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Research Article

Iterative Learning Control of Hysteresis in Piezoelectric Actuators

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We develop convergence criteria of an iterative learning control on the whole desired trajectory to obtain the hysteresis-compensating feedforward input in hysteretic systems. In the analysis, the Prandtl-Ishlinskii model is utilized to capture the nonlinear behavior in piezoelectric actuators. Finally, we apply the control algorithm to an experimental piezoelectric actuator and conclude that the tracking error is reduced to 0.15% of the total displacement, which is approximately the noise level of the sensor measurement.

1. Introduction

Piezoelectric actuators (PEAs) have been widely used in nanopositioning systems due to their fast response and nanometer scale resolution [1–3]. However, the hysteresis existing in PEAs can greatly limit system performance [4, 5]. Control of hysteretic system is an important area of control system research and a challenging problem [6–9]. Research on feedback and model-based feedforward control has been studied to achieve relatively high-precision positioning [10–13]. Iterative methods can be used to improve the positioning performance if the positioning application is repetitive. Therefore, many researchers study the iterative and adaptive control methods to minimize the adverse effect of hysteresis [14–18].

The main challenge in iterative approaches for hysteretic systems is to assure convergence of the iterative algorithm. Leang and Devasia divide a general desired trajectory into some monotonicity partition [15, 16]. Afterwards, they prove the convergence of iterative learning control (ILC) algorithm on each single branch. In this paper, we study the design of (ILC) algorithm to compensate for hysteresis-caused error in PEAs. The main contribution of our work is proving convergence of ILC algorithm on whole tracking trajectory.

The remainder of this paper is organized as follows. First, we state the problem in the next section. Afterwards, we briefly review the Prandtl-Ishlinskii model in the context of this work and prove convergence of the ILC algorithm we designed. Finally, we implement the ILC algorithm on experimental stage and show our experimental results and conclusions.

2. Problem Statement

Consider a hysteretic system of the following form:

$$y(t) = H[v(t)], \tag{1}$$

where $v(t) \in \mathbb{R}$ is the input, $y(t) \in \mathbb{R}$ is the output, and H denotes the hysteresis function $\mathbb{R} \to \mathbb{R}$. For a given desired trajectory $y_d(t)$ defined on the finite time interval $t \in [0, T]$, the objective is to find an input $v_d(t)$ by way of the following iterative learning control (ILC) algorithm:

$$v_{k+1}(t) = v_k(t) + \gamma e_k(t), \tag{2}$$

where $e_k(t) = y_d(t) - y_k(t)$, γ is a constant (to be determined), and $v_k(t)$ and $y_k(t)$ are the input and output at the kth iteration, respectively. Figure 1 depicts the block diagram of the ILC algorithm. The goal of the ILC algorithm is to generate

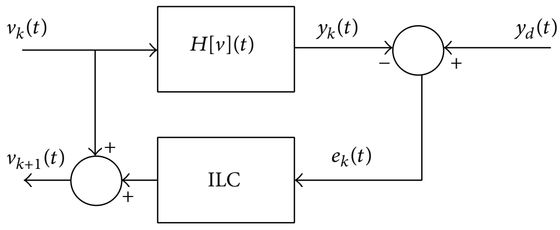


Figure 1: The ILC scheme.

a feedforward control input $v_k(t) \to v_d(t)$ in the $\|\cdot\|_\infty$ norm sense, where

$$\|\cdot\|_\infty \triangleq \sup_{t\in[0,T]} |v(t)|. \quad (3)$$

In this paper, $C[0,T]$ is used to denote the space of continuous functions on $t \in [0,T]$, and $C_m[0,T]$ denotes the space of continuous monotone functions on $t \in [0,T]$.

Definition 1 (incremental strictly increasing operators). An operator $F : C[0,T] \to C[0,T]$ is called incremental and strictly increasing, if, for $v_1, v_2 \in C[0,T]$, considering the ordering $v_1(t) \geq v_2(t)$ for all $t \in [0,T]$, there exist constants $k_1, k_2 > 0$ such that

$$\begin{aligned} k_1 (v_1(t) - v_2(t)) &\leq F[v_1](t) - F[v_2](t) \\ &\leq k_2 (v_1(t) - v_2(t)), \end{aligned} \quad (4)$$

for any $t \in [0,T]$.

