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Research Article

An Optimal Operating Strategy for Battery Life Cycle Costs in Electric Vehicles

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Impact on petroleum based vehicles on the environment, cost, and availability of fuel has led to an increased interest in electric vehicle as a means of transportation. Battery is a major component in an electric vehicle. Economic viability of these vehicles depends on the availability of cost-effective batteries. This paper presents a generalized formulation for determining the optimal operating strategy and cost optimization for battery. Assume that the deterioration of the battery is stochastic. Under the assumptions, the proposed operating strategy for battery is formulated as a nonlinear optimization problem considering reliability and failure number. And an explicit expression of the average cost rate is derived for battery lifetime. Results show that the proposed operating strategy enhances the availability and reliability at a low cost.

1. Introduction

The growth of electric vehicle technologies such as pure electric and hybrid electric vehicles has presented new opportunities, including reduced dependency on nonrenewable energy resources, lowering of CO_2 emissions from transport, and greater public awareness of leading a lower carbon lifestyle. Environmental problems promote the adoption of new-generation electric vehicles for urban transportation.

As it is well known, one of the weakest points of electric vehicles is the battery system. The battery served as energy storage units must be sized so that they store sufficient energy and provide adequate peak power for the vehicle to have a specified acceleration performance and the capability to meet appropriate driving cycles. The battery may be a replacement product for the primary-use vehicle, especially in Europe and Asia, due to its drive performance and safety [1–3]. However, the electric vehicle battery has a cycle cost life, defined as the number of complete charge-discharge cycles the battery can perform before its nominal capacity falls below 80% of its initial rated capacity. The accurate operating model and strategy are not only essential for providing precise battery state information, protecting the battery from harmful charging and discharging, and improving the cost-efficacy, but also for economic viability [4].

Due to manufacturing asymmetries, charge and discharge cycles lead to cell unbalancing, reducing battery capacity, and causing safety troubles or strongly limiting the storage capacity of the full pack. The operating strategy for the battery is useful. So the various literatures have been focusing on the problems of battery management, operation, and maintenance. A battery monitoring and maintenance system based on capacity estimation is proposed in [5]. In [6], a nonchemical based partially linearized input-output battery model is developed to have battery lifetime. Traditional battery maintenance method based on testing charge and discharge is proposed in [7]. This method involved applying a resistive charge to a set of batteries and adjusting a discharge current established by the manufacturer in battery rating tables. Cell-to-cell internal-impedance measurement is analyzed in [8]. These procedures all give exact measures of the condition of the battery. However, the capacity test is not the most viable option for vehicles because, when the fleet of equipment to be maintained is large, the costs arising from vehicle nonavailability are high. Among the published work, the optimal operating strategy for battery cycle costs by frequent charging and discharging has not been studied in detail. This paper has the following objectives.

(1) To study the cost associated with the life cycle of batteries used in electric vehicles.

(2) To establish a correlation between battery life cycle cost and the operating strategy (including repairment and maintenance).

2. Mathematical Models

The battery in the electric vehicle may be described in the following way: it is subject to random failure; upon failure, the battery is either repaired or replaced by a new and identical one. Many factors contributed to the cycle life of an electric vehicle battery in a given application [9]. These include depth of discharge, discharge rate, ambient temperature, charging regime, and battery maintenance procedures. In this paper, the impact of battery maintenance on battery cycle life will be studied in detail. In practice, because of the above factors, the battery is deteriorating. Consequently, the successive charging or discharging times after repair will be decreasing, because of the deterioration. Let X_n be the survival operating time after $(n-1)$th repair. The X_n will be stochastically decreasing and finally dying out. Intuitively, the total life of the battery $\sum_{n=1}^{\infty} X_n$ should be finite.

On the other hand, it would be more reasonable to assume that repair time is not negligible. And the consecutive repair times may be increasing and tending to infinity. Let Z_n be the repair time after nth failure. Then Z_n will be stochastically increasing and tending to infinity. Therefore, a monotone process model should be a natural model for the battery.

Assume that a new battery is installed at the beginning. As commonly, a failure repair is adopted when the battery fails. However, the cost of replacing is high for the user. With the anticipating high penetration of electric vehicles in the near future, the appropriate operation is realizing applications in many of today's state-of-the-art technologies to help mitigate failure. This is achieved by an operating model and policy.

