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Research Article

Hybrid Structure Based Tracking and Consensus for Multiple Motors

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This paper investigates a hybrid structure based synchronous control strategy for multimotor system of shaftless-driven printing press. Many existing algorithms can obtain a stable synchronous system; however, the obtained stable system may encounter a large enough disturbance that can destroy the synchronization. Focusing on this challenging technological problem about how to receive more robust synchronization during steady-state process, this paper first proposes a state-dependent-switching based leader-following control approach, in which synchronization includes two parts, one associated with tracking control for all members, and the other one associated with consensus maintained among followers in the case that one follower loses synchronization with the leader during steady-state motion. By employing the algebra graph theory, matrix theory, and Lyapunov analysis, the convergence and stability of the given multimotor system are proved. Finally, simulation examples are presented to demonstrate the effectiveness and robustness of the theoretical results.

1. Introduction

Recent years have witnessed increasing interests in the study of shaftless-driven systems. Owing to excellent synchronous performance, shaftless-driven printing presses play a central role in printing industry [1, 2]. However, the accuracy of multiaxis synchronization has a direct effect on production quality and efficiency [3]. Accordingly, finding more advanced multiaxis synchronous control technology remains a challenge.

Existing synchronization related literatures for multiaxis printing presses are mostly directed toward coupling control [4, 5] and virtual line shaft control [6, 7]. In the past years, considerable algorithms on synchronization for multiple motors pose advantages as well as limitations. References [4, 5] achieved synchronization by introducing parameter coupling into control strategies. However, the increasing coordinated axes induce intensive online computational work. References [6, 7] presented novel synchronous control laws with a virtual line shaft. However, there is no information exchange among followers, and it is difficult to find a satisfactory solution for the measurement of acceleration.

Meanwhile, we notice that consensus of multiagent system is arousing extensive attention in various disciplines, including biology, computer science, and control engineering [8–10]. Many existing papers have made great contributions in distributed coordinated control. Vicsek et al. first proposed the emergence of self-ordered motion in systems of particles with biologically motivated interaction and received interesting results [11]. Jadbabaie et al. provided a theoretical explanation for this convergence behavior and derived several other similar models [12]. Cortes extended the application of consensus algorithms to general continuous functions [13]. Thereafter, various consensus algorithms were investigated, ranging from single-integrator dynamics to double-integrator and high-order-integrator dynamics [14–16], from continuous time to discrete time [17, 18], from fixed topologies to switching topologies [19], from average consensus to consensus tracking [20–22], and so forth. Extension consensus algorithms that considered many other extra conditions were studied afterwards, and an adaptive synchronization algorithm of coupled oscillators was proposed [23]. In [24], an adaptive algorithm of coupled oscillators with multiple leaders was investigated. In [25], a superior decentralized adaptive cluster synchronization was introduced to investigate the pinning-control problem of complex dynamical networks. In [26, 27], input saturation was taken into account in the leader-following consensus of agents described by general linear systems. Due to different versions of consensus algorithms, various cooperation control capabilities were developed; examples include flocking [28, 29], task assignment [30, 31], containment control [32, 33], formation control [34–36], and rendezvous [37].

To our knowledge, many existing algorithms can obtain a stable synchronous system; however, when the obtained stable system encounters these unanticipated situations, such as parameter perturbations, external load disturbances, and model nonlinearities, the created consistent system may be inconsistent. Moreover, amounts of wastes would be produced in the high speed printing process. Therefore, in industrial manufacturing areas, one of the main challenges is how to find a control strategy that keeps all motors maintaining consensus all the time, especially in the case that one follower loses synchronization with reference signal. If the other followers turn to track the faulty one rather than the given reference in this circumstance, the problem would be solved. Inspired by the aforementioned researches and industrial requirement, based on a state-dependent-switching law, this paper proposes a virtual-leader-based consensus tracking control with hybrid structure for multimotor system.

The remainder of this paper is organized as follows. Section 2 states the problems to be solved according to a multimotor system. Section 3 establishes the control strategy and presents theoretical analysis. Simulation results are given in Section 4. Finally, Section 5 draws a conclusion.

2. Preliminaries and Problem Statement

Consider a shaftless-driven printing press (SDPP), which can be regarded as a multimotor system (MMS). Let each printing roller be driven by a servomotor, and each servomotor system stands for an agent with actual motion ability; multiple servo systems compose a multiagent system.

Let R denote the set of real number. The SDPP consists of n different motors, together with an additional motor labeled $0(L)$, which acts as the unique virtual leader of the group, and motors $1, 2, \ldots, n$ are followers.

