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Research Article

Survival Analysis of a Nonautonomous Logistic Model with Stochastic Perturbation

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Taking white noise into account, a stochastic nonautonomous logistic model is proposed and investigated. Sufficient conditions for extinction, nonpersistence in the mean, weak persistence, stochastic permanence, and global asymptotic stability are established. Moreover, the threshold between weak persistence and extinction is obtained. Finally, we introduce some numerical simulink graphics to illustrate our main results.

1. Introduction

The classical nonautonomous logistic equation can be expressed as follows:

$$\frac{dx(t)}{dt} = x(t)[a(t) - b(t)x(t)]. \tag{1.1}$$

for $t \geq 0$ with initial value $x(0) = x_0 > 0, x(t)$ is the population size at time $t, a(t)$ denotes the rate of growth, and $a(t)/b(t)$ stands for the carrying capacity at time t. We refer the reader to May [1] for a detailed model construction. Obviously, system (1.1) has an equilibrium $x^* = a(t)/b(t)$, see the following (A1). Then, system (1.1) becomes the following equation:

$$\frac{dx(t)}{dt} = a(t)x(t)\left[1 - \frac{x(t)}{x^*}\right]. \tag{1.2}$$

Owing to its theoretical and practical significance, the deterministic system (1.1) and its generalization form have been extensively studied and many important results on the global dynamics of solutions have been founded, for example, Freedman and Wu [2], Golpalsamy [3], Kuang [4], Lisena [5], and the references therein. In particular, the books by Golpalsamy [3] and Kuang [4] are good references in this area.

In the real world, population dynamics is inevitably affected by environmental noise which is an important component in an ecosystem (see e.g., [6–9]). The deterministic models assume that parameters in the systems are all deterministic irrespective environmental fluctuations. Hence, they have some limitations in mathematical modeling of ecological systems, besides they are quite difficult to fitting data perfectly and to predict the future dynamics of the system accurately [8]. May [10] pointed out the fact that due to environmental noise, the birth rate, carrying capacity, competition coefficient, and other parameters involved in the system exhibit random fluctuation to a greater or lesser extent.

Recall that the parameters $a(t)$ represent the intrinsic growth rate. In practice, we usually estimate it by an average value plus an error term. In general, by the well-known central limit theorem, the error term follows a normal distribution and is sometimes dependent on how much the the current population sizes differ from the equilibrium state (see, e.g., [11–13]). In other words, we can replace the rate $a(t)$ by an average value plus a random fluctuation term:

$$a(t) \longrightarrow a(t) + \alpha(t)(x(t) - x^*)\dot{B}(t), \tag{1.3}$$

where $\alpha(t)$ are continuous positive bounded function on $\overline{R}_+$, and $\alpha^2(t)$ represents the intensity of the white noise at time t; $\dot{B}(t)$ are the white noise, namely, $B(t)$ is a Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t\in\overline{R}_+}$ satisfying the usual conditions (i.e., it is right continuous and increasing while $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets). Then, by model (1.2), we obtain an Itǒ stochastic differential equation:

$$dx(t) = a(t)x(t)\left[1 - \frac{x(t)}{x^*}\right]dt + \alpha(t)x(t)(x(t) - x^*)dB(t). \tag{1.4}$$

Owing to the model (1.4) describes a population dynamics, it is necessary to investigate the survival of the logistic population which involves extinction, persistence, and global asymptotical stability (see, e.g., [14–16]). As far as we know, there are few results of this aspect for model (1.4). Furthermore, up to the authors' knowledge, all the publications have not obtained the persistence-extinction threshold for model (1.4). The aims of this work are to deal with the above problems one by one, which generalize the work of Wang and Liu (see, e.g., [17, 18]) where they mainly investigated survival analysis of population model with parameters perturbation.

Throughout the paper, we always have some assumption and notations.

(A1) It holds that $a(t)$ and $b(t)$ are continuous bounded function on R with $b(t) > 0$. Moreover, $a(t)/b(t)$ is a constant.

(A2) It holds that

$$f^u = \sup_{t\in R} f(t), \qquad f^l = \inf_{t\in R} f(t), \qquad \langle x(t)\rangle = \frac{1}{t}\int_0^t x(s)ds,$$
$$f_* = \liminf_{t\to+\infty} f(t), \qquad f^* = \limsup_{t\to+\infty} f(t), \qquad R_+ = (0,+\infty), \tag{1.5}$$
$$\overline{R}_+ = [0,+\infty), \qquad \overline{a} = \limsup_{t\to+\infty}\frac{1}{t}\int_0^t a(s)ds.$$

The following definitions are commonly used and we list them here.

Definition 1.1. (1) The population $x(t)$ is said to be extinctive if $\lim_{t\to+\infty} x(t) = 0$ a.s.

(2) The population $x(t)$ is said to be nonpersistent in the mean (see e.g., Huaping and Zhien [14]) if $\limsup_{t\to+\infty}\langle x(t)\rangle = 0$ a.s.

(3) The population $x(t)$ is said to be weakly persistent (see e.g., Hallam and Ma [15]) if $\limsup_{t\to+\infty} x(t) > 0$ a.s.

(4) The population $x(t)$ is said to be permanence (see e.g., Jiang et al. [19]) if for arbitrary $\varepsilon > 0$, there are constants $\beta > 0$, $M > 0$ such that $\liminf_{t\to+\infty} P\{x(t) \geq \beta\} \geq 1-\varepsilon$ and $\liminf_{t\to+\infty} P\{x(t) \leq M\} \geq 1-\varepsilon$.

It follows from the above definitions that stochastic permanence implies stochastic weak persistence, extinction means stochastic nonpersistence in the mean. But generally, the reverses are not true.

The rest of the paper is arranged as follows. In Section 2, sufficient criteria for extinction, nonpersistence in the mean, weak persistence, and stochastic permanence of the population are established. In Section 3, we study global asymptotic stability of positive equilibrium. In Section 4, we work out some figures to illustrate the various theorems obtained in Section 3 and Section 4. The last section gives the conclusions and future directions of the research.

2. Persistence and Extinction

As $x(t)$ in system (1.2) denotes the population size, it should be nonnegative. So, for further study, we must firstly give some conditions under which system (1.2) has a global positive solution. Similar to Mao et al. [20], we have the following Lemma.