Lemma 2. *Let the operator $F : C[0,T] \to C[0,T]$ be incremental and strictly increasing. If $0 < \gamma \leq 1/k_2$ and $v_0(t) \leq v_d(t)$ in (2) for all $t \in [0,T]$, then $v_k(t) \leq v_d(t)$ for any $t \in [0,T]$.*

Proof. We use the mathematical induction to prove this assertion. First, we prove $v_d(t) - v_1(t) \geq 0$ for any $t \in [0,T]$. Consider

$$\begin{aligned} v_d(t) - v_1(t) & \\ &= v_d(t) - v_0(t) - \gamma (y_d(t) - y_0(t)) \\ &= v_d(t) - v_0(t) - \gamma (F[v_d](t) - F[v_0](t)) \\ &\geq v_d(t) - v_0(t) - \gamma k_2 (v_d(t) - v_0(t)) \\ &= (1 - \gamma k_2)(v_d(t) - v_0(t)), \end{aligned} \quad (5)$$

where $1 - \gamma k_2 \geq 0$. We can obtain $v_d(t) \geq v_1(t)$. Then, we assume $v_d(t) - v_k(t) \geq 0$, for all $t \in [0,T]$, and

$$\begin{aligned} v_d(t) - v_k + 1(t) & \\ &= v_d(t) - v_k(t) - \gamma (y_d(t) - y_k(t)) \\ &= v_d(t) - v_k(t) - \gamma (F[v_d](t) - F[v_k](t)) \\ &\geq v_d(t) - v_k(t) - \gamma k_2 (v_d(t) - v_k(t)) \\ &= (1 - \gamma k_2)(v_d(t) - v_k(t)); \end{aligned} \quad (6)$$

we also get $v_d(t) - v_{k+1}(t) \geq 0$; therefore, the assertion holds. □

Lemma 3. *Let the operator $F : C[0,T] \to C[0,T]$ be incremental and strictly increasing with constants k_1, k_2 as defined by Definition 1. Then,*

$$\begin{aligned} &\|(k_2 v_1(t) - F[v_1](t)) - (k_2 v_2(t) - F[v_2](t))\|_\infty \\ &\quad \leq (k_2 - k_1)\|v_1(t) - v_2(t)\|_\infty \end{aligned} \quad (7)$$

for all $v_1(t), v_2(t) \in C[0,T]$.

Proof. See [19]. □

Theorem 4. *Consider a system of the form $y(t) = F[v](t)$. Let the operator $F : C[0,T] \to C[0,T]$ be incremental and strictly increasing. If the constant $0 < \gamma \leq 1/k_2$ and $v_0, v_d \in C[0,T]$, $v_0(t) \leq v_d(t)$ for all $t \in [0,T]$, then the iterative control algorithm converges; that is,*

$$\|v_d(t) - v_k(t)\|_\infty \leq \rho^k \|v_d(t) - v_0(t)\|_\infty, \quad (8)$$

where $\rho < 1$ and $v_k(t) \to v_d(t)$ uniformly in $t \in [0,T]$ as $k \to \infty$.

Proof. To prove the convergence of the ILC algorithm, we show contraction of the input (2). Subtracting $v_d(t)$ from both sides of (2) and substituting $F[v](t)$ in place of $y(t)$, we get

$$\begin{aligned} v_d(t) - v_{k+1}(t) & \\ &= v_d(t) - v_k(t) - \gamma (y_d(t) - y_k(t)) \\ &= v_d(t) - v_k(t) - \gamma (F[v_d](t) - F[v_k](t)), \end{aligned} \quad (9)$$

for all $t \in [0,T]$. Taking the function $\|\cdot\|_\infty$ norm of (9), we get

$$\begin{aligned} &\|v_d(t) - v_{k+1}(t)\|_\infty \\ &\quad = \|v_d(t) - v_k(t) - \gamma (F[v_d](t) - F[v_k](t))\|_\infty. \end{aligned} \quad (10)$$

