

## Research Article

# Existence and Stability of Positive Periodic Solutions for a Neutral Multispecies Logarithmic Population Model with Feedback Control and Impulse

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We investigate a neutral multispecies logarithmic population model with feedback control and impulse. By applying the contraction mapping principle and some inequality techniques, a set of easily applicable criteria for the existence, uniqueness, and global attractivity of positive periodic solution are established. The conditions we obtained are weaker than the previously known ones and can be easily reduced to several special cases. We also give an example to illustrate the applicability of our results.

## 1. Introduction

As is known to all, ecosystem in the real world is continuously distributed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Of practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In recent years, the qualitative behaviors of the population dynamics with feedback control has attracted the attention of many mathematicians and biologists [1–5]. On the other hand, there are some other perturbations in the real world such as fires and floods, which are not suitable to be considered continually. These perturbations bring sudden changes to the system. Systems with such sudden perturbations involving impulsive differential equations have attracted the interest of many researchers in the past twenty years [6–10], since they provide a natural description of several real processes subject to certain perturbations whose duration is negligible in comparison with the duration of the process. Such processes are often investigated in various fields of science and technology such as physics, population dynamics, ecology, biological systems, and optimal control; for details, see [11–13]. However, to the best of the author's knowledge, to this day, no scholar considered the neutral multispecies logarithmic population model with feedback control and impulse.

The aim of this paper is to investigate the existence, uniqueness, and global attractivity of the positive periodic solution for the following neutral multispecies logarithmic population system with feedback control and impulse:

$$\begin{aligned} & \frac{dN_i(t)}{dt} \\ &= N_i(t) \left[ r_i(t) - \sum_{j=1}^n a_{ij}(t) \ln N_j(t) \right. \\ & \quad - \sum_{j=1}^n b_{ij}(t) \ln N_j(t - \tau_{ij}(t)) \\ & \quad - \sum_{j=1}^n c_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) \ln N_j(s) ds \\ & \quad - \sum_{j=1}^n d_{ij}(t) \frac{d \ln N_j(t - \delta_{ij}(t))}{dt} \\ & \quad \left. - e_i(t) u_i(t) - f_i(t) u_i(t - \sigma_i(t)) \right], \\ & \quad t \neq t_k, \end{aligned}$$

$$\begin{aligned} \frac{du_i(t)}{dt} &= -\alpha_i(t)u_i(t) + \beta_i(t)\ln N_i(t) \\ &\quad + \vartheta_i(t)\ln N_i(t - \gamma_i(t)), \quad t \geq 0, \\ N_i(t_k^+) &= e^{(1+\theta_{ik})}N_i(t_k), \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, \end{aligned} \tag{1}$$

where  $u_i(t)$  denote indirect feedback control variables. For the ecological justification of (1) and the similar types, refer to [14–20].

For the sake of generality and convenience, we always make the following fundamental assumptions:

- (H<sub>1</sub>)  $r_i(t), a_{ij}(t), b_{ij}(t), c_{ij}(t), d_{ij}(t), e_i(t), f_i(t), \tau_{ij}(t), \delta_{ij}(t) \in C^2(\mathbb{R}, \mathbb{R}), \sigma_i(t), \gamma_i(t), \alpha_i(t), \beta_i(t)$ , and  $\eta_i(t)$  are continuous nonnegative  $\omega$ -periodic functions with  $\int_0^\omega r_i(t) > 0, a_{ii}(t) > 0, \delta'_{ij}(t) < 1, \tau = \max_{t \in [0, \omega]} \{\tau_{ij}(t), \delta_{ij}(t), \sigma_i(t), \gamma_i(t)\}$ , and  $\int_0^\infty K_{ij}(s)ds = 1, \int_0^{+\infty} sK_{ij}(s)ds < +\infty, i, j = 1, 2, \dots, n$ ;
- (H<sub>2</sub>)  $0 < t_1 < t_2 < \dots < t_k < \dots$  are fixed impulsive points with  $\lim_{k \rightarrow \infty} t_k = +\infty$ ;
- (H<sub>3</sub>)  $\{\theta_{ik}\}$  is a real sequence,  $\theta_{ik} + 1 > 0$ , and  $\prod_{0 < t_k < t} (1 + \theta_{ik})$  is an  $\omega$ -periodic function.

In the following section, some definitions and some useful lemmas are listed. In the third section, by applying the contraction mapping principle, some sufficient conditions which ensure the existence and uniqueness of positive periodic solution of system (1) are established, and then we get a few sufficient conditions ensuring the global attractivity of the positive periodic solution by employing some inequality techniques. Finally, we give an example to show our results.

## 2. Preliminaries

In order to obtain the existence and uniqueness of a periodic solution for system (1), we first give some definitions and lemmas.

*Definition 1.* A function  $N_i : \mathbb{R} \rightarrow (0, \infty) (i = 1, 2, \dots, n)$  is said to be a positive solution of (1), if the following conditions are satisfied:

- (a)  $N_i(t)$  is absolutely continuous on each  $(t_k, t_{k+1})$ ;
- (b) for each  $k \in \mathbb{Z}_+, N_i(t_k^+)$  and  $N_i(t_k^-)$  exist, and  $N_i(t_k^-) = N_i(t_k)$ ;
- (c)  $N_i(t)$  satisfies the first equation of (1) for almost everywhere (for short a.e.) in  $[0, \infty) \setminus \{t_k\}$  and satisfies  $N_i(t_k^+) = (1 + \theta_{ik})N_i(t_k)$  for  $t = t_k, k \in \mathbb{Z}_+ = \{1, 2, \dots\}$ .

*Definition 2.* System (1) is said to be globally attractive, if there exists a positive solution  $(N_i(t), u_i(t))$  of (1) such that  $\lim_{t \rightarrow +\infty} |N_i(t) - N_i^*(t)| = 0, \lim_{t \rightarrow +\infty} |u_i(t) - u_i^*(t)| = 0$ , for any other positive solution  $(N_i^*(t), u_i^*(t))$  of the system (1).

We can easily get the following lemma.

**Lemma 3.**  $R_+^{2n} = \{(N_i(t), u_i(t)) : N_i(0) > 0, u_i(0) > 0, i = 1, 2, \dots, n\}$  is the positive invariable region of the system (1).

