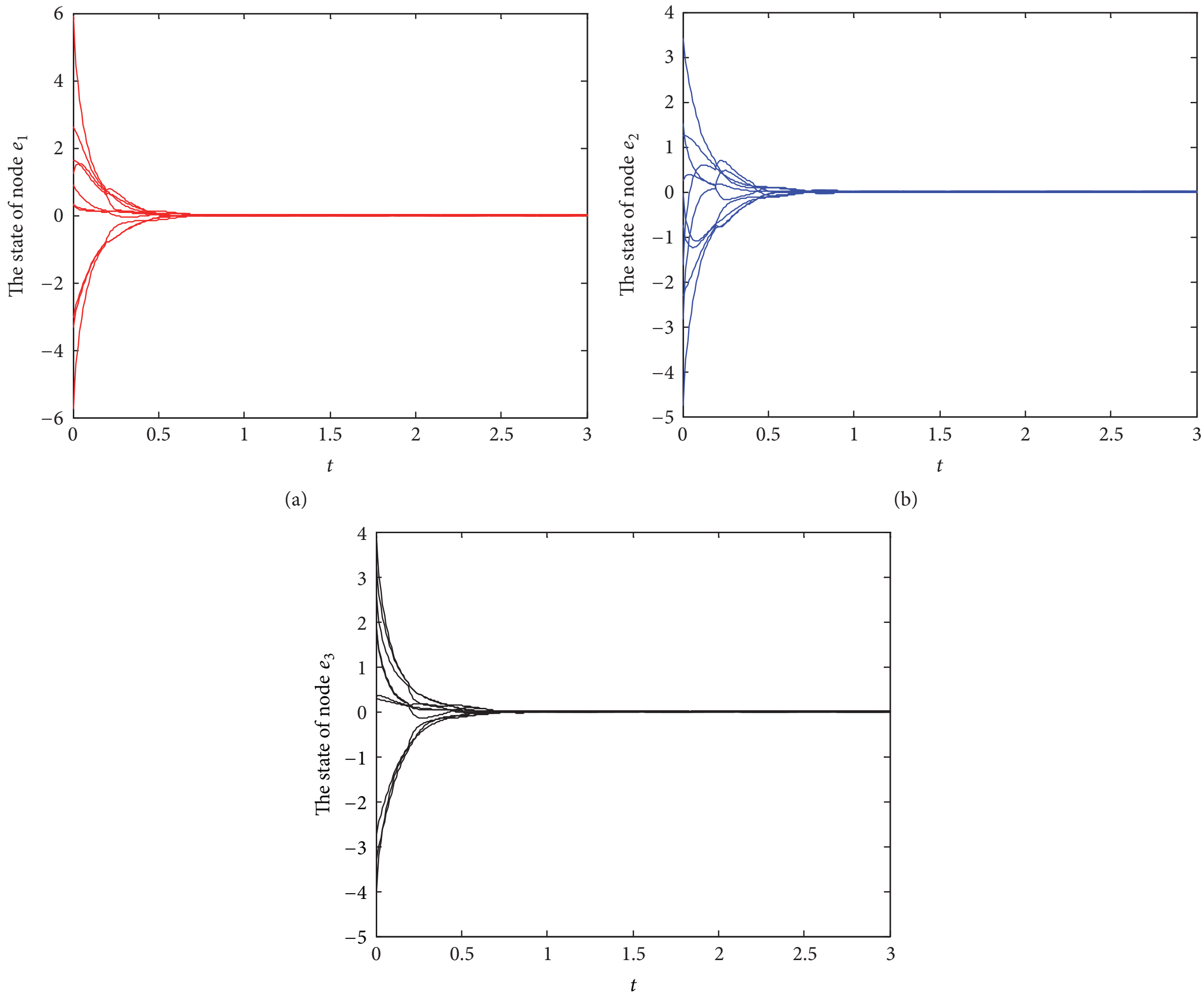

Figure 1: Synchronization error system that satisfies the conditions.

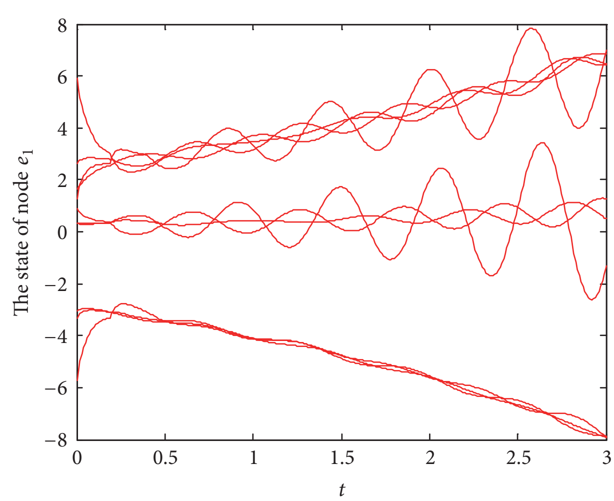

Figure 2: Synchronization error e_{i1} that does not satisfy the conditions.