Lemma 2.1. *For model* (1.4)*, with any given initial value* $x(0) = x_0 > 0$*, there is a unique solution* $x(t)$ *on* $t \geq 0$ *and the solution will remain in* R_+ *with probability 1.*

Proof. Since the coefficients of (1.4) are locally Lipschitz continuous, for any given initial value $x_0 \in R^+$, there is a unique maximal local solution $x(t)$ on $t \in [-\tau^M, \tau_e)$, where τ_e is the explosion time (cf. Mao [21, page 95]). To show this solution is global, we need to show that $\tau_e = +\infty$ a.s. Let $k_0 > 0$ be sufficiently large for

$$\frac{1}{k_0} < x_0 < k_0. \tag{2.1}$$

For each time integer $k \geq k_0$, define the stopping time:

$$\tau_k = \inf\left\{ t \in [0, \tau_e) : x(t) \leq \frac{1}{k} \text{ or } x(t) \geq k \right\}, \tag{2.2}$$

where throughout this paper we set $\inf\ \emptyset = +\infty$ (as usual $\emptyset$ denotes the empty set). Clearly, τ_k is increasing as $k \rightarrow +\infty$. Set $\tau_{+\infty} = \lim_{k\rightarrow+\infty}\tau_k$, whence $\tau_{+\infty} \leq \tau_e$ a.s. for all $t \geq 0$. In other words, to complete the proof, all we need to show is that $\tau_{+\infty} = +\infty$ a.s. To show this statement, let us define a C^2 function $V : R_+ \rightarrow \overline{R}_+$ by

$$V(x) = \left[\sqrt{x} - 1 - 0.5\ \ln\ x\right]. \tag{2.3}$$

The nonnegativity of this function can be seen from

$$\sqrt{u} - 1 - 0.5\ \ln\ u \geq 0 \quad \text{on } u > 0. \tag{2.4}$$

Let $k \geq k_0$ and $T > 0$ be arbitrary. For $0 \leq t \leq \tau_k \wedge T$, we can apply the Itô formula to $V(x(t))$ to obtain that

$$\begin{aligned}
dV(t) &= 0.5\ \left[x^{-0.5}(t) - x^{-1}(t)\right]x(t)\left[\left(a(t) - \frac{a(t)}{x^*}x(t)\right)dt + \alpha(t)(x(t) - x^*)dB(t)\right] \\
&\quad + 0.5\ \left[-0.25x^{-1.5}(t) + 0.5x^{-2}(t)\right]\alpha^2(t)x^2(t)(x(t) - x^*)^2dt \\
&= \left[-0.125\alpha^2(t)x^{2.5}(t) + 0.25\alpha^2(t)x^2(t) + \left(0.25\alpha^2(t)x^* - \frac{0.5a(t)}{x^*}\right)x^{1.5}(t)\right. \\
&\quad - \left(0.125\alpha^2(t)x^* - 0.5a(t)\right)x^{0.5}(t) + \left(\frac{0.5a(t)}{x^*} - 0.5\alpha^2x^*\right)x(t) \\
&\quad \left.+ 0.25\alpha^2(t)(x^*)^2 - 0.5a(t)\right]dt + 0.5\alpha(t)\left[x^{0.5}(t) - 1\right]\ (x(t) - x^*)dB(t) \\
&= F(x(t))dt + 0.5\alpha(t)\left[x^{0.5}(t) - 1\right](x(t) - x^*)dB(t),
\end{aligned} \tag{2.5}$$

where

$$\begin{aligned}
F(x(t)) = &- 0.125\alpha^2(t)x^{2.5}(t) + 0.25\alpha^2(t)x^2(t) + \left(0.25\alpha^2(t)x^* - \frac{0.5a(t)}{x^*}\right)x^{1.5}(t) \\
&+ \left(\frac{0.5a(t)}{x^*} - 0.5\alpha^2x^*\right)x(t) - \left(0.125\alpha^2(t)x^* - 0.5a(t)\right)x^{0.5}(t) \\
&+ 0.25\alpha^2(t)(x^*)^2 - 0.5a(t).
\end{aligned} \tag{2.6}$$

It is easy to see that $F(x(t))$ is bounded, say by K, in R_+. We, therefore, obtain that

$$dVx(t) \leq Kdt + 0.5\alpha(t)\left[x^{0.5}(t) - 1\right](x(t) - x^*)dB(t). \tag{2.7}$$

Integrating both sides from 0 to $\tau_k \wedge T$, and then taking expectations, yields

$$EV(x(\tau_k \wedge T)) \le V(x(0)) + KT. \tag{2.8}$$

Note that, for every $\omega \in \{\tau_k \le T\}$, $x(\tau_k, \omega)$ equals either k or $1/k$, and hence $V(x(\tau_k, \omega))$ is no less than either

$$\sqrt{k} - 1 - 0.5\log(k), \tag{2.9}$$

or

$$\sqrt{\frac{1}{k}} - 1 - 0.5\log\left(\frac{1}{k}\right) = \sqrt{\frac{1}{k}} - 1 + 0.5\log(k). \tag{2.10}$$

Consequently,

$$V(x(\tau_k, \omega)) \ge \left[\sqrt{k} - 1 - 0.5\log(k)\right] \wedge \left[\sqrt{\frac{1}{k}} - 1 + 0.5\log(k)\right]. \tag{2.11}$$

It then follows from (2.15) that

$$\begin{aligned} V(x(0)) + KT &\ge E\left[1_{\{\tau_k \le T\}} V(x(\tau_k, \omega))\right] \\ &\ge P\{\tau_k \le T\}\left(\left[\sqrt{k} - 1 - 0.5\log(k)\right] \wedge \left[\sqrt{\frac{1}{k}} - 1 + 0.5\log(k)\right]\right), \end{aligned} \tag{2.12}$$

where $1_{\{\tau_k \le T\}}$ is the indicator function of $\{\tau_k\}$. Letting $k \to +\infty$ gives

$$P\{\tau_{+\infty} \le T\} = 0. \tag{2.13}$$

Since $T > 0$ is arbitrary, we must have

$$P\{\tau_{+\infty} < +\infty\} = 0, \tag{2.14}$$

so $P\{\tau_{+\infty} = +\infty\} = 1$ as required. □

From Lemma 2.1, we know that solutions of system (1.4) will remain in the positive cone R_+. This nice positive property provides us with a great opportunity to construct different types of Lyapunov functions to discuss how the solutions vary in R_+ in more details. Now we will study the persistence and extinction of system (1.4).

Theorem 2.2. *If $\overline{a} < 0$, then the population $x(t)$ by* (1.4) *goes to extinction.*

Proof. Applying Itŏ's formula to (1.4) leads to

$$d\ln x(t) = \left[a(t) - \frac{a(t)}{x^*}x(t) - \frac{\alpha^2(t)(x(t)-x^*)^2}{2}\right]dt + \alpha(t)(x(t)-x^*)dB(t)\cdot \tag{2.15}$$

Integrating both sides of (2.15) from 0 to t, we have

$$\ln x(t) - \ln x_0 = \int_0^t \left[a(s) - b(s)x(s) - \frac{\alpha^2(s)(x(s)-x^*)^2}{2}\right]ds + M(t), \tag{2.16}$$

where $M(t) = \int_0^t \alpha(s)(x(s)-x^*)dB(s)$. The quadratic variation of $M(s)$ is $\langle M(t), M(t)\rangle = \int_0^t \alpha^2(s)(x(s)-x^*)^2 ds$. By virtue of the exponential martingale inequality (see, e.g., [22] on page 36), for any positive constants T_0, α, and β, we have

$$P\left\{\sup_{0\le t\le T_0}\left[M(t) - \frac{\alpha}{2}\langle M(t), M(t)\rangle\right] > \beta\right\} \le e^{-\alpha\beta}. \tag{2.17}$$

