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Research Article

Mathematical Models of the Simplest Fuzzy Two-Term (PI/PD) Controllers Using Algebraic Product Inference

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This paper reveals mathematical models of the simplest Mamdani PI/PD controllers which employ two fuzzy sets (N: negative and P: positive) on the universe of discourse (UoD) of each of two input variables (displacement and velocity) and three fuzzy sets (N: negative, Z: zero, and P: positive) on the UoD of output variable (control output in the case of PD, and incremental control output in the case of PI). The basic constituents of these models are algebraic product/minimum AND, bounded sum/algebraic sum/maximum OR, algebraic product inference, three linear fuzzy control rules, and Center of Sums (CoS) defuzzification. Properties of all these models are investigated. It is shown that all these controllers are different nonlinear PI/PD controllers with their proportional and derivative gains changing with the inputs. The proposed models are significant and useful to control community as they are completely new and qualitatively different from those reported in the literature.

1. Introduction

Most of the industrial processes are still controlled largely by PID controllers due to their simplicity and low cost. Though the mathematical model (transfer function) of the PID controller is well known to the control community, mathematical modeling of fuzzy PI/PD controllers was not well established until 1993. Availability of mathematical models of fuzzy controllers and their precise understanding are fundamentally important for systematic analysis and synthesis of fuzzy control systems.

Looking at the historical development on fuzzy controller modeling, one would notice from the literature that it was Ying [1] who came up with four different models for the simplest fuzzy PI controller. He revealed that "bounded difference" inference was inappropriate for control. Later, Patel and Mohan [2] and Mohan and Sinha [3] presented three more models for the same using algebraic product/minimum AND and bounded sum/maximum OR. Subsequently, Haj-Ali and Ying [4] developed a mathematical model of fuzzy PI/PD controller using nonlinear input membership functions. Moreover, Mohan and Sinha [5] presented three more models via algebraic sum OR. Very recently, Mohan [6] has made an attempt to dispel some shortcomings that did take place inadvertently in some of the publications which are actually leading to misunderstandings and confusion.

It is very important to note that the output membership functions, shown in Figure 1(a), were considered for obtaining mathematical models in all the aforementioned publications. Though these works help us find the relationship between the models of the so-called the simplest fuzzy controllers and the controllers with multifuzzy sets [7], they cannot truly represent the models of the simplest fuzzy controllers. The output membership functions should be preferably considered as shown in Figure 1(b) to obtain mathematical models of the simplest fuzzy controllers. As a result of this, new mathematical models can be obtained.

The primary objective of this paper is to continue to reveal mathematical models of the simplest fuzzy PI/PD controllers. The results presented in this paper are significant and useful to control community as some of the models developed herein are qualitatively different from the models already reported in the literature. The paper is organized as follows: the next section describes the principal components of fuzzy two-term controllers. Section 3 presents mathematical models of the simplest fuzzy controllers whose properties

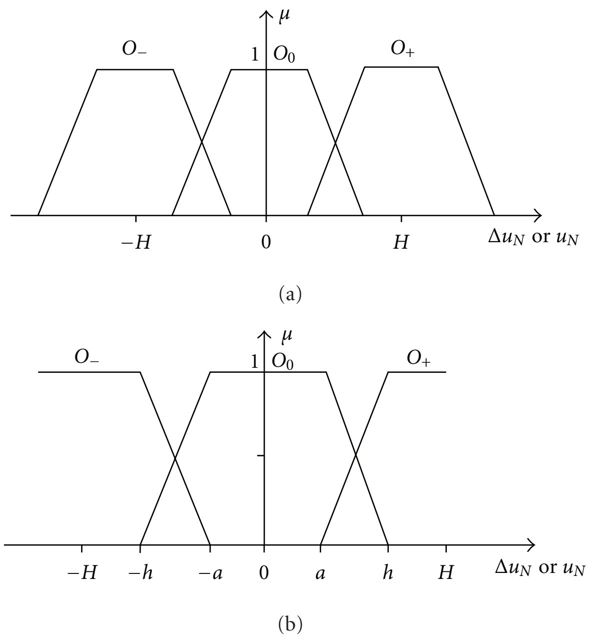


Figure 1: Output membership functions.

are included in Section 4. Finally, Section 5 concludes the paper.