By Lemma 2, we obtain $v_d(t) \geq v_k(t)$ for all $t \in [0,T]$. Since the operator F is incrementally strictly increasing with constants $k_1 \leq k_2$ and $0 < \gamma \leq 1/k_2$, then by Lemma 3 we obtain

$$\begin{aligned} &\|v_d(t) - v_{k+1}(t)\|_\infty \\ &\quad \leq (1 - \gamma k_1)\|v_d(t) - v_k(t)\|_\infty \\ &\quad \triangleq \rho \|v_d(t) - v_k(t)\|_\infty, \end{aligned} \quad (11)$$

where $\rho = 1 - \gamma k_1 < 1$. Because $0 < \rho < 1$, (11) is a contraction. By induction, we obtain that

$$\|v_d(t) - v_k(t)\|_\infty \leq \rho^k \|v_d(t) - v_0(t)\|_\infty; \quad (12)$$

therefore, the sequence $\|v_d(t) - v_d(t)\|_\infty \to 0$ as $k \to \infty$; that is, $v_k(t)$ converges to $v_d(t)$ uniformly in $t \in [0,T]$. Proof is completed. □

3. The Prandtl-Ishlinskii Hysteresis Model

The Prandtl-Ishlinskii (PI) model can be used to capture the rate-independent hysteresis nonlinearity in piezoelectric actuators. In this section, the PI model is presented.

The PI model utilizes the play or stop operators and a density function to characterize the hysteresis behavior. The hysteresis play operator is illustrated in Figure 2, while its detailed formulations have been presented in [20]. For a given input $v(t) \in C_m[0,T]$, the play operator $F_r[v](t)$ with threshold r is defined by

$$\begin{aligned} &F_r[v;\psi](0) = f_r(v(0),\psi), \\ &F_r[v;\psi](t) = f_r(v(t), F_r[v;\psi](t_i)), \\ &\qquad \text{for } t_i < t \le t_{i+1}, \quad 0 \le i \le N, \end{aligned} \tag{13}$$

with

$$f_r(v,w) = \max(v-r, \min(v+r, w)), \tag{14}$$

where ψ is initial value of the operator F_r, and $0 = t_0 < t_1 < \cdots < t_N = T$ is a partition of $[0,T]$, such that the input function v is monotone on each subinterval $[t_i, t_{i+1}]$.

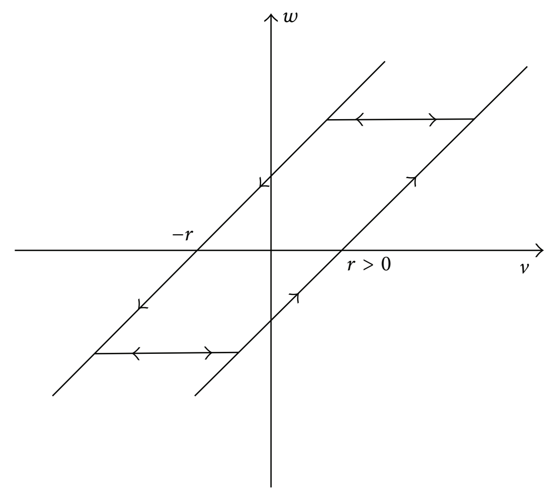


Figure 2: Play operator.