Choose $T_0 = k, \alpha = 1, \beta = 2\ln k$, then it follows that

$$P\left\{\sup_{0\le t\le k}\left[M(t) - \frac{1}{2}\langle M(t), M(t)\rangle\right] > 2\ln k\right\} \le \frac{1}{k^2}. \tag{2.18}$$

Making use of Borel-Cantelli lemma (see, e.g., [22] on page 10) yields that for almost all $\omega \in \Omega$, there is a random integer $k_0 = k_0(\omega)$ such that for $k \ge k_0$,

$$\sup_{0\le t\le k}\left[M(t) - \frac{1}{2}\langle M(t), M(t)\rangle\right] \le 2\ \ln k. \tag{2.19}$$

This is to say

$$M(t) \le 2\ \ln\ k + \frac{1}{2}\langle M(t), M(t)\rangle = 2\ \ln\ k + \frac{1}{2}\int_0^t \alpha(s)(x(s)-x^*)^2 ds, \tag{2.20}$$

for all $0 \le t \le k, k \ge k_0$ a.s. Substituting this inequality into (2.16), we can obtain that

$$\ln x(t) - \ln x_0 \le \int_0^t \left[a(s) - \frac{a(s)}{x^*}x(s)\right]ds + 2\ \ln k, \tag{2.21}$$

for all $0 \le t \le k, k \ge k_0$ a.s. In other words, we have shown that, for $0 < k-1 \le t \le k, k \ge k_0$,

$$t^{-1}\{\ln x(t) - \ln x_0\} \le t^{-1}\int_0^t \left[a(s) - \frac{a(s)}{x^*}x(s)\right]ds + 2t^{-1}\ \ln k. \tag{2.22}$$

Taking superior limit on both sides yields $\limsup_{t\to+\infty} x(t) \le \overline{a}$. That is to say, if $\overline{a} < 0$, one can see that $\lim_{t\to+\infty} x(t) = 0$. □

Theorem 2.3. *If $\overline{a} = 0$, then the population $x(t)$ by* (1.4) *is nonpersistent in the mean.*

Proof. For all $\varepsilon > 0$, $\exists T_1$ such that $t^{-1}\int_0^t a(s)ds \le \overline{a} + \varepsilon/2 = \varepsilon/2$ for $t > T_1$, substituting this inequality into (2.22) gives

$$\begin{aligned} t^{-1}\{\ln x(t) \;-\ln x_0\} &\le t^{-1}\int_0^t \left[a(s) - \frac{a(s)x(s)}{x^*}\right]ds + 2t^{-1}\;\ln k \\ &\le \frac{\varepsilon}{2} - t^{-1}\int_0^t \frac{a(s)x(s)}{x^*}ds + 2t^{-1}\ln k, \end{aligned} \tag{2.23}$$

for all $T_1 < t \le k, k \ge k_0$ a.s. Note that for sufficiently large t satisfying $T_1 < T < k-1 \le t \le k$ and $k \ge k_0$, we have $\ln k/t \le \varepsilon/4$. In other words, we have already shown that

$$\ln x(t) - \ln x_0 < \varepsilon t - \int_0^t \frac{a(s)x(s)}{x^*}ds; \quad t > T. \tag{2.24}$$

Define $h(t) = \int_0^t x(s)ds$ and $N = \inf_{s\in R}\{a(s)/x^*\}$, then we have

$$\ln\left(\frac{dh}{dt}\right) < \varepsilon t - Nh(t) + \ln\; x_0; \quad t > \mathrm{T}. \tag{2.25}$$

Consequently,

$$e^N\left(\frac{dh}{dt}\right) < x_0 e^{\varepsilon t}; \quad t > T. \tag{2.26}$$

Integrating this inequality from T to t results in

$$N^{-1}\left[e^{Nh(t)} - e^{Nh(T)}\right] < x_0\varepsilon^{-1}\left[e^{\varepsilon t} - e^{\varepsilon T}\right]. \tag{2.27}$$

Rewriting this inequality, one then obtains

$$e^{Nh(t)} < e^{Nh(T)} + x_0 N\varepsilon^{-1}e^{\varepsilon t} - x_0 N\varepsilon^{-1}e^{\varepsilon T}. \tag{2.28}$$

Taking the logarithm of both sides leads to

$$h(t) < N^{-1}\ln\left\{x_0 N\varepsilon^{-1}e^{\varepsilon t} + e^{Nh(T)} - x_0 N\varepsilon^{-1}e^{\varepsilon T}\right\}. \tag{2.29}$$

In other words, we have shown that

$$\left\{t^{-1}\int_0^t x(s)ds\right\} \le \left\{t^{-1}N^{-1}\ln\left\{x_0N\varepsilon^{-1}e^{\varepsilon t}+e^{Nh(T)}-x_0N\varepsilon^{-1}e^{\varepsilon T}\right\}\right\}^*. \quad (2.30)$$

An application of the L'Hospital's rule, one can derive

$$\langle x\rangle^* \le N^{-1}\left\{t^{-1}\ln\left[x_0N\varepsilon^{-1}e^{\varepsilon t}\right]\right\}^* = \frac{\varepsilon}{N}. \quad (2.31)$$

Since ε is arbitrary, we have $\langle x\rangle^* \le 0$, which is the required assertion. □

Theorem 2.4. *If* $\overline{a}-\liminf_{t\to+\infty}\langle (x^*\alpha(t))^2/2\rangle > 0$, *then the population* $x(t)$ *by* (1.4) *is weakly persistent.*

Proof. To begin with, let us show that

$$\limsup_{t\to+\infty}\left[t^{-1}\ln x(t)\right] \le 0 \quad \text{a.s.} \quad (2.32)$$

Applying Itŏ's formula to (1.2) results in

$$\begin{aligned} de^t\ \ln x(t) &= e^t\ln x(t)dt + e^t d\ln x(t) \\ &= e^t\left[\ln x(t) + a(t) - \frac{a(t)}{x^*}x(t) - \frac{\alpha^2(t)(x(t)-x^*)^2}{2}\right]dt + e^t\alpha(t)(x(t)-x^*)dB(t). \end{aligned} \quad (2.33)$$

Thus, we have shown that

$$e^t\ \ln\ x(t) - \ln\ x_0 = \int_0^t e^s\left[\ln\ x(s) + a(s) - \frac{a(s)}{x^*}x(s) - \frac{\alpha^2(s)(x(s)-x^*)^2}{2}\right]ds + N(t), \quad (2.34)$$

where

$$N(t) = \int_0^t e^s\alpha(s)(x(t)-x^*)dB(s). \quad (2.35)$$

Note that $N(t)$ is a local martingale with the quadratic form:

$$\langle N(t), N(t)\rangle = \int_0^t e^{2s}\alpha^2(s)(x(t)-x^*)^2ds. \quad (2.36)$$

