Research Article

Existence and Exponential Stability of Equilibrium Point for Fuzzy BAM Neural Networks with Infinitely Distributed Delays and Impulses on Time Scales

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By using the fixed point theorem and constructing a Lyapunov functional, we establish some sufficient conditions on the existence, uniqueness, and exponential stability of equilibrium point for a class of fuzzy BAM neural networks with infinitely distributed delays and impulses on time scales. We also present a numerical example to show the feasibility of obtained results. Our example also shows that the described time and continuous neural time networks have the same dynamic behaviours for the stability.

1. Introduction

The bidirectional associative memory (BAM) neural network models were first introduced by Kosko (see [1]). BAM neural network is a special class of recurrent neural networks that can store bipolar vector pairs. It is composed of neurons arranged in two layers, the X-layer and Y-layer. This class of networks possesses good applications prospects in areas of pattern recognition, signal and image process, and automatic control. Such applications heavily depend on the dynamical behaviors of neural networks. Thus, the analysis of the dynamical behaviors is a necessary step for practical design neural networks. In particular, many researchers have studied the dynamics of BAM neural networks with or without delays including stability and existence of periodic solutions or almost periodic solutions. For the results on BAM neural networks, the reader may see [2–11] and reference therein.

In mathematical modeling of real world problems, we will encounter some inconveniences, for example, the complexity and the uncertainty or vagueness. For the sake of taking vagueness into consideration, fuzzy theory is considered as a suitable method. Yang et al. proposed fuzzy cellular neural network, which integrates fuzzy logic into the structure of traditional cellular neural networks and maintains local connectedness among cells [12]. Fuzzy neural network has

fuzzy logic between its template input and/or output besides the sum of product operation. Studies have been revealed that fuzzy neural network has its potential in imagine processing and pattern recognition and some results have reported on the stability and periodicity of fuzzy neural networks. Besides, in reality, time delays often occur due to finite switching speeds of the amplifiers and communication time and can destroy a stable network or cause sustained oscillations, bifurcation, or chaos. Hence, it is important to consider both the fuzzy logic and delay effect on dynamical behaviors of neural networks. There have been many results on the fuzzy neural networks with time delays [13–21]. For example, in [22], under the assumption that the activation function $f_j(u)$ is a second differentiable bounded function, $j = 1, 2, \ldots, n$, the authors proved the exponential stability and obtained the domain of robust attraction of the equilibrium point of the following interval fuzzy neural network:

$$
x_i'(t) = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right)
$$

$$
+ \bigwedge_{j=1}^{n} \alpha_{ij} f_j\left(x_j(t)\right) + \bigwedge_{j=1}^{n} T_{ij} u_j
$$

$$+ \bigvee_{j=1}^{n} \beta_{ij} f_j \left(x_j(t) \right) + \bigvee_{j=1}^{n} S_{ij} u_j$$

$$+ \sum_{j=1}^{n} c_{ij} \mu_j + I_i, \quad i = 1, 2, \ldots, n. \tag{1}$$

In fact, both continuous and discrete systems are very important in implementation and applications. To avoid the troublesomeness of studying the dynamical properties for continuous and discrete systems, respectively, it is meaningful to study that on time scales, which was initiated by Stefan Hilger in his Ph.D. thesis in order to unify continuous and discrete analysis. Lots of scholars have studied neural networks on time scales and obtained many good results [23–29]. For example, in [30], the authors considered the existence and global exponential stability of an equilibrium point for a class of fuzzy BAM neural networks with time-varying delays in leakage terms on time scales. Moreover, many systems also undergo abrupt changes at certain moments due to instantaneous perturbations, which lead to impulsive effects. Therefore, it is significant to study the dynamics of impulsive systems. For example, in [31], the authors studied the exponential stability of the following impulsive system on time scales:

$$x_i^{\Delta}(t) = -a_i x_i(t) + \sum_{j=1}^{m} b_{ji} f_j \left(y_j(t) \right)$$

$$+ \sum_{j=1}^{m} b_{ji} f_j \left(y_j(t - \sigma_j) \right) + \bigwedge_{j=1}^{m} \alpha_{ji} f_j \left(y_j(t - \sigma_j) \right)$$

$$+ \bigvee_{j=1}^{m} \beta_{ji} f_j \left(y_j(t - \sigma_j) \right) + \bigwedge_{j=1}^{m} T_{ji} \mu_j$$

$$+ \bigvee_{j=1}^{m} H_{ji} \mu_j + I_i, \quad t \ne t_k, \ t \in \mathbb{T}^+, \ i = 1, 2, \ldots, n,$$

$$\Delta x_i(t_k) = I_k \left(x_i(t_k) \right), \quad i = 1, 2, \ldots, n, \ k = 1, 2, \ldots, \tag{2}$$

where $\mathbb{T}^+ = \mathbb{T} \cap (0, +\infty)$ and $\mathbb{T}$ is a time scale, and in [32], authors studied a class of fuzzy BAM neural networks with finite distributed delays and impulses, by using fixed point theorem and differential inequality techniques; they established the existence and global exponential stability of unique equilibrium point to the networks.

However, to the best of our knowledge, few papers were published on the exponential stability of fuzzy BAM neural networks with distributed delays and impulses on time scales. Motivated by the above, in this paper, we study the following fuzzy BAM neural networks with infinitely distributed delays and impulses on time scales:

$$x_i^{\Delta}(t) = -a_i x_i(t) + \sum_{j=1}^{m} c_{ji} \int_0^{\infty} k_{ji}(s) f_j \left(y_j(t - s) \right) \Delta s$$

$$+ \bigwedge_{j=1}^{m} \alpha_{ji} \int_0^{\infty} k_{ji}(s) f_j \left(y_j(t - s) \right) \Delta s$$

$$+ \bigvee_{j=1}^{m} \beta_{ji} \int_0^{\infty} k_{ji}(s) f_j \left(y_j(t - s) \right) \Delta s$$

$$+ \bigwedge_{j=1}^{m} T_{ji} \mu_j + \bigvee_{j=1}^{m} H_{ji} \mu_j + I_i,$$

$$t \ne t_k, \ t \in \mathbb{T}^+, \ i = 1, 2, \ldots, n,$$

$$\Delta x_i(t_k) = P_k \left(x_i(t_k) \right), \quad i = 1, 2, \ldots, n, \ k = 1, 2, \ldots,$$

$$y_j^{\Delta}(t) = -b_j y_j(t) + \sum_{i=1}^{n} d_{ij} \int_0^{\infty} h_{ij}(s) g_i \left(x_i(t - s) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{n} p_{ij} \int_0^{\infty} h_{ij}(s) g_i \left(x_i(t - s) \right) \Delta s$$

$$+ \bigvee_{i=1}^{n} q_{ij} \int_0^{\infty} h_{ij}(s) g_i \left(x_i(t - s) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{n} F_{ij} \nu_i + \bigvee_{i=1}^{n} G_{ij} \nu_i + J_j,$$

$$t \ne t_k, \ t \in \mathbb{T}^+, \ j = 1, 2, \ldots, m,$$

$$\Delta y_j(t_k) = Q_k \left(y_j(t_k) \right), \quad j = 1, 2, \ldots, m, \ k = 1, 2, \ldots, \tag{3}$$

where $\mathbb{T}$ is a time scale that is closed under addition; n, m are the numbers of neurons in layers; $x_i(t)$ and $y_j(t)$ denote the activations of the ith neuron and the jth neuron at time t; $a_i > 0$ and $b_j > 0$ represent the rate with which the ith neuron and the jth neuron will reset their potential to the resting state in isolation when they are disconnected from the network and the external inputs; f_j, g_i are the input-output functions (the activation functions); c_{ji}, d_{ij} are elements of feedback templates; α_{ji}, p_{ij} denote elements of fuzzy feedback MIN templates and β_{ji}, q_{ij} are elements of fuzzy feedback MAX templates; T_{ji}, F_{ij} are fuzzy feed-forward MIN templates and H_{ji}, G_{ij} are fuzzy feed-forward MAX templates; μ_j, ν_i denote the input of the ith neuron and the jth neuron; the delayed feedback $k_{ji}(s)$ and $h_{ij}(s)$ are real valued nonnegative continuous functions defined on $\mathbb{T}^+$ with $\int_0^{\infty} k_{ji}(s)\Delta s \le k_{ji}$ and $\int_0^{\infty} h_{ij}(s)\Delta s \le h_{ij}$, where k_{ji} and h_{ij} are nonnegative constants; I_i, J_j denote biases of the ith neuron and the jth neuron, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$; $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operations, respectively; the impulsive moments t_k satisfy $0 \le t_0 < t_1 < t_2 < \cdots$ and $\lim_{k \to \infty} t_k = \infty$.