2. Principal Components of Fuzzy Two-Term Controllers

The incremental control effort generated by a discrete-time PI controller is given by

$$\Delta u(k) = u(k) - u(k-1) = K_P^d v(k) + K_I^d d(k), \tag{1}$$

while the control effort produced by a discrete-time PD controller is described by

$$u(k) = K_P^d d(k) + K_D^d v(k), \tag{2}$$

where $d(k) = e(k)$, the error signal

$$v(k) = \frac{[d(k) - d(k-1)]}{T}, \tag{3}$$

K_P^d, K_I^d, and K_D^d are, respectively, the proportional, integral, and derivative constants of discrete-time PI and PD controllers, and T is the sampling period. The block-diagram of fuzzy two-term controller is shown in Figure 2 in which N_d, N_v, $N_{\Delta u}$, and N_u represent normalization factors of the fuzzy controller, and $d_N(k)$,$v_N(k)$, $\Delta u_N(k)$, and $u_N(k)$ represent normalized versions of $d(k)$, $v(k)$, $\Delta u(k)$, and $u(k)$, respectively.

The use of normalized UoD requires a scale transformation which maps the physical values of the process input variables (in the present case $d(k)$ and $v(k)$) into a normalized domain. Moreover, denormalization maps the normalized value of the process output variable (here $\Delta u_N(k)$ or $u_N(k)$) into its physical domain. The normalized factors which describe input normalization are denoted by N_d and N_v. Similarly, the denormalization factor describing output denormalization is denoted by $N_{\Delta u}^{-1}$ or N_u^{-1}.

The principal components of fuzzy two-term controllers are fuzzification and defuzzification modules, control rule base, and inference engine which are described in the following.

2.1. Fuzzification Module. This module converts crisp values of process variables (d_N, v_N, Δu_N or u_N) into fuzzy sets in order to make them compatible with the fuzzy set representation of the process variables in the rules. Let d^* and v^* be the crisp values of inputs d_N and v_N, respectively. Then, their fuzzified versions are the degrees of membership $\mu_D(d^*)$ and $\mu_V(v^*)$, where D and V are the linguistic values taken by d_N and v_N, respectively.

The inputs are fuzzified by L-function type and Γ-function type membership functions, shown in Figure 3, whose mathematical description is given by

$$\mu_{X_-} = \begin{cases} 1 & \text{for } -L \le x \le -l \\ \dfrac{(-x_N + l)}{2l} & \text{for } -l \le x \le l \\ 0 & \text{otherwise,} \end{cases}$$
$$\mu_{X_+} = \begin{cases} 0 & \text{for } -L \le x \le -l \\ \dfrac{(x_N + l)}{2l} & \text{for } -l \le x \le l \\ 1 & \text{otherwise,} \end{cases} \tag{4}$$

where $X = D$ or V and $x_N = d_N$ or v_N. Notice that

$$\mu_{X_-} + \mu_{X_+} = 1 \tag{5}$$

always. The membership functions for the normalized output are shown in Figure 1(b). Let

$$\rho = \frac{a}{h} \tag{6}$$

such that

$$0 \le \rho < 1. \tag{7}$$

Then, the trapezoid shrinks to a triangle when $\rho = a = 0$.