It then follows from the exponential martingale inequality (2.17) by choosing $T_0 = \mu k, \alpha = e^{-\mu k}, \beta = \rho e^{\mu k} \ln k$ that

$$P\left[\sup_{0\leq t\leq \mu k}\left[N(t) - 0.5e^{-\mu k}\langle N(t), N(t)\rangle\right] > \rho e^{\mu k} \ \ln k\right] \leq k^{-\rho}, \tag{2.37}$$

where $\rho > 1$ and $\mu > 1$. In view of Borel-Cantelli lemma, for almost all $\omega \in \Omega$, there exists a $k_0(\omega)$ such that, for every $k \geq k_0(\omega)$,

$$N(t) \leq 0.5e^{-\mu k}\langle N(t), N(t)\rangle + \rho e^{\mu k} \ \ln k, \quad 0 \leq t \leq \mu k. \tag{2.38}$$

Substituting the above inequalities into (2.34) yields

$$\begin{aligned} &e^t \ \ln x(t) - \ln \ x_0 \\ &\leq \int_0^t e^s\left[\ln x(s) + a(s) - \frac{a(s)}{x^*}x(s) - \frac{\alpha^2(s)(x(s)-x^*)^2}{2}\right]ds \\ &\quad + \frac{e^{-\mu k}}{2}\int_0^t e^{2s}\alpha^2(s)(x(s)-x^*)^2 ds + \rho e^{\mu k}\ln k \\ &= \int_0^t e^s\left[\ln \ x(s) + a(s) - \frac{a(s)}{x^*}x(s) - \frac{\alpha^2(s)(x(s)-x^*)^2\left(1-e^{s-\mu k}\right)}{2}\right]ds + \rho e^{\mu k} \ \ln k. \end{aligned} \tag{2.39}$$

It is easy to see that there exists a constant C independent of k such that

$$\ln \ x(s) + a(s) - \frac{a(s)}{x^*}x(s) - \frac{\alpha^2(s)(x(s)-x^*)^2\left(1-e^{s-\mu k}\right)}{2} \leq C \tag{2.40}$$

for any $0 \leq s \leq \mu k$ and $x > 0$. In other words, we have

$$e^t \ \ln x(t) - \ln x_0 \leq C\left(e^t - 1\right) + \rho e^{\mu k} \ \ln k \tag{2.41}$$

for any $0 \leq t \leq \mu k$.

This is to say

$$\ln x(t) \leq e^{-t} \ \ln \ x_0 + C\left(1-e^{-t}\right) + \rho e^{-t+\mu k} \ \ln k. \tag{2.42}$$

Consequently, if $\mu(k-1) < t \leq \mu k$ and $k \geq k_0(\omega)$, one can observe that

$$t^{-1} \ \ln x(t) \leq t^{-1}e^{-t} \ \ln \ x_0 + t^{-1}C\left(1-e^{-t}\right) + t^{-1}\rho e^{-\mu(k-1)+\mu k} \ \ln k, \tag{2.43}$$

which becomes the desired assertion (2.32) by letting $k \to +\infty$.

Now suppose that $\overline{a} > 0$, we will prove $\limsup_{t\to+\infty} x(t) > 0$ a.s. If this assertion is not true, let $F = \{\limsup_{t\to+\infty} x(t) = 0\}$ and suppose $P(F) > 0$. In view of (2.16),

$$\frac{\ln x(t)}{t} = \frac{(\ln x_0)}{t} + t^{-1}\int_0^t \left[a(s) - \frac{a(s)}{x^*}x(s) - \frac{\alpha^2(s)(x(s)-x^*)^2}{2}\right]ds, + \frac{M(t)}{t}. \tag{2.44}$$

On the other hand, for all $\omega \in F$, we have $\lim_{t\to+\infty} x(t,\omega) = 0$. Then, the law of large numbers for local martingales (see, e.g., [22, page 12]) indicates that $\lim_{t\to+\infty}(M(t)/t) = 0$. Substituting this equality into (2.44) results in

$$\limsup_{t\to+\infty}\left[t^{-1}\ \ln\ x(t,\omega)\right] = \overline{a} - \liminf_{t\to+\infty}\left\langle \frac{(x^*\alpha(t))^2}{2}\right\rangle > 0. \tag{2.45}$$

Then, $P(\limsup_{t\to+\infty}[t^{-1}\ln x(t)] > 0) > 0$, which contradicts (2.32). □

Remark 2.5. Theorems 2.2–2.4 have an obvious and interesting biological interpretation. It is easy to see that the extinction and persistence of population $x(t)$ modeled by (1.4) depend on $\overline{a}$ and $\overline{a} - \liminf_{t\to+\infty}\langle (x^*\alpha(t))^2/2\rangle$. If $\overline{a} - \liminf_{t\to+\infty}\langle (x^*\alpha(t))^2/2\rangle > 0$, the population $x(t)$ will be weakly persistent; If $\overline{a} < 0$, the population $x(t)$ will go to extinction. That is to say, if $\liminf_{t\to+\infty}\langle (x^*\alpha(t))^2/2\rangle = 0$, then $\overline{a}$ is the threshold between weak persistence and extinction for the population $x(t)$.

Remark 2.6. From the condition $\overline{a} - \liminf_{t\to+\infty}\langle (x^*\alpha(t))^2/2\rangle > 0$ in Theorems 2.4, we know that the white noise $\alpha(t)$ is of disadvantage to the survival of the population.

On the other hand, it is well known that in the study of population systems, stochastic permanence, which means that the population will survive forever, is one of the most important and interesting topic owing to its theoretical and practical significance. So now let us show that population $x(t)$ modeled by (1.4) is stochastically permanent in some cases.

Theorem 2.7. *If* $\liminf_{t\to+\infty}\{a(t) - (\alpha^u)^2(x^*)^2/2\} > 0$, *then the population* $x(t)$ *by* (1.4) *will be stochastic permanent.*

Proof. First, we prove that for arbitrary $\varepsilon > 0$, there is a constant $M > 0$ such that $\liminf_{t\to+\infty} P\{x(t) \le M\} \ge 1 - \varepsilon$. Define $V(x) = x^p$ for $x \in R_+$, where $0 < p < 1$. Then, it follows from Itô formula that

$$\begin{aligned} dV(x) &= px^{p-1}dx + \frac{p(p-1)}{2}x^{p-2}(dx)^2 \\ &= px^{p-1}\left[a(t)x\left(1-\frac{x}{x^*}\right)dt + \alpha(t)x(x-x^*)dB(t)\right] \\ &\quad + \frac{p(p-1)}{2}x^{p-2}\alpha^2(t)x^2(x-x^*)^2 \\ &= px^{p-1}\left[a(t)x\left(1-\frac{x}{x^*}\right)dt + \frac{p(p-1)}{2}\alpha^2(t)x(x-x^*)^2\right] \\ &\quad + \alpha(t)x^p(t)(x-x^*)dB(t). \end{aligned} \tag{2.46}$$