For $z = (x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m) \in \mathbb{R}^{n+m}$, we define the norm as $\|z\| = \sum_{i=1}^{n} |x_i| + \sum_{j=1}^{m} |y_j|$. For the sake of convenience, we denote the $\mathbb{T}$-interval $[a, b]_{\mathbb{T}}$ as $[a, b]_{\mathbb{T}} := \{t \in \mathbb{T} \mid a \le t \le b\}$. The initial condition of (3) is of the form

$$x_i(s) = \varphi_i(s), \quad s \in (-\infty, 0]_{\mathbb{T}}, \ i = 1, 2, \ldots, n,$$

$$y_j(s) = \psi_j(s), \quad s \in (-\infty, 0]_{\mathbb{T}}, \ j = 1, 2, \ldots, m, \tag{4}$$

where $\varphi_i(\cdot)$, $\psi_j(\cdot)$ denote positive real-valued continuous functions on $(-\infty, 0]_{\mathbb{T}}$.

Throughout this paper, we make the following assumptions:

(H_1) for $t, s \in \mathbb{T}, t \pm s \in \mathbb{T}$;

(H_2) $f_j, g_i \in C(\mathbb{R}, \mathbb{R})$ and there exist positive constants L_j^f, L_i^g such that

$$\left| f_j(u) - f_j(v) \right| \le L_j^f |u - v|,$$
$$\left| g_i(u) - g_i(v) \right| \le L_i^g |u - v|,$$

$$\tag{5}$$

for all $u, v \in \mathbb{R}, i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m$;

(H_3) there exists a positive constant p such that for $i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, m$,

$$\int_0^\infty e_p(t+s, t) k_{ji}(s) \Delta s < +\infty,$$
$$\int_0^\infty e_p(t+s, t) h_{ij}(s) \Delta s < +\infty,$$

$$\tag{6}$$

$$\forall t \in \mathbb{T}.$$

Remark 1. It is oblivious that if $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = \mathbb{Z}$, then (H_1) holds and that (1) and (2) are all special cases of (3). Our methods used in this paper are different from those used in [22, 30–32].

The organization of the rest of this paper is as follows. In Section 2, we introduce some preliminary results which are needed in later sections. In Section 3, we establish some sufficient conditions for the existence and uniqueness of the equilibrium point of (3). In Section 4, we prove the equilibrium point of (3) is exponentially stable. In Section 5, we give an example to illustrate the feasibility of our results obtained in previous sections.

2. Preliminaries

In this section, we state some preliminary results.

Definition 2 (see [33]). Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}_+$ are defined, respectively, by

$$\sigma(t) = \inf \{ s \in \mathbb{T} : s > t \},$$
$$\rho(t) = \sup \{ s \in \mathbb{T} : s < t \},$$
$$\mu(t) = \sigma(t) - t.$$

$$\tag{7}$$

Definition 3 (see [33]). A function $r : \mathbb{T} \to \mathbb{R}$ is called regressive if

$$1 + \mu(t) r(t) \ne 0, \tag{8}$$

for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $r : \mathbb{T} \to \mathbb{R}$ will be denoted by $\mathscr{R}$. We define the set $\mathscr{R}^+ = \{ r \in \mathscr{R} : 1 + \mu(t)r(t) > 0, \ \forall t \in \mathbb{T} \}$.

Lemma 4 (see [33]). *Assume that $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions; then*

(i) $e_0(t, s) \equiv 1$ *and* $e_p(t, t) \equiv 1$;

(ii) $e_p(t, s) = 1/e_p(s, t) = e_{\ominus p}(s, t)$;

(iii) $e_p(t, s) e_p(s, r) = e_p(t, r)$;

(iv) $(e_p(t, s))^\Delta = p(t) e_p(t, s)$.

Definition 5 (see [33]). A function $F : \mathbb{T}^k \to \mathbb{R}$ is called a delta antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^\Delta = f$ holds for all $t \in \mathbb{T}^k$. In this case we defined the integral of f by

$$\int_a^t f(s) \Delta s = F(t) - F(a), \quad t \in \mathbb{T}, \tag{9}$$

and we have the following formula:

$$\int_t^{\sigma(t)} f(s) \Delta s = \mu(t) f(t), \quad t \in \mathbb{T}^k. \tag{10}$$

Lemma 6 (see [33]). *Let f, g be Δ-differentiable functions on T; then*

(i) $(\nu_1 f + \nu_2 g)^\Delta = \nu_1 f^\Delta + \nu_2 g^\Delta$, *for any constants ν_1, ν_2;*

(ii) $(fg)^\Delta(t) = f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g(\sigma(t))$.

Lemma 7 (see [33]). *Assume that $p(t) \ge 0$ for $t \ge s$; then $e_p(t, s) \ge 1$.*

Lemma 8 (see [33]). *Suppose that $p \in \mathscr{R}^+$; then*

(i) $e_p(t, s) > 0$, *for all $t, s \in \mathbb{T}$;*

(ii) *if $p(t) \le q(t)$ for all $t \ge s, t, s \in \mathbb{T}$, then $e_p(t, s) \le e_q(t, s)$ for all $t \ge s$.*

Lemma 9 (see [33]). *If $p \in \mathscr{R}$ and $a, b, c \in \mathbb{T}$, then*

$$\left[e_p(c, \cdot) \right]^\Delta = -p \left[e_p(c, \cdot) \right]^\sigma,$$
$$\int_a^b p(t) e_p(c, \sigma(t)) \Delta t = e_p(c, a) - e_p(c, b).$$

$$\tag{11}$$

Lemma 10 (see [33]). *Let $a \in \mathbb{T}^k$, $b \in \mathbb{T}$ and assume that $f : \mathbb{T} \times \mathbb{T}^k \to \mathbb{R}$ is continuous at (t, t), where $t \in \mathbb{T}^k$ with $t > a$. Also assume that $f^\Delta(t, \cdot)$ is rd-continuous on $[a, \sigma(t)]$. Suppose that for each $\varepsilon > 0$, there exists a neighborhood U of $\tau \in [a, \sigma(t)]$ such that*

$$\left| f(\sigma(t), \tau) - f(s, \tau) - f^\Delta(t, \tau)(\sigma(t) - s) \right|$$
$$\le \varepsilon |\sigma(t) - s|, \quad \forall s \in U,$$

$$\tag{12}$$

where f^Δ denotes the derivative of f with respect to the first variable. Then

(i) $g(t) := \int_a^t f(t, \tau)\Delta\tau$ *implies* $g^\Delta(t) := \int_a^t f^\Delta(t, \tau)\Delta\tau + f(\sigma(t), t)$;

(ii) $h(t) := \int_t^b f(t, \tau)\Delta\tau$ *implies* $h^\Delta(t) := \int_t^b f^\Delta(t, \tau)\Delta\tau - f(\sigma(t), t)$.

Definition 11 (see [33]). For each $t \in \mathbb{T}$, let N be a neighborhood of t; then we define the generalized derivative (or Dini derivative), $D^+u^\Delta(t)$, to mean that, given $\epsilon > 0$, there exists a right neighborhood $N(\epsilon) \subset N$ of t such that

$$\frac{u(\sigma(t)) - u(s)}{\sigma(t) - s} \leq D^+u^\Delta(t) + \epsilon, \tag{13}$$

for each $s \in N(\epsilon)$, $s > t$. In case t is right-scattered and $u(t)$ is continuous at t, this reduces to

$$D^+u^\Delta(t) = \frac{u(\sigma(t)) - u(t)}{\sigma(t) - t}. \tag{14}$$

Definition 12 (see [33]). If $a \in \mathbb{T}$, $\sup \mathbb{T} = \infty$, and f is rd-continuous on $[a, \infty)$, then we define the improper integral by

$$\int_a^\infty f(t)\,\Delta t = \lim_{b \to \infty} \int_a^b f(t)\,\Delta t, \tag{15}$$

provided this limit exists, and we say that the improper integral converges in this case. If this limit does not exist, then we say that the improper integral diverges.

Definition 13 (see [33]). If $a \in \mathbb{T}$, $\inf \mathbb{T} = -\infty$, and f is rd-continuous on $(-\infty, a)$, then we define the improper integral by

$$\int_{-\infty}^a f(t)\,\Delta t = \lim_{b \to -\infty} \int_b^a f(t)\,\Delta t, \tag{16}$$

provided this limit exists, and we say that the improper integral converges in this case. If this limit does not exist, then we say that the improper integral diverges.

Lemma 14 (see [33]). *Let $f, g, h \in C_{rd}([a,b]_\mathbb{T}, \mathbb{R})$ and $1/p + 1/q = 1$ with $p > 1$; then*

$$\int_a^b |h(x)| |f(x)g(x)|\,\Delta x$$
$$\leq \left(\int_a^b |h(x)| |f(x)|^p \Delta x\right)^{1/p} \left(\int_a^b |h(x)| |g(x)|^q \Delta x\right)^{1/q}. \tag{17}$$

Definition 15. A point $z^* = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^T \in \mathbb{R}^{n+m}$ is said to be an equilibrium point of (3) if $z(t) = z^*$ is a solution of (3).

Lemma 16 (see [19]). *Let f_j be defined on $\mathbb{R}$, $j = 1, 2, \ldots, m$. Then for any $a_{ij} \in \mathbb{R}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, we have the following estimations:*

$$\left| \bigwedge_{j=1}^m a_{ij} f_j(u_j) - \bigwedge_{j=1}^m a_{ij} f_j(v_j) \right| \leq \sum_{j=1}^m |a_{ij}| |f_j(u_j) - f_j(v_j)|,$$
$$\left| \bigvee_{j=1}^m a_{ij} f_j(u_j) - \bigvee_{j=1}^m a_{ij} f_j(v_j) \right| \leq \sum_{j=1}^m |a_{ij}| |f_j(u_j) - f_j(v_j)|, \tag{18}$$

where $u_j, v_j \in \mathbb{R}$, $j = 1, 2, \ldots, m$.