2.2. Control Rule Base. The following linear control rules are considered for fuzzy PI control.

$$\begin{aligned} &R_1\text{: IF } d_N \text{ is } D_- \text{ AND } v_N \text{ is } V_- \text{ THEN } \Delta u_N \text{ is } 0_- \\ &R_2\text{: (IF } d_N \text{ is } D_- \text{ AND } v_N \text{ is } V_+) \\ &\qquad \text{OR (IF } d_N \text{ is } D_+ \text{ AND } v_N \text{ is } V_-) \text{ THEN } \Delta u_N \text{ is } 0_0 \\ &R_3\text{: IF } d_N \text{ is } D_+ \text{ AND } v_N \text{ is } V_+ \text{ THEN } \Delta u_N \text{ is } 0_+. \end{aligned} \tag{8}$$

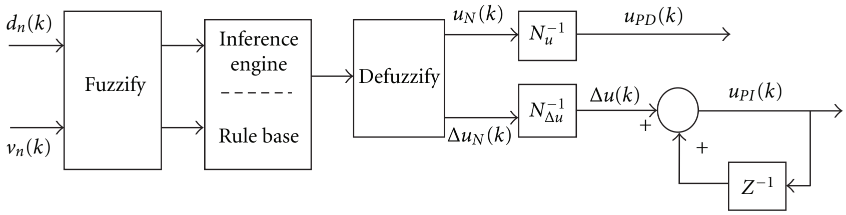


FIGURE 2: Block diagram of a typical fuzzy PI/PD controller.

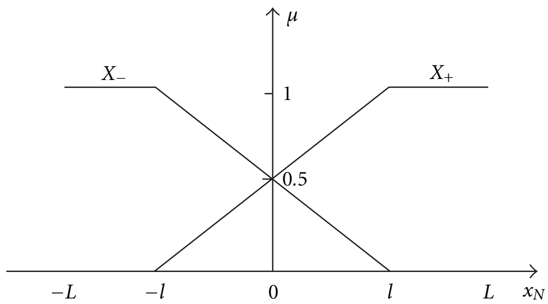


FIGURE 3: Input membership functions.

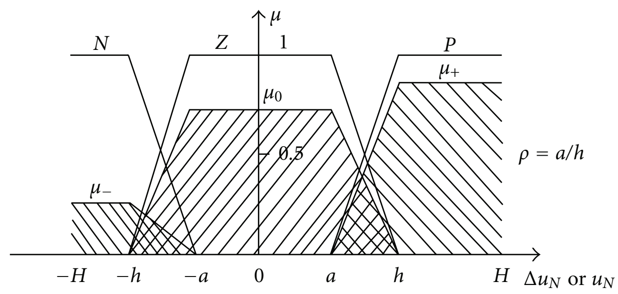


FIGURE 4: Inferred output membership functions via algebraic product inference.

The justification of the above rule base can be seen in Mohan and Sinha [5]. As a matter of fact, the above rule base can also be written in the following manner.

$$\begin{aligned}
&R_1\text{: IF } d_N \text{ is } D_- \text{ AND } v_N \text{ is } V_- \text{ THEN } \Delta u_N \text{ is } 0_- \\
&R_2\text{: IF } d_N \text{ is } D_- \text{ AND } v_N \text{ is } V_+ \text{ THEN } \Delta u_N \text{ is } 0_0 \\
&R_3\text{: IF } d_N \text{ is } D_+ \text{ AND } v_N \text{ is } V_- \text{ THEN } \Delta u_N \text{ is } 0_0 \\
&R_4\text{: IF } d_N \text{ is } D_+ \text{ AND } v_N \text{ is } V_+ \text{ THEN } \Delta u_N \text{ is } 0_+.
\end{aligned} \tag{9}$$

The rule base in the above form was considered in the current literature [1–5]. Δu_N is replaced by u_N for fuzzy PD control in the above rule base. We consider "algebraic product" or "minimum" AND operation, and they are defined as

$$\begin{aligned}
\mu_{\text{ap}}(d_N, v_N) &= \mu_i(d_N) \cdot \mu_j(v_N), \\
\mu_{\text{mn}}(d_N, v_N) &= \min\left\{\mu_i(d_N), \mu_j(v_N)\right\},
\end{aligned} \tag{10}$$

where $i \in \{D_-, D_+\}$ and $j \in \{V_-, V_+\}$. The OR operation in rule 2 may be performed by employing one of the following.