*Proof.* In view of biological population, we obtain  $N_i(0) > 0, u_i(0) > 0$ . By the system (1), we have

$$\begin{aligned} N_i(t) &= N_i(0) \\ &\quad \times \exp \left\{ \int_0^t \left[ r_i(\eta) - \sum_{j=1}^n a_{ij}(\eta)\ln N_j(\eta) \right. \right. \\ &\quad \left. \left. - \sum_{j=1}^n b_{ij}(\eta)\ln N_j(\eta - \tau_{ij}(\eta)) \right. \right. \\ &\quad \left. \left. - \sum_{j=1}^n c_{ij}(\eta) \int_{-\infty}^t K_{ij}(\eta - s)\ln N_j(s) ds \right. \right. \\ &\quad \left. \left. - \sum_{j=1}^n d_{ij}(\eta) \frac{d \ln N_j(\eta - \delta_{ij}(\eta))}{dt} \right. \right. \\ &\quad \left. \left. - e_i(\eta)u_i(\eta) \right. \right. \\ &\quad \left. \left. - f_i(\eta)u_i(\eta - \sigma_i(\eta)) \right] d\eta \right\}, \\ &\quad t \in [0, t_1], \quad i = 1, 2, \dots, n, \end{aligned}$$

$$\begin{aligned} N_i(t) &= N_i(t_k) \\ &\quad \times \exp \left\{ \int_{t_k}^t \left[ r_i(\eta) \right. \right. \\ &\quad \left. \left. - \sum_{j=1}^n a_{ij}(\eta)\ln N_j(\eta) \right. \right. \\ &\quad \left. \left. - \sum_{j=1}^n b_{ij}(\eta)\ln N_j(\eta - \tau_{ij}(\eta)) \right. \right. \\ &\quad \left. \left. - \sum_{j=1}^n c_{ij}(\eta) \int_{-\infty}^t K_{ij}(\eta - s)\ln N_j(s) ds \right. \right. \\ &\quad \left. \left. - \sum_{j=1}^n d_{ij}(\eta) \frac{d \ln N_j(\eta - \delta_{ij}(\eta))}{dt} \right. \right. \\ &\quad \left. \left. - e_i(\eta)u_i(\eta) \right. \right. \\ &\quad \left. \left. - f_i(\eta)u_i(\eta - \sigma_i(\eta)) \right] d\eta \right\}, \\ &\quad t \in (t_k, t_{k+1}], \quad i = 1, 2, \dots, n, \end{aligned}$$

$$N_i(t_k^+) = e^{(1+p_{ik})}N_i(t_k) > 0, \quad k \in \mathbb{N}, \quad i = 1, 2, \dots, n,$$

$$\begin{aligned} u_i(t) &= \int_t^{t+\omega} G(t, s) [\beta_i(s)\ln N_i(s) \\ &\quad + \vartheta_i(s)\ln N_i(s - \gamma_i(s))] ds \\ &:= (\phi_i \ln N_i)(t), \end{aligned} \tag{2}$$

where

$$G_i(t, s) = \frac{\exp \left\{ \int_t^s \alpha_i(\xi) d\xi \right\}}{\exp \left\{ \int_t^s \alpha_i(\xi) d\xi \right\} - 1}. \tag{3}$$

Then the solution of the system (1) is positive.

Under the above hypotheses  $(H_1)$ – $(H_3)$ , we consider the neutral nonimpulsive system:

$$\begin{aligned} \frac{dy_i(t)}{dt} = y_i(t) & \left[ r_i(t) - \sum_{j=1}^n A_{ij}(t) \ln y_j(t) \right. \\ & - \sum_{j=1}^n B_{ij}(t) \ln y_j(t - \tau_{ij}(t)) \\ & - \sum_{j=1}^n C_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) \ln y_j(s) ds \\ & - \sum_{j=1}^n D_{ij}(t) \frac{d \ln y_j(t - \delta_{ij}(t))}{dt} \\ & \left. - e_i(t) u_i(t) - f_i(t) u_i(t - \sigma_i(t)) \right], \tag{4} \end{aligned}$$

$$\begin{aligned} \frac{du_i(t)}{dt} = -\alpha_i(t) u_i(t) + \beta_i^*(t) \ln y_i(t) \\ + \vartheta_i^*(t) \ln y_i(t - \gamma_i(t)), \end{aligned}$$

where

$$\begin{aligned} A_{ij}(t) &= a_{ij}(t) \prod_{0 < t_k < t} (1 + \theta_{ik}), \\ B_{ij}(t) &= b_{ij}(t) \prod_{0 < t_k < t - \tau_{ij}(t)} (1 + \theta_{ik}), \\ C_{ij}(t) &= c_{ij}(t) \prod_{0 < t_k < t} (1 + \theta_{ik}), \\ D_{ij}(t) &= d_{ij}(t) \prod_{0 < t_k < t - \delta_{ij}(t)} (1 + \theta_{ik}), \\ \beta_i^*(t) &= \beta_i(t) \prod_{0 < t_k < t} (1 + \theta_{ik}), \\ \vartheta_i^*(t) &= \theta_i(t) \prod_{0 < t_k < t - \gamma_i(t)} (1 + \theta_{ik}). \end{aligned} \tag{5}$$

By a solution  $(y_i(t), u_i(t))$  of (4), it means an absolutely continuous function  $(y_i(t), u_i(t))$  defined on  $[-\tau, 0]$  that satisfies (4) a.e., for  $t \geq 0$ , and  $y(\xi) = \varphi(\xi)$ ,  $y'(\xi) = \varphi'(\xi)$  on  $[-\tau, 0]$ .  $\square$

The following lemma will be used in the proofs of our results, and the proof of the lemma is similar to that of Theorem 1 in [6].

**Lemma 4.** *Suppose that  $(H_1)$ – $(H_4)$  hold. Then*

- (i) *if  $(y_i(t), u_i(t))$  is a solution of (4) on  $[-\tau, +\infty)$ , then  $(N_i(t), u_i(t)) = (\prod_{0 < t_k < t} e^{(1+\theta_{ik})} y_i(t), u_i(t))$  is a solution of (1) on  $[-\tau, +\infty)$ ,*

- (ii) *if  $(N_i(t), u_i(t))$  is a solution of (1) on  $[-\tau, +\infty)$ , then  $(y_i(t), u_i(t)) = (\prod_{0 < t_k < t} (e^{(1+\theta_{ik})})^{-1} N_i(t), u_i(t))$  is a solution of (4) on  $[-\tau, +\infty)$ .*

