

## Research Article

# Transient Stability Improvement of a Power System with Parametric Uncertainties Using a Robust Optimal $H_2$ State Feedback Controller

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In recent years, improvement of dynamic behavior of power systems has interested many researchers and to achieve it, various control methods are proposed. In this paper, in order to improve transient stability of power system, a robust optimal  $H_2$  state feedback is employed. In order to appropriate formulation of the problem, linear matrix inequality (LMI) theory is used. To achieve the best answer, controller parameters are tuned using particle swarm algorithm. The obtained results of the proposed method are compared to conventional power system stabilizer.

## 1. Introduction

With the development of power grids, low-frequency oscillations appeared in power system. Small and sudden disturbances cause such oscillations. In more cases, these oscillations are damped rapidly and the amplitude of the oscillation is below a definite value, but, depending on the operating point conditions and system parameters values, these oscillations may become continuing for a long time and, in the worst case, their amplitudes are increased. The transient stability of the power system is an important factor in development of power grids. In [1], a robust controller is proposed for SVC control to improve the damping of synchronous machine oscillations. The obtained results in this work are compared to the ones from a conventional power system stabilizer (PSS). In [2], the effect of injected reactive power of STATCOM on grid voltage and the damping of synchronous machine oscillations are investigated. In [3], using fuzzy logic laws, a controller is designed for STATCOM and the improvement of power system transient stability is studied. In [4], to improve the transient stability, a UPFC is employed and two control methods are proposed. In this work, the effect of UPFC

capacitance value on transient stability is investigated. There are various PSS structures, but conventional PSS is still interesting because of its simple structure and good flexibility and feasibility. However, the performance of conventional PSS is sensitive to operating point of the system which is changed by load variation; thus the conventional PSS may be failed or may lose its capability [5]. Most of the controllers proposed for this purpose need a perfect model of power system with good precision. It is worthwhile to note that the power system is a nonlinear coupled system. Most of the models used in controller design are a linear approximation around the operating point. Usually the design of the controller is based on the worst operating point and simply the damping torque is increased. With a change in load or system parameters, the good performance of the system is not guaranteed. In this research paper, a robust optimal  $H_2$  state feedback control to improve transient stability of power system in the presence of parametric uncertainties is introduced. This controller overcomes the mentioned difficulty in power system. To achieve the best controller tuning, the particle swarm optimization (PSO) is employed. The obtained results are compared to the results with a conventional PSS.

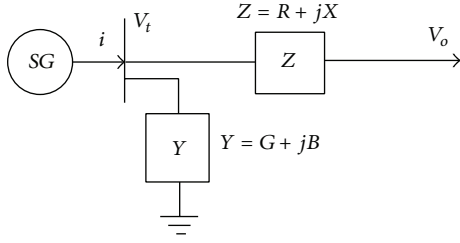


FIGURE 1: Selected power system.

It has to be noted that PSO is an efficient and reliable optimization method that has the following advantages compared to the other well-known methods [6].

- (1) PSO has no overlapping and mutation calculation. The search can be carried out by the speed of the particle. During the development of several generations, only the most optimist particle can transmit information to the other particles, and the speed of the researching is very fast.
- (2) The calculation in PSO is very simple. Compared with the other developing calculations, it occupies the bigger optimization ability and it can be completed easily.
- (3) PSO adopts the real number code, and it is decided directly by the solution. The number of the dimension is equal to the constant of the solution.

## 2. Selected Power System and Its Modeling

The power system under study in this paper is presented in Figure 1. In this single machine infinite bus (SMIB) system,  $V_t$  and  $V_o$  are terminal voltage and infinite bus voltage, respectively. A local load with  $Y = G + jB$  admittance is on the generator bus and the transmission line is presented with  $Z = R + jX$  total impedance.