The motion of each DC motor is described by [38]

$$\begin{aligned} \dot{\theta}_i &= \omega_i \\ \dot{\omega}_i &= -\frac{K_{ti}K_{ei}}{J_iR_i}\omega_i + \frac{K_{ti}}{J_iR_i}u_i - \frac{1}{J_i}T_{Li} \quad (i = 0, 1, \ldots, n), \end{aligned} \tag{1}$$

where θ_i is the position of motor i, ω_i is the speed of motor i, u_i is the control input of motor i, R_i is total armature resistance of motor i, K_{ei} is the voltage feedback coefficient of motor i, J_i is inertia of motor i, K_{ti} is the electromagnetic torque coefficient of motor i, and T_{Li} is the load torque of motor i.

Let $a_i = -K_{ti}K_{ei}/J_iR_i$, $b_i = K_{ti}/J_iR_i$, and $c_i = -1/J_i$; here, $a_i = \overline{a}_i + \Delta a_i$, $b_i = \overline{b}_i + \Delta b_i$, and $c_i = \overline{c}_i + \Delta c_i$. $\overline{a}_i$, $\overline{b}_i$, and $\overline{c}_i$ denote the nominal value of a_i, b_i, and c_i, respectively. Δa_i, Δb_i, and Δc_i denote the uncertain value.

Equation (1) can be rewritten as

$$\begin{aligned} \dot{\theta}_i &= \omega_i \\ \dot{\omega}_i &= \overline{a}_i\omega_i + \overline{b}_iu_i + F_i. \end{aligned} \tag{2}$$

Here, $i = 0, 1, \ldots, n$, $F_i = \Delta a_i\omega_i + \Delta b_iu_i + c_iT_{Li}$ is a time-varying uncertainty.

Define

$$S = \begin{bmatrix}\theta_0 & \theta_1 & \cdots & \theta_n\end{bmatrix}^T \tag{3}$$

to be the actual output position of each motor in MMS (2). Let θ^d be the given reference, which is only transmitted to virtual leader. The output of leader $\theta_0(\theta_L)$ acts as the reference of followers. Since the power of inverter is limited, the driven control system provides a limited torque; that is, θ_i $(i = 0, 1, \ldots, n)$ is bounded. Our results will rely on the following assumption.

Assumption 1. The uncertainty F_i $(i = 0, 1, \ldots, n)$, θ_i, $\dot{\theta}_i$, and $\ddot{\theta}_i$ $(i = 1, \ldots, n)$ are bounded in an actual SDPP.

The interaction topology of MMS (2) is represented by graph $G_{n+1} = (v_{n+1}, \varepsilon_{n+1})$, where $v_{n+1} = \{0, 1, \ldots, n\}$ is the set of nodes and $\varepsilon_{n+1} \subseteq v_{n+1} \times v_{n+1}$ is the set of edges. Let $A_{n+1} = [a_{ij}] \in R^{(n+1)\times(n+1)}$ be the adjacency matrix related to G_{n+1}; $a_{ij} > 0$ implies that information flows from node j to i; otherwise, $a_{ij} = 0$. Here we assume $a_{ii} = 0$, $\forall i = 0, 1, \ldots, n$. a_{i0} is positive if $(0, i) \in \varepsilon_{n+1}$; otherwise, $a_{i0} = 0$, $\forall i = 1, 2, \ldots, n$. Consider $a_{0j} = 0$, $\forall j = 0, 1, \ldots, n$. When we need to focus only on followers 1 to n, we use a follower graph $G_n = (v_n, \varepsilon_n)$, letting $\overline{A}_n = [a_{ij}] \in R^{n\times n}$ be the adjacency matrix associated with G_n. The Laplacian matrix $\overline{L}_n = [l_{ij}] \in R^{n\times n}$ associated with $\overline{A}_n$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$. In this paper, we suppose $\theta_0(\theta_L)$ is available to all the followers, and the followers have only local interactions with their neighbors and graphs G_{n+1} and G_n are fixed and have a spanning tree.

Problems to be addressed in this paper can be divided into two parts. One is the tracking control of all group members, under normal circumstances (i.e., the followers can track the motion trajectory of virtual leader with the given tracking control law), designing a tracking control algorithm to achieve $\|\theta_i - \theta_L\| \rightarrow 0$ $(i = 1, 2, \ldots, n)$, such that all followers will track the trajectory of virtual leader. The other one is consensus control among followers, under abnormal circumstance (i.e., at least one follower cannot follow the virtual leader during large enough disturbances), designing a consensus algorithm to guarantee that the other followers can track the faulty one; if it satisfies $\|\theta_i - \theta_j\| \rightarrow 0$ $(i, j = 1, 2, \ldots, n)$, then followers would maintain consensus. It contributes to solving the technological problem about how to reduce the production of vast amounts of wastes due to the existent printing-registration deviation when a high speed SDPP is in response to local environment disturbances.

