

*Research Article*

# **Chaos and Control of Game Model Based on Heterogeneous Expectations in Electric Power Triopoly**

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A dynamic repeated game model has been established based on heterogeneous expectations in electric power triopoly. Theoretical analysis and numerical simulation show the complexity of this model; suppose that the producers make decisions with naive expectation and bounded rationality. The straight-line stabilization chaos control method was successfully applied to the dynamic repeated game model. The results have important practical value for the producers in the electric power oligopoly.

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## **1. Introduction**

In economic system research on chaos theory, bifurcation theory based on differential equations is the earliest chaotic dynamics branch with the most widespread application and the deepest advancement [1, 2]. Whether the economic system is chaos or not has been a new hot topic in economic studies. A series of research on the dynamic game model of the output decision has been conducted. In references [3–7], the outputs would take on dynamic characters including chaos when the feedback rates of producers changed. In 2003, Agiza and Elsadany [7] studied the duopoly game model based on heterogeneous expectations, that is, one player applied naive expectation rule and the other used bounded rationality rule.

In this paper, the model of duopoly [8] was modified to triopoly in the electric power market. Supposing two producers determined on the optimal outputs with naive expectation rule while the other with bounded rationality rule for the maximal profits. Theoretical analysis and numerical simulations were performed in detail and the parameters leading to the chaos were obtained in the definite range. Then the straight-line stabilization method was used to control the period-doubling bifurcation and unstable periodic orbits in chaotic attractor of the discrete nonlinear dynamic system [9]. The research results help the electric power producers to decide appropriate outputs.

## 2. The Model in Electric Power Triopoly

Suppose that there are three electric power producers in the given region, and they provide electric power to consumers through the electric utilities. Each electric power producer makes the optimal output decision and  $q_i(t)$  is the  $t$ -output of producer  $i$ . At each period  $t$ , the price  $p$  is determined by the total outputs  $Q(t) = q_1(t) + q_2(t) + q_3(t)$ . Suppose that the inverse demand function  $p = p(Q) = m - nQ^2$ , then  $m$  is the maximal price (corresponding to  $Q = 0$ ) and  $n$  is parameter of the price. The variable cost function  $VC_i = VC_i(q_i) = b_i q_i + c_i q_i^2$  ( $i = 1, 2, 3$ ) is nonlinear [10], too. Thereinto  $b_i$  and  $c_i$  are cost parameters. So the profit of the producer  $i$  in period  $t$  would be

$$\pi_i(t) = q_i(t) \left[ m - nQ^2(t) \right] - \left[ b_i q_i(t) + c_i q_i^2(t) \right] - r q_i(t), \quad i = 1, 2, 3, \quad (2.1)$$

where  $r$  is the wheeling rate [11].

In this game, the producers make the optimal output decisions for the maximal profits; one of the methods to find out the Nash equilibrium is to let the partial differentiation of the profit equal zero as

$$\frac{\partial \pi_i}{\partial q_i} = m - nQ^2(t) - 2nq_i Q(t) - b_i - 2c_i q_i - r = 0, \quad i = 1, 2, 3. \quad (2.2)$$

According to the Rational Expectation Theory, expectation rules include naive expectation, adaptive expectation, and bounded rationality. Naive expectation is the simplest and the most common expectation form. Suppose that the players decide the future outputs according to the past [12] and take the output expectation in electric power market as an example, then the expectant output of  $t + 1$  equals the optimal output of  $t$ :

$$q_i(t + 1) = q_i^*(t). \quad (2.3)$$

The boundedly rational company makes its output decision on the basis of a local estimate of the marginal income ratio. The company decides to increase its production if it has a positive marginal income ratio or to decrease its production if the marginal income ratio is negative. According to the rules above, the dynamical equation of producer  $i$  has the form as following [3]:

$$q_i(t + 1) = q_i(t) + \alpha q_i(t) \frac{\partial \pi_i}{\partial q_i}, \quad 0 < \alpha < 1, \quad (2.4)$$

where  $\alpha$  is the relative speed of adjustment; it is a positive parameter.