3.1. Property of Play Operator

Lemma 5. *For any $r \ge 0$, the operator F_r can be extended uniquely to a Lipschitz continuous operator $F_r : C[0,T]\times\mathbb{R} \to C[0,T]$. And it holds for all $v_1, v_2 \in C[0,T]$, for all initial values $\psi_1, \psi_2 \in \mathbb{R}$, and for all $t \in [0,T]$. Consider*

$$\begin{aligned} &\left|F_r[v_1;\psi_1](t) - F_r[v_2;\psi_2](t)\right| \\ &\quad \le \max\left(\sup_{\tau\in[0,t]}\left|v_1(\tau)-v_2(\tau)\right|, \left|\psi_1-\psi_2\right|\right) \end{aligned} \tag{15}$$

$$F_r[v_1;\psi_1](t) \le F_r[v_2;\psi_2](t), \quad \text{if } v_1 \le v_2, \quad \psi_1 \le \psi_2. \tag{16}$$

Proof. See [20] Section 2.3. □

Lemma 6. *If $v_1(t) \ge v_2(t)$, for all $t \in [0,T]$, and initial value $\psi_1 = \psi_2 = 0$, a Lipschitz continuous operator $F_r : C[0,T] \times R \to C[0,T]$ is incremental and strictly increasing; that is, there exist constants $k_1, k_2 > 0$ such that $k_1(v_1(t)-v_2(t)) \le F_r[v_1](t) - F_r[v_2](t) \le k_2(v_1(t)-v_2(t))$.*

Proof. Consider

$$\begin{aligned} &F_r[v_1;\psi_1](t) - F_r[v_2;\psi_2](t) \\ &\quad = F_r[v_1](t) - F_r[v_2](t) \\ &\quad = f_r(v_1(t), F_r[v_1](t-1)) - f_r(v_2(t), F_r[v_2](t-1)). \end{aligned} \tag{17}$$

Since $v_1(t) \ge v_2(t)$, for all $t \in [0,T]$, and initial value $\psi_1 = \psi_2 = 0$, then $F_r[v_1](0) \ge F_r[v_2](0)$. By induction, $F_r[v_1](t) \ge F_r[v_2](t)$. There must exist a constant $k_1 > 0$ such that

$$F_r[v_1](t) - F_r[v_2](t) \ge k_1(v_1(t)-v_2(t)). \tag{18}$$

Because $F_r[v_1](t) - F_r[v_2](t) \ge 0$ and $\psi_1 = \psi_2 = 0$, (15) can be simplified as

$$F_r[v_1](t) - F_r[v_2](t) \le \sup_{\tau\in[0,t]}(v_1(\tau)-v_2(\tau)). \tag{19}$$

Let $\alpha(t) > 0$ and $\sup_{\tau\in[0,t]}(v_1(\tau)-v_2(\tau)) = \alpha(t)(v_1(t)-v_2(t))$. We get

$$F_r[v_1](t) - F_r[v_2](t) \le \sup_{\tau\in[0,t]}(\alpha(\tau))(v_1(t)-v_2(t)). \tag{20}$$

Let $k_2 = \sup_{\tau\in[0,t]}(\alpha(\tau))$. From (18) and (20), we obtain

$$\begin{aligned} k_1(v_1(t)-v_2(t)) &\le F_r[v_1](t) - F_r[v_2](t) \\ &\le k_2(v_1(t)-v_2(t)); \end{aligned} \tag{21}$$

therefore, the assertion holds. □

3.2. Property of the Prandtl-Ishlinskii Model. The PI model assumes that the output $y(t)$ of a piezoelectric actuator is a weighted superposition of basic $F_r[v]$. The output $y(t)$ is written as

$$y(t) = H[v](t) = \int_0^\infty F_r[v](t)\,p(r)\,dr, \tag{22}$$

where $p(r)$ is a given density function, satisfying $p(r) \ge 0$ with $\int_0^\infty p(r)dr < \infty$.

Lemma 7. *If $v_1(t) \ge v_2(t)$, for all $t \in [0,T]$, and all initial values $\psi_1 = \psi_2$ in all operators F_r, then PI operator $H : C[0,T] \to C[0,T]$ is incremental and strictly increasing; that is, there exist constants $\lambda_1, \lambda_2 > 0$ such that*

$$\begin{aligned} \lambda_1(v_1(t)-v_2(t)) &\le H[v_1](t) - H[v_2](t) \\ &\le \lambda_2(v_1(t)-v_2(t)) \end{aligned} \tag{23}$$

for any $t \in C[0,T]$.