3. Main Results

In this section, we design control algorithms to satisfy the conditions proposed in Section 2 and give the theoretical analysis.

Lemma 2 (see [39]). *Given a matrix $L_n = [l_{ij}] \in R^{n\times n}$, where $l_{ii} \le 0$, $l_{ij} \ge 0$, $\forall i \ne j$, and $\sum_{j=1}^{n} l_{ij} = 0$ for each i, then L_n has at least one zero eigenvalue with an associated eigenvector 1_n, and all nonzero eigenvalues are in the open left half plane. Furthermore, L_n has exactly one zero eigenvalue if and only if the directed graph of L_n has a directed spanning tree.*

Corollary 3 (see [39]). *The nonsymmetrical Laplacian matrix L_n of a directed graph has a simple zero eigenvalue with an associated eigenvector 1_n and all of the other eigenvalues are in the open right half plane if and only if the directed graph has a directed spanning tree.*

Define synchronous coordinated matrix $\overline{D} = [\overline{D}_{ij}] \in R^{n\times n}$ $(i, j = 1, 2, \dots, n)$, where

$$\overline{D}_{ij} = \theta_i - \theta_j. \tag{4}$$

Let $\overline{e} = [e_1\ e_2\ \cdots\ e_n]^T$ represent the tracking error of each member, where

$$e_i = \theta_i - \theta_L \quad (i = 1, 2, \dots, n). \tag{5}$$

Let $E_i = \sum_{j=1, j\ne i}^{n} a_{ij}(\theta_i - \theta_j)$ be the element of $E^{n\times 1}$, which represents the synchronous coordinated error of each follower, and $E^{n\times 1}$ can be expressed in a vector form

$$E = \left(\overline{A} \circ \overline{D}\right) 1_{n\times 1}. \tag{6}$$

Here, operator $\circ$ expresses the Hadamard product of matrix.

Theorem 4. *Consider the MMS (2) and suppose graphs G_{n+1} and G_n are connected and both have a spanning tree, under normal circumstance; n followers track the virtual leader if and only if $\overline{e} = 0$; under abnormal circumstance, the followers maintain consensus if and only if $E = 0$.*

Proof. From (5), $\overline{e} = [e_i]_{n\times 1} = [\theta_i - \theta_L]_{n\times 1}$, and we have $\overline{e} = 0 \Leftrightarrow \theta_i = \theta_L$ $(i = 1, 2, \dots, n)$. Thus, each follower can follow the motion state of virtual leader if and only if $\overline{e} = 0$.

Consider the related properties of adjacency matrix and (5); (6) is equivalent to

$$E = \left(\overline{A} \circ \overline{D}\right) 1_{n\times 1} = \overline{L}\overline{e}. \tag{7}$$

If we have $E = 0$, according to Lemma 2 and Corollary 3, we can also have $\overline{e} \in \operatorname{span}\{1\}$, it means that each element of $\overline{e}$ is nonzero and the same as the others; that is, $\theta_1 = \cdots = \theta_i = \cdots = \theta_n \ne \theta_L$. It follows that $E = 0$ infers $\theta_1 = \cdots = \theta_i = \cdots = \theta_n$ and vice versa. Therefore, we can conclude that followers maintain consensus under abnormal circumstance if and only if synchronous coordination error E asymptotically converges to zero.

This completes the proof. □

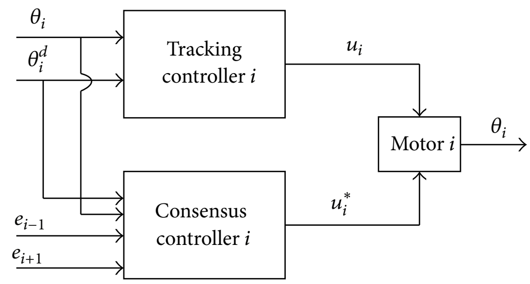

Figure 1: Structure diagram of the position controller.

Our control scheme is composed by tracking control under normal circumstance and coordinated consensus control under abnormal circumstance. The position controller structure of the follower servomotor i $(i = 1, 2, \dots, n)$ is shown in Figure 1. Define a state-dependent-switching law

$$U_i = \begin{cases} u_i & \dfrac{1}{2}\displaystyle\sum_{i,j=1}^{n} \left|e_i - e_j\right| < \delta \\ u_i^* & \dfrac{1}{2}\displaystyle\sum_{i,j=1}^{n} \left|e_i - e_j\right| \ge \delta, \end{cases} \tag{8}$$

where δ is a positive constant, and u_i and u_i^* are designed as equations (9) and (10).