Bounded sum:

$$\mu_{\text{bs}}(\mu_1, \mu_2) = \min\{1, \mu_1 + \mu_2\}. \tag{11}$$

Algebraic sum:

$$\mu_{\text{as}}(\mu_1, \mu_2) = \min\{1, \mu_1 + \mu_2 - \mu_1 \cdot \mu_2\}. \tag{12}$$

Maximum:

$$\mu_{\text{mx}}(\mu_1, \mu_2) = \max\{\mu_1, \mu_2\}. \tag{13}$$

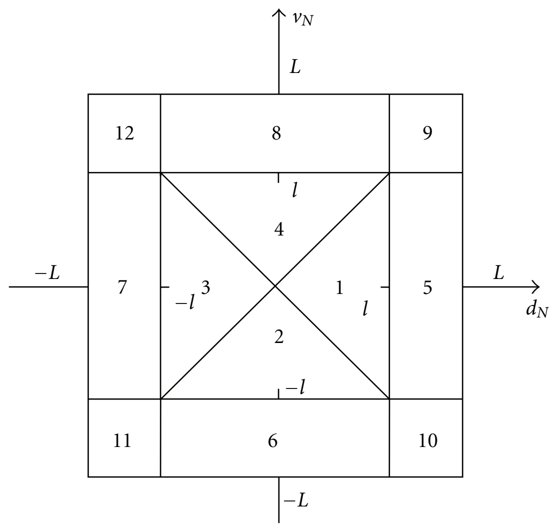


FIGURE 5: Regions of normalized input plane.

2.3. Inference Engine. The basic function of inference engine is to compute overall control output value based on the individual contribution of each rule in the rule base. For each rule, a degree of match is found by using any AND operator (and also any OR operator in the case of rule 2). Based on this degree of match, the control output value in the rule consequent is modified by using "algebraic product" inference. The reference membership functions and the inferred membership functions (shown with hatching) after employing algebraic product inference are shown in Figure 4.

All possible combinations of normalized inputs d_N and v_N are shown in Figure 5. The control rules R_1–R_3 are used to evaluate appropriate control law in each region.

The outcomes of fuzzy control rules with algebraic product/minimum AND and bounded sum/algebraic sum/maximum OR are given for all the regions in Table 1.

2.4. Defuzzification Module. It converts the set of all inferred control output values into a pointwise value. Defuzzification is done here using the well-known CoS method. According to this method, the crisp value of normalized control output is given by

$$\Delta u_N \text{ or } u_N = \frac{A(\mu_-)C(\mu_-) + A(\mu_+)C(\mu_+)}{A(\mu_-) + A(0) + A(\mu_+)}, \tag{14}$$

where $A(\cdot)$ and $C(\cdot)$ are, respectively, the area and centroid corresponding to the inferred output membership function. It is easy to compute the following from Figure 4:

$$\begin{aligned} A(N) &= 0.5[2H - h(1+\rho)]\mu_-, \\ A(Z) &= h(1+\rho)\mu_0, \\ A(P) &= 0.5[2H - h(1+\rho)]\mu_+, \\ C(N) &= \frac{-[3H^2 - h^2(1+\rho+\rho^2)]}{3[2H - h(1+\rho)]}, \\ C(z) &= 0, \\ C(P) &= \frac{3H^2 - h^2(1+\rho+\rho^2)}{3[2H - h(1+\rho)]}. \end{aligned} \tag{15}$$

3. Mathematical Models of the Simplest Fuzzy PI/PD Controllers

Here, we present mathematical models of the simplest fuzzy controllers which are derived using algebraic product/minimum AND, bounded sum/algebraic sum/maximum OR. These controllers are classified into six different classes depending upon the AND and OR operators used.