Batteries for high-performances electric vehicles should depend on high reliability. The reliability includes a long lifetime, high degree of safety, and energy regeneration capabilities. During the lifetime of a battery, its performance or "health" tends to deteriorate gradually due to irreversible physical and chemical changes which take place with usage and age until eventually the battery is no longer usable or dead. The state of health (SOH) is an indication of the point which has been reached in the life cycle of the battery and a measure of its condition relative to a fresh battery. Here, SOH, depth of discharge, and discharge rate are weighted and used to measuring the reliability, which is given by

$$R = \omega_1 \times \mathrm{SOH} + \omega_2 \times D_D + \omega_3 \times D_R, \tag{1}$$

where ω_1, ω_2, and ω_3 are weighted values, and D_D, D_R are depth of discharge rate and discharge rate, respectively.

First of all, the battery has a new class of lifetime distribution. Let F be the distribution of the operating time X of a battery. The preventive repair will be adopted as soon as the reliability fails to R. The battery will be replaced by a new and identical one at the time following Nth failure.

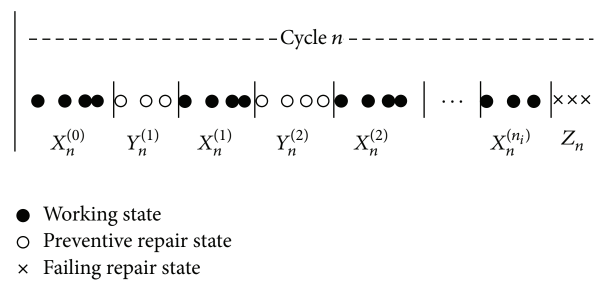

Figure 1: A possible course of the battery in a cycle.

The time interval between the installation of a battery and the first replacement or two successive replacements of the battery is called a cycle. The time interval between the completion of the $(n-1)$th failure repair and the nth failure repair in a cycle is called the nth period of the cycle, and $n = 1, 2, \ldots, N-1$. The battery will be replaced by a new and identical one following the Nth failure. Let $X_n^{(i)}$, $n = 1, 2, \ldots, N-1$, $i = 1, 2, \ldots, M_n$ be the operating time of the battery after the $(i-1)$th preventive repair in the nth period, and $X_n^{(i)}$, $n = 1, 2, \ldots, N-1$, $i = 1, 2, \ldots, M_n$ are independent and identically distributed (i.i.d.) random variables. And let $Y_n^{(i)}$, $n = 1, 2, \ldots$, $i = 1, 2, \ldots, M_n - 1$ be the ith preventive repair time of the battery in the nth period. $Y_n^{(i)}$, $n = 1, 2, \ldots$, $i = 1, 2, \ldots, M_n - 1$ are also i.i.d. random variables. Denote the failure repair time of the battery in the nth period by Z_n. Figure 1 shows a possible course of the battery process in cycle n.

Assumption 1. The preventive repair is adopted as soon as the reliability falls to R and the battery is still working; the battery is repaired as soon as it fails, before the working time of the battery reaches T_i; the failure repair is not as good as new.

The successive operating times $\{X_n^{(1)}, n = 1, 2, \ldots\}$ after preventive repair form a geometric process with radio a and $E(X_1^{(1)}) = \lambda$. On the other hand, the preventive repair times $\{Y_n^{(1)}, n = 1, 2, \ldots\}$ in successive periods form also a geometric process with radio b and $E(Y_1^{(1)}) = \mu$. The consecutive failure repair times in the nth period $\{Z_n, n = 1, 2, \ldots\}$ constitute a geometric process with radio b_1 and $E(Z_1) = \mu_1$.

Assumption 2. The processes $\{X_n^{(i)}, i = 1, 2, \ldots\}$, $\{Y_n^{(i)}, n = 1, 2, \ldots\}$, and $\{Z_n, n = 1, 2, \ldots\}$ are independent.

3. The Long-Run Average Cost per Unit Time

Now, we give the following results as lemmas and theorems.

Lemma 3. *One has*

$$1 - F(T_1) = 1 - F(T_n) = R, \tag{2}$$

where T_i is preventive repair period. $F(T_i)$ is the distribution function. The proof could be obtained by the definitions of probability and distribution function.

