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Research Article

Rayleigh Waves in a Rotating Orthotropic Micropolar Elastic Solid Half-Space

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A problem on Rayleigh wave in a rotating half-space of an orthotropic micropolar material is considered. The governing equations are solved for surface wave solutions in the half space of the material. These solutions satisfy the boundary conditions at free surface of the half-space to obtain the frequency equation of the Rayleigh wave. For numerical purpose, the frequency equation is approximated. The nondimensional speed of Rayleigh wave is computed and shown graphically versus nondimensional frequency and rotation-frequency ratio for both orthotropic micropolar elastic and isotropic micropolar elastic cases. The numerical results show the effects of rotation, orthotropy, and nondimensional frequency on the nondimensional speed of the Rayleigh wave.

1. Introduction

Material response to external stimuli depends heavily on the motions of its inner structures. Classical elasticity does not include this effect, where only translation degrees of freedom of material point of body are considered. Eringen [1] developed the linear micropolar theory of elasticity, which included the intrinsic rotations of the microstructure. It provides a model which can support body and surface couples and display high frequency optical branch of the wave spectrum. For engineering applications, it can model composites with rigid chopped fibres, elastic solid with rigid granular inclusions, and other industrial materials such as liquid crystals.

The assumptions of isotropy in a solid medium may not capture some of significant features of the continuum responses of soils, geological materials, and composites. Iesan [2–4] studied some static problems in orthotropic micropolar elasticity. Kumar and Choudhary [5, 6] studied the mechanical sources and dynamic behaviour of orthotropic micropolar elastic medium. Kumar and Chaudhary [7] studied the plane strain problem in a homogeneous orthotropic micropolar elastic solid. Kumar and Ailawalia [8] studied the response of a micropolar cubic crystal due to various sources. Kumar and Gupta [9] studied the propagation of waves in transversely isotropic micropolar generalized thermoelastic half-space. Singh [10] investigated the two-dimensional plane wave propagation in an orthotropic micropolar elastic solid.

Surface waves in elastic solids were first studied by Rayleigh [11] for an isotropic elastic solid. The extension of surface wave analysis and other wave propagation problems to anisotropic elastic materials has been the subject of many studies; see, for example, [12–21]. The aim of the present paper is to study the propagation of Rayleigh wave in a rotating orthotropic micropolar elastic solid half space. The frequency equation of the Rayleigh wave is obtained. The speed of Rayleigh wave is computed with the help of approximated frequency equation. The effects of orthotropy, non-dimensional frequency, and rotation are shown graphically on the non-dimensional speed of the Rayleigh wave.

2. Formulation of the Problem and Solution

We consider a homogeneous and orthotropic medium of an infinite extent with Cartesian coordinate system (x, y, z). We restrict our study to the plane strain parallel to xy-plane, with the displacement vector $\mathbf{u} = (u_1, u_2, 0)$ and microrotation vector $\phi = (0, 0, \phi_3)$. Following Eringen [22] and Schoenberg and Censor [23], the field equations in xy-plane for homogeneous and rotating orthotropic micropolar solid in absence of body forces and couples are written as

$$A_{11}u_{1,11} + (A_{12} + A_{78})\,u_{2,12} + A_{88}u_{1,22} - K_1\phi_{3,2} = \rho\left[\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 - 2\Omega\frac{\partial u_2}{\partial t}\right], \quad (1)$$

$$(A_{12} + A_{78})\,u_{1,12} + A_{77}u_{2,11} + A_{22}u_{2,22} - K_2\phi_{3,1} = \rho\left[\frac{\partial^2 u_2}{\partial t^2} - \Omega^2 u_2 + 2\Omega\frac{\partial u_1}{\partial t}\right], \quad (2)$$

$$B_{66}\phi_{3,11} + B_{44}\phi_{3,22} - \chi\phi_3 + K_1 u_{1,2} + K_2 u_{2,1} = \rho j\ddot{\phi}_3, \quad (3)$$

where

$$K_1 = A_{78} - A_{88}, \qquad K_2 = A_{77} - A_{78}, \qquad \chi = K_2 - K_1. \quad (4)$$

We consider the following surface wave solutions

$$\{u_1, u_2, \phi_3\} = \{\overline{u}_1(y), \overline{u}_2(y), \overline{\phi}_3(y)\}\,e^{ik(x-ct)}, \quad (5)$$

where k is the wave number, c is phase velocity of the wave, and $\omega = kc$ is the angular frequency. Making use of (5) in (1) to (3), we obtain three homogeneous equations in $\overline{u}_1(y)$, $\overline{u}_2(y)$, and $\overline{\phi}_3(y)$, which have nontrivial solutions if

$$\alpha D^6 - \beta D^4 + \gamma D^2 - \delta = 0, \quad (6)$$

where $D = d/dy$ and α, β, γ, and δ are given in Appendix.