Considering Heffron-Phillips model for synchronous generator, the model of the system is given by the following equations [7]:

$$\begin{aligned}\dot{\delta} &= \omega_0 \omega, \\ \dot{\omega} &= \frac{1}{M} (T_m - T_e - D\omega), \\ \dot{E}'_q &= \frac{1}{T'_{d0}} (E_{fd} - E'_q - (X_d - X'_d) i_d), \\ i_d &= \frac{E'_q - V_o \cos \delta}{X'_d}, \\ T_e &= C_3 E'_q \sin \delta + C_4 \sin 2\delta,\end{aligned}$$

$$C_3 = \frac{V'_o}{X'_d},$$

$$C_4 = \frac{V_o^2}{2} \left( \frac{1}{X_q} - \frac{1}{X'_d} \right). \quad (1)$$

After linearization, considering state variables as  $X_1 = \delta$ ,  $X_2 = \omega$ , and  $X_3 = E'_q$  and input variables as  $u_1 = E_{fd}$  and  $u_2 = T_m$ , these equations can be written as

$$\Delta \dot{X}_1 = \omega_0 \Delta X_2, \quad (2)$$

$$\Delta \dot{X}_2 = \frac{1}{M} (\Delta u_1 - \Delta T_e - D \Delta X_2), \quad (3)$$

$$\Delta X_3 = \frac{1}{T'_{d0}} (\Delta u_2 - \Delta X_3 - (X_d - X'_d) \Delta i_d), \quad (4)$$

$$\Delta i_d = \frac{\Delta X_3}{X'_d} + \frac{V_o \sin X_{10} \Delta X_2}{X'_d} = Y_d \Delta X_3 + F_d \Delta X_2, \quad (5)$$

$$\begin{aligned}\Delta T_e &= (C_3 \sin X_{10}) \Delta X_3 \\ &+ (C_3 X_{30} \cos X_{10} + 2C_4 \cos 2X_{10}) \Delta X_1.\end{aligned} \quad (6)$$

Substituting (5) and (6) in (3) and (4), the following linear state equation is obtained:

$$\begin{bmatrix} \Delta \dot{X}_1 \\ \Delta \dot{X}_2 \\ \Delta \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 \\ \frac{-k_1}{M} & \frac{-D}{M} & \frac{-k_2}{M} \\ \frac{F_d (X'_d - X_d)}{T'_{d0}} & 0 & \frac{Y_d (X'_d - X_d) - 1}{T'_{d0}} \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \Delta X_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M} & 0 \\ 0 & \frac{1}{T'_{d0}} \end{bmatrix} \begin{bmatrix} u_2 \\ u_1 \end{bmatrix}, \quad (7)$$

where

$$\begin{aligned}k_1 &= C_3 X_{30} \cos X_{10} + 2C_4 \cos 2X_{10}, \\ k_2 &= C_3 \sin X_{10},\end{aligned} \quad (8)$$

$T_m$  is the mechanical input torque,  $T_e$  is the electromagnetic torque of machine,  $M$  and  $D$  are inertia constant and damping coefficient of machine, respectively,  $\omega_0$  is synchronous speed,  $E_{fd}$  is excitation voltage,  $X_d$  and  $X'_d$  are synchronous and transient  $d$ -axis machine reactance, respectively,  $X_q$  is synchronous  $q$ -axis machine reactance,  $\delta$  is power angle,  $T'_{d0}$  is open circuit time constant of the machine, and  $X_{10}$  is the initial value of power angle.

## 3. Robust Control

Using a suitable robust control, the closed-loop system remains stable even in the presence of system uncertainties.

Most of these uncertainties are due the approximation in modeling the system. Usually in system modeling, small time constants and some nonlinear and time varying terms are neglected. To retain the good performance of the closed-loop system despite these approximations, a robust controller is designed for introduced power system [8].