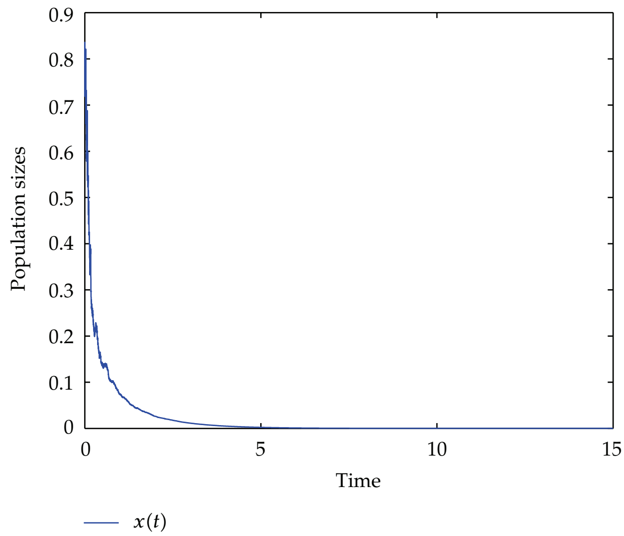

Figure 1: Stochastic extinction. The horizontal axis and the vertical axis in this and following figures represent the time t and the populations size $x(t)$, step size $\Delta t = 0.001$ and $x(0) = 0.8$.

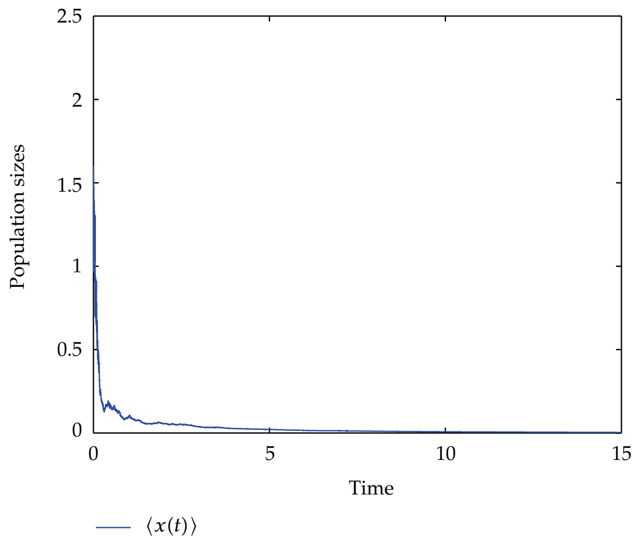

Figure 2: Stochastically nonpersistent in the mean, step size $\Delta t = 0.001$ and $x(0) = 1.6$.

Let $k_0 > 0$ be so large that x_0 lying within the interval $[1/k_0, k_0]$. For each integer $k \geq k_0$, define the stopping time $\tau_k = \inf\{t \geq 0 : x(t) \notin (1/k, k)\}$. Clearly, $\tau_k \longrightarrow +\infty$ almost surely as $k \longrightarrow +\infty$. Applying Itô formula again to $e^t V(x)$ gives

$$\begin{aligned} d\left(e^t V(x)\right) &= e^t V(x)dt + e^t dV(x) \\ &= e^t x^p \left[1 + p(a(t) - b(t)x) - \frac{p(1-p)\alpha^2(t)}{2}(x - x^*)^2\right]dt \\ &\quad + e^t \alpha(t) x^p(t)(x - x^*)dB(t) \\ &= e^t x^p \left[1 + pa(t) - \frac{p(1-p)\alpha^2(t)}{2}(x^*)^2 - pb(t)x + p(1-p)\alpha^2(t)xx^* \right. \\ &\quad \left. -\frac{p(1-p)\alpha^2(t)}{2}x^2\right]dt + e^t\alpha(t)x^p(t)(x - x^*)dB(t) \\ &\leq e^t K_1 dt + e^t \alpha(t) x^p(t)(x - x^*)dB(t), \end{aligned} \tag{2.47}$$

where K_1 is a positive constant. Integrating this inequality and then taking expectations on both sides, one can see that

$$E\left[e^{t\wedge\tau_k} x^p(t \wedge \tau_k)\right] - x_0^p \leq \int_0^{t\wedge\tau_k} e^s K_1 ds \leq K_1\left(e^t - 1\right). \tag{2.48}$$

Letting $k \longrightarrow +\infty$ yields

$$E[x^p(t)] \leq K_1 + e^{-t} x_0^p. \tag{2.49}$$

In other words, we have already shown that

$$\limsup_{t\to+\infty} Ex^p \leq K_1. \tag{2.50}$$

Thus, for any given $\varepsilon > 0$, let $M = K_1^{1/p}/\varepsilon^{1/p}$, by virtue of Chebyshevs inequality, we can derive that

$$P\{x(t) > M\} = P\{x^p(t) > M^p\} \leq \frac{E[x^p(t)]}{M^p}. \tag{2.51}$$

That is to say $P^*\{x(t) > M\} \leq \varepsilon$. Consequently, $P_*\{x(t) \leq M\} \geq 1 - \varepsilon$.

Next, we claim that, for arbitrary $\varepsilon > 0$, there is constant $\beta > 0$ such that $\liminf_{t\to+\infty} P\{x(t) \geq \beta\} \geq 1 - \varepsilon$.

Define $V_1(x) = 1/x^2$ for $x \in R_+$. Applying Itô formula to (1.2), we can obtain that

$$dV_1(x(t)) = -2x^{-3}dx + 3x^{-4}(dx)^2$$
$$= 2V_1(x)\left[1.5\alpha^2(t)(x-x^*)^2 + \frac{a(t)}{x^*}x - a(t)\right]dt - \frac{2\alpha(t)(x-x^*)}{x^2}dB(t). \tag{2.52}$$

Since $\liminf_{t\to+\infty}\{a(t)-(\alpha^u)^2(x^*)^2/2\} > 0$, we can choose a sufficient small constant $0 < \theta < 1$ and $\varepsilon > 0$ such that $a_* - (\alpha^u)^2(x^*)^2/2 - \theta(\alpha^u)^2(x^*)^2 - \varepsilon > 0$.