Definition 17. Let $z^* = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^T$ be an equilibrium point of (3). If there exists a positive constant λ with $-\lambda \in \mathcal{R}^+$ such that, for $t_0 \in (-\infty, 0]_\mathbb{T}$, there exists $M > 1$ such that for an arbitrary solution $z(t) = (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^T$ of (3) with initial value $\phi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s), \psi_1(s), \psi_2(s), \ldots, \psi_m(s))^T$ satisfies

$$|z(t) - z^*|_1 \leq M \|\phi - z^*\| e_{\ominus\lambda}(t, t_0),$$
$$t \in [t_0, \infty)_\mathbb{T}, \ t \geq t_0, \tag{19}$$

where $|z(t) - z^*|_1 = \sum_{i=1}^n |x_i(t) - x_i^*|^2 + \sum_{j=1}^m |y_j(t) - y_j^*|^2$, $\|\phi - z^*\| = \sum_{i=1}^n \sup_{s \in (-\infty, 0]_\mathbb{T}} \{|\varphi_i(s) - x_i^*|^2 + \sum_{j=1}^m \sup_{s \in (-\infty, 0]_\mathbb{T}} \{|\psi_j(s) - y_j^*|^2\}\}$. Then the equilibrium point z^* is said to be exponentially stable.

3. Existence and Uniqueness of the Equilibrium Point

In this section, we discuss the existence and uniqueness of the equilibrium point of (3).

Without loss of generality, we assume that the impulsive jump vectors P and Q satisfy

$$P(x^*) = (P_1(x_1^*), P_2(x_2^*), \ldots, P_n(x_n^*))^T = 0,$$
$$Q(y^*) = (Q_1(y_1^*), Q_2(y_2^*), \ldots, Q_n(y_n^*))^T = 0. \tag{20}$$

That is, if $(x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^T$ is an equilibrium point of the following nonimpulsive system

$$x_i^\Delta(t) = -a_i x_i(t) + \sum_{j=1}^m c_{ji} \int_0^\infty k_{ji}(s) f_j(y_j(t-s))\,\Delta s$$
$$+ \bigwedge_{j=1}^m \alpha_{ji} \int_0^\infty k_{ji}(s) f_j(y_j(t-s))\,\Delta s$$
$$+ \bigvee_{j=1}^m \beta_{ji} \int_0^\infty k_{ji}(s) f_j(y_j(t-s))\,\Delta s$$
$$+ \bigwedge_{j=1}^m T_{ji}\mu_j + \bigvee_{j=1}^m H_{ji}\mu_j + I_i,$$
$$t \in \mathbb{T}^+, \ i = 1, 2, \ldots, n,$$

$$y_j^\Delta(t) = -b_j y_j(t) + \sum_{i=1}^n d_{ij} \int_0^\infty h_{ij}(s) g_i(x_i(t-s))\,\Delta s$$
$$+ \bigwedge_{i=1}^n p_{ij} \int_0^\infty h_{ij}(s) g_i(x_i(t-s))\,\Delta s$$
$$+ \bigvee_{i=1}^n q_{ij} \int_0^\infty h_{ij}(s) g_i(x_i(t-s))\,\Delta s$$

$$+ \bigwedge_{i=1}^{n} F_{ij} \nu_i + \bigvee_{i=1}^{n} G_{ij} \nu_i + J_j,$$

$$t \in \mathbb{T}^+, \quad j = 1, 2, \ldots, m, \tag{21}$$

then it is also the equilibrium point of impulsive system (3).

Theorem 18. *Let* (H_1) *and* (H_2) *hold. Suppose further that* (H_4) $\theta < 1$, *where*

$$\theta = \max \left\{ \max_{1 \le j \le m} \left\{ \sum_{i=1}^{n} a_i^{-1} L_j^f k_{ji} \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) \right\}, \right.$$

$$\left. \max_{1 \le i \le n} \left\{ \sum_{j=1}^{m} b_j^{-1} L_i^g h_{ij} \left(|d_{ij}| + |p_{ij}| + |q_{ij}| \right) \right\} \right\}; \tag{22}$$

then (3) *has one unique equilibrium point.*

Proof. If $z^* = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^T$ is an equilibrium point of (3), then we have

$$a_i x_i^* = \sum_{j=1}^{m} c_{ji} \int_0^{\infty} k_{ji}(s) f_j(y_j^*) \Delta s$$

$$+ \bigwedge_{j=1}^{m} \alpha_{ji} \int_0^{\infty} k_{ji}(s) f_j(y_j^*) \Delta s$$

$$+ \bigvee_{j=1}^{m} \beta_{ji} \int_0^{\infty} k_{ji}(s) f_j(y_j^*) \Delta s$$

$$+ \bigwedge_{j=1}^{m} T_{ji} \mu_j + \bigvee_{j=1}^{m} H_{ji} \mu_j + I_i,$$

$$b_j y_j^* = \sum_{i=1}^{n} d_{ij} \int_0^{\infty} h_{ij}(s) g_i(x_i^*) \Delta s$$

$$+ \bigwedge_{i=1}^{n} p_{ij} \int_0^{\infty} h_{ij}(s) g_i(x_i^*) \Delta s$$

$$+ \bigvee_{i=1}^{n} q_{ij} \int_0^{\infty} h_{ij}(s) g_i(x_i^*) \Delta s$$

$$+ \bigwedge_{i=1}^{n} F_{ij} \nu_i + \bigvee_{i=1}^{n} G_{ij} \nu_i + J_j, \tag{23}$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. To finish the proof, it suffices to prove that (23) has a unique solution. Define a mapping $\Phi : \mathbb{R}^{n+m} \to \mathbb{R}^{n+m}$ as follows:

$$\Phi(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m)$$

$$= (\Phi_1(x_1), \Phi_2(x_2), \ldots, \Phi_n(x_n), \tag{24}$$

$$\Phi_{n+1}(y_1), \ldots, \Phi_{n+m}(y_m))^T,$$

where

$$\Phi_i(x_i) = a_i^{-1} \left[\sum_{j=1}^{m} c_{ji} \int_0^{\infty} k_{ji}(s) f_j(y_j) \Delta s \right]$$

$$+ a_i^{-1} \left[\bigwedge_{j=1}^{m} \alpha_{ji} \int_0^{\infty} k_{ji}(s) f_j(y_j) \Delta s \right]$$

$$+ a_i^{-1} \left[\bigvee_{j=1}^{m} \beta_{ji} \int_0^{\infty} k_{ji}(s) f_j(y_j) \Delta s \right]$$

$$+ a_i^{-1} \left[\bigwedge_{j=1}^{m} T_{ji} \mu_j + \bigvee_{j=1}^{m} H_{ji} \mu_j + I_i \right],$$

$$\Phi_{n+j}(y_j) = b_j^{-1} \left[\sum_{i=1}^{n} d_{ij} \int_0^{\infty} h_{ij}(s) g_i(x_i) \Delta s \right]$$

$$+ b_j^{-1} \left[\bigwedge_{i=1}^{n} p_{ij} \int_0^{\infty} h_{ij}(s) g_i(x_i) \Delta s \right]$$

$$+ b_j^{-1} \left[\sum_{i=1}^{n} p_{ij} \int_0^{\infty} h_{ij}(s) g_i(x_i) \Delta s \right]$$

$$+ b_j^{-1} \left[\bigwedge_{i=1}^{n} F_{ij} \nu_i + \bigvee_{j=1}^{m} G_{ij} \nu_i + B_j \right], \tag{25}$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. Obviously, we need to show that Φ is a contraction mapping on $\mathbb{R}^{n+m}$. In fact, for any $\vartheta = (h_1, h_2, \ldots, h_n, \nu_1, \nu_2, \ldots, \nu_m)$ and $\overline{\vartheta} = (\overline{h}_1, \overline{h}_2, \ldots, \overline{h}_n, \overline{\nu}_1, \overline{\nu}_2, \ldots, \overline{\nu}_m) \in \mathbb{R}^{n+m}$, we have

$$\left| \Phi_i(h_i) - \Phi_i(\overline{h}_i) \right|$$

$$= \left| a_i^{-1} \left[\sum_{j=1}^{m} c_{ji} \int_0^{\infty} k_{ji}(s) \left(f_j(\nu_j) - f_j(\overline{\nu}_j) \right) \Delta s \right] \right.$$

$$+ a_i^{-1} \left[\bigwedge_{j=1}^{m} \alpha_{ji} \int_0^{\infty} k_{ji}(s) \left(f_j(\nu_j) - f_j(\overline{\nu}_j) \right) \Delta s \right]$$

$$\left. + a_i^{-1} \left[\bigvee_{j=1}^{m} \beta_{ji} \int_0^{\infty} k_{ji}(s) \left(f_j(\nu_j) - f_j(\overline{\nu}_j) \right) \Delta s \right] \right|$$