*Proof.* (i) It is easy to see that  $(N_i(t), u_i(t)) = (\prod_{0 < t_k < t} e^{(1+\theta_{ik})} y_i(t), u_i(t))$  is absolutely continuous on every interval  $(t_k, t_{k+1}]$ ,  $t \neq t_k$ ,  $k = 1, 2, \dots$ ,

$$\begin{aligned} & N_i'(t) - N_i(t) \\ & \times \left[ r_i(t) - \sum_{j=1}^n a_{ij}(t) \ln N_j(t) - \sum_{j=1}^n b_{ij}(t) \ln N_j(t - \tau_{ij}(t)) \right. \\ & - \sum_{j=1}^n c_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) \ln N_j(s) ds \\ & - \sum_{j=1}^n d_{ij}(t) \frac{d \ln N_j(t - \delta_{ij}(t))}{dt} \\ & \left. - e_i(t) u_i(t) - f_i(t) u_i(t - \sigma_i(t)) \right] \\ & = \prod_{0 < t_k < t} e^{(1+\theta_{ik})} y_i'(t) - \prod_{0 < t_k < t} e^{(1+\theta_{ik})} y_i(t) \\ & \times \left[ r_i(t) - \sum_{j=1}^n a_{ij}(t) \prod_{0 < t_k < t} (1 + \theta_{ik}) \ln y_j(t) \right. \\ & - \sum_{j=1}^n b_{ij}(t) \prod_{0 < t_k < t - \tau_{ij}(t)} (1 + \theta_{ik}) \ln y_j(t - \tau_{ij}(t)) \\ & - \sum_{j=1}^n c_{ij}(t) \int_{-\infty}^t K_j(t-s) \prod_{0 < t_k < t} (1 + \theta_{ik}) \ln y_j(s) ds \\ & - \sum_{j=1}^n d_{ij}(t) \prod_{0 < t_k < t - \delta_{ij}(t)} (1 + \theta_{ik}) \frac{dy_j(t - \delta_{ij}(t))}{dt} \\ & \left. - e_i(t) u_i(t) - f_i(t) u_i(t - \sigma_i(t)) \right] \\ & = \prod_{0 < t_k < t} e^{(1+\theta_k)} \\ & \times \left\{ y'(t) - y(t) \right. \\ & \times \left[ r_i(t) - \sum_{j=1}^n A_{ij}(t) \ln y_j(t) \right. \\ & \left. - \sum_{j=1}^n B_{ij}(t) \ln y_j(t - \tau_{ij}(t)) \right] \end{aligned}$$

$$\begin{aligned}
 & - \sum_{j=1}^n C_{ij}(t) \times \int_{-\infty}^t K_{ij}(t-s) \ln y_j(s) ds \\
 & - \sum_{j=1}^n D_{ij}(t) \frac{d \ln y_j(t - \delta_{ij}(t))}{dt} \\
 & \left. - e_i(t) u_i(t) - f_i(t) u_i(t - \sigma_i(t)) \right\} = 0,
 \end{aligned}$$

$$\begin{aligned}
 & u_i'(t) + \alpha_i(t) u_i(t) - \beta_i(t) \ln N_i(t) - \vartheta_i(t) \ln N_i(t - \gamma_i(t)) \\
 & = u_i'(t) + \alpha_i(t) u_i(t) - \beta_i^*(t) \ln y_i(t) \\
 & - \vartheta_i^*(t) \ln y_i(t - \gamma_i(t)) = 0.
 \end{aligned} \tag{6}$$

On the other hand, for any  $t = t_k, k = 1, 2, \dots$ ,

$$\begin{aligned}
 N_i(t_k^+) &= \lim_{t \rightarrow t_k^+} \prod_{0 < t_j < t} e^{(1+\theta_{ik})} y_i(t) \\
 &= \prod_{0 < t_j \leq t_k} e^{(1+\theta_{ik})} y_i(t_k),
 \end{aligned} \tag{7}$$

$$N_i(t_k) = \prod_{0 < t_j < t_k} e^{(1+\theta_{ik})} y_i(t_k).$$

Thus

$$N(t_k^+) = e^{(1+\theta_{ik})} N(t_k), \quad k = 1, 2, \dots \tag{8}$$

It follows from (6)–(8) that  $(N_i(t), u_i(t))$  is a solution of (1).

(ii) Since  $N_i(t) = \prod_{0 < t_k < t} e^{(1+\theta_{ik})} y_i(t)$  is absolutely continuous on every interval  $(t_k, t_{k+1}]$ ,  $t \neq t_k, k = 1, 2, \dots$ , and in view of (8), it follows that for any  $k = 1, 2, \dots$ ,

$$\begin{aligned}
 y_i(t_k^+) &= \prod_{0 < t_j \leq t_k} (e^{(1+\theta_{ik})})^{-1} N_i(t_k^+) \\
 &= \prod_{0 < t_j < t_k} (e^{(1+\theta_{ik})})^{-1} N_i(t_k) = y_i(t_k), \\
 y_i(t_k^-) &= \prod_{0 < t_j < t_k} (e^{(1+\theta_{ik})})^{-1} N_i(t_k^-) \\
 &= \prod_{0 < t_j \leq t_k^-} (e^{(1+\theta_{ik})})^{-1} N_i(t_k^-) = y_i(t_k),
 \end{aligned} \tag{9}$$

which implies that  $y_i(t)$  is continuous on  $[-\tau, +\infty)$ . It is easy to prove that  $y_i(t)$  is absolutely continuous on  $[-\tau, +\infty)$ . Similar to the proof of (i), we can check that  $(y_i(t), u_i(t)) = (\prod_{0 < t_k < t} (e^{(1+\theta_{ik})})^{-1} N_i(t), u_i(t))$  are solutions of (4) on  $[-\tau, +\infty)$ . The proof of Lemma 4 is completed.  $\square$

**Lemma 5.**  $(y_i(t), u_i(t))$  is a  $\omega$ -periodic solution of (4) if and only if  $y_i(t)$  is a  $\omega$ -periodic solution of the following system:

$$\begin{aligned}
 & \frac{dy_i(t)}{dt} \\
 & = y_i(t) \left[ r_i(t) - \sum_{j=1}^n A_{ij}(t) \ln y_j(t) \right. \\
 & \quad - \sum_{j=1}^n B_{ij}(t) \ln y_j(t - \tau_{ij}(t)) \\
 & \quad - \sum_{j=1}^n C_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) \ln y_j(s) ds \\
 & \quad - \sum_{j=1}^n D_{ij}(t) \frac{d \ln y_j(t - \delta_{ij}(t))}{dt} \\
 & \quad - e_i(t) (\phi_i \ln y_i)(t) \\
 & \quad \left. - f_i(t) (\phi_i \ln y_i)(t) (t - \sigma_i(t)) \right],
 \end{aligned} \tag{10}$$

where

$$\begin{aligned}
 & (\phi_i \ln y_i)(t) \\
 & := \int_t^{t+\omega} G_i(t, s) [\beta_i^*(s) \ln y_i(s) + \vartheta_i^*(s) \ln y_i(s - \gamma_i(s))] ds,
 \end{aligned} \tag{11}$$

and  $G_i(t, s)$  is defined by (3).