Considering model uncertainties, the state equation is given as

$$\dot{X}(t) = AX(t) + BU(t). \quad (9)$$

It can be written as

$$\dot{X}(t) = (A + D\Delta(t)E_1)X(t) + (B + D\Delta(t)E_2)U(t), \quad (10)$$

where  $\Delta(t)$  represents a scalar or matrix including uncertainties, which is satisfied by  $\Delta^T(t)\Delta(t) \leq I$ , and  $D, E_1$ , and  $E_2$  are scalars or matrices relating to uncertainties coefficients.

In control theory, the main aim is the obtaining of a stabilizing feedback gain (state feedback) as follows:

$$U = KX. \quad (11)$$

To solve this control problem, the following matrix inequalities are used:

$$\begin{bmatrix} G & (E_1Q + E_2Y)^T & QR_1^{1/2} & Y^TR_2^{1/2} \\ (E_1Q + E_2Y) & -MI & 0 & 0 \\ QR_1^{1/2} & 0 & -I & 0 \\ YR_2^{1/2} & 0 & 0 & -I \end{bmatrix} < 0, \quad (12)$$

where

$$G = QA^T + AQ + BY + Y^TB^T + DMD, \quad (13)$$

$$M > 0, \quad (14)$$

$$Q > 0.$$

In these LMIs,  $Y$  is a variable without sign, and  $R_1$  and  $R_2$  are positive definite constants relating to the following  $H_2$  cost function:

$$J = \int_0^\infty (X^TR_1X + U^TR_2U) dt. \quad (15)$$

$M$  is a diagonal positive definite matrix as follows:

$$M = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \varepsilon_i \end{bmatrix}, \quad \varepsilon_i > 0, \quad i = 1, 2, \dots, k. \quad (16)$$

Using MATLAB coding, the following state feedback gain can be obtained:

$$K = YQ^{-1}. \quad (17)$$

Now, if one wants to obtain the optimal robust control, he or she should solve the following optimization problem Min trace ( $Q^{-1}$ ) under the conditions dictated with LMIs given in (12) and (14).

#### 4. Tuning by Particle Swarm Optimization

Particle swarm optimization (PSO) was introduced in 1995 by Kennedy and Eberhart [9]. In PSO algorithm, a random population of points is generated. Each point represents a member of the population. In PSO algorithm, there is no sudden jump or confusion; each point is a solution. Considering  $X$  and  $V$  as particle position and velocity, respectively, the position of  $n$ th particle in a space with  $m$  dime is represented with  $X_n = [X_{n1}, X_{n2}, \dots, X_{nm}]$ .

The position of each particle is changed in the next stage and it reaches a new position. The best position of  $n$ th particle which is corresponding to the lowest cost function for that particle is saved in  $P_{best,n}$ . In addition,  $P_{best}$  of all particles are compared and the position of particle which has the lowest cost function is saved in  $G_{best}$ . The next vector of each particle is depending on its position and its distance to its  $P_{best}$  and its distance to  $G_{best}$ . The relations of particles movements are as follows:

$$\begin{aligned} V_{nm}^{i+1} &= w \times V_{nm}^i + C_1 \times \text{rand}() \times (P_{best, nm} - X_{nm}^i) \\ &+ C_2 \times \text{rand}() \times (G_{best} - X_{nm}^i), \\ X_{nm}^{i+1} &= X_{nm}^i + CV_{nm}^{i+1}, \\ |V_{nm}^{i+1}| &\leq V_{max}, \end{aligned} \quad (18)$$

where  $V_{max}$  is a parameter that prevents going out of suitable search space which causes the solution to be in acceptable region and  $C_1$  and  $C_2$  are constants which represent the speed of learning or pulling to  $P_{best}$  and  $G_{best}$ ; the weighing function  $w$  is given by

$$w = w_{max} - \frac{w_{max} - w_{min}}{\text{iter}_{max}} \times \text{iter}, \quad (19)$$

where  $w_{min}$  and  $w_{max}$  are the minimum and maximum of weighing function and  $\text{iter}$  is the number of iterations.