$$\le a_i^{-1} \sum_{j=1}^{m} L_j^f k_{ji} \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) |\nu_j - \overline{\nu}_j|,$$

$$i = 1, 2, \ldots, n,$$

$$\left| \Phi_{n+j}\left(v_j\right) - \Phi_{n+j}\left(\bar{v}_j\right) \right|$$

$$= \left| b_j^{-1} \left[\sum_{i=1}^{n} d_{ij} \int_0^\infty h_{ij}(s) \left(g_i(h_i) - g_i(\bar{h}_i) \right) \Delta s \right] \right.$$

$$+ b_j^{-1} \left[\bigwedge_{i=1}^{n} p_{ij} \int_0^\infty h_{ij}(s) \left(g_i(h_i) - g_i(\bar{h}_i) \right) \Delta s \right]$$

$$+ b_j^{-1} \left. \left[\bigvee_{i=1}^{n} q_{ij} \int_0^\infty h_{ij}(s) \left(g_i(h_i) - g_i(\bar{h}_i) \right) \Delta s \right] \right|$$

$$\leq b_j^{-1} \sum_{i=1}^{n} L_i^g h_{ij} \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right) \left| h_i - \bar{h}_i \right|,$$

$$j = 1, 2, \ldots, m.$$

$$(26)$$

Therefore, we have

$$\left\| \Phi(\vartheta) - \Phi(\bar{\vartheta}) \right\|$$

$$= \sum_{i=1}^{n} \left| \Phi_i(h_i) - \Phi_i(\bar{h}_i) \right|$$

$$+ \sum_{j=1}^{m} \left| \Phi_{n+j}(v_j) - \Phi_{n+j}(\bar{v}_j) \right|$$

$$\leq \sum_{i=1}^{n} a_i^{-1} \sum_{j=1}^{m} L_j^f k_{ji} \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right) \left| v_j - \bar{v}_j \right|$$

$$+ \sum_{j=1}^{m} b_j^{-1} \sum_{i=1}^{n} L_i^g h_{ij} \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right) \left| h_i - \bar{h}_i \right|$$

$$\leq \max_{1 \leq j \leq m} \left\{ \sum_{i=1}^{n} a_i^{-1} L_j^f k_{ji} \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right) \right\}$$

$$\times \sum_{j=1}^{m} \left| v_j - \bar{v}_j \right|$$

$$+ \max_{1 \leq i \leq n} \left\{ \sum_{j=1}^{m} a_i^{-1} L_i^g h_{ij} \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right) \right\}$$

$$\times \sum_{i=1}^{n} \left| h_i - \bar{h}_i \right|$$

$$\leq \theta \left(\sum_{i=1}^{n} \left| h_i - \bar{h}_i \right| + \sum_{j=1}^{m} \left| v_j - \bar{v}_j \right| \right)$$

$$= \theta \left\| \vartheta - \bar{\vartheta} \right\|.$$

$$(27)$$

By (H_3), we obtain that $\Phi : \mathbb{R}^{n+m} \to \mathbb{R}^{n+m}$ is a contraction mapping. By the fixed point theorem of Banach space, there exists a unique fixed point of Φ, which is a solution of (21).

Therefore, (3) has exactly one equilibrium point. The proof of Theorem 18 is completed. $\square$

4. Exponential Stability of Equilibrium Point

In this section, we consider the impulsive fuzzy BAM neural networks of the following type:

$$x_i^\Delta(t) = -a_i x_i(t)$$

$$+ \sum_{j=1}^{m} c_{ji} \int_0^\infty k_{ji}(s) f_j \left(y_j(t-s) \right) \Delta s$$

$$+ \bigwedge_{j=1}^{m} \alpha_{ji} \int_0^\infty k_{ji}(s) f_j \left(y_j(t-s) \right) \Delta s$$

$$+ \bigvee_{j=1}^{m} \beta_{ji} \int_0^\infty k_{ji}(s) f_j \left(y_j(t-s) \right) \Delta s$$

$$+ \bigwedge_{j=1}^{m} T_{ji} \mu_j + \bigvee_{j=1}^{m} H_{ji} \mu_j + I_i,$$

$$t \in \mathbb{T}^+, \ i = 1, 2, \ldots, n,$$

$$\Delta x_i(t_k) = -\gamma_{ik} \left(x_i(t_k) - x_i^* \right), \quad k = 1, 2, \ldots, \ i = 1, 2, \ldots, n,$$

$$y_j^\Delta(t) = -b_j y_j(t) + \sum_{i=1}^{n} d_{ij} \int_0^\infty h_{ij}(s) g_i \left(x_i(t-s) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{n} p_{ij} \int_0^\infty h_{ij}(s) g_i \left(x_i(t-s) \right) \Delta s$$

$$+ \bigvee_{i=1}^{n} q_{ij} \int_0^\infty h_{ij}(s) g_i \left(x_i(t-s) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{n} F_{ij} v_i + \bigvee_{i=1}^{n} G_{ij} v_i + J_j,$$

$$t \in \mathbb{T}^+, \ j = 1, 2, \ldots, m,$$

$$\Delta y_j(t_k) = -\bar{\gamma}_{jk} \left(y_j(t_k) - y_j^* \right),$$

$$k = 1, 2, \ldots, \ j = 1, 2, \ldots, m,$$

$$(28)$$

where $a_i, b_j, c_{ji}, \beta_{ji}, \alpha_{ji}, d_{ij}, p_{ij}, q_{ij}, k_{ji}, h_{ij}, f_j, g_i$ are defined as those in (3). The initial conditions associated with (28) are given by (4). In the following, we study the exponential stability of the unique equilibrium point for (28) on time scales by using Lyapunov method.

Theorem 19. Let (H_1)–(H_4) hold. Suppose further that (H_4) there exists a constant $p > 0$ such that

$$p + (1 + p\mu)$$

$$\times \left(4a_i^2 \mu - 2a_i + \sum_{j=1}^{m} L_j^f k_{ji} \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right) \right)$$

$$+ \sum_{j=1}^{m} L_i^g (1 + p\mu)$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^\infty e_p (t + s, t) h_{ij} (s) \Delta s < 0, \quad i = 1, 2, \ldots, n,$$

$$p + (1 + p\mu)$$

$$\times \left(4b_j^2 \mu - 2b_j + \sum_{i=1}^{n} L_i^g h_{ij} \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right) \right)$$

$$+ \sum_{i=1}^{n} L_j^f (1 + p\mu)$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^\infty e_p (t + s, t) k_{ji} (s) \Delta s < 0, \quad j = 1, 2, \ldots, m,$$

$$(29)$$

where $\mu = \sup_{t \in \mathbb{T}} \mu(t) < \infty$.

(H_5) $0 < \gamma_{ik} < 2$ and $0 < \overline{\gamma}_{jk} < 2$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $k = 1, 2, \ldots$.

Then the equilibrium point $z^* = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^T$ of (28) is exponentially stable.

Proof. By Theorem 18, (28) has one unique equilibrium point $z^* = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_m^*)^T$. Let $z(t) = (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^T$ be an arbitrary solution of (28). Denote $u_i(t) = x_i(t) - x_i^*$, $v_j(t) = y_j(t) - y_j^*$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. Then from (28), we have the following:

$$u_i^\Delta (t) = - a_i u_i (t)$$

$$+ \sum_{j=1}^{m} c_{ji} \int_0^\infty k_{ji} (s) \left(f_j \left(y_j (t - s) \right) - f_j \left(y_j^* \right) \right) \Delta s$$

$$+ \bigwedge_{j=1}^{m} \alpha_{ji} \int_0^\infty k_{ji} (s) \left(f_j \left(y_j (t - s) \right) - f_j \left(y_j^* \right) \right) \Delta s$$

$$+ \bigvee_{j=1}^{m} \beta_{ji} \int_0^\infty k_{ji} (s) \left(f_j \left(y_j (t - s) \right) - f_j \left(y_j^* \right) \right) \Delta s,$$

$$t \in \mathbb{T}^+, \quad i = 1, 2, \ldots, n,$$

$$\Delta u_i (t_k) = -\gamma_{ik} \left(u_i (t_k) \right), \quad k = 1, 2, \ldots, \quad i = 1, 2, \ldots, n,$$

$$v_j^\Delta (t) = - b_j v_j (t)$$

$$+ \sum_{i=1}^{n} d_{ij} \int_0^\infty h_{ij} (s) \left(g_i \left(x_i (t - s) \right) - g_i \left(x_i^* \right) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{n} p_{ij} \int_0^\infty h_{ij} (s) \left(g_i \left(x_i (t - s) \right) - g_i \left(x_i^* \right) \right) \Delta s$$

$$+ \bigvee_{i=1}^{n} q_{ij} \int_0^\infty h_{ij} (s) \left(g_i \left(x_i (t - s) \right) - g_i \left(x_i^* \right) \right) \Delta s,$$

$$t \in \mathbb{T}^+, \quad j = 1, 2, \ldots, m,$$

$$\Delta v_j (t_k) = -\overline{\gamma}_{jk} \left(v_j (t_k) \right), \quad k = 1, 2, \ldots, \quad j = 1, 2, \ldots, m.$$