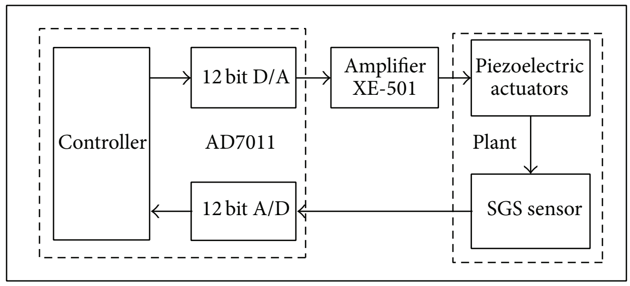


FIGURE 3: Structure of the experimental setup.

Proof. Consider

$$\begin{aligned} H[v_1](t) - H[v_2](t) &= \int_0^\infty F_r[v_1](t)\,p(r)\,dr - \int_0^\infty F_r[v_2](t)\,p(r)\,dr \\ &= \int_0^\infty (F_r[v_1](t) - F_r[v_2](t))\,p(r)\,dr \\ &\le \int_0^\infty k_2 (v_1(t) - v_2(t))\,p(r)\,dr \\ &= k_2 \int_0^\infty p(r)\,dr\,(v_1(t) - v_2(t)) \\ &\triangleq \lambda_2 (v_1(t) - v_2(t)), \end{aligned} \tag{24}$$

for any $t \in C[0,T]$, where $\lambda_2 = k_2 \int_0^\infty p(r)dr$. Similarly, we can get

$$\begin{aligned} H[v_1](t) - H[v_2](t) &= \int_0^\infty (F_r[v_1](t) - F_r[v_2](t))\,p(r)\,dr \\ &\ge k_1 \int_0^\infty p(r)\,dr\,(v_1(t) - v_2(t)) \\ &\triangleq \lambda_1 (v_1(t) - v_2(t)), \end{aligned} \tag{25}$$

for any $t \in C[0,T]$, where $\lambda_1 = k_1 \int_0^\infty p(r)dr$. Proof is completed. □

Theorem 8. *Consider a hysteretic system of the form $y(t) = H[v](t)$. Let the PI hysteresis operator H satisfy the condition of Lemmas 6 and 7. If the constant $0 < \gamma \le 1/\lambda_2$ and $v_0, v_d \in C[0,T]$ in (2), $v_0(t) \le v_d(t)$ for all $t \in [0,T]$, then the iterative control algorithm converges; that is,*

$$\|v_d(t) - v_k(t)\|_\infty \le \rho^k \|v_d(t) - v_0(t)\|_\infty, \tag{26}$$

where $0 < \rho < 1$ and $v_k(t) \to v_d(t)$ uniformly in $t \in [0,T]$ as $k \to \infty$.

Proof. The proof is identical to the proof of Theorem 4. □

4. Experimental Results

4.1. Experimental Setup. The ILC algorithm is applied on an experimental piezoelectric actuator PST150/7/40VS12, which is a preloaded PZT from Piezomechanik in Germany. The natural frequency of the actuator is 20 kHz. The actuator provides a maximum displacement of 40 μm and includes an integrated high-resolution strain gauge position sensor (SGS). The excitation module comprises a voltage amplifier (HVPZT XE-501.B) with a fixed gain of 15, which provides the excitation voltage for the actuator. The AD7011-EVA controller board equipped with 12-bit ADC and12-bit DAC is utilized to generate and acquire input voltage and output displacement signals. The experimental data are acquired at a sampling frequency of 1 kHz. Figure 3 illustrates the structure of the experimental system.

4.2. Experimental Result. We apply the ILC algorithm to track a sinusoidal trajectory $y_d(t) = 5\sin((1/2\pi)t - (\pi/2)) + 7$. The constant γ in (2) is chosen to be $\gamma = 0.5$ and the initial input $v_0(t) = 5\sin((1/2\pi)t - (\pi/2)) + 5$. The experimental results are shown in Figure 4.