In order to optimize the parameters of the controller with PSO, the following cost function is used:

$$\text{Cost Function} = \int_0^{t_1} t_s |e(t)| dt, \quad (20)$$

where  $t_1$  is the final time of simulation,  $e$  is the error signal, and  $t_s$  is the settling time of the system.

#### 5. Simulation and Results

To show the effectiveness of the proposed controller, simulation results are demonstrated in this section. The simulation results are obtained for two cases: without considering uncertainties and with considering uncertainties.

Using the model and system parameters, the following state space and control matrices are obtained:

$$A = \begin{bmatrix} 0 & 376.9911 & 0 \\ -0.0286 & 0 & -0.0748 \\ -0.1683 & 0 & -0.4371 \end{bmatrix}, \quad (21)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0.0893 & 0 \\ 0 & 0.1874 \end{bmatrix}.$$

To obtain the coefficients and parameters of (10), two cases are considered.

*5.1. Without Considering Uncertainties.* In this case, these coefficients and parameters are given by

$$E_1 = E_2 = 0, \quad (22)$$

$$D = 0.$$

The parameters of the proposed controllers  $R_1$  and  $R_2$  should be positive definite. In the first step, they selected as unitary matrices. Using these matrices, some machine input variables become out of reasonable range. With PSO algorithm, the optimal matrices are obtained as

$$R_1 = \begin{bmatrix} 35.97 & 0 & 0 \\ 0 & 390.5 & 0 \\ 0 & 0 & 0.000001 \end{bmatrix}, \quad (23)$$

$$R_2 = \begin{bmatrix} 575.74 & 0 \\ 0 & 64.87 \end{bmatrix}.$$

PSO Parameters are iteration = 100 and population = 30.

With MATLAB coding, the following state feedback gain is obtained:

$$K = \begin{bmatrix} -0.0609 & -32.7277 & 0.1612 \\ -0.134 & 3.0028 & -0.8812 \end{bmatrix}. \quad (24)$$

The obtained results for three-phase short circuit are represented as follows.

The initial conditions of the system are obtained with AC load flow. Assuming a three-phase short circuit fault at  $t = 0.1$  sec on synchronous machine terminal for 200 ms, the results of Figures 2, 3, and 4 are obtained. The controller used here with  $D = 0$  and  $E_1 = E_2 = 0$  is called optimal  $H_2$  state feedback.

Referring to Figures 2, 3, and 4, it can be seen that the designed robust controller with optimized coefficients using PSO has considerably lower oscillations compared to conventional PSS. Using the designed controller, the oscillations are damped very rapidly; in fact, the settling time of the closed-loop system is very small which is very important in dynamic stability.

*5.2. Power System with Parametric Uncertainties.* In this section, it is assumed that there are uncertainties in damping

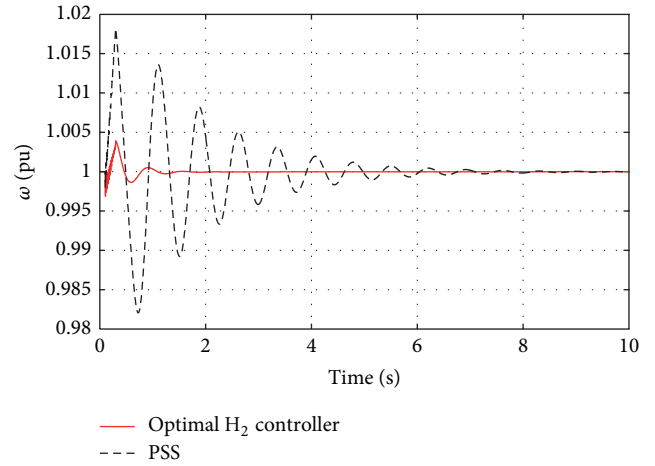


FIGURE 2: Machine rotor speed for three-phase short circuit fault.