Lemma 4. *One has*

$$E\left[X_n\right] = \frac{\lambda}{a^{n-1}}, \qquad E\left[Z_n\right] = \frac{\mu_1}{b_1^{n-1}}. \tag{3}$$

Lemma 5. *One has*

$$R_1\left(T_1\right) = R_2\left(T_2\right) = \cdots = R_{N-1}\left(T_{N-1}\right) = R. \tag{4}$$

Theorem 6. *The number of preventive repairs in the period can be expressed as*

$$n_i = \sum_{i=1}^{\infty} i p_i\left(t\right) = \int_0^{T_i} \lambda\left(\mu\right) d\mu = -\ln R\left(T_i\right) = -\ln R. \tag{5}$$

Theorem 7. *One has*

$$E\left[\sum_{j=1}^{n_n} Y_n^{(j)}\right] = \frac{\mu}{b^{n-1}}\left(-\ln R\right). \tag{6}$$

Proof.

$$\begin{aligned} E\left[\sum_{j=1}^{n_n} Y_n^{(j)}\right] &= E\left[E\left[\sum_{j=1}^{n_n} Y_n^{(j)} \mid n_n\right]\right] \\ &= \sum_{m=0}^{\infty}\sum_{j=1}^{m} E\left[Y_n^{(j)}\right]\cdot P\left\{n_n = m\right\} \\ &= \frac{\mu}{b^{n-1}}\cdot\sum_{m=0}^{\infty} m\cdot P\left\{n_n = m\right\} \\ &= \frac{\mu}{b^{n-1}}\left(-\ln R\right). \end{aligned} \tag{7}$$

To determine the long-run average cost per unit time $C(R,N)$, we first note that as the successive cycles will form a renewal process, the successive cycles together with the costs incurred in each cycle will constitute a renewal process. Suppose a maintenance policy (R,N) is adopted, by applying the standard result in renewal reward process, the average cost $C(R,N)$ is given by

$$C\left(R,N\right) = \frac{\text{Expected cost incurred in a cycle}}{\text{Expected length of a cycle}} \tag{8}$$

$L \equiv$ length of a renewal cycle under policy $C(R,N)$ with preventive repair for the battery,

$$L = \sum_{n=1}^{N} X_n + \sum_{i=1}^{N}\sum_{j=1}^{n_i} Y_n^{(i)} + \sum_{i=1}^{N-1} Z_n. \tag{9}$$

Thus, according to (8) and (9) and the assumption, we have

$$\begin{aligned} &C\left(R,N\right) \\ &= \left(E\left(c_p\sum_{i=1}^{N}\sum_{j=1}^{n_i} Y_{ij} + c_m\sum_{i=1}^{N-1} Z_i + r\left(\sum_{i=1}^{N}\sum_{j=1}^{n_i} Y_{ij} + \sum_{i=1}^{N-1} Z_i\right) + \gamma\right)\right) \\ &\quad\times\left(E\left(\sum_{n=1}^{N} X_n + \sum_{i=1}^{N}\sum_{j=1}^{n_i} Y_{ij} + \sum_{i=1}^{N-1} Z_i\right)\right)^{-1} \\ &= \left(\left(\left(c_p + r\right)\left(-\ln\left(R\right)\right)\sum_{i=1}^{N} E\left(Y_i\right)\right) + \left(c_m + r\right)\sum_{i=1}^{N-1} E\left(Z_i\right)\right) \\ &\quad\times\left(\sum_{n=1}^{N} E\left(X_n\right) + \sum_{i=1}^{N} n_i E\left(Y_i\right) + \sum_{i=1}^{N-1} E\left(Z_i\right)\right)^{-1}, \end{aligned} \tag{10}$$

$$\begin{aligned} C\left(R,N\right) &= \frac{\psi_1 + \psi_2 - \psi_3 + \gamma}{\psi_4 + \psi_5 + \psi_6} \\ \psi_1 &= c_m\sum_{n=1}^{N-1}\frac{\mu_1}{b_1^{\,n-1}}; \\ \psi_2 &= c_p\sum_{n=1}^{N-1}\left(-\frac{\ln\left(R\right)\mu}{b^{n-1}}\right); \\ \psi_3 &= r\sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}; \\ \psi_4 &= \sum_{n=1}^{N-1}\frac{\mu_1}{b_1^{\,n-1}}; \\ \psi_5 &= \sum_{n=1}^{N-1}\left(-\frac{\ln\left(R\right)\mu}{b^{n-1}}\right); \\ \psi_6 &= \sum_{n=1}^{N}\frac{\lambda}{a^{n-1}}, \end{aligned} \tag{11}$$

where the preventive repair cost rate is c_p, the reward rate when the battery is operating is r, and the failure repair cost rate is c_m. The replacement cost is γ. Thus, consider the following.