$$(30)$$

For $p > 0$, construct Lyapunov functional $V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$, where

$$V_1 (t) = \sum_{i=1}^{n} u_i^2 (t) e_p (t, 0),$$

$$V_2 (t) = \sum_{j=1}^{m} v_j^2 (t) e_p (t, 0),$$

$$V_3 (t) = \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f (1 + p\mu)$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^\infty \int_{t-s}^{t} e_p (r + s, 0) k_{ji} (s) v_j^2 (r) \Delta r \Delta s,$$

$$V_4 (t) = \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g (1 + p\mu)$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^\infty \int_{t-s}^{t} e_p (r + s, 0) h_{ij} (s) u_i^2 (r) \Delta r \Delta s.$$

$$(31)$$

Calculating the Δ-derivative $V^\Delta(t)$ of $V(t)$ along the solution of (30), we have

$$V_1^\Delta (t) = \sum_{i=1}^{n} \left[e_p (\sigma (t), 0) \left(u_i^2 (t) \right)^\Delta + u_i^2 (t) e_p^\Delta (t, 0) \right]$$

$$= \sum_{i=1}^{n} \left[e_p (\sigma (t), 0) \left(2u_i (t) u_i^\Delta (t) + \mu (t) \left(u_i^\Delta (t) \right)^2 \right) + p u_i^2 (t) e_p (t, 0) \right]$$

$$= \sum_{i=1}^{n} \left\{ e_p (\sigma (t), 0) \right.$$

$$\times \left[2u_i (t) \right.$$

$$\times \left(- a_i u_i (t) \right.$$

$$+ \sum_{j=1}^{m} c_{ji} \int_0^\infty k_{ji} (s)$$

$$
\times \left(f_j \left(y_j \left(t - s \right) \right) - f_j \left(y_j^* \right) \right) \Delta s
$$

$$
+ \bigwedge_{j=1}^{m} \alpha_{ji} \int_0^\infty k_{ji}(s)
$$

$$
\times \left(f_j \left(y_j \left(t - s \right) \right) - f_j \left(y_j^* \right) \right) \Delta s
$$

$$
+ \bigvee_{j=1}^{m} \beta_{ji} \int_0^\infty k_{ji}(s) \left(f_j \left(y_j \left(t - s \right) \right) \right.
$$

$$
\left. - f_j \left(y_j^* \right) \right) \Delta s \Big)
$$

$$
+ \mu(t) \left(-a_i u_i(t) + \sum_{j=1}^{m} c_{ji} \right.
$$

$$
\times \int_0^\infty k_{ji}(s)
$$

$$
\times \left(f_j \left(y_j \left(t - s \right) \right) - f_j \left(y_j^* \right) \right) \Delta s
$$

$$
+ \bigwedge_{j=1}^{m} \alpha_{ji}
$$

$$
\times \int_0^\infty k_{ji}(s)
$$

$$
\times \left(f_j \left(y_j \left(t - s \right) \right) \right.
$$

$$
\left. - f_j \left(y_j^* \right) \right) \Delta s
$$

$$
+ \bigvee_{j=1}^{m} \beta_{ji}
$$

$$
\times \int_0^\infty k_{ji}(s)
$$

$$
\times \left(f_j \left(y_j \left(t - s \right) \right) \right.
$$

$$
\left. \left. - f_j \left(y_j^* \right) \right) \Delta s \right)^2 \Big]
$$

$$
+ p u_i^2(t) e_p(t, 0) \Bigg\}
$$

$$
\leq \sum_{i=1}^{n} \left\{ p e_p(t, 0) u_i^2(t) + e_p(\sigma(t), 0) \right.
$$

$$
\times \left[-2 a_i u_i^2(t) \right.
$$

$$
+ 2 \sum_{j=1}^{m} \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right) L_j^f
$$

$$
\times \int_0^\infty k_{ji}(s) \left| v_j \left(t - s \right) \right| \left| u_i(t) \right| \Delta s
$$

$$
+ \mu(t) \left(4 a_i^2 u_i^2(t) \right.
$$

$$
+ 4 \left(\sum_{j=1}^{m} \left| c_{ji} \right| L_j^f \right.
$$

$$
\times \int_0^\infty k_{ji}(s) \left| v_j \left(t - s \right) \right| \Delta s \right)^2
$$

$$
+ 4 \left(\sum_{j=1}^{m} \left| \alpha_{ji} \right| L_j^f \right.
$$

$$
\times \int_0^\infty k_{ji}(s) \left| v_j \left(t - s \right) \right| \Delta s \right)^2
$$

$$
+ 4 \left(\sum_{j=1}^{m} \left| \beta_{ji} \right| L_j^f \right.
$$

$$
\times \int_0^\infty k_{ji}(s)
$$

$$
\left. \left. \left. \times \left| v_j \left(t - s \right) \right| \Delta s \right)^2 \right) \right] \Bigg\}
$$

$$
\leq \sum_{i=1}^{n} \left\{ p e_p(t, 0) u_i^2(t) + e_p(\sigma(t), 0) \right.
$$

$$
\times \left[-2 a_i u_i^2(t) + \sum_{j=1}^{m} \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right) L_j^f \right.
$$

$$
\times \int_0^\infty k_{ji}(s) \left(v_j^2 \left(t - s \right) + u_i^2(t) \right) \Delta s
$$

$$
+ \mu(t) \left(4 a_i^2 u_i^2(t) + 4m \sum_{j=1}^{m} c_{ji}^2 \left(L_j^f \right)^2 \right.
$$

$$
\times \left(\int_0^\infty k_{ji}(s) \left| v_j \left(t - s \right) \right| \Delta s \right)^2
$$

$$
+ 4m \sum_{j=1}^{m} \alpha_{ji}^2 \left(L_j^f \right)^2
$$

$$
\times \left(\int_0^\infty k_{ji}(s) \left| v_j \left(t - s \right) \right| \Delta s \right)^2
$$

$$
+ 4m \sum_{j=1}^{m} \beta_{ji}^2 \left(L_j^f \right)^2
$$

$$
\left. \left. \times \left(\int_0^\infty k_{ji}(s) \left| v_j \left(t - s \right) \right| \Delta s \right)^2 \right) \right] \Bigg\}
$$

$$
\leq \sum_{i=1}^{n} \left\{ p e_p(t, 0) u_i^2(t) + e_p(\sigma(t), 0) \right.
$$

$$
\times \left[-2 a_i u_i^2(t) \right.
$$

$$+ \sum_{j=1}^{m} \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) L_j^f$$

$$\times \int_0^\infty k_{ji}(s) \left(v_j^2(t-s) + u_i^2(t) \right) \Delta s + \mu(t)$$

$$\times \left(4a_i^2 u_i^2(t) \right.$$

$$+ 4m \sum_{j=1}^{m} c_{ji}^2 \left(L_j^f\right)^2 \int_0^\infty k_{ji}(s) \, \Delta s$$

$$\times \int_0^\infty k_{ji}(s) \, v_j^2(t-s) \, \Delta s$$

$$+ 4m \sum_{j=1}^{m} \alpha_{ji}^2 \left(L_j^f\right)^2 \int_0^\infty k_{ji}(s) \, \Delta s$$

$$\times \int_0^\infty k_{ji}(s) \, v_j^2(t-s) \, \Delta s$$

$$+ 4m \sum_{j=1}^{m} \beta_{ji}^2 \left(L_j^f\right)^2$$

$$\times \int_0^\infty k_{ji}(s) \, \Delta s$$

$$\left. \left. \times \int_0^\infty k_{ji}(s) \, v_j^2(t-s) \, \Delta s \right) \right] \Bigg\}$$

$$\leq \sum_{i=1}^{n} \left\{ p e_p(t,0) u_i^2(t) + \left(1 + p\mu(t)\right) e_p(t,0) \right.$$

$$\times \left[-2a_i u_i^2(t) \right.$$

$$+ \sum_{j=1}^{m} \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) L_j^f k_{ji} u_i^2(t)$$

$$+ \sum_{j=1}^{m} \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) L_j^f$$

$$\times \int_0^\infty k_{ji}(s) \, v_j^2(t-s) \, \Delta s$$

$$+ 4a_i^2 \mu(t) u_i^2(t) + 4m\mu(t)$$

$$\times \sum_{j=1}^{m} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \left(L_j^f\right)^2 k_{ji}$$

$$\left. \left. \times \int_0^\infty k_{ji}(s) \, v_j^2(t-s) \, \Delta s \right] \right\}$$

$$= \sum_{i=1}^{n} e_p(t,0)$$

$$\times \left\{ p + \left(1 + p\mu(t)\right) \right.$$

$$\times \left[4a_i^2 \mu(t) - 2a_i \right.$$

$$+ \sum_{j=1}^{m} L_j^f k_{ji}$$

$$\left. \times \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) \right] \Bigg\}$$

$$\times u_i^2(t) + e_p(t,0) \sum_{i=1}^{n} \sum_{j=1}^{m} \left(1 + p\mu(t)\right)$$

$$\times \left[L_j^f \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) \right.$$

$$\left. + 4m\mu(t) \left(L_j^f\right)^2 k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right]$$