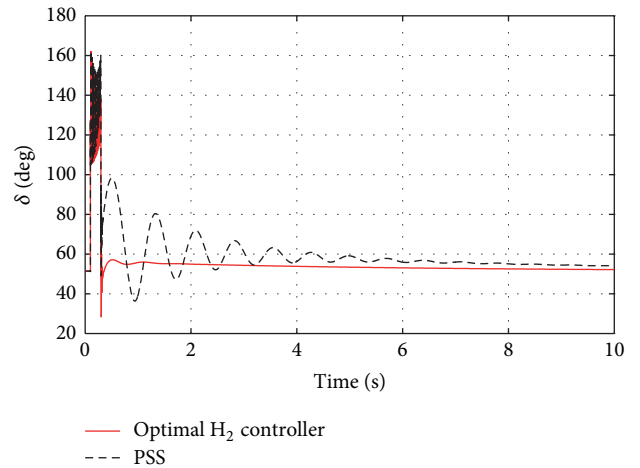


FIGURE 3: Machine power angle for three-phase short circuit fault.

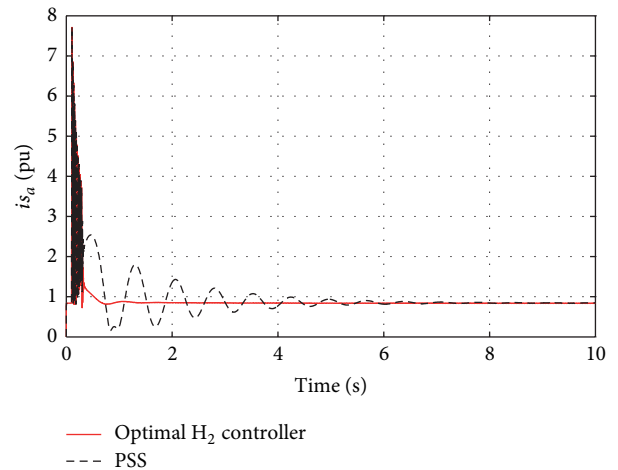


FIGURE 4: Machine phase "a" current for three-phase short circuit fault.

coefficient ( $D$ ) and inertia constant ( $H$ ) of synchronous machine. The uncertainty in  $D$  is modeled as

$$\frac{D + \delta_D}{M} = \frac{D}{M} + \frac{\delta_D}{M}. \quad (25)$$

Using the following geometric series:

$$\frac{1}{1 - a} = 1 + a + a^2 + a^3 + \dots, \quad (26)$$

the uncertainty in  $H$  is modeled as follows:

$$\frac{-k_1}{2(H + \delta_H)} = \frac{-k_1}{2H(1 + (\delta_H/H))} = \frac{-k_1}{2H} + \frac{k_1}{2H^2} \delta_H. \quad (27)$$

In (25) and (27),  $\delta_D$  and  $\delta_H$  represent the uncertainties in  $D$  and  $H$ , respectively.

The matrices of (10) are given by

$$E_1 = \begin{bmatrix} 0 & -\frac{1}{2H} & 0 \\ -\frac{k_1}{2H^2} & \frac{D}{2H} & -\frac{k_2}{2H^2} \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0 & 0 \\ -\frac{1}{2H^2} & 0 \end{bmatrix}, \quad (28)$$

$$D = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

The matrices  $R_1$  and  $R_2$  using PSO algorithm after 100 iterations are obtained as

$$R_1 = \begin{bmatrix} 6.6907 & 0 & 0 \\ 0 & 45.2011 & 0 \\ 0 & 0 & 0.1026 \end{bmatrix}, \quad (29)$$

$$R_2 = \begin{bmatrix} 78.0133 & 0 \\ 0 & 35.3664 \end{bmatrix}.$$

Using MATLAB software, the state feedback gain is obtained as

$$K = \begin{bmatrix} -0.1549 & -57.0393 & 0.0819 \\ -0.0781 & 1.0032 & -0.3686 \end{bmatrix}. \quad (30)$$

**5.3. Three-Phase Short Circuit.** Assuming a three-phase short circuit fault on synchronous machine terminal at  $t = 0.1$  sec for 200 msec, the capability of the proposed controller is investigated with uncertainties. Figures 5, 6, and 7 show the simulation results of this test. Figure 8 shows the convergence of PSO for the best cost.