3.1. Mathematical Models in the Regions 1–4

Class 1. Algebraic product AND and algebraic sum OR:

$$\Delta u_N(k) \text{ or } u_N(k) = \frac{\text{I}}{\text{II}}, \tag{16}$$

where

$$\begin{aligned} \text{I} &= 4l^3[3H^2 - h^2 - h^2\rho(1+\rho)](d_N(k) + v_N(k)), \\ \text{II} &= 3\{4l^2[2H - h(1+\rho)](d_N(k)v_N(k) + l^2) \\ &\quad + h(1+\rho)[4l^2((d_N(k)+l)(-v_N(k)+l) \\ &\quad +(-d_N(k)+l)(v_N(k)+l)) \\ &\quad -(l^2 - d_N^2(k))(l^2 - v_N^2(k))]\}. \end{aligned} \tag{17}$$

Class 2. Algebraic product AND and bounded sum OR:

$$\Delta u_N(k) \text{ or } u_N(k) = \frac{\text{I}}{\text{II}}, \tag{18}$$

where

$$\begin{aligned} \text{I} &= l[3H^2 - h^2 - h^2\rho(1+\rho)](d_N(k) + v_N(k)), \\ \text{II} &= 3\{[2H - h(1+\rho)](d_N(k)v_N(k) + l^2) \\ &\quad + h(1+\rho)[(d_N(k)+l)(-v_N(k)+l) \\ &\quad +(-d_N(k)+l)(v_N(k)+l)]\}. \end{aligned} \tag{19}$$

Class 3. Algebraic product AND and maximum OR:

$$\Delta u_N(k) \text{ or } u_N(k) = \frac{\text{I}}{\text{II}}, \tag{20}$$

where

$$\begin{aligned} \text{I} &= l[3H^2 - h^2 - h^2\rho(1+\rho)](d_N(k) + v_N(k)), \\ \text{II} &= 3\{[2H - h(1+\rho)](d_N(k)v_N(k) + l^2) \\ &\quad +h(1+\rho)(s_1 d_N(k) + l)(s_2 v_N(k) + l)\} \end{aligned} \tag{21}$$

with s_1 and s_2 as defined in Table 2(a).

Class 4. Minimum AND and algebraic sum OR:

$$\Delta u_N(k) \text{ or } u_N(k) = \frac{\text{I}}{\text{II}}, \tag{22}$$

where

$$\begin{aligned} \text{I} &= l[3H^2 - h^2 - h^2\rho(1+\rho)](d_N(k) + v_N(k)), \\ \text{II} &= 3\{l[2H - h(1+\rho)](s_1 d_N(k) + s_2 v_N(k) + 2l) + h(1+\rho) \\ &\quad \times [2l(2l + s_3(d_N(k) + v_N(k))) \\ &\quad -(d_N(k)v_N(k) + s_3 l(d_N(k) + v_N(k)) + l^2)]\} \end{aligned} \tag{23}$$

with s_1, s_2, and s_3 as defined in Table 2(b).

Class 5. Minimum AND and bounded sum OR:

$$\Delta u_N(k) \text{ or } u_N(k) = \frac{\text{I}}{\text{II}}, \tag{24}$$

where

$$\begin{aligned} \text{I} &= [3H^2 - h^2 - h^2\rho(1+\rho)](d_N(k) + v_N(k)), \\ \text{II} &= 3\{[2H - h(1+\rho)](s_1 d_N(k) + s_2 v_N(k) + 2l) \\ &\quad +2h(1+\rho)(2l + s_3(d_N(k) + v_N(k)))\} \end{aligned} \tag{25}$$

with s_1, s_2, and s_3 as defined in Table 2(b).

Class 6. Minimum AND and maximum OR:

$$\Delta u_N(k) \text{ or } u_N(k) = \frac{\text{I}}{\text{II}}, \tag{26}$$

where

$$\begin{aligned} \text{I} &= [3H^2 - h^2 - h^2\rho(1+\rho)](d_N(k) + v_N(k)), \\ \text{II} &= 3\{[2H - h(1+\rho)](s_1 d_N(k) + s_2 v_N(k) + 2l) \\ &\quad +2h(1+\rho)(x_N(k) + l)\} \end{aligned} \tag{27}$$

with s_1, s_2, and $x_N(k)$ as defined in Table 2(c).

Table 1: (a) Outcomes of antecedent part of control rules: using ap AND. (b) Outcomes of antecedent part of control rules: using mn AND. (c) Outcomes of antecedent part of control rules: using ap/mn AND.