Define

$$V_2(x) = (1 + V_1(x))^{\theta}. \tag{2.53}$$

Making use of Itŏ's formula again leads to

$$dV_2 = \theta(1+V_1(x))^{\theta-1}dV_1 + 0.5\theta(\theta-1)(1+V_1(x))^{\theta-2}(dV_1)^2$$
$$= \theta(1+V_1(x))^{\theta-2}\left\{(1+V_1(x))2V_1(x)\left[1.5\alpha^2(t)(x-x^*)^2 + \frac{a(t)}{x^*}x - a(t)\right]\right.$$
$$\left. + 2\alpha^2(t)(\theta-1)\frac{(x-x^*)^2}{x^4}\right\}dt - \theta(1+V_1(x))^{\theta-1}\frac{2\alpha(t)(x-x^*)}{x^2}dB(t)$$
$$= \theta(1+V_1(x))^{\theta-2}\left\{(1+V_1(x))2V_1(x)\left[1.5\alpha^2(t)\left(x^2 - 2xx^* + (x^*)^2\right) + \frac{a(t)}{x^*}x - a(t)\right]\right.$$
$$\left. + 2\alpha^2(t)(\theta-1)\left(x^{-2} - 2x^{-3}x^* + x^{-4}(x^*)^2\right)\right\}dt$$
$$- \theta(1+V_1(x))^{\theta-1}\frac{2\alpha(t)(x-x^*)}{x^2}dB(t)$$
$$= \theta(1+V_1(x))^{\theta-2}\left\{-2\left(a(t) - (\theta+0.5)\alpha^2(t)(x^*)^2\right)V_1^2(x)\right.$$
$$+ \left(\frac{2a(t)}{x^*} - 2\alpha^2(t)x^* - 4\theta\alpha^2(t)x^*\right)V_1^{1.5}(x)$$
$$+ \left(-2a(t) + 3\alpha^2(t)(x^*)^2 + 2\alpha^2(t)(\theta+0.5)\right)V_1(x)$$
$$\left. + \left(\frac{2a(t)}{x^*} - 6\alpha^2(t)(x^*)^2\right)V_1^{0.5}(x) + 3\alpha^2(t)\right\}$$

$$
\begin{aligned}
&- \theta(1+V_1(x))^{\theta-1}\frac{2\alpha(t)(x-x^*)}{x^2}dB(t)\\
&\le \theta(1+V_1(x))^{\theta-2}\Bigg\{-2\left(a_* - \frac{(\alpha^u)^2(x^*)^2}{2} - \theta(\alpha^u)^2(x^*)^2 - \varepsilon\right)V_1^2(x)\\
&\qquad + \left(\frac{2a(t)}{x^*} - 2\alpha^2(t)x^* - 4\theta\alpha^2(t)x^*\right)V_1^{1.5}(x)\\
&\qquad + \left(-2a(t) + 3\alpha^2(t)(x^*)^2 + 2\alpha^2(t)(\theta+0.5)\right)V_1(x)\\
&\qquad + \left(\frac{2a(t)}{x^*} - 6\alpha^2(t)(x^*)^2\right)V_1^{0.5}(x) + 3\alpha^2(t)\Bigg\}\\
&- \theta(1+V_1(x))^{\theta-1}\frac{2\alpha(t)(x-x^*)}{x^2}dB(t)
\end{aligned}
\tag{2.54}
$$

for sufficiently large $t \ge T$. Now, let $\eta > 0$ be sufficiently small satisfying

$$
0 < \frac{\eta}{2\theta} < a_* - \frac{(\alpha^u)^2(x^*)^2}{2} - \theta(\alpha^u)^2(x^*)^2 - \varepsilon. \tag{2.55}
$$

Define $V_3(x) = e^{\eta t}V_2(x)$. By virtue of Itǒ's formula,

$$
\begin{aligned}
dV_3(x(t)) &= \eta e^{\eta t}V_2(x) + e^{\eta t}dV_2(x)\\
&\le \theta e^{\eta t}(1+V_1(x))^{\theta-2}\Bigg\{\eta\frac{(1+V_1(x))^2}{\theta} - 2\left(a_* - \frac{(\alpha^u)^2(x^*)^2}{2} - \theta(\alpha^u)^2(x^*)^2 - \varepsilon\right)V_1^2(x)\\
&\qquad + \left(2b(t) - 2\alpha^2(t)x^* - 4\theta\alpha^2(t)x^*\right)V_1^{1.5}(x)\\
&\qquad + \left(-2a(t) + 3\alpha^2(t)(x^*)^2 + 2\alpha^2(t)(\theta+0.5)\right)V_1(x)\\
&\qquad + \left(2b(t) - 6\alpha^2(t)(x^*)^2\right)V_1^{0.5}(x) + 3\alpha^2(t)\Bigg\}dt\\
&\quad - \theta e^{\eta t}(1+V_1(x))^{\theta-1}\frac{2\alpha(t)(x-x^*)}{x^2}dB(t)
\end{aligned}
$$

$$
\begin{aligned}
&\le \theta e^{\eta t}(1+V_1(x))^{\theta-2}\Bigg\{-2\left(a_* - \frac{(\alpha^u)^2(x^*)^2}{2} - \theta(\alpha^u)^2(x^*)^2 - \varepsilon - \frac{\eta}{2\theta}\right)V_1^2(x) \\
&\qquad + \left(2b(t) - 2\alpha^2(t)x^* - 4\theta\alpha^2(t)x^*\right)V_1^{1.5}(x) \\
&\qquad + \left(-2a(t) + 3\alpha^2(t)(x^*)^2 + 2\alpha^2(t)(\theta+0.5) + \frac{2\eta}{\theta}\right)V_1(x) \\
&\qquad + \left(2b(t) - 6\alpha^2(t)(x^*)^2\right)V_1^{0.5}(x) + 3\alpha^2(t) + \frac{\eta}{\theta}\Bigg\}dt \\
&\quad - \theta e^{\eta t}(1+V_1(x))^{\theta-1}\frac{2\alpha(t)(x-x^*)}{x^2}dB(t) \\
&= e^{\eta t}H(x)dt - 2\theta e^{\eta t}(1+V_1(x))^{\theta-1}\frac{\alpha(t)(x-x^*)}{x^2}dB(t)
\end{aligned}
\tag{2.56}
$$

for $t \ge T$. Note that $H(x)$ is upper bounded in R_+, namely, $H = \sup_{x\in R_+} H(x) < +\infty$. Consequently,

$$
dV_3(x(t)) = He^{\eta t}dt - 2\theta e^{\eta t}(1+V_1(x))^{\theta-1}\frac{\alpha(t)(x-x^*)}{x^2}dB(t) \tag{2.57}
$$

for sufficiently large t. Integrating both sides of the above inequality and then taking expectations give

$$
E[V_3(x(t))] = E\left[e^{\eta t}(1+V_1(x(t)))^{\theta}\right] \le e^{\eta T}(1+V_1(x(T)))^{\theta} + \frac{H}{\eta}\left(e^{\eta t} - e^{\eta T}\right). \tag{2.58}
$$

That is to say

$$
\limsup_{t\to+\infty} E\left[V_1^{\theta}(x(t))\right] \le \limsup_{t\to+\infty} E\left[(1+V_1(x(t)))^{\theta}\right] < \frac{H}{\eta}. \tag{2.59}
$$

In other words, we have already shown that

$$
\limsup_{t\to+\infty} E\left[\frac{1}{x^{2\theta}(t)}\right] \le \frac{H}{\eta} = M_4. \tag{2.60}
$$