*Proof.* The proof of Lemma 5 is similar to that of Lemma 2.2 in [2], and we omit the details here.  $\square$

Obviously, the existence, uniqueness, and global attractivity of positive periodic solution of system (1) is equivalent to the existence, uniqueness, and global attractivity of periodic solution of system (10).

**Lemma 6.** Assume that  $u(t), \tau(t)$  are all continuously differentiable  $\omega$ -periodic functions and  $a(t)$  is a nonnegative continuous  $\omega$ -periodic function such that  $\int_0^\omega a(t) dt > 0$ ; then

$$\begin{aligned}
 & \int_{-\infty}^t e^{-\int_s^t a(\xi) d\xi} b(s) u'(s - \tau(s)) ds \\
 & = c(t) u(t - \tau(t)) \\
 & - \int_{-\infty}^t e^{-\int_s^t a(\xi) d\xi} [a(s) c(s) + c'(s)] u(s - \tau(s)) ds,
 \end{aligned} \tag{12}$$

where  $c(t) = b(t)/(1 - \tau'(t))$ .

*Proof.* As

$$\begin{aligned}
 & \int_{-\infty}^t e^{-\int_s^t a(\xi)d\xi} b(s) u'(s - \tau(s)) ds \\
 &= \int_{-\infty}^t e^{-\int_s^t a(\xi)d\xi} c(s) du(s - \tau(s)) \\
 &= e^{-\int_s^t a(\xi)d\xi} c(s) u(s - \tau(s)) \Big|_{-\infty}^t \\
 &\quad - \int_{-\infty}^t u(s - \tau(s)) d\left(e^{-\int_s^t a(\xi)d\xi} c(s)\right) \\
 &= e^{-\int_s^t a(\xi)d\xi} c(s) u(s - \tau(s)) \Big|_{-\infty}^t \\
 &\quad - \int_{-\infty}^t u(s - \tau(s)) [a(s)c(s) + c'(s)] e^{-\int_s^t a(\xi)d\xi} ds.
 \end{aligned} \tag{13}$$

Denote  $m = e^{-\int_0^\omega a(t)dt}$ ; then from  $a(t) \geq 0$ ,  $\int_0^\omega a(t)dt > 0$ , it follows that  $m < 1$ . Also, when  $t \geq s$  without loss of generality, we may assume that  $s + n\omega \leq t \leq s + (n + 1)\omega$ ; thus

$$\begin{aligned}
 & \left| e^{-\int_s^t a(\xi)d\xi} c(s) u(s - \tau(s)) \right| \\
 &\leq e^{-\int_s^t a(\xi)d\xi} \|c\| \|u\| \\
 &= e^{-\sum_{j=1}^{n-1} \int_{s+j\omega}^{s+(j+1)\omega} a(\xi)d\xi - \int_{s+n\omega}^t a(\xi)d\xi} \|c\| \|u\| \\
 &= m^n e^{-\int_{s+n\omega}^t a(\xi)d\xi} \|c\| \|u\| \\
 &\leq m^n \|c\| \|u\|.
 \end{aligned} \tag{14}$$

Therefore,

$$\lim_{s \rightarrow -\infty} e^{-\int_s^t a(\xi)d\xi} c(s) u(s - \tau(s)) = 0, \tag{15}$$

and so from (13)–(15) it follows that

$$\begin{aligned}
 & \int_{-\infty}^t e^{-\int_s^t a(\xi)d\xi} b(s) u'(s - \tau(s)) ds \\
 &= c(t) u(t - \tau(t)) \\
 &\quad - \int_{-\infty}^t e^{-\int_s^t a(\xi)d\xi} [a(s)c(s) + c'(s)] u(s - \tau(s)) ds.
 \end{aligned} \tag{16}$$

The proof Lemma 6 is complete. □

### 3. Main Theorem

In this section, by using contraction principle and some inequality techniques, several conditions on the existence, uniqueness, and global attractivity of periodic solution for system (1) are presented.

Let  $y_i(t) = e^{h_i x_i(t)}$ ; the system (10) can be reduced to

$$\begin{aligned}
 \frac{dx_i}{dt} &= -A_{ii}(t) x_i(t) - h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) x_j(t) \\
 &\quad - h_i^{-1} \sum_{j=1}^n h_j B_{ij}(t) x_j(t - \tau_{ij}(t)) \\
 &\quad - h_i^{-1} \sum_{j=1}^n h_j C_{ij}(t) \int_{-\infty}^t K_{ij}(t - s) x_j(s) ds \\
 &\quad - h_i^{-1} \sum_{j=1}^n h_j D_{ij}(t) (1 - \delta'_{ij}(t)) x'_j(t - \delta_{ij}(t)) \\
 &\quad - e_i(t) (\phi_i x_i)(t) - f_i(t) (\phi_i x_i)(t - \sigma_i(t)) + h_i^{-1} r_i(t),
 \end{aligned} \tag{17}$$

where  $h_i > 0$  ( $i = 1, 2, \dots, n$ ) are  $n$  positive real numbers.

Obviously, the existence, uniqueness, and global attractivity of positive periodic solution of system (10) is equivalent to the existence, uniqueness, and global attractivity of periodic solution of system (17).

For the rest of this paper, we will devote ourselves to study the existence, uniqueness, and global attractivity of periodic solution of (17). We denote

$$\begin{aligned}
 \Gamma_i^1(t) &:= e_i(t) (\phi_i 1)(t), & \Gamma_i^2(t) &:= f_i(t) (\phi_i 1)(t - \sigma_i(t)), \\
 \Delta_i(t) &:= h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) \\
 &\quad + h_i^{-1} \sum_{j=1}^n h_j [B_{ij}(t) + C_{ij}(t) \\
 &\quad\quad + (A_{ii}(s) D_{ij}(t) + |D'_{ij}(t)|)] \\
 &\quad + \Gamma_i^1(t) + \Gamma_i^2(t), \quad i = 1, 2, \dots, n.
 \end{aligned} \tag{18}$$

Our first result on the global existence of a periodic solution of system (1) is stated in the following theorem.