The tracking control law of each motor is given by

$$\begin{aligned} u_i = \frac{1}{\overline{b}_i} \big(& -\lambda_i \dot{e}_i - \overline{a}_i \dot{\theta}_i + \ddot{\theta}_L - \overline{F}_i \operatorname{sgn}(\sigma_i) \\ & - h_i \left(\sigma_i + \beta_i \operatorname{sgn}(\sigma_i)\right)\big) \\ & (i = 0, 1, \dots, n), \end{aligned} \tag{9}$$

where the gains λ_i, h_i, and β_i are to be designed, $\overline{F}_i$ is the upper bound of F_i, and $\operatorname{sgn}(\sigma_i)$ denotes a signum function, where $\sigma_i = \lambda_i e_i + \dot{e}_i$.

The consensus control law is given by

$$u_i^* = k_p E_i + k_i \int E_i dt + k_d \frac{dE_i}{dt} \quad (i = 1, 2, \dots, n), \tag{10}$$

where the gains k_p, k_i, and k_d are to be designed.

Theorem 5. *Consider the MMS (2). Let each motor be steered by the control input (8). Then choose arbitrary proper gains k_p, k_i, k_d, and δ and positive gains λ_i, h_i, and β_i satisfying $\lambda_i h_i > 1/4$; the multimotor system asymptotically synchronizes with a common evolution with the following:*

(i) *under normal circumstances, $\|\theta_i - \theta_L\| \to 0$ $(i = 1, 2, \dots, n)$, as $t \to \infty$;*

(ii) *under abnormal circumstances, $\|\theta_i - \theta_j\| \to 0$ $(i = 1, 2, \dots, n)$, as $t \to \infty$.*

Proof. Case (i). Under the normal circumstances (i.e., when unanticipated environmental disturbances can be tolerated by followers, that is, $(1/2)\sum_{i,j=1}^{n}|e_i - e_j| < \delta$), define vector

$$\alpha_i^T = [e_i \ \ \sigma_i]. \tag{11}$$

Consider a Lyapunov function candidate

$$V(t) = \sum_{i=1}^{n} V_i(t), \tag{12}$$

where

$$V_i(t) = \frac{1}{2}\alpha_i^T \alpha_i \tag{13}$$

which is a positive definite function and radially unbounded with respect to α_i^T.

Differentiating $V(t)$ with respect to time gives

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{n} \alpha_i^T \dot{\alpha}_i \\ &= \sum_{i=1}^{n} (e_i\dot{e}_i + \sigma_i\dot{\sigma}_i) \\ &= \sum_{i=1}^{n} (e_i\dot{e}_i + \sigma_i(\lambda_i\dot{e}_i + \ddot{e}_i)).\end{aligned} \tag{14}$$

From (2) and (5), we obtain

$$\ddot{e}_i = \ddot{\theta}_i - \ddot{\theta}_L = \overline{a}_i\dot{\theta}_i + \overline{b}_i u_i + F_i - \ddot{\theta}_L. \tag{15}$$

Substituting (15) and (9) into (14) yields

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{n} \left(e_i\dot{e}_i + \sigma_i\left(\lambda_i\dot{e}_i + \overline{a}_i\dot{\theta}_i + \overline{b}_i u_i + F_i - \ddot{\theta}_L\right)\right) \\ &= \sum_{i=1}^{n} \Big[e_i\dot{e}_i + \sigma_i\Big(\lambda_i\dot{e}_i + \overline{a}_i\dot{\theta}_i + F_i - \ddot{\theta}_L \\ &\qquad + \left(-\lambda_i\dot{e}_i - \overline{a}_i\dot{\theta}_i + \ddot{\theta}_L\right) \\ &\qquad - \overline{F}_i\,\mathrm{sgn}(\sigma_i) - h_i(\sigma_i + \beta_i\,\mathrm{sgn}(\sigma_i))\Big)\Big] \\ &= \sum_{i=1}^{n} \left(e_i\dot{e}_i + F_i\sigma_i - \overline{F}_i|\sigma_i| - h_i\sigma_i^2 - h_i\beta_i|\sigma_i|\right) \\ &\le \sum_{i=1}^{n} \left(e_i(\sigma_i - \lambda_i e_i) + \left(|F_i| - \overline{F}_i\right)|\sigma_i| - h_i\sigma_i^2 - h_i\beta_i|\sigma_i|\right) \\ &\le \sum_{i=1}^{n} \left(-\lambda_i e_i^2 + e_i\sigma_i - h_i\sigma_i^2 - h_i\beta_i|\sigma_i|\right).\end{aligned} \tag{16}$$

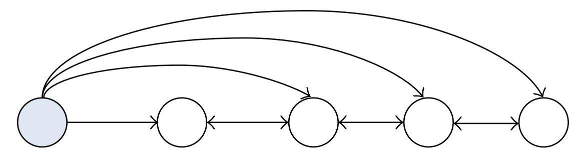

FIGURE 2: Interaction topology among five motors where θ_L is available to all the followers.