So, for any $\varepsilon > 0$, set $\beta = \varepsilon^{1/2\theta}/M_4^{1/2\theta}$, by Chebyshev's inequality, one can derive that

$$
P\{x(t) < \beta\} = P\left\{\frac{1}{x^{2\theta}(t)} > \frac{1}{\beta^{2\theta}}\right\} \le \frac{E\left[1/x^{2\theta}(t)\right]}{1/\beta^{2\theta}}, \tag{2.61}
$$

this is to say that

$$\limsup_{t\to+\infty}\{x(t)<\beta\}\le\beta^{2\theta}M_4=\varepsilon. \tag{2.62}$$

Consequently,

$$\liminf_{t\to+\infty}\{x(t)\ge\beta\}\ge 1-\varepsilon. \tag{2.63}$$

This completes the whole proof. □

Remark 2.8. It is easy to see that, if $x^*=0$, our Theorems 2.2–2.7 will become Theorem 2–5 in [18], respectively. At the same time, Theorem 2.7 improves and generalizes the work of Liu and Wang [17] and Jiang et al. [19] in some cases.

Remark 2.9. Generally, from the biological viewpoint, Theorem 2.2 means the population will go to extinction which is the worst case. Theorem 2.3 indicates the population is rare. Theorem 2.4 means the species will be survival, but it admits the case that $\liminf_{t\to+\infty}x(t)=0$, which implies that the population size is closed to zero even if the time is sufficiently large. That is to say the survival of species could be dangerous in reality. Theorem 2.7 is more desired than Theorems 2.2–2.4. Theorem 2.7 means that the population size will be neither too small nor too large with large probability if the time is sufficiently large. That is to say, with large probability, the population will stably exist, which is the most desired case. In other words, that is the reasons why $\liminf_{t\to+\infty}\{a(t)-\alpha^2(t)(x^*)^2/2\}>0$ is used by Theorem 2.7 whereas $\overline{a}-\liminf_{t\to+\infty}\langle(x^*(\alpha^u)^2/2\rangle>0$ and $\overline{a}$ are used in Theorem 2.2–2.3, respectively.

3. Global Stability

In this section, we suppose that the equilibrium $x^*=a(t)/b(t)$ is a positive constant. When studying biologic dynamical system, one important topic is when the population will survive forever. Since model (1.4) is the perturbation system of model (1.2) which has a positive equilibrium x_2^*, it seems reasonable to consider that the population will have chance to survive forever if the solution of model (1.4) is going around x^* at the most time. We get following results.

Theorem 3.1. *If* $\inf_{t\in R}\{a(t)-\alpha^2(t)(x^*)^2/2\}>0$*, then* x^* *in* (1.4) *is global asymptotical stability almost surely (a.s.), that is,* $\lim_{t\to+\infty}x(t)=x^*$ *a.s., almost surely.*

Proof. From the stability theory of stochastic functional differential equations, we only need to find a Lyapunov function $V(x,t)$ satisfying $LV(x,t)\le 0$, and the identity holds if and only if $z=z^*$(see, e.g., [21, 23]), where $z=z(t)$ is the solution of the one-dimensional stochastic functional differential equation:

$$dz(t)=f(z(t),t)dt+g(z(t),t)dB(t),\quad t\ge 0. \tag{3.1}$$

Here, let $f : R \times \overline{R}_+ \to R$ and $g : R \times \overline{R}_+ \to R$. $B(t)$ be a one-dimensional Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. z^* is the positive equilibrium position of (3.1) and

$$LV(t) = V_t + V_z(z) f + 0.5 \text{ trace } \left[g^T V_{zz}(z) g\right]. \tag{3.2}$$

For $t \in \overline{R}_+$, define Lyapunov functions:

$$V(t) = x(t) - x^* - x^* \ln\left(\frac{x(t)}{x^*}\right). \tag{3.3}$$

Applying Itŏ's formula leads to

$$\begin{aligned} LV(t) &= (x(t) - x^*)\left[a(t) - \frac{a(t)}{x^*}x(t)\right]dt + \frac{\alpha^2(t)x^*(x(t) - x^*)^2}{2} \\ &= (x(t) - x^*)\left[-\frac{a(t)}{x^*}(x(t) - x^*)\right] + \frac{\alpha^2(t)x^*(x(t) - x^*)^2}{2}dt \\ &= \left[-\frac{a(t)}{x^*} + \frac{\alpha^2(t)x^*}{2}\right](x(t) - x^*)^2 \\ &\le -\inf_{t\in R}\left\{a(t) - \frac{\alpha^2(t)(x^*)^2}{2}\right\}\frac{(x(t) - x^*)^2}{x^*}. \end{aligned} \tag{3.4}$$

The assumption of $\inf_{t\in R}\{a(t) - \alpha^2(t)(x^*)^2/2\} > 0$ implies that $LV(x,t) < 0$ along all trajectories in $\overline{R}_+$ except x^*. Then, the desired assertion follows immediately.

Now, let us return back to system (1.2). □

Corollary 3.2. *If* $\inf_{t\in R} a(t) > 0$*, then* x^* *in* (1.2) *is global asymptotic stability.*

Remark 3.3. By comparing Theorem 3.1 with Corollary 3.2, we can find that if the positive equilibrium of the deterministic model is global asymptotic stability, then the stochastic system will keep this nice property provided the noise is not very large.

4. Examples and Numerical Simulations

In order to conform to the results above, we numerically simulate the solution of system (1.4). By the Milstein scheme mentioned in [24], we consider the discretized equation:

$$\begin{aligned} x_{k+1} &= x_k + x_k[a(k\Delta t) - b(k\Delta t)x_k]\Delta t + \alpha(k\Delta t)x_k(x_k - x_2^*)\sqrt{\Delta t}\xi_k \\ &\quad + 0.5\alpha^2(k\Delta t)(x_k - x_2^*)^k\left(\xi_k^2 - 1\right)\Delta t, \end{aligned} \tag{4.1}$$

where ξ_k are Gaussian random variable that follows $N(0,1)$.

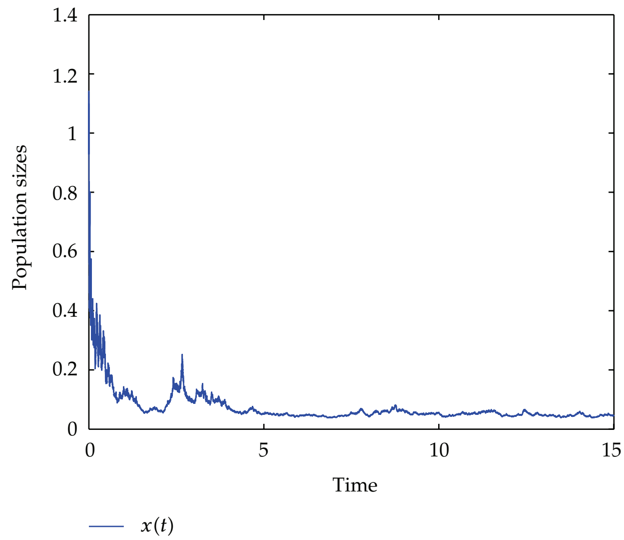

Figure 3: Stochastically weakly persistent, step size $\Delta t = 0.001$ and $x(0) = 1.1$.