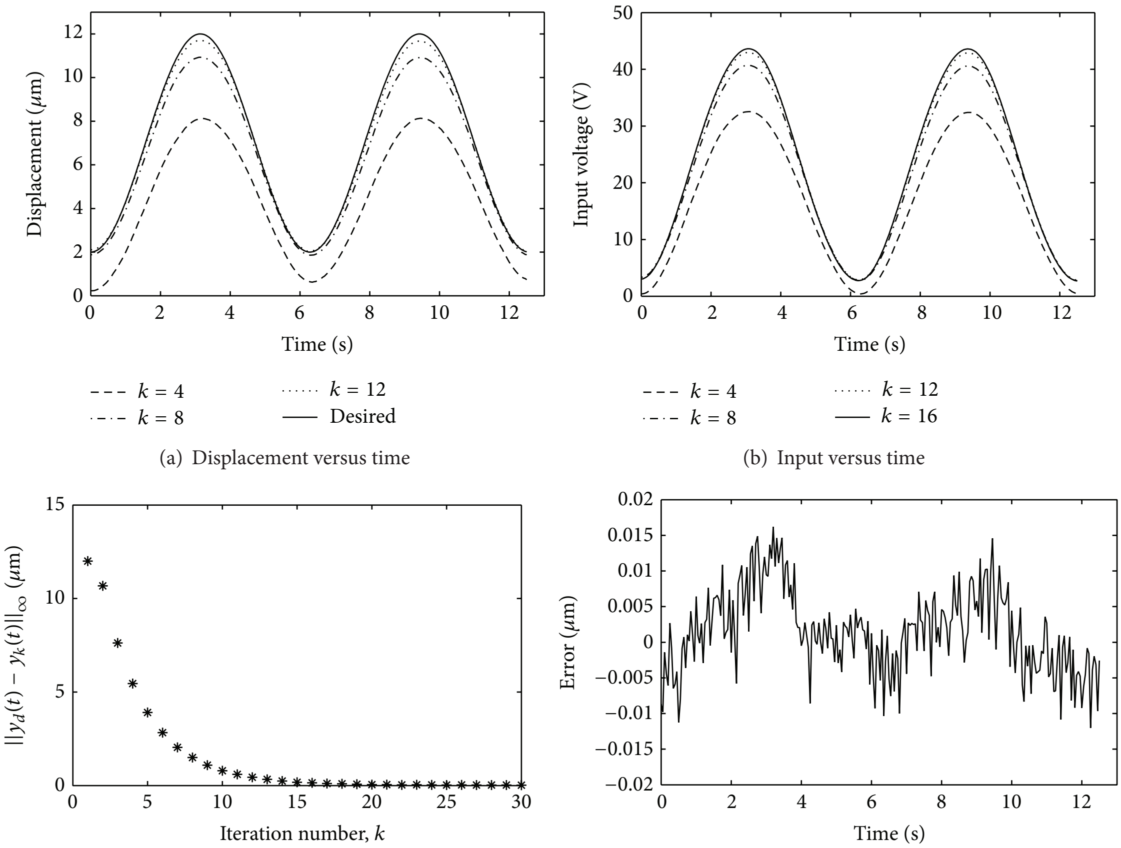
Figure 4(a) shows the results of the ILC algorithm to track the desired trajectory, and the feedforward input of the ILC algorithm is depicted in Figure 4(b). The maximum error at the kth iteration $\|e_k(t)\|_\infty$ is shown in Figure 4(c), and it demonstrated that convergence of ILC algorithm is achieved. Figure 4(d) shows the tracking error at the 30th iteration. The maximum error is approximately 0.015 μm, which is 0.15% of the total displacement range.

5. Conclusions

In this paper, we designed an ILC algorithm to compensate for hysteresis-caused tracking error in piezoelectric actuators and proved convergence of this algorithm on the whole tracking trajectory. Experiments were carried out to verify the effectiveness of the ILC algorithm. The experimental results show that the tracking error can be reduced to the noise level of the sensor measurements.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.



(a) Displacement versus time

(b) Input versus time

(c) $\|e_k(t)\|_\infty$ versus k

(d) $e_{30}(t)$ versus time

Figure 4: Experiment results.

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