(a) For a given R, the optimal operating policy N^* is determined by analytically, or numerically, minimizing $C(R,N)$.

(b) If R is unknown, then (11) is a bivariate function about R and N. When N is fixed, then (11) is a function of R. Thus, R^* is found by analytic or numerical methods.

Because the total lifetime of the system is limited; the minimum of average cost rate exists. Obviously, the given R_0 might not be R^*.

□

Consequently, an optimal operating policy (R^*, N^*) could be obtained numerically or analytically by minimizing $C(R, N)$.

The average cost is a function of $a, \lambda, b, \mu, b_1, \mu_1$. However, it is difficult to obtain the above parameters. Here, the parameters can be estimated by minimizing the least-squared error on X_i, Y_i, and Z_i. For example, assume that a data set $\{X_i,\ i = 1, \ldots, n\}$ is consistent with a renew process. Lam derived the following nonparametric estimators [10]. By taking logarithm of $W_i = a^{i-1} X_i$, $i = 1, 2, \ldots, n$ at both sides, it results

$$\ln X_i = \ln W_i - (i-1)\ln a, \quad i = 1, 2, \ldots, n. \tag{12}$$

Since the renewal process of the battery contains i.i.d variables, using the Simple Linear Regression method, W_i can be written in the form

$$\ln W_i = \tilde{\lambda} + \varepsilon_i, \tag{13}$$

where $E[(\ln W_i)] = \tilde{\lambda}$ and $\operatorname{var}(\ln W_i) = \operatorname{var}(\varepsilon_i) = \tau^2$. Combining (12) and (13), we have

$$\ln X_i = \tilde{\lambda} - (i-1)\ln a + \varepsilon_i. \tag{14}$$

The linear regression could be applied to have the estimates of squared errors of $\tilde{\lambda}$, $\beta = \ln a$ and τ^2,

$$\begin{aligned}
\widehat{\beta} &= -\frac{6}{n(n^2-1)}\left\{2\sum_{i=1}^{n} i \ln X_i - (n+1)\sum_{i=1}^{n} \ln X_i\right\} \\
\widehat{\tilde{\lambda}} &= \frac{2}{n(n+1)}\left\{(2n+2)\sum_{i=1}^{n} \ln X_i - 3\sum_{i=1}^{n} i \ln X_i\right\} \\
\widehat{\tau}^2 &= \frac{1}{n-2}\left\{\left(\sum_{i=1}^{n} \ln X_i\right)^2 - \frac{1}{n}\left(\sum_{i=1}^{n} \ln X_i\right)^2\right. \\
&\qquad \left. - \frac{\widehat{\beta}}{2}\sum_{i=1}^{n} (n-2i+1)\ln X_i\right\}.
\end{aligned} \tag{15}$$

The estimates of square error are as follows:

$$\begin{aligned}
\widehat{a} &= \exp\left(\widehat{\beta}\right) \\
\widehat{\lambda} &= \exp\left(\widehat{\tilde{\lambda}} + \frac{\widehat{\tau}^2}{2}\right).
\end{aligned} \tag{16}$$

4. Simulation Results

In this section, we provide a numerical example to illustrate an optimal operating policy for battery using for electric vehicles.

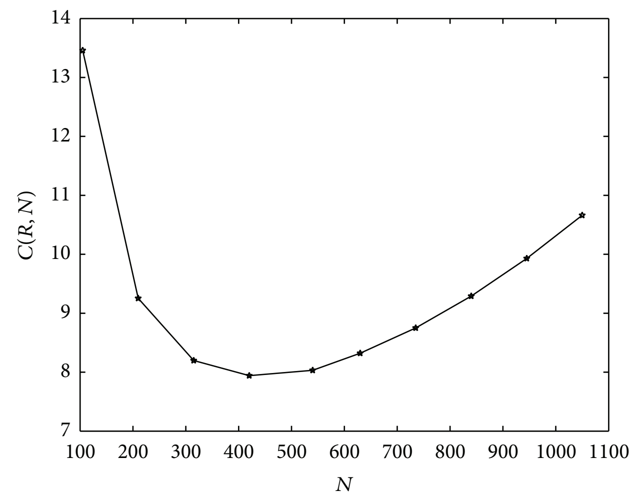

Figure 2: The average cost against N when $R = 0.9$.