We suppose that two producers adopt naive expectation rule and the other bounded rationality. Let  $q_1(t) = x$ ,  $q_2(t) = y$ ,  $q_3(t) = z$ ,  $q_1(t + 1) = X$ ,  $q_2(t + 1) = Y$ , and  $q_3(t + 1) = Z$ ,

then the nonlinear dynamic game model based on heterogeneous expectations in the electric power market can be expressed as

$$\begin{aligned} X &= \frac{1}{6n} \left( -4ny - 4nz - 2c_1 + 2\sqrt{n^2y^2 + n^2z^2 + 2n^2yz + 4nc_1y + 4nc_1z + c_1^2 + 3mn - 3nb_1 - 3nr} \right), \\ Y &= \frac{1}{6n} \left( -4nx - 4nz - 2c_2 + 2\sqrt{n^2x^2 + n^2z^2 + 2n^2xz + 4nc_2x + 4nc_2z + c_2^2 + 3mn - 3nb_2 - 3nr} \right), \\ Z &= z + \alpha z \left( -3nz^2 - nx^2 - ny^2 - 2nxy - 4myz - 4nxz - 2c_3z + m - b_3 - r \right). \end{aligned} \quad (2.5)$$

### 3. Numerical Simulations of the System

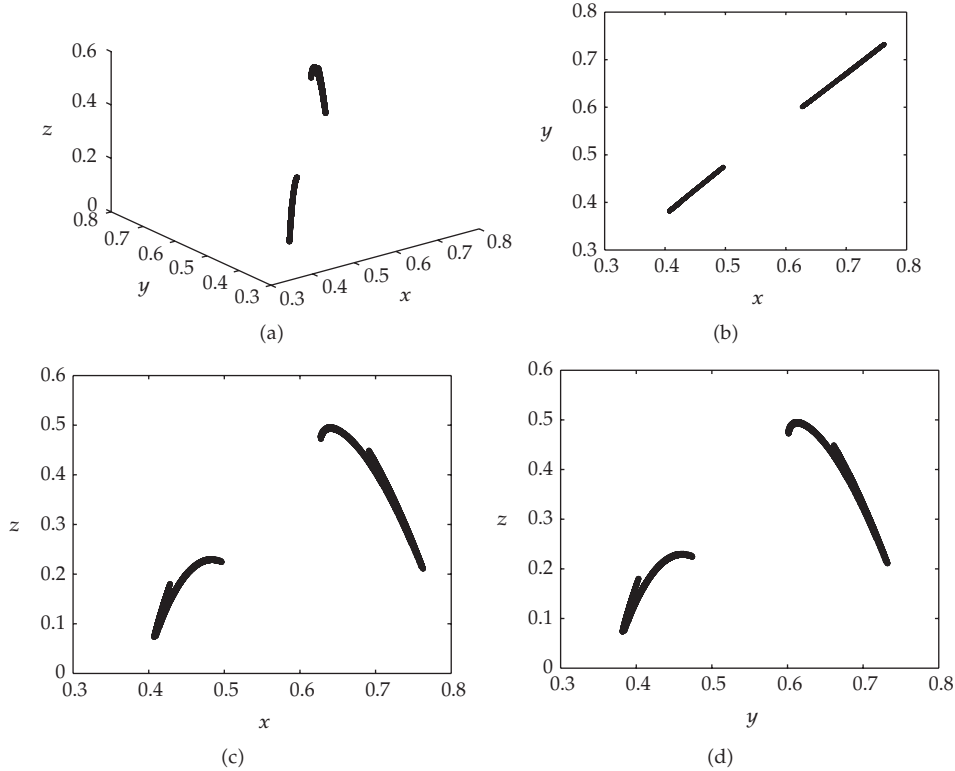
Now only a few of simple dynamic systems can be concluded as the chaotic characters by the analytical method, so the numerical analysis is the prime method in the research on the chaotic phenomenon. In order to deduce some numerical chaotic evidences of system (2.5), the inverse demand function and three cost functions will be taken as constants. The parameters take the values of  $m = 5$ ,  $n = 1$ ,  $b_1 = 0.8$ ,  $c_1 = 0.3$ ,  $b_2 = 0.6$ ,  $c_2 = 0.5$ ,  $b_3 = 0.7$ , and  $c_3 = 0.4$ . So the inverse function is  $p(Q) = 5 - Q^2$ , and variable cost functions are  $VC_1(q_1) = 0.8q_1 + 0.3q_1^2$ ,  $VC_2(q_2) = 0.6q_2 + 0.5q_2^2$ , and  $VC_3(q_3) = 0.7q_3 + 0.4q_3^2$ , respectively. Several numerical results have been concluded, such as bifurcation diagrams, strange attractors, Lyapunov exponents, and the influence on the system with the changing parameters.