**Theorem 7.** *In addition to (H<sub>1</sub>)–(H<sub>3</sub>), assume further that there exist positive constants  $h_i$  ( $i = 1, 2, \dots, n$ ) and a positive constant  $M < 1$  such that*

$$(H_4) \sup_{t \in [0, \omega]} \max_{i \in [1, n]} \left\{ \sum_{j=1}^n (h_j/h_i) D_{ij}(t) + \int_{-\infty}^t \exp\{-\int_s^t A_{ii}(\xi)d\xi\} \Delta_i(s) ds \right\} \leq M.$$

*Then, system (1) has a unique  $\omega$ -periodic solution with strictly positive components, where  $\Delta_i(t)$  is defined by (18).*

*Proof.* From the above analysis, to finish the proof of Theorem 7, it is enough to prove under the conditions of Theorem 7 that system (17) has a unique  $\omega$ -periodic solution. Let

$$\Omega = \{u(t) \mid u \in C(R^n, R), u(t + \omega) = u(t)\}; \tag{19}$$

under the norm  $\|u\| = \max_{1 \leq i \leq n} \max_{t \in [0, \omega]} \{|u_i(t)|\}$ ,  $\Omega$  is a Banach space. For any  $u(t) = (u_1(t), \dots, u_n(t))^T \in \Omega$ , we consider the periodic solution  $x_u(t)$  of periodic differential equation

$$\begin{aligned} \frac{dx_i}{dt} = & -A_{ii}(t)x_i(t) - h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) u_j(t) \\ & - h_i^{-1} \sum_{j=1}^n h_j B_{ij}(t) u_j(t - \tau_{ij}(t)) \\ & - h_i^{-1} \sum_{j=1}^n h_j C_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) u_j(s) ds \\ & - h_i^{-1} \sum_{j=1}^n h_j D_{ij}(t) (1 - \delta'_{ij}(t)) u'_j(t - \delta_{ij}(t)) \\ & - e_i(t) (\phi_i u_i)(t) - f_i(t) (\phi_i u_i)(t) (t - \sigma_i(t)) \\ & + h_i^{-1} r_i(t), \quad i = 1, 2, \dots, n. \end{aligned} \tag{20}$$

Since  $A_{ii}(t) > 0$ , we know that the linear system of system (20)

$$\frac{dx_i}{dt} = -A_{ii}(t)x_i(t), \quad i = 1, 2, \dots, n \tag{21}$$

admits exponential dichotomies on  $R$ , and so system (20) has a unique periodic solution  $x_u(t)$ , which can be expressed as

$$\begin{aligned} x_u(t) = & (x_{1u}(t), \dots, x_{nu}(t))^T \\ = & \left( \int_{-\infty}^t \exp \left\{ - \int_s^t A_{11}(\xi) d\xi \right\} F_{1u}(s) ds, \dots, \right. \\ & \left. \int_{-\infty}^t \exp \left\{ - \int_s^t A_{mm}(\xi) d\xi \right\} F_{mu}(s) ds \right)^T, \end{aligned} \tag{22}$$

where

$$\begin{aligned} F_{iu}(t) = & -h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) x_j(t) \\ & - h_i^{-1} \sum_{j=1}^n h_j B_{ij}(t) x_j(t - \tau_{ij}(t)) \\ & - h_i^{-1} \sum_{j=1}^n h_j C_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) x_j(s) ds \\ & - h_i^{-1} \sum_{j=1}^n h_j D_{ij}(t) (1 - \delta'_{ij}(t)) x'_j(t - \delta_{ij}(t)) \\ & - e_i(t) (\phi_i x_i)(t) - f_i(t) (\phi_i x_i)(t) (t - \sigma_i(t)) \\ & + h_i^{-1} r_i(t); \end{aligned} \tag{23}$$

its proof is similar to that of Theorem 1 in [18]; here we omit it.

Now, by using Lemma 6,  $x_{iu}(t)$  can also be expressed as

$$\begin{aligned} x_{iu}(t) = & - \sum_{j=1}^n \frac{h_j}{h_i} D_{ij}(t) u_j(t - \delta_{ij}(t)) \\ & + \int_{-\infty}^t \exp \left\{ - \int_s^t A_{ii}(\xi) d\xi \right\} G_{iu}(s) ds, \end{aligned} \tag{24}$$

$i = 1, 2, \dots, n,$

where

$$\begin{aligned} G_{iu}(t) = & -h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) u_j(t) \\ & - h_i^{-1} \sum_{j=1}^n h_j B_{ij}(t) u_j(t - \tau_{ij}(t)) \\ & - h_i^{-1} \sum_{j=1}^n h_j \times C_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) u_j(s) ds \\ & + h_i^{-1} \sum_{j=1}^n h_j (A_{ii}(s) D_{ij}(t) + D'_{ij}(t)) u_j(t - \delta_{ij}(t)) \\ & + e_i(t) (\phi_i u_i)(t) - f_i(t) (\phi_i u_i)(t) (t - \sigma_i(t)) \\ & + h_i^{-1} r_i(t). \end{aligned} \tag{25}$$

Now we define mapping  $T : \Omega \rightarrow \Omega$ ,  $Tu(t) = x_u(t)$ . Following this we will prove that  $T$  is a contraction mapping; that is, there exists a constant  $\beta \in (0, 1)$ , such that  $\|Tu - Tv\| \leq \beta \|u - v\|$ , for all  $u, v \in \Omega$ . In fact, for any  $u(t) = (u_1(t), \dots, u_n(t))^T$  and  $v(t) = (v_1(t), \dots, v_n(t))^T$ , we have

$$\begin{aligned} & \|G_{iu}(t) - G_{iv}(t)\| \\ & \leq h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) |u_j(t) - v_j(t)| \\ & \quad + h_i^{-1} \sum_{j=1}^n h_j \left[ B_{ij}(t) |u_j(t - \tau_{ij}(t)) - v_j(t - \tau_{ij}(t))| \right. \\ & \quad \left. + C_{ij}(t) \int_{-\infty}^t K_{ij}(t-s) |u_j(s) - v_j(s)| ds \right. \\ & \quad \left. + (A_{ii}(s) D_{ij}(t) + D'_{ij}(t)) \right. \\ & \quad \left. \times |u_j(t - \delta_{ij}(t)) - v_j(t - \delta_{ij}(t))| \right] \\ & \quad + e_i(t) (\phi_i 1) \|u - v\| + f_i(t) (\phi_i 2) (t - \sigma_i(t)) \|u - v\| \\ & \leq \left\{ h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) \right. \\ & \quad \left. + h_i^{-1} \sum_{j=1}^n h_j [B_{ij}(t) + C_{ij}(t) \right. \\ & \quad \left. + (A_{ii}(s) D_{ij}(t) + |D'_{ij}(t)|)] \right. \\ & \quad \left. + \Gamma_i^1(t) + \Gamma_i^2(t) \right\} \|u - v\| = \Delta_i(t) \|u - v\|. \end{aligned} \tag{26}$$