(a)

Region	R_1 μ_-	R_2 μ_0 using bs OR	as OR	mx OR	R_3 μ_+
1, 2	$\mu_{D_-}\cdot\mu_{V_-}$	$\mu_{D_-}\cdot\mu_{V_+}+\mu_{D_+}\cdot\mu_{V_-}$	$\mu_{D_-}\cdot\mu_{V_+}+\mu_{D_+}\cdot\mu_{V_-}-\mu_{D_-}\cdot\mu_{D_+}\cdot\mu_{V_-}\cdot\mu_{V_+}$	$\mu_{D_+}\cdot\mu_{V_-}$	$\mu_{D_+}\cdot\mu_{V_+}$
3, 4	$\mu_{D_-}\cdot\mu_{V_-}$	$\mu_{D_-}\cdot\mu_{V_+}+\mu_{D_+}\cdot\mu_{V_-}$	$\mu_{D_-}\cdot\mu_{V_+}+\mu_{D_+}\cdot\mu_{V_-}-\mu_{D_-}\cdot\mu_{D_+}\cdot\mu_{V_-}\cdot\mu_{V_+}$	$\mu_{D_-}\cdot\mu_{V_+}$	$\mu_{D_+}\cdot\mu_{V_+}$

(b)

Region	R_1 μ_-	R_2 μ_0 using bs·OR	as·OR	mx·OR	R_3 μ_+
1	μ_{D_-}	$\mu_{D_-}+\mu_{V_-}$	$\mu_{D_-}+\mu_{V_-}-\mu_{D_-}\cdot\mu_{V_-}$	μ_{V_-}	μ_{V_+}
2	μ_{D_-}	$\mu_{D_+}+\mu_{V_+}$	$\mu_{D_+}+\mu_{V_+}-\mu_{D_+}\cdot\mu_{V_+}$	μ_{D_+}	μ_{V_+}
3	μ_{V_-}	$\mu_{D_+}+\mu_{V_+}$	$\mu_{D_+}+\mu_{V_+}-\mu_{D_+}\cdot\mu_{V_+}$	μ_{V_+}	μ_{D_+}
4	μ_{V_-}	$\mu_{D_-}+\mu_{V_-}$	$\mu_{D_-}+\mu_{V_-}-\mu_{D_-}\cdot\mu_{V_-}$	μ_{D_-}	μ_{D_+}

(c)

Region	R_1 μ_-	R_2 μ_0 using bs/as/mx OR	R_3 μ_+
5	0	μ_{V_-}	μ_{V_+}
6	μ_{D_-}	μ_{D_+}	0
7	μ_{V_-}	μ_{V_+}	0
8	0	μ_{D_-}	μ_{D_+}
9	0	0	1
10, 12	0	1	0
11	1	0	0

Table 2: (a) Signs of s_1 and s_2. (b) Signs of s_1, s_2 and s_3. (c) Attributes of s_1, s_2, and $x_N(k)$.

(a)

Region	s_1	s_2
1, 2	+	−
3, 4	−	+

(b)

Region	s_1	s_2	s_3
1	−	+	−
2	−	+	+
3	+	−	+
4	+	−	−

(c)