Letting $Q_i = \begin{bmatrix} \lambda_i & -1/2 \\ -1/2 & h_i \end{bmatrix}$, we have $|Q_i| = \lambda_i h_i - 1/4$ and supposing $\lambda_i h_i > 1/4$, then, $|Q_i| > 0$, Q_i is a positive definite matrix

$$\begin{aligned}\alpha_i^T Q_i \alpha_i &= [e_i \ \ \sigma_i]\begin{bmatrix} \lambda_i & -\frac{1}{2} \\ -\frac{1}{2} & h_i \end{bmatrix}\begin{bmatrix} e_i \\ \sigma_i \end{bmatrix} \\ &= \lambda_i e_i^2 - e_i\sigma_i + h_i\sigma_i^2.\end{aligned} \tag{17}$$

Substituting (17) into (16) yields

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=1}^{n} \left(-\alpha_i^T Q_i \alpha_i - h_i\beta_i|\sigma_i|\right) \\ &\le \sum_{i=1}^{n} -\alpha_i^T Q_i \alpha_i \\ &\le \sum_{i=1}^{n} -\lambda_{\min}(Q_i)\left\|\alpha_i^T\right\|^2 \le 0,\end{aligned} \tag{18}$$

where $\lambda_{\min}(Q_i)$ is the minimum eigenvalue of Q_i.

It implies that $V(t) \le V(0)$; that is, e_i, δ_i are bounded; $\dot{e}_i$ is bounded since δ_i is bounded. $\ddot{e}_i$ is bounded from (15) and Assumption 1, and thus $\dot{e}_i$, $\dot{\sigma}_i$ are bounded, and then e_i, σ_i are uniformly continuous; from Barbalat lemma, $\alpha_i^T = [e_i \ \ \sigma_i] \to 0$ as $t \to \infty$; that is, $\theta_i \to \theta_L$, which is equivalent to $\|\theta_i - \theta_L\| \to 0$.

Case (ii). Under the abnormal circumstances (i.e., when unanticipated environmental disturbances cannot be tolerated by followers, at least one member fails to follow the reference motion state; that is, it satisfies $(1/2)\sum_{i,j=1}^{n}|e_i - e_j| \ge \delta$), when $(1/2)\sum_{i,j=1}^{n}|e_i - e_j|$ is above a specified threshold δ, the system will switch to consensus control law u_i^* which is shown in (10), controller u_i^* provides $E \to 0$, as the proof of Theorem 4, and $E \to 0$ is equivalent to $e_1 = \cdots = e_i = \cdots = e_n \ne 0$ (i.e., $\theta_1 = \cdots = \theta_i = \cdots = \theta_n$). Thus $\|\theta_i - \theta_j\| \to 0$, and all the followers maintain consensus. In the regulation of switching law and consensus control law, the system ultimately switches to tracking control and converges to the leader again.

This completes the proof. □

4. Simulations

In this section, three different cases are considered to validate the theoretical results. The network topology of MMS (2)

Table 1: Parameters of five driven motors.

Motor	0 (L)	1	2	3	4
R (Ω)	0.2	0.5	0.4	0.6	0.7
K_t	0.005	0.01	0.008	0.015	0.02
J (Kg·m^2)	0.02	0.03	0.025	0.05	0.04
K_e	0.1	0.2	0.2	0.18	0.25
Initial position (rad)	0.5	−0.8	1.3	−2.0	−2.4