(26)

Hence,

$$\begin{aligned} & \|Tu - Tv\| \\ &= \sup_{t \in [0, \omega]} \max \left\{ \left| \sum_{j=1}^n \frac{h_j}{h_1} D_{1j}(t) \right. \right. \\ &\quad \times \left. \left[ u_j(t - \delta_{1j}(t)) - v_j(t - \delta_{1j}(t)) \right] \right| \\ &\quad + \int_{-\infty}^t \exp \left\{ - \int_s^t A_{11}(\xi) d\xi \right\} \\ &\quad \times |G_{1u}(s) - G_{1v}(s)| ds, \dots, \\ &\quad \left| \sum_{j=1}^n \frac{h_j}{h_n} D_{nj}(t) \right. \\ &\quad \times \left. \left[ u_j(t - \delta_{nj}(t)) - v_j(t - \delta_{nj}(t)) \right] \right| \\ &\quad + \int_{-\infty}^t \exp \left\{ - \int_s^t A_{nn}(\xi) d\xi \right\} \\ &\quad \times |G_{nu}(s) - G_{nv}(s)| ds \left. \right\} \\ &\leq \sup_{t \in [0, \omega]} \max \left\{ \left[ \sum_{j=1}^n \frac{h_j}{h_1} D_{1j}(t) \right. \right. \\ &\quad \left. \left. + \int_{-\infty}^t \exp \left\{ - \int_s^t A_{11}(\xi) d\xi \right\} \Delta_1(s) ds \right] \right. \\ &\quad \times \|u - v\|, \dots, \\ &\quad \left[ \sum_{j=1}^n \frac{h_j}{h_n} D_{nj}(t) \right. \\ &\quad \left. + \int_{-\infty}^t \exp \left\{ - \int_s^t A_{nn}(\xi) d\xi \right\} \Delta_n(s) ds \right] \\ &\quad \left. \times \|u - v\| \right\}. \end{aligned} \tag{27}$$

It follows from  $(H_4)$  that  $\|Tu - Tv\| \leq \|u - v\|$  for all  $u, v \in \Omega$ . That is,  $T$  is a contraction mapping. Hence, there exists a unique fixed point  $x^*(t) = (x_1^*(t), \dots, x_n^*(t))^T \in \Omega$ ; that is,  $Tx^*(t) = x^*(t)$ . Therefore,  $x^*(t)$  is the unique periodic solution of system (17). It follows from (1), (4), (10), and (17) that  $(N^*(t), u^*(t))^T = (N_1^*(t), \dots, N_n^*(t), u_1^*(t), \dots, u_n^*(t))^T$  is the unique positive periodic solution of system (1). The proof of Theorem 7 is completed.  $\square$

Our next theorem is concerned with the global stability of periodic solution for system (1).

**Theorem 8.** *In addition to  $(H_1)$ – $(H_4)$ , suppose further that the following condition holds:*

$$(H_5) \exp \left\{ - \int_0^t A_{ii}(\xi) d\xi \right\} \rightarrow 0, \text{ as } t \rightarrow +\infty, i = 1, 2, \dots, n.$$

*Then system (1) has a unique periodic solution which is globally attractive.*

*Proof.* Let  $N^*(t) = (N_1^*(t), N_2^*(t), \dots, N_n^*(t))^T$  be the unique positive periodic solution of system (1), whose existence and uniqueness are guaranteed by Theorem 7, and let  $N(t) = (N_1(t), N_2(t), \dots, N_n(t))^T$  be any other solution of system (1). Let  $N_i^*(t) = \exp \{ \prod_{0 < t_k < t} h_i(1 + p_{ik}) x_i^*(t) \}$ ,  $N_i(t) = \exp \{ \prod_{0 < t_k < t} \times h_i(1 + p_{ik}) x_i(t) \}$ ; then, similar to (17), we have

$$\begin{aligned} \frac{dx_i^*}{dt} &= -A_{ii}(t) x_i^*(t) - h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) x_j^*(t) \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j B_{ij}(t) x_j^*(t - \tau_{ij}(t)) \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j C_{ij}(t) \int_{-\infty}^t K_{ij}(t - s) x_j^*(s) ds \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j D_{ij}(t) (1 - \delta'_{ij}(t)) x_j^*(t - \delta_{ij}(t)) \\ &\quad - e_i(t) (\phi_i x_i^*)(t) - f_i(t) (\phi_i x_i^*)(t - \sigma_i(t)) \\ &\quad + h_i^{-1} r_i(t), \\ \frac{dx_i}{dt} &= -A_{ii}(t) x_i(t) - h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) x_j(t) \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j B_{ij}(t) x_j(t - \tau_{ij}(t)) \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j C_{ij}(t) \int_{-\infty}^t K_{ij}(t - s) x_j(s) ds \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j D_{ij}(t) (1 - \delta'_{ij}(t)) x_j(t - \delta_{ij}(t)) \\ &\quad - e_i(t) (\phi_i x_i)(t) - f_i(t) (\phi_i x_i)(t - \sigma_i(t)) + h_i^{-1} r_i(t). \end{aligned} \tag{28}$$

Let  $x_i^*(t) - x_i(t) = w_i(t)$ ; then

$$\begin{aligned} \frac{dw_i}{dt} &= -A_{ii}(t) w_i(t) - h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(t) w_j(t) \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j B_{ij}(t) w_j(t - \tau_{ij}(t)) \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j C_{ij}(t) \int_{-\infty}^t K_{ij}(t - s) w_j(s) ds \\ &\quad - h_i^{-1} \sum_{j=1}^n h_j D_{ij}(t) (1 - \delta'_{ij}(t)) w_j(t - \delta_{ij}(t)) \\ &\quad - e_i(t) (\phi_i w_i)(t) - f_i(t) (\phi_i w_i)(t - \sigma_i(t)). \end{aligned} \tag{29}$$