#### 3.1. The Chaotic Attractors of the System

When the parameters take the values of  $r = 0.4$ ,  $\alpha = 0.63$  and the initial outputs are  $(0.2, 0.5, 0.8)$ , the system (2.5) has the chaotic attractor as shown in Figure 1. The Lyapunov exponents of the system (2.5) are  $\lambda_1 = 0.1125$ ,  $\lambda_2 = -0.6467$ , as well as  $\lambda_3 = -0.6656$ , and the Lyapunov dimension is  $d = 1 - \lambda_1/\lambda_2 = 1 + 0.1125/0.6467 = 1.1740$ . The fractal dimension is less than 2, so the occupied space is small and the structure is sparse, which can be seen in the chaotic attractor diagrams.

#### 3.2. The Influence on the System of the Parameters Changing

Figure 2 shows the outputs bifurcation diagram when the parameters were set as  $r = 0.4$ ,  $\alpha = [0, 0.63]$ . It is obvious that the outputs change from the initial status  $(0.2, 0.5, 0.8)$  to the stable point then exhibit period-doubling bifurcations till chaos with the increase of output modification speed parameter. It is concluded [13] that the market presents the out-of-order phenomenon more easily when the modification speed parameter is larger. That is to say that the feedback rate of producers to market can cause the outputs game model to show complicated characteristics. The output modification speed parameter has limited value and  $\alpha_{\max} = 0.6326$  in system (2.5). In this study, two of the producers adopt the same decision rules and the variable cost functions are similar quadratic function, so the evolvments of



**Figure 1:** (a) Three-dimensional view of the chaotic attractor, (b)  $x$ - $y$  phase plane strange attractor, (c)  $x$ - $z$  phase plane strange attractor, (d)  $y$ - $z$  phase plane strange attractor with  $r = 0.4$ ,  $\alpha = 0.63$ .

their outputs are similar as shown in the bifurcation figures. The range of  $\alpha$  emerging chaos in the system (2.5) is  $[0.6185, 0.6326]$ .

When  $r = 0.4$ , the diagram of the largest Lyapunov exponent is changing with the parameter  $\alpha$  in Figure 3. Different parameters  $\alpha$  are accompanied by different Lyapunov exponents. In addition, Figure 3 is in the same essence as that of Figure 2. When  $\alpha < \alpha_\infty$ , the system is regular periodic, so the largest Lyapunov exponent is less than zero except at the critical point ( $\lambda = 0$ ). When  $\alpha$  is larger than  $\alpha_\infty$ ,  $\lambda > 0$  which means that the system is chaos while  $\lambda < 0$  at limited range of  $\alpha$  corresponds to periodic windows in the chaotic region.

#### 4. The Chaos Control of the System

The chaos control methods include perturbation feedback chaos control method and non-feedback control method. The feedback objects are system parameters in the perturbation feedback chaos control method, system variable, and external parameters. Small continuous perturbation is regarded as control signal in feedback chaos control method. Even if the perturbation is very small, the stable control of the specific goal can be realized.

Recently, Yang and Xu, and so forth, proposed a new control method called the straight-line stabilization method [14, 15]. So we adopt this method to control the chaos