Let m_1, m_2, and m_3 be the roots of auxiliary equation (6). Then, the general solutions of (6) are written as

$$\begin{aligned} u_1 &= \left(A_1 e^{-m_1 y} + A_2 e^{-m_2 y} + A_3 e^{-m_3 y} + A_4 e^{m_1 y} + A_5 e^{m_2 y} + A_6 e^{m_3 y}\right) e^{ik(x-ct)}, \\ u_2 &= \left(\zeta_1 A_1 e^{-m_1 y} + \zeta_2 A_2 e^{-m_2 y} + \zeta_3 A_3 e^{-m_3 y} + \zeta_1 A_4 e^{m_1 y} + \zeta_2 A_5 e^{m_2 y} + \zeta_3 A_6 e^{m_3 y}\right) e^{ik(x-ct)}, \\ \phi_3 &= \left(\eta_1 A_1 e^{-m_1 y} + \eta_2 A_2 e^{-m_2 y} + \eta_3 A_3 e^{-m_3 y} + \eta_1 A_4 e^{m_1 y} + \eta_2 A_5 e^{m_2 y} + \eta_3 A_6 e^{m_3 y}\right) e^{ik(x-ct)}, \end{aligned} \quad (7)$$

where

$$m_1^2 + m_2^2 + m_3^2 = \frac{\beta}{\alpha}, \quad (8)$$

$$m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2 = \frac{\gamma}{\alpha}, \quad (9)$$

$$m_1^2 m_2^2 m_3^2 = \frac{\delta}{\alpha}, \quad (10)$$

and the expressions for ζ_1, ζ_2, and ζ_3 and η_1, η_2, and η_3 are given in the Appendix.

With the use of the radiation conditions $u_1 \to 0$, $u_2 \to 0$, $\varphi_3 \to 0$ as $y \to \infty$, we obtain the particular solutions for medium $(y > 0)$ as

$$u_1 = \left(A_1 e^{-m_1 y} + A_2 e^{-m_2 y} + A_3 e^{-m_3 y}\right) e^{ik(x-ct)}, \quad (11)$$

$$u_2 = \left(\zeta_1 A_1 e^{-m_1 y} + \zeta_2 A_2 e^{-m_2 y} + \zeta_3 A_3 e^{-m_3 y}\right) e^{ik(x-ct)}, \quad (12)$$

$$\phi_3 = \left(\eta_1 A_1 e^{-m_1 y} + \eta_2 A_2 e^{-m_2 y} + \eta_3 A_3 e^{-m_3 y}\right) e^{ik(x-ct)}. \quad (13)$$

3. Boundary Conditions

The mechanical boundary conditions at $y = 0$ are the vanishing of normal force stress tangential force stress; and tangential couple stress that is,

$$t_{22} = 0, \qquad t_{21} = 0, \qquad m_{23} = 0, \quad (14)$$

where

$$\begin{aligned} t_{22} &= A_{12}\frac{\partial u_1}{\partial x} + A_{22}\frac{\partial u_2}{\partial y}, \\ t_{21} &= A_{78}\frac{\partial u_2}{\partial x} + A_{88}\frac{\partial u_1}{\partial y} + (A_{88} - A_{78})\,\phi_3, \\ m_{23} &= B_{44}\frac{\partial \phi_3}{\partial y}. \end{aligned} \quad (15)$$

The solutions given by (11) to (13) satisfy the boundary conditions (14) at $y = 0$, and we obtain the following frequency equation:

$$\begin{aligned} &\frac{A_{12}}{A_{22}}\frac{A_{78}}{A_{88}}\sum\frac{m_1}{k}\frac{\eta_1}{k}(\zeta_2 - \zeta_3) + i\frac{A_{12}}{A_{22}}\sum\frac{m_1}{k}\frac{m_2}{k}\left(\frac{\eta_1}{k} - \frac{\eta_2}{k}\right) \\ &\quad + i\frac{A_{12}}{A_{22}}\frac{K_1}{A_{88}}\sum\frac{m_1}{k}\frac{\eta_1}{k}\left(\frac{\eta_2}{k} - \frac{\eta_3}{k}\right) \\ &\quad + i\frac{A_{78}}{A_{88}}\sum\frac{m_1}{k}\frac{m_2}{k}\zeta_3\left(\frac{\eta_2}{k}\zeta_1 - \frac{\eta_1}{k}\zeta_2\right) \\ &\quad + \frac{K_1}{A_{88}}\sum\frac{m_1}{k}\frac{m_2}{k}\frac{\eta_3}{k}\left(\frac{\eta_1}{k}\zeta_2 - \frac{\eta_2}{k}\zeta_1\right) \\ &\quad + \sum\frac{m_1}{k}\frac{m_2}{k}\frac{m_3}{k}\left(\frac{\eta_1}{k}\zeta_2 - \frac{\eta_2}{k}\zeta_1\right) = 0. \end{aligned} \quad (16)$$