Multiply both sides of (29) with  $\exp\{\int_0^t A_{ii}(\xi)d\xi\}$ , and then integrate from 0 to  $t$  to obtain

$$\begin{aligned} & \int_0^t \left[ w_i(u) \exp \left\{ \int_0^u A_{ii}(\xi) d\xi \right\} \right]' du \\ &= - \int_0^t \left[ h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(u) w_j(u) \right. \\ & \quad + h_i^{-1} \sum_{j=1}^n h_j B_{ij}(u) w_j(u - \tau_{ij}(u)) \\ & \quad + h_i^{-1} \sum_{j=1}^n h_j C_{ij}(u) \int_{-\infty}^t K_{ij}(u-s) w_j(s) ds \\ & \quad + h_i^{-1} \sum_{j=1}^n h_j D_{ij}(u) (1 - \delta'_{ij}(u)) w'_j(u - \delta_{ij}(u)) \\ & \quad \left. + e_i(u) (\phi_i w_i)(u) + f_i(u) (\phi_i w_i)(u - \sigma_i(t)) \right] \\ & \quad \times \exp \left\{ \int_0^u A_{ii}(\xi) d\xi \right\} du, \quad i = 1, 2, \dots, n; \end{aligned} \tag{30}$$

then

$$\begin{aligned} w_i(t) &= w_i(0) \exp \left\{ - \int_0^t A_{ii}(\xi) d\xi \right\} \\ & \quad - \int_0^t \left[ h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(u) w_j(u) \right. \\ & \quad + h_i^{-1} \sum_{j=1}^n h_j B_{ij}(u) w_j(u - \tau_{ij}(u)) \\ & \quad + h_i^{-1} \sum_{j=1}^n h_j C_{ij}(u) \\ & \quad \quad \times \int_{-\infty}^t K_{ij}(u-s) w_j(s) ds \\ & \quad + h_i^{-1} \sum_{j=1}^n h_j D_{ij}(u) (1 - \delta'_{ij}(u)) \\ & \quad \quad \times w'_j(u - \delta_{ij}(u)) \\ & \quad + e_i(u) (\phi_i w_i)(u) \\ & \quad \left. + f_i(u) (\phi_i w_i)(u - \sigma_i(t)) \right] \\ & \quad \times \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} du, \quad i = 1, 2, \dots, n. \end{aligned} \tag{31}$$

Let  $D_{0ij}(t) = D_{ij}(t)(1 - \delta'_{ij}(t))$ ; we see that

$$\begin{aligned} & \int_0^t D_{0ij}(u) w'_j(u - \delta_{ij}(u)) \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} du \\ &= \int_0^t \frac{D_{0ij}(u) w'_j(u - \delta_{ij}(u)) (1 - \delta'_{ij}(u))}{1 - \delta_{ij}(u)} \\ & \quad \times \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} du \\ &= \int_0^t \left[ \frac{D_{0ij}(u) \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\}}{1 - \delta'_{ij}(u)} \right] \\ & \quad \times [w'_j(u - \delta_{ij}(u)) (1 - \delta'_{ij}(u))] du \\ &= \left[ \frac{D_{0ij}(t)}{1 - \delta'_{ij}(t)} w_j(t - \delta_{ij}(t)) \right. \\ & \quad \left. - \frac{D_{0ij}(0)}{1 - \delta'_{ij}(0)} w_j(-\delta_{ij}(0)) \exp \left\{ - \int_0^t A_{ii}(\xi) d\xi \right\} \right] \\ & \quad - \int_0^t (A_{ii}(u) D_{ij}(u) + D'_{ij}(u)) \\ & \quad \times \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} w_j(u - \delta_{ij}(u)) du \\ &= \left[ D_{ij}(t) w_j(t - \delta_{ij}(t)) \right. \\ & \quad \left. - D_{ij}(0) w_j(-\delta_{ij}(0)) \exp \left\{ - \int_0^t A_{ii}(\xi) d\xi \right\} \right] \\ & \quad - \int_0^t (A_{ii}(u) D_{ij}(u) + D'_{ij}(u)) \\ & \quad \times \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} w_j(u - \delta_{ij}(u)) du. \end{aligned} \tag{32}$$

Substituting (32) into (31), we get

$$\begin{aligned} w_i(t) &= \left[ w_i(0) + h_i^{-1} \sum_{j=1}^n h_j D_{ij}(0) w_j(-\delta_{ij}(0)) \right] \\ & \quad \times \exp \left\{ - \int_0^t A_{ii}(\xi) d\xi \right\} \\ & \quad + \int_0^t \left\{ h_i^{-1} \sum_{j=1}^n h_j \left[ (A_{ii}(u) D_{ij}(u) \right. \right. \right. \\ & \quad \quad \left. \left. + D'_{ij}(u)) w_j(u - \delta_{ij}(u)) \right. \right. \\ & \quad \left. \left. - B_{ij}(u) w_j(u - \tau_{ij}(u)) \right. \right. \\ & \quad \left. \left. - C_{ij}(u) \int_{-\infty}^t K_{ij}(u-s) w_j(s) ds \right] \right\} \end{aligned}$$



$$\begin{aligned}
 & -h_i^{-1} \sum_{j=1, j \neq i}^n h_j A_{ij}(u) w_j(u) \\
 & -e_i(u) (\phi_i w_i)(u) - f_i(u) (\phi_i w_i)(u - \sigma_i(t)) \Big\} \\
 & \times \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} du \\
 & + h_i^{-1} \sum_{j=1}^n h_j D_{ij}(t) w_j(t - \delta_{ij}(t));
 \end{aligned} \tag{33}$$

therefore, we have

$$\begin{aligned}
 \|w\| & \leq \left| w_i(0) + h_i^{-1} \sum_{j=1}^n h_j D_{ij}(0) w_j(-\delta_{ij}(0)) \right| \\
 & \times \exp \left\{ - \int_0^t A_{ii}(\xi) d\xi \right\} \\
 & + \left\{ \int_0^t \left[ h_i^{-1} \sum_{j=1}^n h_j [(A_{ii}(u) D_{ij}(u) + |D'_{ij}(u)|) \right. \right. \\
 & \quad \left. \left. \times |B_{ij}(u)| + |C_{ij}(u)|] \right. \right. \\
 & \quad \left. \left. + h_i^{-1} \sum_{j=1, j \neq i}^n h_j |A_{ij}(u)| + |\Gamma_i^1(u)| + |\Gamma_i^2(u)| \right\} \\
 & \times \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} du \\
 & + h_i^{-1} \sum_{j=1}^n h_j |D_{ij}(t)| \Big\} \|w\| \\
 & = \left| w_i(0) + h_i^{-1} \sum_{j=1}^n h_j D_{ij}(0) w_j(-\delta_{ij}(0)) \right| \\
 & \times \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} \\
 & + \left[ \sum_{j=1}^n \frac{h_j}{h_i} D_{ij}(t) \right. \\
 & \quad \left. + \int_0^t \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} \Delta_i(u) du \right] \|w\|,
 \end{aligned} \tag{34}$$

where  $\Delta_i(t)$  is defined by (18). From  $(H_4)$ , we have

$$\begin{aligned}
 \|w\| & \leq \frac{|w_i(0) + h_i^{-1} \sum_{j=1}^n h_j D_{ij}(0) w_j(-\delta_{ij}(0))| \exp \left\{ - \int_0^t A_{ii}(\xi) d\xi \right\}}{1 - h_i^{-1} \sum_{j=1}^n h_j |D_{ij}(t)| - \int_0^t \Delta_i(u) \exp \left\{ - \int_u^t A_{ii}(\xi) d\xi \right\} du} \\
 & \leq \frac{|w_i(0) + h_i^{-1} \sum_{j=1}^n h_j D_{ij}(0) w_j(-\delta_{ij}(0))| \exp \left\{ - \int_0^t A_{ii}(\xi) d\xi \right\}}{1 - M}.
 \end{aligned} \tag{35}$$