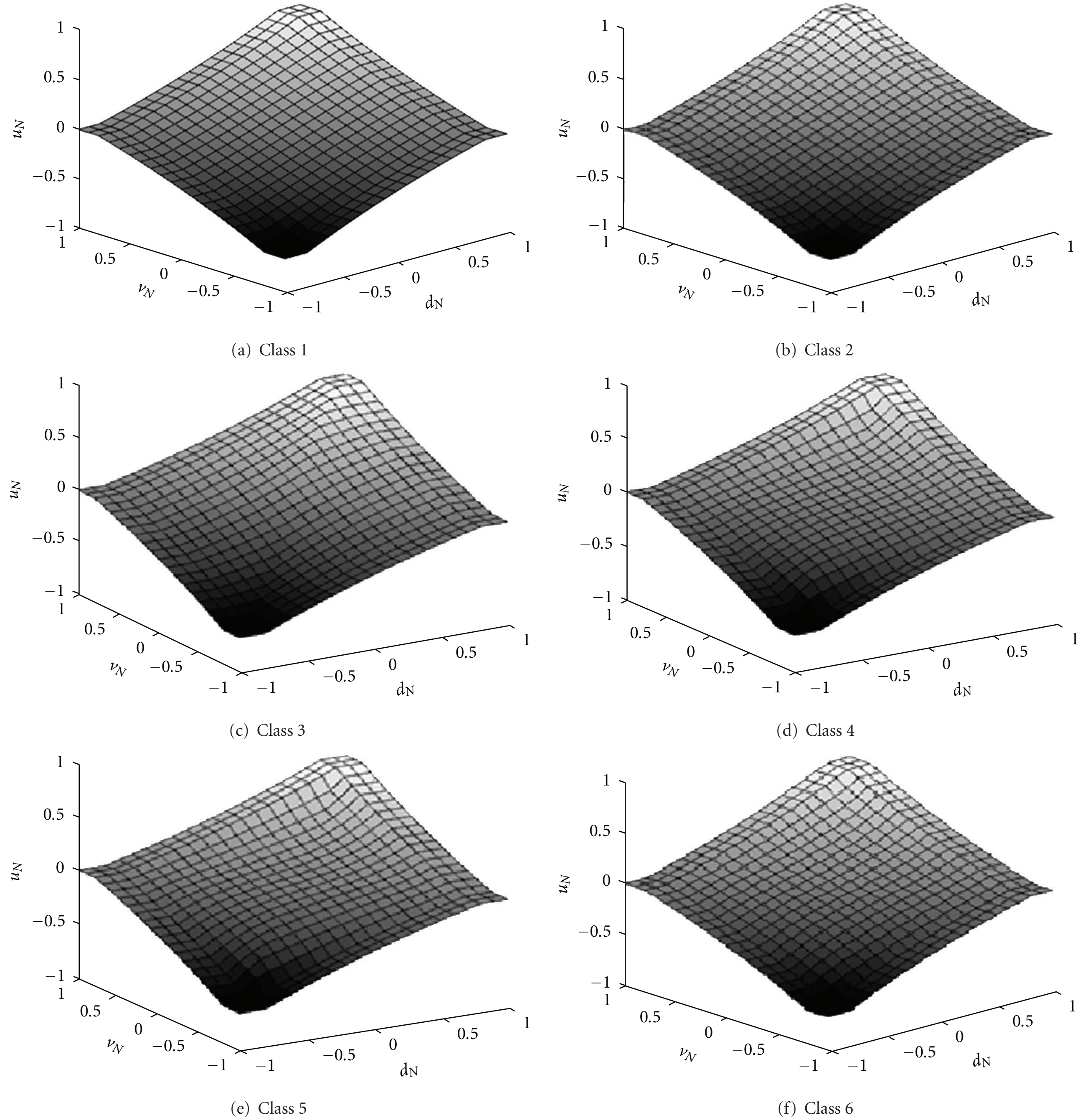
Region	s_1	s_2	$x_N(k)$
1	−	+	$-v_N(k)$
2	−	+	$d_N(k)$
3	+	−	$v_N(k)$
4	+	−	$-d_N(k)$

Table 3: Attributes of $d_N(k)$ and $v_N(k)$.

Region	$d_N(k)$	$v_N(k)$
5	l	Remains
6	Remains	$-l$
7	$-l$	Remains
8	Remains	l

3.2. Mathematical Models in the Regions 5–8. As can be seen from Table 1(c), μ_0 is independent of OR operator used. Therefore, the models are also independent of OR operation. We obtain two distinct models, one for algebraic product AND (Class 1/Class 2/Class 3) and the other for minimum AND (Class 4/Class 5/Class 6), by substituting appropriate values for $d_N(k)$ and $v_N(k)$ in the expressions of Class 3 and Class 6 models as shown in Table 3.