4. Particular Case

The frequency equation (16) reduces to the frequency equation for an isotropic rotating micropolar elastic case, if we take

$$\begin{aligned} &A_{11} = A_{22} = \lambda + 2\mu + \kappa, \\ &A_{77} = A_{88} = \mu + \kappa, \qquad A_{12} = \lambda, \qquad A_{78} = \mu, \\ &B_{44} = B_{66} = \gamma, \qquad -K_1 = K_2 = \frac{\chi}{2} = \kappa. \end{aligned} \quad (17)$$

5. Numerical Results and Discussion

From relations (8) to (10), we obtain the following approximated roots:

$$\frac{m_1^2}{k^2} \cong \frac{\left(A_{11} - \rho c^2 \Omega^*\right)}{A_{88}}, \quad (18)$$

$$\frac{m_2^2}{k^2} \cong \frac{\left(A_{77} - \rho c^2 \Omega^*\right)}{A_{22}}, \quad (19)$$

$$\frac{m_3^2}{k^2} \cong \frac{\left(B_{66} + \rho j c^2 + \left(\chi/k^2\right)\right)}{B_{44}}. \quad (20)$$

With the help of (18) to (20), the frequency equation (16) reduces to the approximated frequency equation for an orthotropic rotating micropolar elastic case. The approximated frequency equation is used to compute the non-dimensional speed of the Rayleigh wave in orthotropic micropolar solid half-space for the following arbitrary physical constants:

$$\begin{aligned} A_{11} &= 11.65 \times 10^{10}\,\mathrm{Nm}^{-2}, \\ A_{22} &= 11.71 \times 10^{10}\,\mathrm{Nm}^{-2}, \\ A_{12} &= 7.69 \times 10^{10}\,\mathrm{Nm}^{-2}, \\ A_{77} &= 1.99 \times 10^{10}\,\mathrm{Nm}^{-2}, \\ A_{78} &= 1.98 \times 10^{10}\,\mathrm{Nm}^{-2}, \\ A_{88} &= 2.01 \times 10^{10}\,\mathrm{Nm}^{-2}, \\ B_{44} = 0.036 \times 10^{10}\,\mathrm{N}, &\qquad B_{66} = 0.037 \times 10^{10}\,\mathrm{N}, \\ \rho = 2.19 \times 10^{3}\,\mathrm{Kg\,m}^{-3}, &\qquad j = 0.000196\,\mathrm{m}^2. \end{aligned} \quad (21)$$

The non-dimensional speed of the Rayleigh wave is also computed for isotropic micropolar elastic case with following relevant parameters [24]:

$$\begin{aligned} \lambda = 7.59 \times 10^{10}\,\mathrm{Nm}^{-2}, &\qquad \mu = 1.89 \times 10^{10}\,\mathrm{Nm}^{-2}, \\ \kappa = 0.0149 \times 10^{10}\,\mathrm{Nm}^{-2}, &\qquad \gamma = 0.0268 \times 10^{9}\,\mathrm{N}, \\ \rho = 2.19 \times 10^{3}\,\mathrm{Kg\,m}^{-3}, &\qquad j = 0.000196\,\mathrm{m}^2. \end{aligned} \quad (22)$$

The non-dimensional speed c^* $(= \rho c^2/A_{22})$ of Rayleigh wave is computed for orthotropic micropolar elastic case and isotropic micropolar elastic case for different values of non-dimensional frequency ω^* $(= \omega^2/[\chi/\rho j])$ and rotation-frequency ratio Ω/ω.

The non-dimensional speed c^* is plotted against the rotation-frequency ratio Ω/ω, when non-dimensional frequency $\omega^* = 5, 10$, and 20. The speed c^* decreases with the increase in value of rotation-frequency ratio Ω/ω. For each value of rotation-frequency ratio Ω/ω, the speed c^* increases with the increase in value of non-dimensional frequency ω^*. The effect of non-dimensional frequency ω^* on non-dimensional speed c^* decreases with the increase in values of Ω/ω. The variations showing the effect of orthotropy on non-dimensional speed of Rayleigh wave are shown in Figure 1, where solid lines and dotted lines correspond to the orthotropic micropolar elastic case and isotropic micropolar elastic case, respectively.