$$\times \int_0^\infty k_{ji}(s) \, v_j^2(t-s) \, \Delta s$$

$$\leq \sum_{i=1}^{n} e_p(t,0)$$

$$\times \left\{ p + \left(1 + p\mu\right) \right.$$

$$\times \left[4a_i^2 \mu - 2a_i \right.$$

$$\left. + \sum_{j=1}^{m} L_j^f k_{ji} \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) \right] \Bigg\}$$

$$\times u_i^2(t) + e_p(t,0) \sum_{i=1}^{n} \sum_{j=1}^{m} \left(1 + p\mu\right)$$

$$\times \left\{ \left[L_j^f \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right) + 4m\mu \left(L_j^f\right)^2 \right. \right.$$

$$\left. \times k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right]$$

$$\left. \times \int_0^\infty k_{ji}(s) \, v_j^2(t-s) \, \Delta s \right\}.$$

$$\tag{32}$$

Similarly, we have that

$$V_2^\Delta(t) = \sum_{j=1}^{m} \left[e_p(\sigma(t),0) \left(v_j^2(t)\right)^\Delta + v_j^2(t) e_p^\Delta(t,0) \right]$$

$$= \sum_{j=1}^{m} \left[e_p(\sigma(t),0) \left(2v_j(t) v_j^\Delta(t) + \mu(t) \left(v_j^\Delta(t)\right)^2 \right) \right.$$

$$\left. + p v_i^2(t) e_p(t,0) \right]$$

$$= \sum_{j=1}^{m} e_p(\sigma(t),0)$$

Left column:

$$\times \left[2v_i(t) \right.$$

$$\times \left(-b_j v_i(t) \right.$$

$$+ \sum_{j=1}^{m} d_{ij} \int_0^\infty h_{ij}(s)$$

$$\times \left(g_i(x_i(t-s)) - g_i(x_i^*) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{n} p_{ij}$$

$$\times \int_0^\infty h_{ij}(s) \left(g_i(x_i(t-s)) - g_i(x_i^*) \right) \Delta s$$

$$+ \bigvee_{i=1}^{n} \beta_{ij} \int_0^\infty h_{ij}(s) \left(g_i(x_i(t-s)) \right.$$

$$\left. -g_i(x_i^*) \right) \Delta s \Bigg)$$

$$+ \mu(t)$$

$$\times \left(-b_j v_j(t) + \sum_{i=1}^{n} d_{ij} \right.$$

$$\times \int_0^\infty h_{ij}(s) \left(g_i(x_i(t-s)) - g_i(x_i^*) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{n} p_{ij} \int_0^\infty h_{ij}(s)$$

$$\times \left(g_i(x_i(t-s)) - g_i(x_i^*) \right) \Delta s$$

$$+ \bigvee_{i=1}^{n} q_{ij}$$

$$\times \int_0^\infty h_{ij}(s)$$

$$\times \left. \left(g_i(x_i(t-s)) - g_i(x_i^*) \right) \Delta s \right)^2 \Bigg]$$

$$+ p v_j^2(t) e_p(t,0)$$

$$\leq \sum_{j=1}^{m} e_p(t,0) \left\{ p + (1 + p\mu(t)) \right.$$

$$\times \left[4b_j^2 \mu(t) - 2b_j \right.$$

$$\left. \left. + \sum_{i=1}^{n} L_i^g h_{ij} \left(|d_{ij}| + |p_{ij}| + |q_{ij}| \right) \right] \right\}$$

Right column:

$$\times v_j^2(t) + e_p(t,0)$$

$$\times \sum_{j=1}^{m} \sum_{i=1}^{n} (1 + p\mu(t))$$

$$\times \left[L_i^g \left(|d_{ij}| + |p_{ij}| + |q_{ij}| \right) \right.$$

$$\left. + 4n\mu(t) \left(L_i^g \right)^2 \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right]$$

$$\times \int_0^\infty h_{ij}(s) u_i^2(t-s) \Delta s$$

$$\leq \sum_{j=1}^{m} e_p(t,0) \left\{ p + (1 + p\mu) \right.$$

$$\times \left[4b_j^2 \mu - 2b_j \right.$$

$$\left. \left. + \sum_{i=1}^{n} L_i^g h_{ij} \left(|d_{ij}| + |p_{ij}| + |q_{ij}| \right) \right] \right\}$$

$$\times v_j^2(t) + e_p(t,0)$$

$$\times \sum_{j=1}^{m} \sum_{i=1}^{n} (1 + p\mu)$$

$$\times \left[L_i^g \left(|d_{ij}| + |p_{ij}| + |q_{ij}| \right) \right.$$

$$\left. + 4n\mu (L_i^g)^2 h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right]$$

$$\times \int_0^\infty h_{ij}(s) u_i^2(t-s) \Delta s.$$

$$(33)$$

Besides, we can obtain that

$$V_3^\Delta(t) = \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f (1 + p\mu)$$

$$\times \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^\infty e_p(t+s,0) k_{ji}(s) v_j^2(t) \Delta s$$

$$- \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f (1 + p\mu)$$

$$\times \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^\infty e_p(t,0) k_{ji}(s) v_j^2(t-s) \Delta s$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f (1 + p\mu)$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times e_p(t, 0) \int_0^\infty e_p(t + s, t) k_{ji}(s) \Delta s v_j^2(t)$$

$$- e_p(t, 0)(1 + p\mu)$$

$$\times \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right.$$

$$\left. + 4m L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^\infty k_{ji}(s) v_j^2(t - s) \Delta s,$$

$$V_4^\Delta(t) = \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g (1 + p\mu)$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^\infty e_p(t + s, 0) h_{ij}(s) u_i^2(t) \Delta s$$

$$- \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g (1 + p\mu)$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^\infty e_p(t, 0) h_{ij}(s) u_i^2(t - s) \Delta s$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g (1 + p\mu)$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times e_p(t, 0) \int_0^\infty e_p(t + s, t) h_{ij}(s) \Delta s u_i^2(t)$$

$$- e_p(t, 0)(1 + p\mu)$$

$$\times \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^\infty h_{ji}(s) u_i^2(t - s) \Delta s.$$

$$(34)$$

Hence, we have that

$$V^\Delta(t) = V_1^\Delta(t) + V_2^\Delta(t) + V_3^\Delta(t) + V_4^\Delta(t)$$

$$\leq \sum_{i=1}^{n} e_p(t, 0)$$

$$\times \left\{ p + (1 + p\mu) \right.$$

$$\times \left[4a_i^2 \mu - 2a_i \right.$$

$$+ \sum_{j=1}^{m} L_j^f k_{ji}$$

$$\left. \times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right) \right] \right\}$$

$$\times u_i^2(t) + e_p(t, 0) \sum_{i=1}^{n} \sum_{j=1}^{m} (1 + p\mu)$$

$$\times \left\{ \left[L_j^f \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right) \right. \right.$$

$$\left. + 4m\mu \left(L_j^f \right)^2 k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right]$$

$$\left. \times \int_0^\infty k_{ji}(s) v_j^2(t - s) \Delta s \right\}$$

$$+ \sum_{j=1}^{m} e_p(t, 0)$$

$$\times \left\{ p + (1 + p\mu) \right.$$

$$\times \left[4b_j^2 \mu - 2b_j \right.$$

$$\left. + \sum_{i=1}^{n} L_i^g h_{ij} \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right) \right] \right\} v_j^2(t)$$

$$+ e_p(t, 0) \sum_{j=1}^{m} \sum_{i=1}^{n} (1 + p\mu)$$

$$\times \left[L_i^g \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right) \right.$$

$$\left. + 4n\mu \left(L_i^g \right)^2 h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right]$$

$$\times \int_0^\infty h_{ij}(s)\, u_i^2\,(t-s)\,\Delta s$$

$$+ \sum_{i=1}^n \sum_{j=1}^m L_j^f\,(1+p\mu)$$

$$\times \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right.$$

$$\left. + 4m\mu L_j^f k_{ji}\left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2\right)\right) e_p\,(t,0)$$

$$\times \int_0^\infty e_p\,(t+s,t)\, k_{ji}(s)\,\Delta s v_j^2\,(t) - e_p\,(t,0)\,(1+p\mu)$$

$$\times \sum_{i=1}^n \sum_{j=1}^m L_j^f \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right.$$

$$\left. + 4m L_j^f k_{ji}\left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2\right)\right)$$

$$\times \int_0^\infty k_{ji}(s)\, v_j^2\,(t-s)\,\Delta s$$

$$+ \sum_{j=1}^m \sum_{i=1}^n L_i^g\,(1+p\mu)\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right.$$

$$\left. + 4n\mu L_i^g h_{ij}\left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2\right)\right)$$

$$\times e_p\,(t,0) \int_0^\infty e_p\,(t+s,t)\, h_{ij}(s)\,\Delta s u_i^2\,(t)$$

$$- e_p\,(t,0)\,(1+p\mu)$$

$$\times \sum_{j=1}^m \sum_{i=1}^n L_i^g \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right.$$

$$\left. + 4n\mu L_i^g h_{ij}\left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2\right)\right)$$