3.3. Mathematical Models in the Regions 9–12. In this case, all six classes of controllers have the same unique models as stated below.



(a) Class 1 (b) Class 2

(c) Class 3 (d) Class 4

(e) Class 5 (f) Class 6

Figure 6: Control Output.

$$\Delta u_N(k) \text{ or } u_N(k)$$

$$= \frac{\{3H^2 - h^2 - h^2\rho(1+\rho)\}}{3[2H - h(1+\rho)]} \quad \text{in region 9}$$

$$= 0 \quad \text{in regions 10 and 12}$$

$$= \frac{-\{3H^2 - h^2 - h^2\rho(1+\rho)\}}{3[2H - h(1+\rho)]} \quad \text{in region 11.} \tag{28}$$

4. Properties of the Simplest Fuzzy PI/PD Controllers

All the above six controllers are found to possess certain interesting properties which are discussed here.

(1) As can be seen from Section 3.3, all the controllers produce the same minimum control effort, given by

$$\frac{-\{3H^2 - h^2 - h^2\rho(1+\rho)\}}{3[2H - h(1+\rho)]}, \quad \text{at } (d_N(k), v_N(k)) = (-l, -l). \tag{29}$$

(2) All the controllers produce zero control effort at $(d_N(k),\ v_N(k)) = (0,0)$.

(3) All the controllers produce the same maximum control effort, given by

$$\frac{\{3H^2 - h^2 - h^2\rho(1+\rho)\}}{3[2H - h(1+\rho)]}, \quad \text{at } (d_N(k), v_N(k)) = (l, l). \tag{30}$$

(4) All the controllers produce zero control effort on the line $v_N(k) = -d_N(k)$.

(5) Control plots of all the controllers are shown in Figure 6. The control effort produced by each controller is continuous at any input point and symmetric about the line $v_N(k) = d_N(k)$. Further, the magnitude of control effort increases monotonically as the distance of the input point moves away from the origin of the input plane.

(6) The model equations of all six controllers in the regions 1–4 can be rewritten as

$$\Delta u_N(k) \text{ or } u_N(k) = \gamma(d_N(k),\ v_N(k)) \cdot (d_N(k) + v_N(k)), \tag{31}$$

where $\gamma(d_N(k),\ v_N(k))$ is called dynamic gain of the controller because γ varies as the normalized inputs $d_N(k)$ and $v_N(k)$ vary. Since γ is a nonlinear function (different for different controllers) of $d_N(k)$ and $v_N(k)$; every fuzzy controller considered in this paper is a different nonlinear controller. Moreover, each model (controller) has its own minimum γ value (call it $\gamma_{\min}$), maximum γ value ($\gamma_{\max}$), and $\gamma(0,\ 0)$ value. As a result of this, the control performance of each model is different.

5. Conclusions

Mathematical models of six different classes of the simplest fuzzy PI/PD controllers have been developed via L-function type and Γ-function type input membership functions, algebraic product/minimum AND operation, bounded sum/algebraic sum/maximum OR operation, algebraic product inference method, and CoS defuzzification method. As $\rho = 0$ gives rise to triangular membership function, the models derived are also applicable to triangular output membership function.

It is believed that Class 1 controller has never been reported in the literature. It is very important to note that the controller corresponding to the Class 2 case, reported in Patel and Mohan [2], was shown to be linear. This is, however, not the case with the Class 2 controller in this paper. It is also very important to note that the controller corresponding to the Class 3 case, reported in Patel and Mohan [2], is not suitable for control as it has been found to become indeterminate at certain points in the input plane. This is, however, not the case with the Class 3 controller in this paper. As a matter of fact, Class 3 controller also has all desirable control properties of the rest of the five controllers. So, it is very much suitable for control. The output membership functions shown in Figure 1(a) (used in [2]) and Figure 1(b) (used in this paper) could make all this difference in the end results.

Mathematical models of fuzzy controllers also depend on the type of inference method used. Hence, the important task that remains to be done is finding mathematical models via "minimum" inference and studying their properties.

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