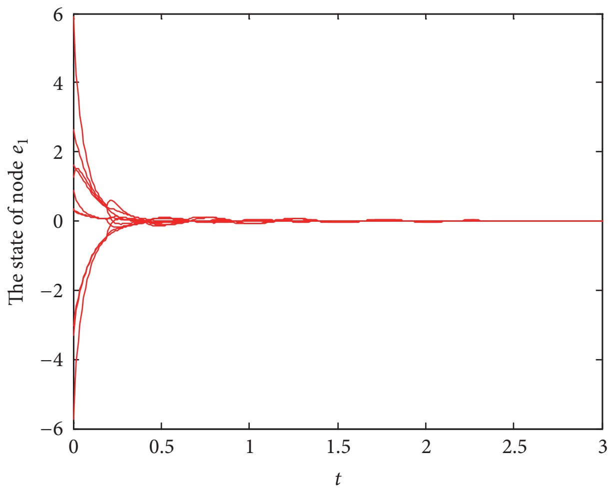

Figure 3: Synchronization error e_{i1} of adding controllers.

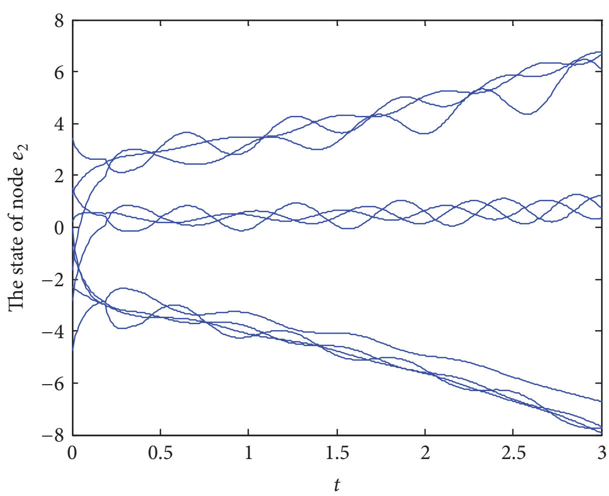

Figure 4: Synchronization error e_{i2} that does not satisfy the conditions.

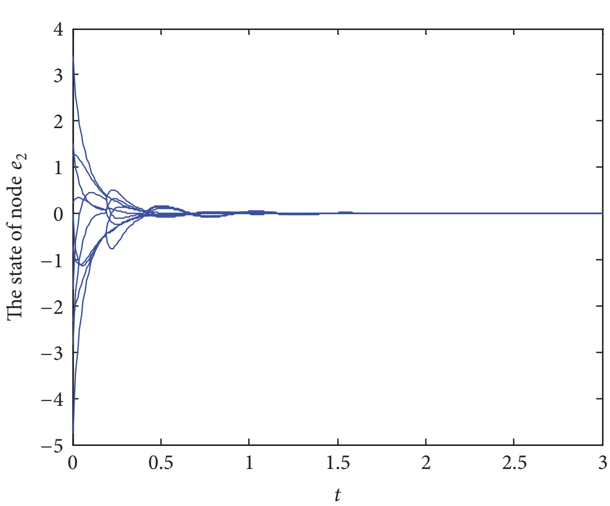

Figure 5: Synchronization error e_{i1} of adding controllers.

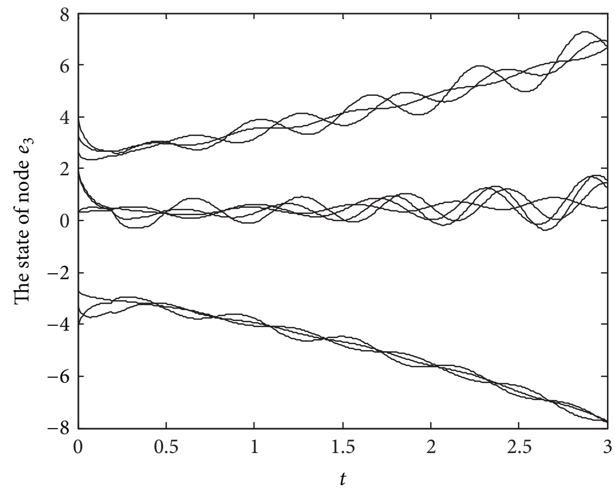

Figure 6: Synchronization error e_{i3} that does not satisfy the conditions.

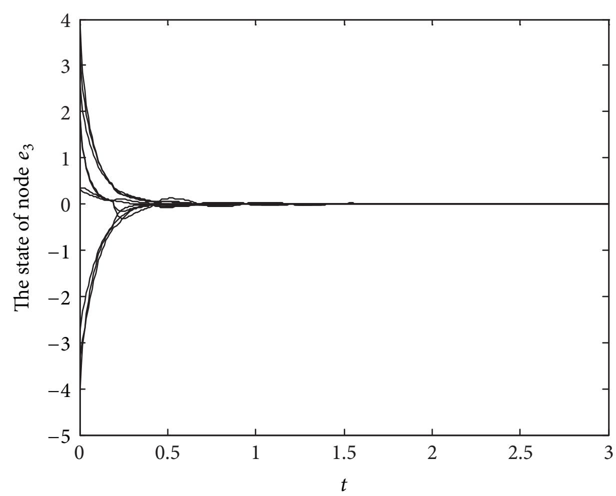

Figure 7: Synchronization error e_{i3} of adding controllers.

Conflicts of Interest

The authors (Yi-Ping Luo, Li Shu, and Bi-Feng Zhou) declare that there are no conflicts of interest regarding the publication of this paper.

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