These results show that when there are uncertainties in the system, the conventional PSS does not have a suitable response, but the proposed controller has a good response and good dynamic stability.

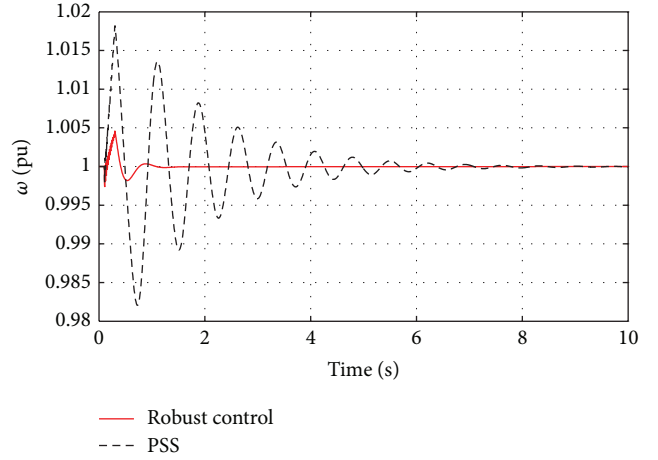


FIGURE 5: Machine rotor speed for three-phase short circuit fault.

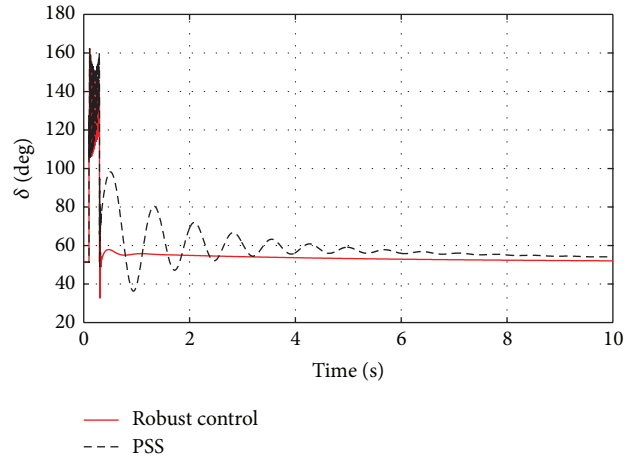


FIGURE 6: Machine power angle for three-phase short circuit fault.

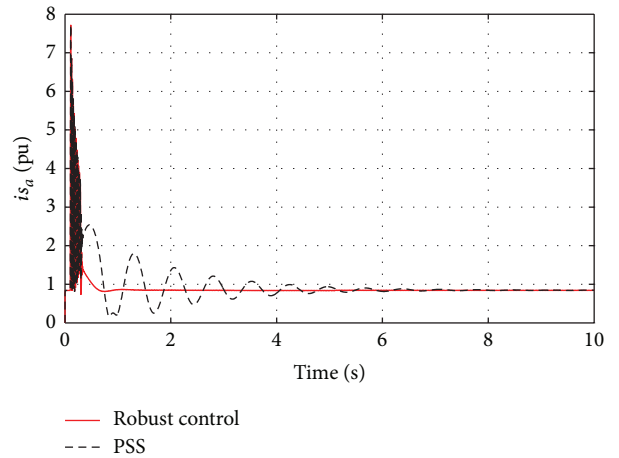


FIGURE 7: Machine phase "a" current for three-phase short circuit fault.