$$\times \int_0^\infty h_{ji}(s)\, u_i^2\,(t-s)\,\Delta s$$

$$\leq \sum_{i=1}^n e_p\,(t,0)\left\{\left[p + (1+p\mu)\right.\right.$$

$$\times \left(4a_i^2 \mu - 2a_i\right.$$

$$+ \sum_{j=1}^m L_j^f k_{ji}$$

$$\left.\left.\left. \times \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right)\right)\right]\right.$$

$$+ \sum_{j=1}^m L_i^g\,(1+p\mu)$$

$$\times \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right.$$

$$\left. + 4n\mu L_i^g h_{ij}\left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2\right)\right)$$

$$\times \int_0^\infty e_p\,(t+s,t)\, h_{ij}(s)\,\Delta s\Bigg\}\, u_i^2\,(t)$$

$$+ \sum_{j=1}^m e_p\,(t,0)\left\{\left[p + (1+p\mu)\right.\right.$$

$$\times \left(4b_j^2 \mu - 2b_j\right.$$

$$\left.\left. + \sum_{i=1}^n L_i^g h_{ij}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right)\right)\right]$$

$$+ \sum_{i=1}^n L_j^f\,(1+p\mu)$$

$$\times \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right.$$

$$+ 4m\mu L_j^f$$

$$\left. \times k_{ji}\left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2\right)\right)$$

$$\times \int_0^\infty e_p\,(t+s,t)\, k_{ji}(s)\,\Delta s \Bigg\}\, v_j^2\,(t).$$

$$(35)$$

By (H_4), we can conclude that $V^\Delta(t) \leq 0$, for $t \in \mathbb{T}^+$, $t \neq t_k$, $k = 1, 2, \ldots$, which implies that $V(t) \leq V(0)$, for $t \in \mathbb{T}^+$, $t \neq t_k$, $k = 1, 2, \ldots$. For $t = t_k$, $k = 1, 2, \ldots$, we have

$$V\left(t_k^+\right) = V_1\left(t_k^+\right) + V_2\left(t_k^+\right) + V_3\left(t_k^+\right) + V_4\left(t_k^+\right)$$

$$= \sum_{i=1}^n u_i^2\left(t_k^+\right) e_p\left(t_k^+,0\right) + \sum_{j=1}^m v_j^2\left(t_k^+\right) e_p\left(t_k^+,0\right)$$

$$+ \sum_{i=1}^n \sum_{j=1}^m L_j^f\,(1+p\mu)$$

$$\times \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right.$$

$$\left. + 4m\mu L_j^f k_{ji}\left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2\right)\right)$$

$$\times \int_0^\infty \int_{t_k^+-s}^{t_k^+} e_p\,(r+s,0)\, k_{ji}(s)\, v_j^2\,(r)\,\Delta r\,\Delta s$$

$$+ \sum_{j=1}^m \sum_{i=1}^n L_i^g\,(1+p\mu)$$

$$\times \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right.$$

$$\left. + 4n\mu L_i^g h_{ij}\left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2\right)\right)$$

$$\times \int_0^\infty \int_{t_k^+-s}^{t_k^+} e_p\,(r+s,0)\, h_{ij}(s)\, u_i^2\,(r)\,\Delta r\,\Delta s$$

$$\leq \sum_{i=1}^{n} \left((1 - \gamma_{ik}) u_i (t_k) \right)^2 e_p (t_k, 0)$$

$$+ \sum_{j=1}^{m} \left((1 - \overline{\gamma}_{jk}) v_j (t_k) \right)^2 e_p (t_k, 0)$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f (1 + p\mu)$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^{\infty} \int_{t_k - s}^{t_k} e_p (r + s, 0) k_{ji} (s) v_j^2 (r) \Delta r \, \Delta s$$

$$+ \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g (1 + p\mu)$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^{\infty} \int_{t_k - s}^{t_k} e_p (r + s, 0) h_{ij} (s) u_i^2 (r) \Delta r \, \Delta s$$

$$\leq \sum_{i=1}^{n} u_i^2 (t_k) e_p (t_k, 0) + \sum_{j=1}^{m} v_j^2 (t_k) e_p (t_k, 0)$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f (1 + p\mu) \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^{\infty} \int_{t_k - s}^{t_k} e_p (r + s, 0) k_{ji} (s) v_j^2 (r) \Delta r \, \Delta s$$

$$+ \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g (1 + p\mu) \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^{\infty} \int_{t_k - s}^{t_k} e_p (r + s, 0) h_{ij} (s) u_i^2 (r) \Delta r \, \Delta s$$

$$= V (t_k).$$

$$(36)$$

So we get $V(t) \leq V(0)$, for all $t \in \mathbb{T}^+$. Now, we estimate the value of $V(0)$. We have that

$$V_1 (0) = \sum_{i=1}^{n} u_i^2 (0) e_p (0, 0) = \sum_{i=1}^{n} u_i^2 (0)$$

$$\leq \sum_{i=1}^{n} \sup_{t \in (-\infty, 0]_{\mathbb{T}}} u_i^2 (t),$$

$$V_2 (0) = \sum_{j=1}^{m} v_j^2 (0) e_p (0, 0) = \sum_{j=1}^{m} v_j^2 (0)$$

$$\leq \sum_{j=1}^{m} \sup_{t \in (-\infty, 0]_{\mathbb{T}}} v_j^2 (t),$$

$$V_3 (0) = \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f (1 + p\mu)$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^{\infty} \int_{-s}^{0} e_p (r + s, 0) k_{ji} (s) v_j^2 (r) \Delta r \, \Delta s$$

$$\leq \sum_{i=1}^{n} \sum_{j=1}^{m} L_j^f (1 + p\mu)$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\times \int_0^{\infty} \int_{-s}^{0} e_p (r + s, 0) k_{ji} (s) \Delta r \, \Delta s$$

$$\times \sup_{t \in (-\infty, 0]_{\mathbb{T}}} v_j^2 (t)$$

$$\leq \max_{1 \leq j \leq m} \left\{ \sum_{i=1}^{n} L_j^f (1 + p\mu) \right.$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\left. \times \int_0^{\infty} \int_{-s}^{0} e_p (r + s, 0) k_{ji} (s) \Delta r \, \Delta s \right\}$$

$$\times \sum_{j=1}^{m} \sup_{t \in (-\infty, 0]_{\mathbb{T}}} v_j^2 (t),$$

$$V_4 (0) = \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g (1 + p\mu)$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^{\infty} \int_{-s}^{0} e_p (r + s, 0) h_{ij} (s) u_i^2 (r) \Delta r \, \Delta s$$

$$\leq \sum_{j=1}^{m} \sum_{i=1}^{n} L_i^g (1 + p\mu)$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\times \int_0^\infty \int_{-s}^0 e_p(r+s,0) h_{ij}(s) \Delta r \Delta s$$

$$\times \sup_{t \in (-\infty,0]_{\mathbb{T}}} u_i^2(t)$$

$$\leq \max_{1 \leq i \leq n} \left\{ \sum_{j=1}^m \sum_{i=1}^n L_i^g (1+p\mu) \right.$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\left. \times \int_0^\infty \int_{-s}^0 e_p(r+s,0) h_{ji}(s) \Delta r \Delta s \right\}$$

$$\times \sum_{i=1}^n \sup_{t \in (-\infty,0]_{\mathbb{T}}} u_i^2(t).$$

$$(37)$$

It follows that

$$V(0) \leq \max_{1 \leq i \leq n} \left\{ 1 + \sum_{j=1}^m \sum_{i=1}^n L_i^g (1+p\mu) \right.$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\left. \times \int_0^\infty \int_{-s}^0 e_p(r+s,0) h_{ji}(s) \Delta r \Delta s \right\}$$

$$\times \sum_{i=1}^n \sup_{t \in (-\infty,0]_{\mathbb{T}}} u_i^2(t)$$

$$+ \max_{1 \leq j \leq m} \left\{ 1 + \sum_{i=1}^n L_j^f (1+p\mu) \right.$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| \right.$$

$$\left. + 4m\mu L_j^f k_{ji} \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\left. \times \int_0^\infty \int_{-s}^0 e_p(r+s,0) k_{ji}(s) \Delta r \Delta s \right\}$$

$$\times \sum_{j=1}^m \sup_{t \in (-\infty,0]_{\mathbb{T}}} v_j^2(t).$$

$$(38)$$

Observe that

$$V(t) \geq V_1(t) + V_2(t)$$

$$\geq e_p(t,0) \left(\sum_{i=1}^n u_i^2(t) + \sum_{j=1}^m v_j^2(t) \right).$$

$$(39)$$

So we have

$$\sum_{i=1}^n u_i^2(t) + \sum_{j=1}^m v_j^2(t)$$

$$\leq M e_{\ominus p}(t,0) \left(\sum_{i=1}^n \sup_{t \in (-\infty,0]} u_i^2(t) + \sum_{j=1}^m \sup_{t \in (-\infty,0]} v_j^2(t) \right),$$

$$\forall t \in \mathbb{T}^+,$$

$$(40)$$

where

$$M = \max \left\{ \max_{1 \leq i \leq n} \left\{ 1 + \sum_{j=1}^m \sum_{i=1}^n L_i^g (1+p\mu) \right. \right.$$

$$\times \left(\left| d_{ij} \right| + \left| p_{ij} \right| + \left| q_{ij} \right| \right.$$

$$\left. + 4n\mu L_i^g h_{ij} \left(d_{ij}^2 + p_{ij}^2 + q_{ij}^2 \right) \right)$$

$$\left. \left. \times \int_0^\infty \int_{-s}^0 e_p(r+s,0) h_{ji}(s) \Delta r \Delta s \right\}, \right.$$

$$\max_{1 \leq j \leq m} \left\{ 1 + \sum_{i=1}^n L_j^f (1+p\mu) \right.$$

$$\times \left(\left| c_{ji} \right| + \left| \alpha_{ji} \right| + \left| \beta_{ji} \right| + 4m\mu L_j^f k_{ji} \right.$$

$$\times \left(c_{ji}^2 + \alpha_{ji}^2 + \beta_{ji}^2 \right) \right)$$

$$\left. \left. \times \int_0^\infty \int_{-s}^0 e_p(r+s,0) k_{ji}(s) \Delta r \Delta s \right\} \right\}.$$

$$(41)$$

It is obvious that $M > 1$. Therefore, the equilibrium point z^* of (28) is exponentially stable. This completes the proof of Theorem 19. □

5. An Example

In this section, an example is given to verify the feasibility of our results obtained in previous sections.