From  $(H_5)$ , we have

$$\begin{aligned}
 \|w\| & = \max_{i \in [0, n]} |w_i(t)| \\
 & = \max_{i \in [0, n]} |x_i^*(t) - x_i(t)| = 0, \quad \text{as } t \rightarrow +\infty;
 \end{aligned} \tag{36}$$

thus,  $x_i(t) \rightarrow x_i^*(t)$ , as  $t \rightarrow +\infty, i = 1, 2, \dots, n$ . Hence, the positive  $\omega$ -periodic solution of (17) is globally attractive; accordingly,  $N_i(t) = \exp\{\prod_{0 < t_k < t} h_i(1 + p_{ik})x_i(t)\} \rightarrow \exp\{\prod_{0 < t_k < t} h_i(1 + p_{ik})x_i^*(t)\} = N_i^*(t), u_i(t) = (\phi_i \ln N_i)(t) \rightarrow (\phi_i \ln N_i^*)(t) = u_i^*(t)$  as  $t \rightarrow +\infty, i = 1, 2, \dots, n$ , and by Definition 2, the positive  $\omega$ -periodic solution of (1) is globally attractive. The proof of Theorem 8 is completed.  $\square$

*Remark 9.* If  $e_i(t) = f_i(t) = \alpha_i(t) = \beta_i(t) = \vartheta_i(t) = 0, \theta_{ik} + 1 = 0, i = 1, 2, \dots, n, k = 1, 2, \dots$ , then system (1) is studied by [3]. Hence, Theorems 7 and 8 generalize the corresponding results in [3].

*Remark 10.* If  $\theta_{ik} + 1 = 0, i = 1, 2, \dots, n, k = 1, 2, \dots$ , then system (1) is studied by [4]. Hence Theorems 7 and 8 also generalize the corresponding results in [4].

### 4. Example

Consider the following impulsive model:

$$\begin{aligned}
 \frac{dN(t)}{dt} & = N(t) \left[ r(t) - a(t) \ln N(t) - b(t) \ln N(t - \tau(t)) \right. \\
 & \quad - c(t) \int_{-\infty}^t K(t-s) \ln N(s) ds \\
 & \quad - d(t) \frac{d \ln N(t - \delta(t))}{dt} - e(t) u(t) \\
 & \quad \left. - f(t) u(t - \sigma(t)) \right], \quad t \neq t_k,
 \end{aligned} \tag{37}$$

$$\begin{aligned}
 \frac{du(t)}{dt} & = -\alpha(t) u(t) + \beta(t) \ln N(t) \\
 & \quad + \theta(t) \ln N(t - \gamma(t)), \quad t \geq 0,
 \end{aligned}$$

$$N(t_k^+) = e^{(1+p_k)} N(t_k), \quad k = 1, 2, \dots,$$

where  $r(t), a(t), b(t), c(t), d(t), e(t), f(t), \delta(t) \in C^2(\mathbb{R}, \mathbb{R}), \sigma(t), \gamma(t), \alpha(t), \beta(t), \theta(t)$  are all nonnegative  $\omega$ -periodic continuous functions with  $\int_0^\omega r(t) > 0, a(t) > 0, \delta'(t) < 1$  and  $p_k$  is a real sequence, and  $\prod_{0 < t_k < t} (1 + p_k)$  is a positive  $\omega$ -periodic function with  $k = 1, 2, \dots$ . Furthermore,  $\int_0^\infty K(s) ds = 1, \int_0^{+\infty} sK(s) ds < +\infty$ .

We denote

$$\begin{aligned}
 A(t) &= a(t) \prod_{0 < t_k < t} (1 + p_k), \\
 B(t) &= b(t) \prod_{0 < t_k < t - \tau(t)} (1 + p_k), \\
 C(t) &= c(t) \prod_{0 < t_k < t} (1 + p_k), \\
 D(t) &= d(t) \prod_{0 < t_k < t - \delta(t)} (1 + p_k), \\
 D_0(t) &= D(t) (1 - \delta'(t)), \\
 \beta^*(t) &= \beta(t) \prod_{0 < t_k < t} (1 + p_k) \\
 \theta^*(t) &= \theta(t) \prod_{0 < t_k < t - \gamma(t)} (1 + p_k), \\
 \Gamma^1(t) &:= e(t) (\phi 1)(t), \\
 \Gamma^2(t) &:= f(t) (\phi 1)(t - \sigma(t)),
 \end{aligned}$$

$$\Delta(t) := B(t) + C(t) + A(t) D(t) + |D'(t)| + \Gamma^1(t) + \Gamma^2(t). \quad (38)$$

Similar to Theorems 7 and 8, we can get the following results.

**Corollary 11.** *In addition to conditions  $(H_1)$ – $(H_3)$ , assume further that there exists a positive constant  $M < 1$  such that*

$$(H_6) \quad D(t) + \int_{-\infty}^t \exp\{-\int_s^t A(\xi) d\xi\} \Delta(s) ds \leq M.$$

*Then, (37) has a unique  $\omega$ -periodic solution with strictly positive components, where  $\Delta(t)$  is defined by (38).*

**Corollary 12.** *In addition to conditions  $(H_1)$ – $(H_3)$  and  $(H_6)$ , suppose further that the following condition holds:*

$$(H_7) \quad \exp\{-\int_0^t A(\xi) d\xi\} \rightarrow 0, \text{ as } t \rightarrow +\infty.$$

*Then, system (37) has a unique periodic solution which is globally attractive.*

**Remark 13.** The results in the work show that by means of appropriate impulsive perturbations and feedback control we can control the dynamics of these equations.

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