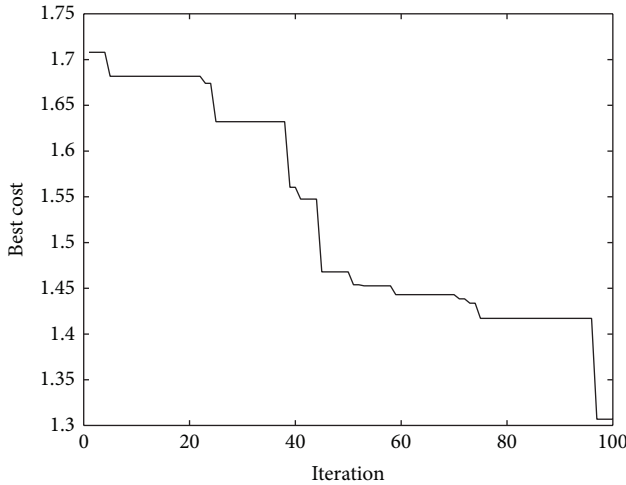


FIGURE 8: Best cost of PSO for optimal  $H_2$  controller.

## 6. Conclusion

Considering the capability of robust control in the presence of uncertainties, in this paper, a robust optimal  $H_2$  state feedback controller is designed for a power system. Parameters of the controller are tuned using PSO algorithm. The performance of the closed-loop system is investigated without and with uncertainties in the mechanical parameters with three-phase short circuit fault and sudden load variation.

The obtained simulation results show the superiority of the proposed controller over conventional PSS. Using the proposed controller, the oscillations have a small amplitude and they are damped rapidly.

## Appendix

System data for single machine infinite bus power system are shown as follows.

Generator is

$$\begin{aligned} H = 5.6, \quad X_d = 1.8 \text{ pu}, \quad X_q = 1.8 \text{ pu}, \quad D = 1 \text{ pu}, \\ X'_d = 0.32 \text{ pu}, \quad T'_{d0} = 5.3371 \text{ sec}, \quad f = 60 \text{ Hz}. \end{aligned} \quad (\text{A.1})$$

Transmission Line and Load are

$$\begin{aligned} R = 0.1273 \text{ pu}, \quad X = 0.85 \text{ pu}, \quad G = 0.27027 \text{ pu}, \\ B = 0. \end{aligned} \quad (\text{A.2})$$

Exciter (simplified IEEE type-ST1) is

$$K_A = 10, \quad T_A = 0.01 \text{ sec}. \quad (\text{A.3})$$

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## References

- [1] Q. Zhao and J. Jiang, "Robust SVC controller design for improving power system damping," *IEEE Transactions on Power Systems*, vol. 10, no. 4, pp. 1927–1932, 1995.
- [2] S. S. Kanojia and V. K. Chandrakar, "Coordinated tuning of POD and PSS controllers with STATCOM in increasing the oscillation stability of single and multi-machine power system," in *Proceedings of the Nirma University International Conference on Engineering (NUiCONE '11)*, pp. 1–5, December 2011.
- [3] S. Datta and A. K. Roy, "Fuzzy logic based STATCOM controller for enhancement of power system dynamic stability," in *Proceedings of the 6th International Conference on Electrical and Computer Engineering (ICECE '10)*, pp. 294–297, December 2010.
- [4] K. Sreenivasachar, S. Jayaram, and M. M. A. Salama, "Dynamic stability improvement of multi-machine power system with UPFC," *Electric Power Systems Research*, vol. 55, no. 1, pp. 27–37, 2000.
- [5] Z. Wang, C. Y. Chung, K. P. Wong, and C. T. Tse, "Robust power system stabiliser design under multi-operating conditions using differential evolution," *IET Generation, Transmission and Distribution*, vol. 2, no. 5, pp. 690–700, 2008.
- [6] Q. Bai, "Analysis of particle swarm optimization algorithm," *Computer and Information Science*, vol. 3, no. 1, 2010.
- [7] P. Kundur, *Power System Stability and Control*, McGraw-Hill, New York, NY, USA, 1994.
- [8] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, Pa, USA, 1994.
- [9] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proceedings of the IEEE International Conference on Neural Networks*, pp. 1942–1948, December 1995.



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