Let $a(t) = -0.03, \alpha(t) = 0.1 + 0.01\cos t$, and $x^* = -0.04$. Then, the conditions of Theorem 2.2 are satisfied, which means that the population $x(t)$ by (1.4) will be extinction (see Figure 1).

Let $a(t) = 0, \alpha(t) = 0.4 + 0.2\cos t$, and $x^* = 0$. Then, the conditions of Theorem 2.3 hold, which implies that the population $x(t)$ by (1.4) will be nonpersistent in the mean (see Figure 2).

Let $a(t) = 0.15, \alpha(t) = 0.03 + 0.01\sin t$, and $x^* = 0.2$. Then, the conditions of Theorem 2.4 are satisfied. One can see that $x(t)$ by (1.4) will be weakly persistent (see Figure 3).

Let $a(t) = 0.96, \alpha(t) = 0.05 + 0.01\cos t$, and $x^* = 1.3$. Then, the conditions of Theorem 2.7 are satisfied. That is to say, the population $x(t)$ by (1.4) will be stochastic permanent (see Figure 4).

Let $a(t) = 1.2$ and $x^* = 1$. In Figure 5, we consider $\alpha(t) = 0.2$. Then the corresponding conditions of Theorem 3.1 are satisfied, which means that the positive equilibrium $x^* = 1$ in (1.4) is global asymptotic stability almost surely. In Figure 6, the parameters are same as in Figure 5 except $\alpha(t) = 0$. Then the conditions of Corollary 3.2 hold, which shows that the positive equilibrium $x^* = 1$ of (1.2) is global asymptotic stability. By comparing Figure 5 with Figure 6, one can see that if the positive equilibrium of the deterministic model is asymptotically stable, then the stochastic system will keep this nice property provided the noise is sufficiently small.

5. Conclusions and Future Directions

In the real world, the natural growth of population is inevitably affected by random disturbances. In this paper, we are concerned with the effects of white noise on the survival analysis of logistic model. Firstly, we show that the system has a unique positive global solution. Afterward, sufficient criteria for extinction, nonpersistence in the mean, weak persistence, stochastic permanence, and global asymptotic stability of a positive equilibrium are established. Further, the threshold between weak persistence and extinction is obtained.

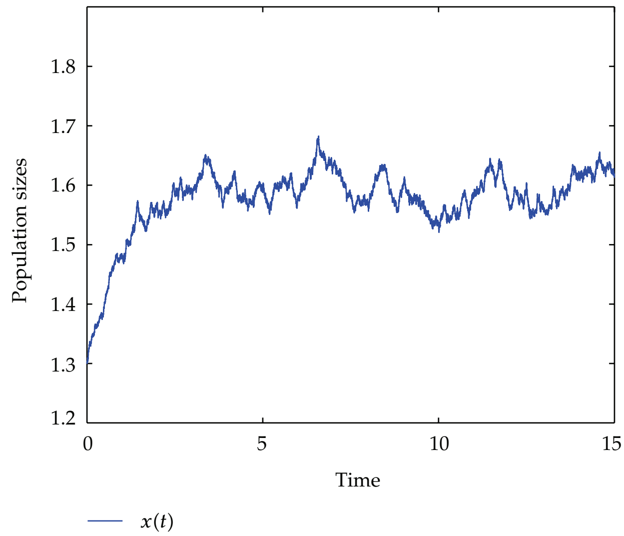

Figure 4: Seen from the two-dimensional spaces, the population $x(t)$ will be stochastic permanent, step size $\Delta t = 0.001$ and $x(0) = 1.3$.

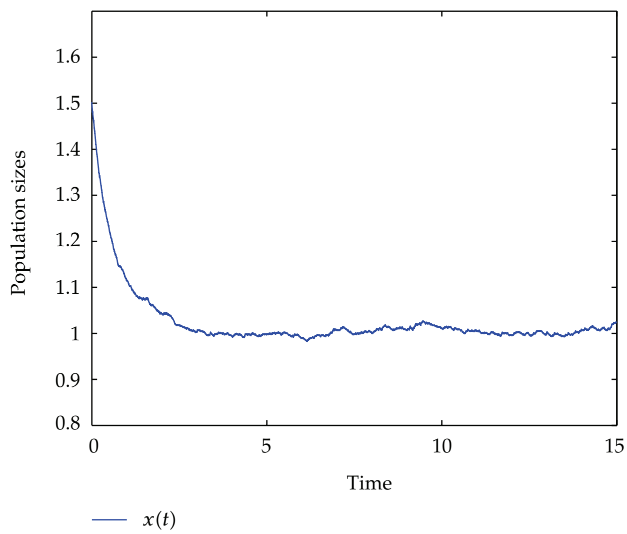

Figure 5: Global asymptotical stability of model (1.2), step size $\Delta t = 0.001$ and $x(0) = 1.5$.

Some interesting topics deserve further investigation. One may propose some realistic but complex models. An example is to incorporate the colored noise, such as continuous-time Markov chain, into the system. The motivation is that the population may suffer sudden environmental changes, for example, rain falls and changes in nutrition or food resources, and so forth. Frequently, the switching among different environments is memoryless and the waiting time for the next switch is exponentially distributed, then the sudden environmental changes can be modeled by a continuous-time Markov chain (see, e.g., [25–27]), and these investigations are in progress.

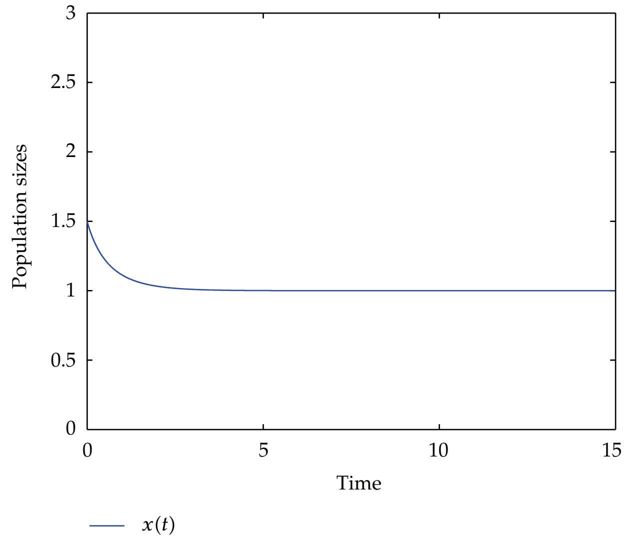

Figure 6: Global asymptotical stability of model (1.1), step size $\Delta t = 0.001$ and $x(0) = 1.5$.

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