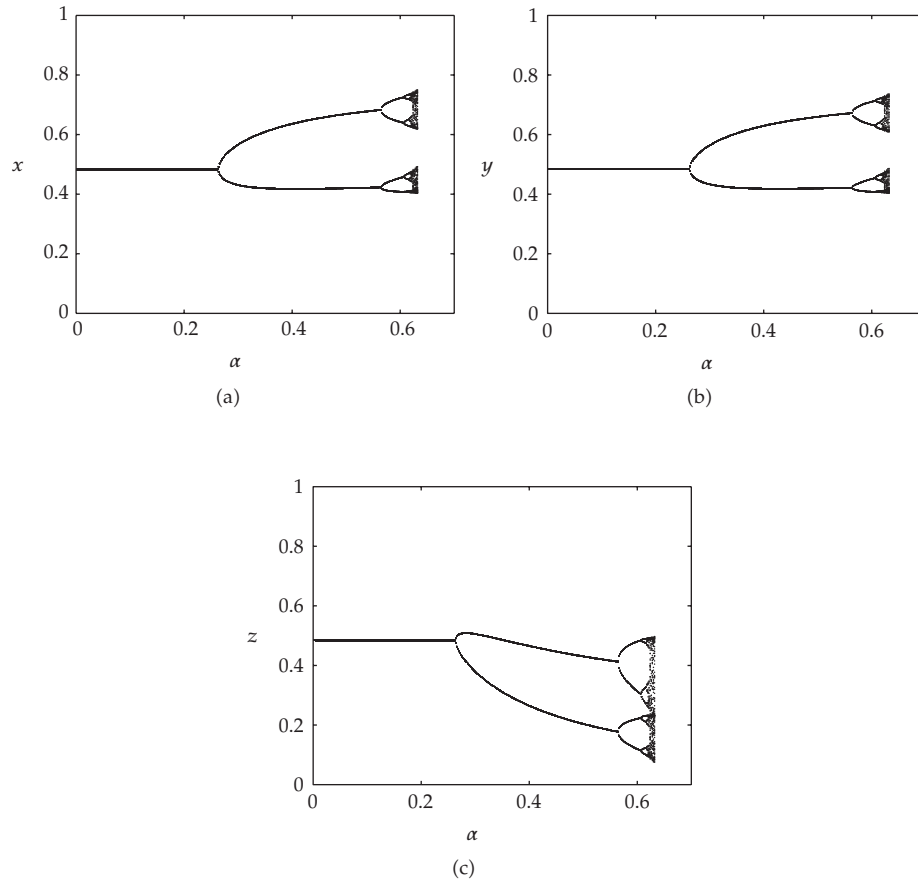


Figure 2: The state of the outputs with  $r = 0.4$ ,  $\alpha = 0 - 0.63$ .

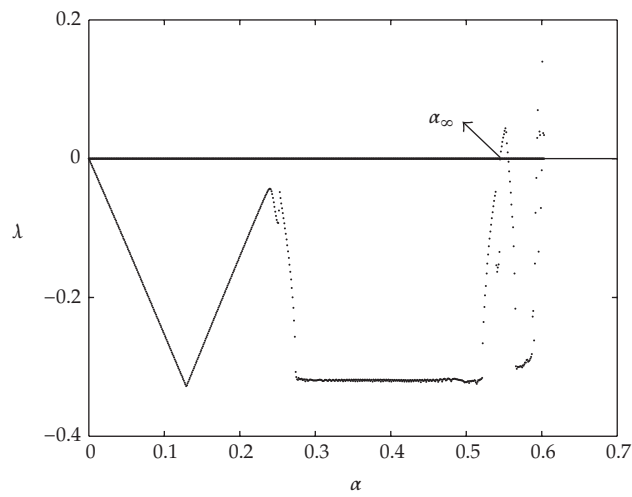


Figure 3: The largest Lyapunov exponent of the system with  $r = 0.4$ ,  $\alpha = [0, 0.63]$ .

in the triopoly game model. Impose the external control signal  $e_n$  on the primary system, therefore, we obtain

$$e_n = \begin{pmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{pmatrix} = (\eta I - J) \begin{pmatrix} q_1(t) - q_1^* \\ q_2(t) - q_2^* \\ q_3(t) - q_3^* \end{pmatrix}, \quad (4.1)$$

where  $|\eta| < 1$  is the feedback control parameter, and  $q_1^*, q_2^*, q_3^*$  are the outputs at the fixed points. Suppose that  $q_1^* = x^*, q_2^* = y^*,$  and  $q_3^* = z^*,$  then the controlled game model is attained as

$$\begin{aligned} X &= \frac{1}{6} \left( -4y - 4z - 0.6 + 2\sqrt{y^2 + z^2 + 2yz + 1.2y + 1.2z + 12.69 - 3r} \right) + (\eta I - J)(x - x^*), \\ Y &= \frac{1}{6} \left( -4x - 4z - 1 + 2\sqrt{x^2 + z^2 + 2xz + 2x + 2z + 13.45 - 3r} \right) + (\eta I - J)(y - y^*), \\ Z &= z + \alpha z \left( -3z^2 - x^2 - y^2 - 2xy - 4yz - 4xz - 0.8z + 4.3 - r \right) + (\eta I - J)(z - z^*), \end{aligned} \quad (4.2)$$

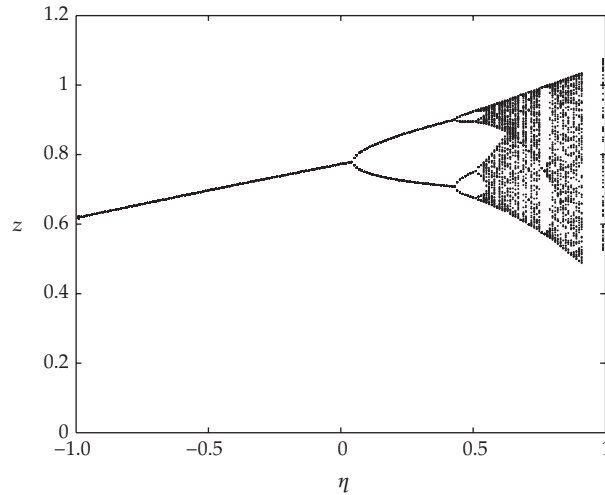
where  $J$  is the Jacobian matrix of the primary system with  $r = 0.40$  and  $\alpha = 0.63$  given as

$$J = \begin{bmatrix} 0 & -0.5017 & -0.5017 \\ -0.4960 & 0 & -0.4960 \\ -2.3184 & -2.3184 & -4.1093 \end{bmatrix}. \quad (4.3)$$