Example 20. Let $n = m = 2$. Consider the following fuzzy BAM neural networks without impulses on time scale $\mathbb{T} = \mathbb{R}$ or $\mathbb{Z}$:

$$x_i^\Delta(t) = -a_i x_i(t)$$

$$+ \sum_{j=1}^2 c_{ji} \int_0^\infty k_{ji}(s) f_j \left(y_j(t-s) \right) \Delta s$$

$$+ \bigwedge_{j=1}^2 \alpha_{ji} \int_0^\infty k_{ji}(s) f_j \left(y_j(t-s) \right) \Delta s$$

$$+ \bigvee_{j=1}^{2} \beta_{ji} \int_{0}^{\infty} k_{ji}(s) f_j \left(y_j (t-s) \right) \Delta s$$

$$+ \bigwedge_{j=1}^{2} T_{ji} \mu_j + \bigvee_{j=1}^{2} H_{ji} \mu_j + I_i, \quad t \in \mathbb{T}^+, \; i = 1, 2,$$

$$y_j^{\Delta}(t) = -b_j y_j(t)$$

$$+ \sum_{i=1}^{2} d_{ij} \int_{0}^{\infty} h_{ij}(s) g_i \left(x_i (t-s) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{2} p_{ij} \int_{0}^{\infty} h_{ij}(s) g_i \left(x_i (t-s) \right) \Delta s$$

$$+ \bigvee_{i=1}^{2} q_{ij} \int_{0}^{\infty} h_{ij}(s) g_i \left(x_i (t-s) \right) \Delta s$$

$$+ \bigwedge_{i=1}^{2} F_{ij} \nu_i + \bigvee_{i=1}^{2} G_{ij} \nu_i + J_j, \quad t \in \mathbb{T}^+, \; j = 1, 2,$$

$$\tag{42}$$

where $(a_1, a_2)^T = (0.2, 0.3)^T$, $(b_1, b_2)^T = (0.2, 0.5)^T$,

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0.01 \\ 0.02 & 0 \end{pmatrix},$$

$$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0.03 \\ 0.01 & 0 \end{pmatrix},$$

$$f_1(u) = f_2(u) = g_1(u) = g_2(u) = 0.1 |u|,$$

$$k_{ji}(s) = h_{ij}(s) = e_2(0, s), \quad i, j = 1, 2.$$

$$\tag{43}$$

We can easily see that $L_1^f = L_2^f = L_1^g = L_2^g = 0.1$. If $\mathbb{T} = \mathbb{R}$, then $\mu(t) = 0$; if $\mathbb{T} = \mathbb{Z}$ then $\mu(t) = 1$. Hence, we have

$$\int_{0}^{\infty} e_2(0, s) \Delta s = \begin{cases} \dfrac{1}{2}, & \text{if } \mathbb{T} = \mathbb{R}, \\ \dfrac{1}{3 \ln 3}, & \text{if } \mathbb{T} = \mathbb{Z}. \end{cases} \tag{44}$$

So, we can take $k_{ji} = h_{ij} = 1$, $i, j = 1, 2$. We can obtain that $\theta = 0.06 < 1$. Moreover, if $\mu(t) = 0$, we can obtain that

$$- 2a_1 + \sum_{j=1}^{2} L_j^f k_{j1} \left(|c_{j1}| + |\alpha_{j1}| + |\beta_{j1}| \right)$$

$$+ \sum_{j=1}^{2} L_1^g \left(|d_{1j}| + |p_{1j}| + |q_{1j}| + h_{1j} \right)$$

$$= -0.17 < 0,$$

$$- 2a_2 + \sum_{j=1}^{2} L_j^f k_{j2} \left(|c_{j2}| + |\alpha_{j2}| + |\beta_{j2}| \right)$$

$$+ \sum_{j=1}^{2} L_1^g \left(|d_{2j}| + |p_{2j}| + |q_{2j}| + h_{2j} \right)$$

$$= -0.37 < 0,$$

$$- 2b_1 + \sum_{i=1}^{2} L_i^g h_{i1} \left(|d_{i1}| + |p_{i1}| + |q_{i1}| \right)$$

$$+ \sum_{i=1}^{2} L_j^f \left(|c_{1i}| + |\alpha_{1i}| + |\beta_{1i}| + k_{1i} \right)$$

$$= -0.17 < 0,$$

$$- 2b_2 + \sum_{i=1}^{2} L_i^g h_{i2} \left(|d_{i2}| + |p_{i2}| + |q_{i2}| \right)$$

$$+ \sum_{i=1}^{2} L_j^f \left(|c_{2i}| + |\alpha_{2i}| + |\beta_{2i}| + k_{2i} \right)$$

$$= -0.77 < 0.$$

$$\tag{45}$$

If $\mu(t) = 1$, we have that

$$4a_1^2 - 2a_1 + \sum_{j=1}^{2} L_j^f k_{j1} \left(|c_{j1}| + |\alpha_{j1}| + |\beta_{j1}| \right)$$

$$+ \sum_{j=1}^{2} L_1^g h_{1j} \left(|d_{1j}| + |p_{1j}| + |q_{1j}| \right.$$

$$\left. + 4n L_1^g \left(d_{1j}^2 + p_{1j}^2 + q_{1j}^2 \right) + h_{1j} \right)$$

$$\approx -0.0336 < 0,$$

$$4a_2^2 - 2a_2 + \sum_{j=1}^{2} L_j^f k_{j2} \left(|c_{j2}| + |\alpha_{j2}| + |\beta_{j2}| \right)$$

$$+ \sum_{j=1}^{2} L_2^g h_{2j} \left(|d_{2j}| + |p_{2j}| + |q_{2j}| \right.$$

$$\left. + 4n L_2^g \left(d_{2j}^2 + p_{2j}^2 + q_{2j}^2 \right) + h_{2j} \right)$$

$$\approx -0.0374 < 0,$$

$$4b_1^2 - 2b_1 + \sum_{i=1}^{2} L_i^g h_{i1} \left(|d_{i1}| + |p_{i1}| + |q_{i1}| \right)$$

$$+ \sum_{i=1}^{2} L_1^f k_{1i} \left(|c_{1i}| + |\alpha_{1i}| + |\beta_{1i}| \right.$$

$$\left. + 4m L_1^f k_{1i} \left(c_{1i}^2 + \alpha_{1i}^2 + \beta_{1i}^2 \right) + k_{1i} \right)$$

$$\approx -0.0336 < 0,$$

$$4b_2^2 - 2b_2 + \sum_{i=1}^{2} L_i^g h_{i12} \left(|d_{i2}| + |p_{i2}| + |q_{i2}| \right)$$

$$+ \sum_{i=1}^{2} L_2^f k_{2i} \left(|c_{2i}| + |\alpha_{2i}| + |\beta_{2i}| \right.$$

$$+ 4m L_2^f k_{2i} \left(c_{2i}^2 + \alpha_{2i}^2 + \beta_{2i}^2 \right) + k_{2i} \Big)$$

$$\approx -0.0381 < 0,$$

(46)

which imply that all assumptions in Theorems 18 and 19 are satisfied. Thus, it follows from Theorems 18 and 19 that (42) has one unique equilibrium point, which is exponentially stable.

Remark 21. The conclusion of Example 1 cannot be obtained by the results obtained in [22, 30–32].

6. Conclusion

In this paper, using the time scale calculus theory, the fixed point theory, and Lyapunov functional method, some sufficient conditions are obtained to ensure the existence and the exponential stability of unique equilibrium point of a class of fuzzy BAM neural networks with continuously distributed delays and impulses on time scales. Since it is troublesome to study the existence and stability of equilibrium points for continuous and discrete delay systems, respectively, it is significant to study that on time scales which can unify the continuous and discrete time situations. The sufficient conditions we obtained can easily be checked in practice by simple algebraic methods.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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