Table 1: P values of testing.

H_0: it is a geometric process			
P_T^U	P_D^U	P_T^V	P_D^V
0.9082	0.9207	0.9961	0.9207

First of all, we will test if the operating time of the battery in electric vehicle will agree with a geometric process according to the theorems in [11]. The data set was originally studied by 220Ah Lithium-ion cell.

The P values P_T^U, P_D^U, P_T^V, and P_D^V are all insignificant in Table 1. And hence we conclude that the operating time of the battery could be modeled by a geometric process.

Assume that the distribution of the operating time of the battery is Weibull; that is, the distribution function of X_n is

$$F_n(t) = 1 - e^{-(\lambda a^{n-1} t)^{\alpha}}, \quad t > 0, \tag{17}$$

where α and λ are the parameters of the Weibull distribution.

For the deteriorating characters of the battery, let $a = 1.1$, $b = 0.95$, $b_1 = 0.9$, $\lambda = 200$, $\mu = 6$, $\mu_1 = 5$, $c_p = 30$, $c_m = 10$, $r = 90$, and $\alpha = 10$.

When $R = 0.9$, substituting the above values into (11), we can obtain the results presented in Table 2 and Figure 2. It is easy to find that $C(0.9, 420)$ is the minimum of the average cost rate of the battery. In other words, an optimal operating policy is $N^* = 420$.

In the other words, R is a continuous variable. For plotting the average cost rate, we select 0.1 units as a step of R from 0.55 to 0.95. After numerical calculation, it is easy to find that $C(R^*, N^*) = (0.70, 4)$, such that $C(R, N)$ is minimized at (R^*, N^*); that is, $C(R, N) = 7.5944$ is the minimum of the average of rate of the battery. It is seen from Table 3 and Figure 3 that the optimal operating policy (R^*, N^*) is also unique.

Table 2: Some results obtained from proposed policy when $R = 0.9$.

N	105	210	315	420	540	630	735	840	945	1050
$C(R, N)$	13.4584	9.2533	8.1978	**7.9419**	8.0319	8.3212	8.7498	9.2901	9.9293	10.6614

Table 3: Some results obtained from proposed policy.

N/R	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
105	10.5279	10.6948	10.8786	11.0859	11.3268	11.6183	11.9925	12.5237	13.4584
210	8.1228	8.1392	8.1735	8.2292	8.3123	8.4335	8.6141	8.9038	9.4691
315	7.6764	7.6383	7.6194	7.6201	7.6513	7.7163	7.8352	8.0517	8.5122
420	7.7374	7.6684	7.6201	**7.5944**	7.5957	7.6327	7.7228	7.9080	8.3294
540	8.0274	7.9372	7.8690	7.8247	7.8086	7.8292	7.9041	8.0754	8.4851
630	8.4574	8.3507	8.2676	8.2099	8.1818	8.1922	8.2591	8.4258	8.8389
735	8.9919	8.8715	8.7764	8.7083	8.6717	8.6757	8.7392	8.9074	9.3888
840	9.6151	9.4830	9.3779	9.3016	9.2588	9.2591	9.3225	9.4967	9.9439
945	10.3204	10.1781	10.0645	9.9816	9.9345	9.9333	9.9992	10.1831	10.6573
1050	11.1055	10.9542	10.8334	10.7454	10.6956	10.6948	10.7656	10.9624	11.4690

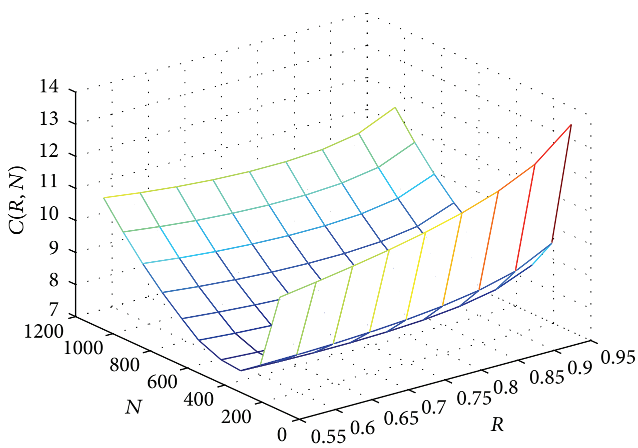

Figure 3: The average cost against (R, N).