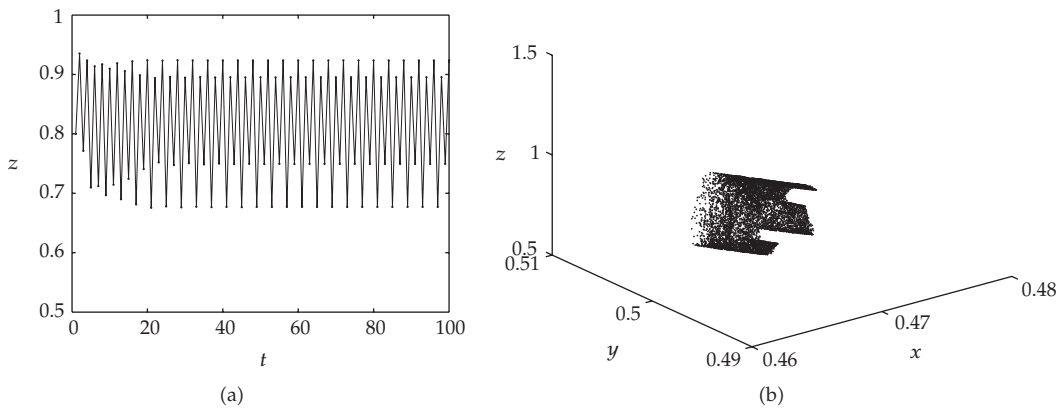
The fixed point of the system is  $(0.4830, 0.4848, 0.4840),$  which means that  $q_1^* = 0.4830,$   $q_2^* = 0.4848,$  and  $q_3^* = 0.4840,$  so the external control signal is

$$e_n = \begin{pmatrix} \eta & 0.5017 & 0.5017 \\ 0.4960 & \eta & 0.4960 \\ 2.3184 & 2.3184 & \eta + 4.1093 \end{pmatrix} \begin{pmatrix} x - 0.4830 \\ y - 0.4848 \\ z - 0.4840 \end{pmatrix}. \quad (4.4)$$

The primary system is chaotic with  $r = 0.40$  and  $\alpha = 0.63.$  Figure 4 is the third output of controlled system with the initial outputs  $(0.2, 0.5, 0.8)$  and the external control signal  $\eta \in [-1, 1].$  As can be seen from the figure, the controlling strategy is applied on the system by the external control signal, and the controlled system is steady with  $\eta \in [-1, 0.0442].$  The range of  $\eta$  corresponding to the stable region is larger than that corresponding to the unstable region. Figure 5(a) is the 4-period time series of the third output with  $r = 0.40, \alpha = 0.63,$  and  $\eta = 0.50.$  Figure 5(b) is the chaotic attractor of the controlled system with  $r = 0.40, \alpha = 0.63,$  and  $\eta = 0.80.$



**Figure 4:** The output of the controlled system with  $r = 0.40$ ,  $\alpha = 0.63$ , and  $\eta = [-1, 1]$ .



**Figure 5:** (a) The time series of the output with  $r = 0.40$ ,  $\alpha = 0.63$ , and  $\eta = 0.50$ ; (b) the chaotic attractor of the controlled system with  $r = 0.40$ ,  $\alpha = 0.63$ , and  $\eta = 0.80$ .

### 5. Conclusions

The chaotic dynamic theory is successfully applied to the electric power market. The discrete game model is obtained based on heterogeneous expectation rules. Suppose that the inverse demand function and cost functions are all nonlinear and take the wheeling rate into account, then theoretic analysis and numerical simulation results show that the system is chaotic. The influences on the market of the outputs modification speed parameters and the range of the parameters leading to the chaos have been discussed. Then the straight-line stabilization method has been used to control the period-doubling bifurcation and unstable periodic orbits in chaotic attractor. Once a small perturbation is applied in the chaos region, the equilibrium point arrived quickly, so the producer’s anticipated target will be achieved. The model is closer to the economic reality and worth spreading in oligopoly though the dynamic analysis is more complicated.

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