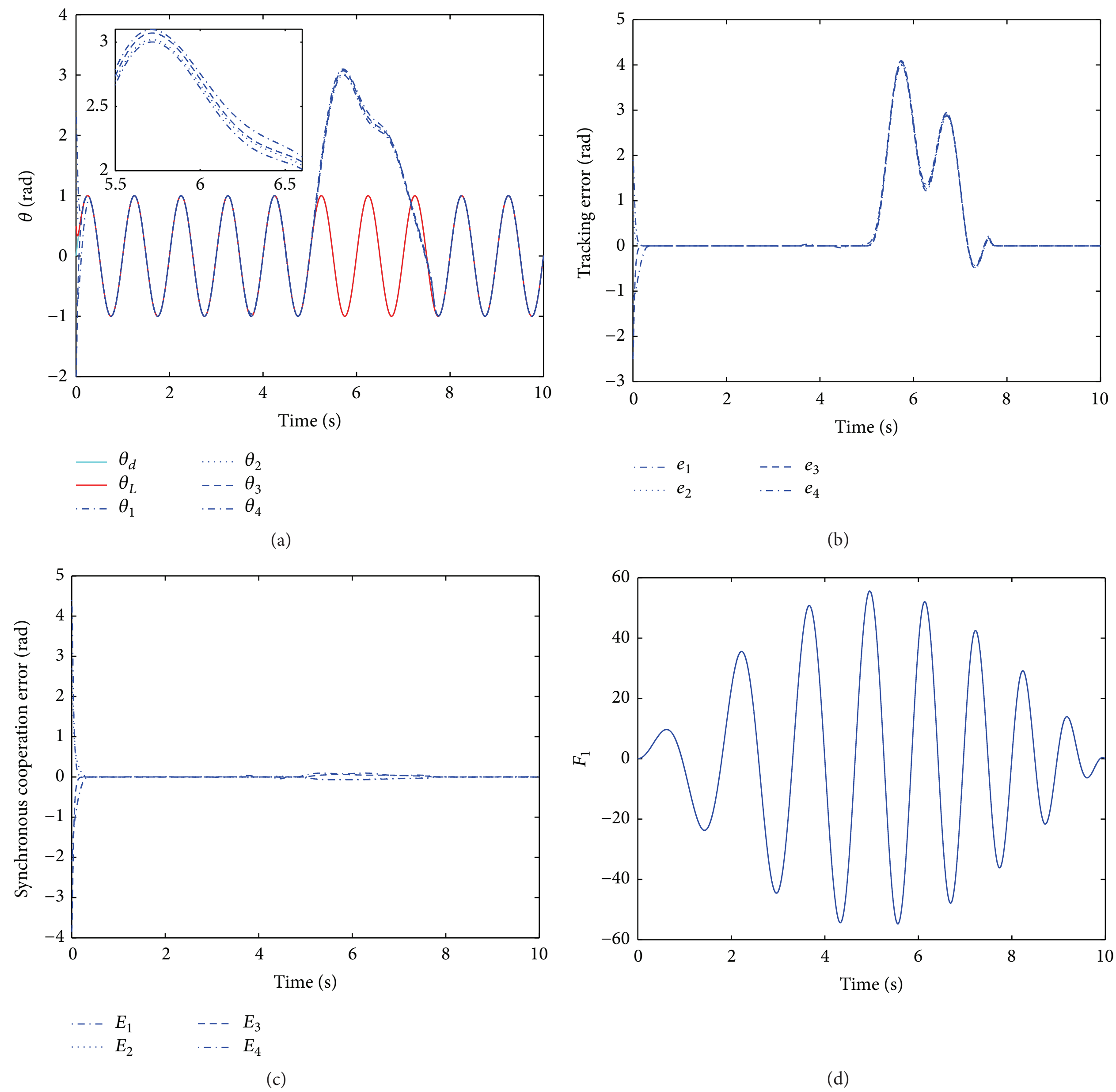

Figure 3: Simulation results for motor 1 with a slowly varying disturbance under protocol (8) corresponding to topology in Figure 2.

is shown in Figure 2. Apparently, it has a spanning tree in this graph, for which the virtual leader $0(L)$ acts as the root node, and the reference state θ_L is available to all the followers. Follower motor 1 is selected to be added to various disturbances. The parameters of the five driven motors are given in Table 1. Let each motor be steered by the control input (8), set $\delta = 0.075$, and let the consensus reference state of virtual leader be $\theta^d = \sin(2\pi t)$rad.