5. Conclusions

In this paper, a model expressed battery life cycle depending on reliability is proposed for the ageing effect and the accumulated wearing as well as the environment influence. An operating policy is obtained by which the battery will be repaired when it fails or its reliability reaches a threshold, and the battery will be replaced by a new and identical one following some failures. The proposed operating method for battery could not only extend battery lifetime but also improve the electric vehicle reliability and economical efficiency.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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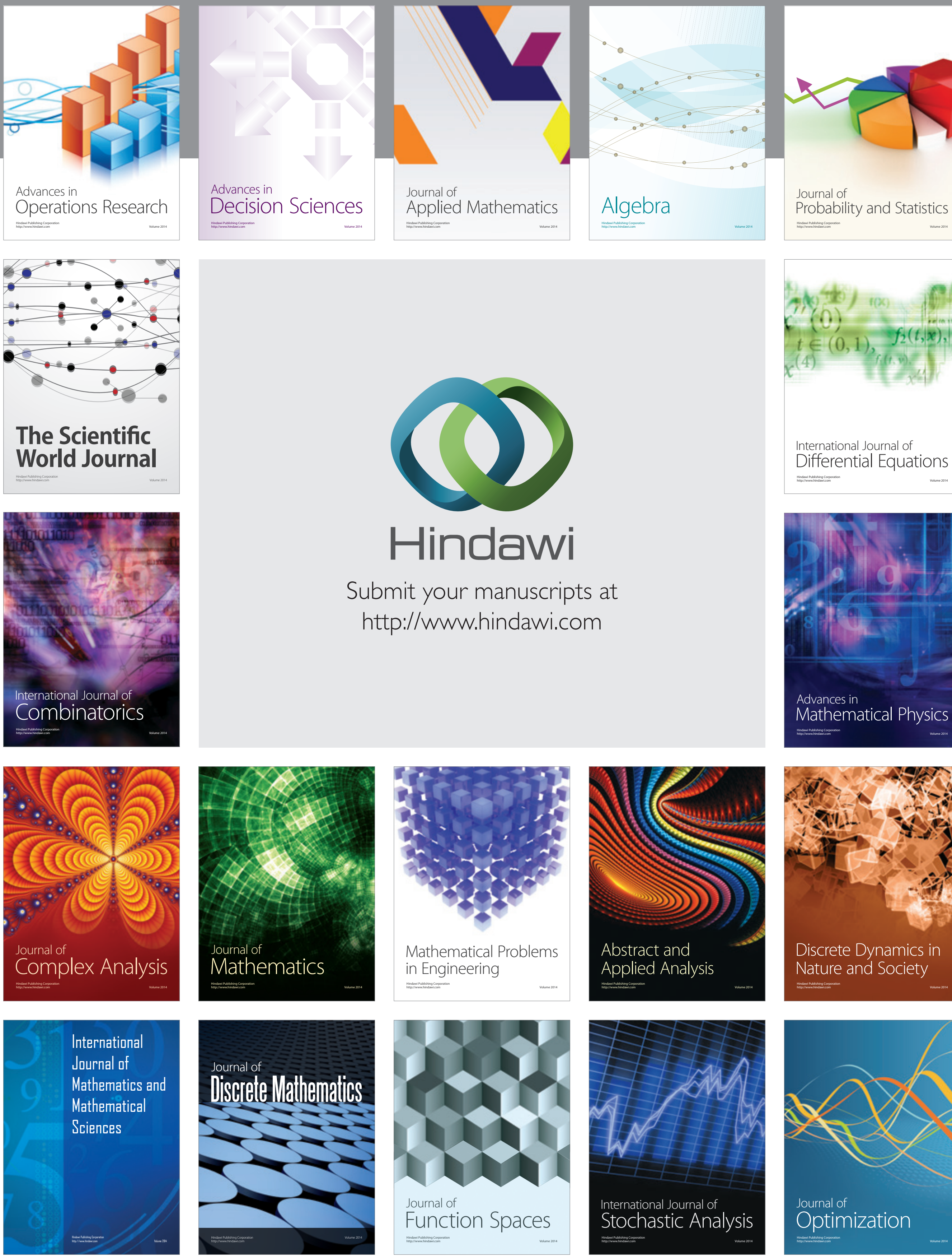
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