From Figure 2, let

$$A = \begin{bmatrix} 0 & B \\ C & A \end{bmatrix}, \tag{19}$$

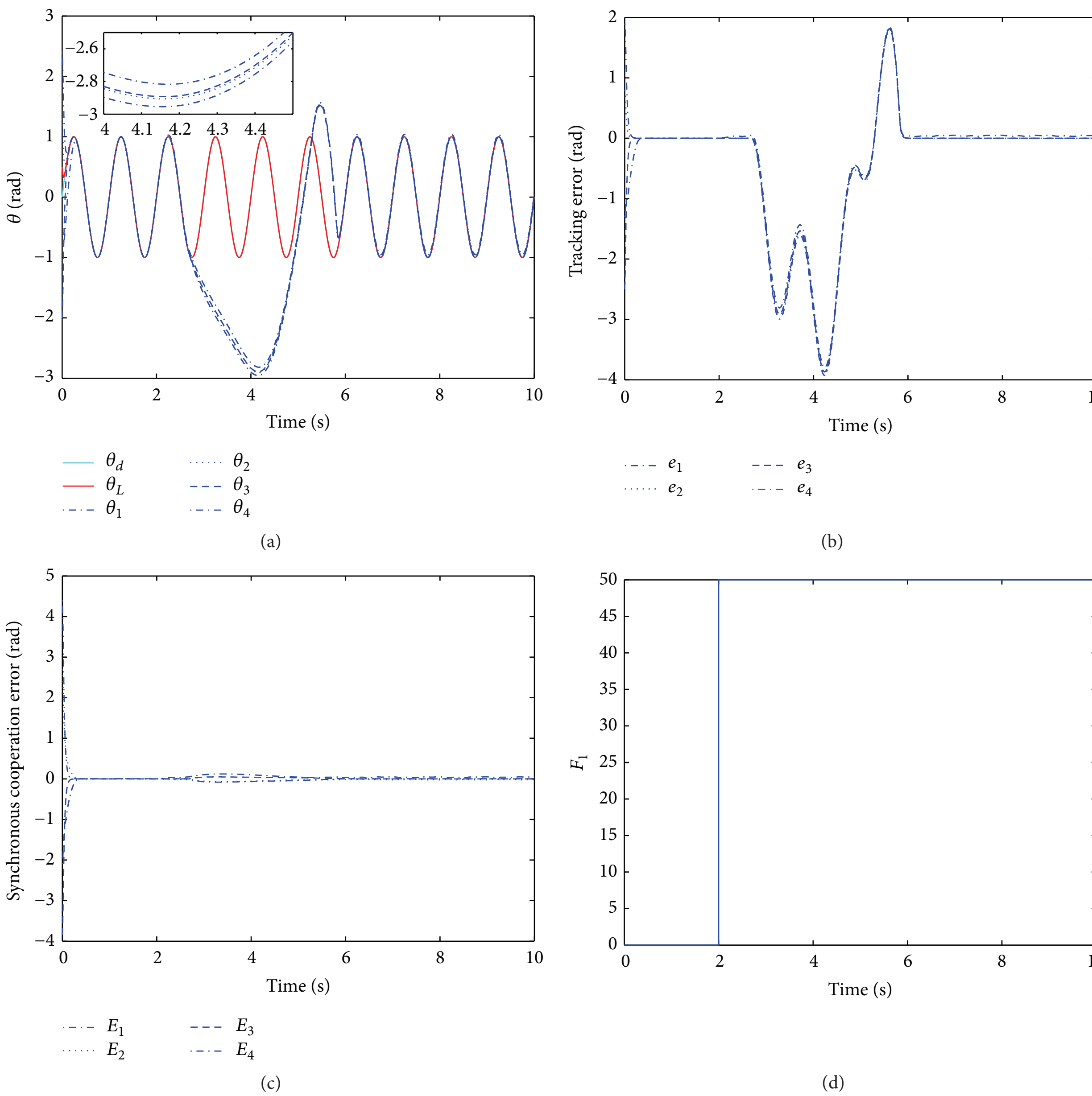

FIGURE 4: Simulation results for motor 1 with a step disturbance under protocol (8) corresponding to topology in Figure 2.

and supposing all the neighbors have the same effect on each motor, we have

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T,$$
$$\overline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{20}$$

Case (1). In the first case, we choose a slowly varying disturbance that is shown in subplot 3(d). Subplot 3(a) shows the position tracking of each motor, subplot 3(b) is the tracking error of each team member, and subplot (c) in Figure 3 shows the synchronous cooperation error of each follower. From subplot 3(a), each follower can follow the reference state at first. As the disturbance F_1 increases gradually, motor 1 will lose tracking. Simultaneously, from subplots 3(a) and 3(b), we can see that other followers maintain consensus with motor 1 in this situation.

Case (2). In the second case, we consider a step disturbance

$$F_1 = \begin{cases} 0N \cdot m & t < 2s \\ 50N \cdot m & t \geq 2s, \end{cases} \tag{21}$$

which is shown in subplot 4(d). It is obvious from subplot 4(a) that the control protocol (8) is capable of tracking the virtual leader and that the followers maintain a consensus state even

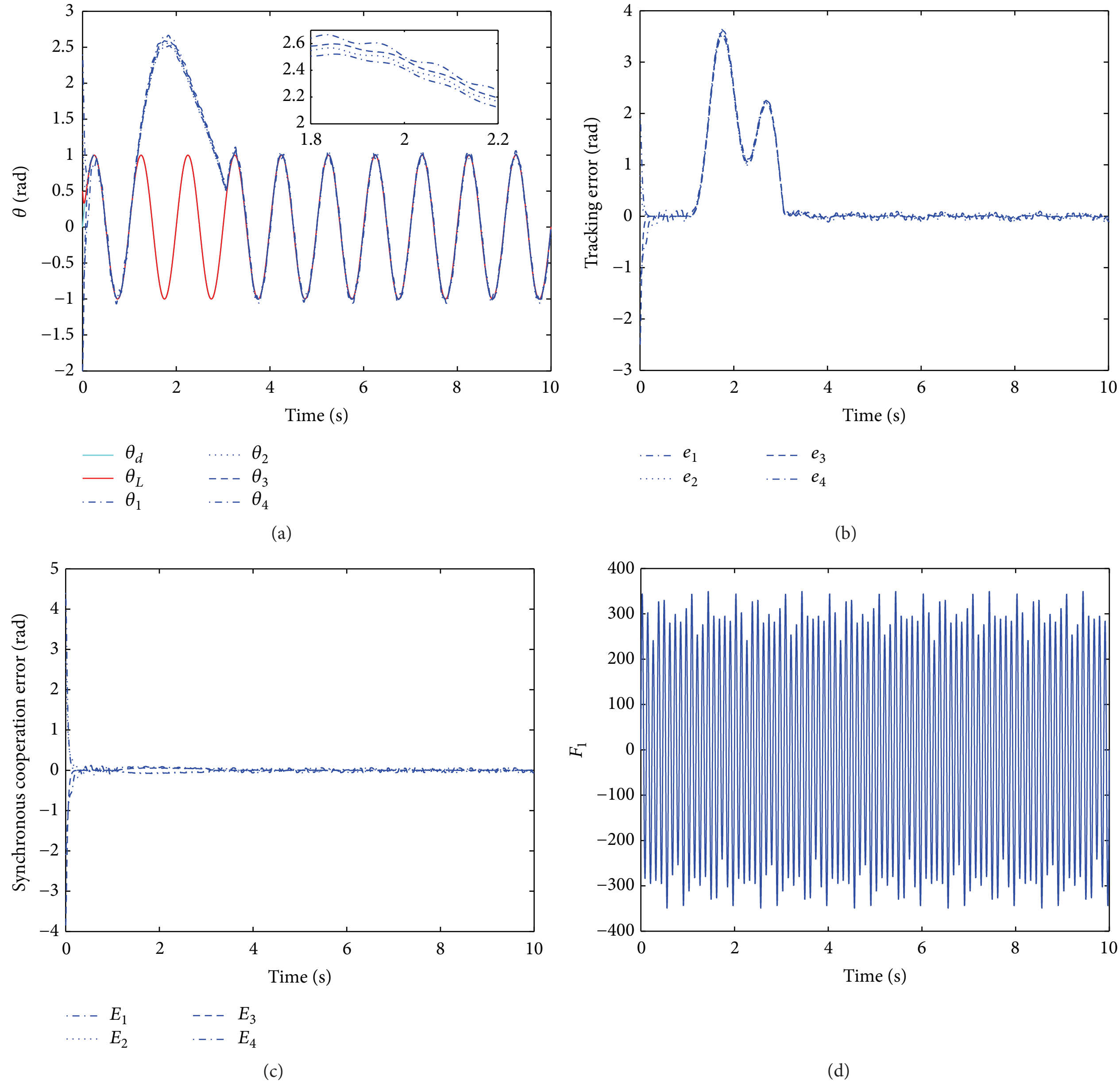

FIGURE 5: Simulation results for motor 1 with a high-frequency disturbance under protocol (8) corresponding to topology in Figure 2.

during a step disturbance. Subplot 4(b) corresponds to the tracking error of each motor and subplot 4(c) corresponds to the synchronous cooperation error of each follower.

Case (3). The third case considers a high-frequency disturbance shown in subplot 5(d). It is clear from subplot (a) in Figure 5 that the control protocol (8) is capable of tracking the virtual leader at first and followers maintain consensus even during high-frequency disturbance F_1 in 5(d). Subplot 5(b) presents the tracking error of each motor, and subplot 5(c) shows the synchronous cooperation error of each follower.

5. Conclusion

In this paper, we have studied approaches of improving synchronous accuracy for multiple motors. Compared with other relevant results which refer only to a final synchronization, our novel control strategy in this paper additionally considers the synchronous process by integrating tracking with consensus control based on hybrid structure. Theoretical analysis has shown that all followers asymptotically converge to a consistent state even when one follower fails to follow the virtual leader during a large enough disturbance. Simulation results show good performance of synchronization control accuracy, interference immunity, and convergence for the suggested algorithms.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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