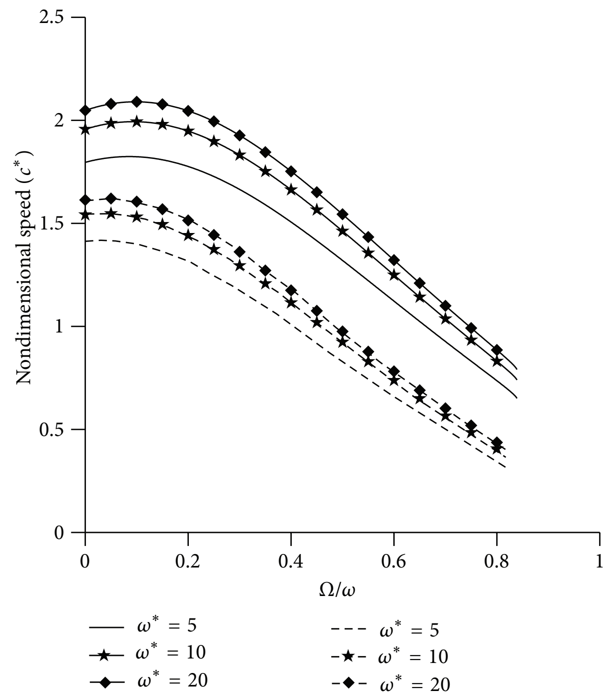

Figure 1: Variations of the nondimensional speed of Rayleigh wave against rotation-frequency ratio (solid lines:orthotropic micropolar elastic case, dotted lines:isotropic micropolar elastic case).

The speed c^* is also plotted against the non-dimensional frequency ω^*, when rotation-frequency ratio $\Omega/\omega = 0.2, 0.4$, and 0.6. The speed c^* of Rayleigh wave increases sharply with the increase in value of non-dimensional frequency ω^* for both orthotropic micropolar elastic case and isotropic micropolar elastic case. Beyond $\omega^* = 4$, it increases slowly in the both cases. Here, the speed c^* decreases with the increase in values of Ω/ω in each case. The effect of rotation on non-dimensional speed c^* increases with the increase in value of non-dimensional frequency ω^*. The variations showing the effect of orthotropy on non-dimensional speed of Rayleigh wave are shown in Figure 2, where solid lines and dotted lines correspond to the orthotropic micropolar elastic case and isotropic micropolar elastic case, respectively.

6. Conclusion

The propagation of Rayleigh wave is studied in an orthotropic micropolar elastic solid half-space, where we obtained the required approximated frequency equation of Rayleigh wave. The non-dimensional speed c^* $(= \rho c^2/A_{22})$ is computed for certain ranges of non-dimensional frequency ω^* and rotation-frequency ratio Ω/ω. The comparison of solid and dotted line curves in the figures reveals the effect of orthotropy,

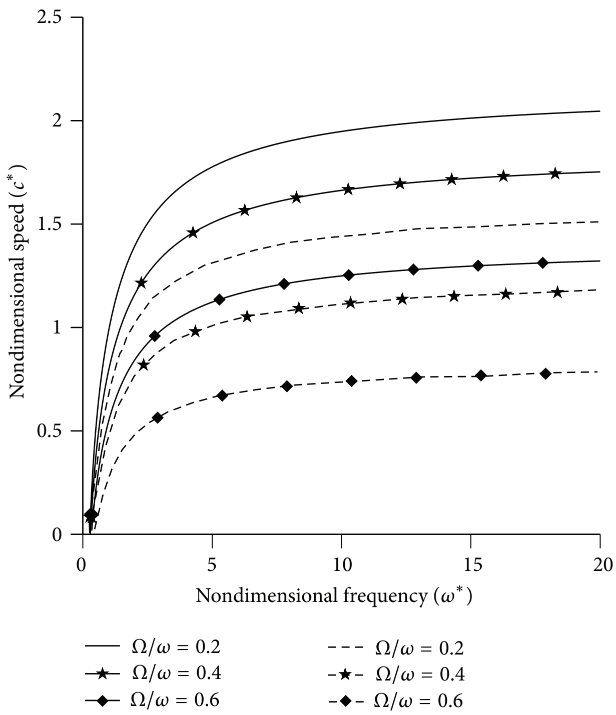

FIGURE 2: Variations of the non-dimensional speed of Rayleigh wave against non-dimensional frequency ω^* (solid lines:orthotropic micropolar elastic case, dotted lines:isotropic micropolar elastic case).

rotation, and non-dimensional frequency ω^* on the non-dimensional speed c^* of the Rayleigh wave in an orthotropic micropolar elastic solid half-space.

Appendix

The values of α, β, γ, and δ are given as

$$\begin{gathered}
\alpha = A_{22}A_{88}B_{44}, \\
\beta = k^2\left(A_{22}A_{88}L_4 + A_{88}B_{44}L_3 + A_{22}B_{44}L_1 - B_{44}L_2^2\right) - A_{22}K_1^2, \\
\gamma = k^4\left(-L_2^2L_4 + L_3L_4A_{88} + L_1L_4A_{22} + L_1L_3B_{44} - 4\left(\rho c^2\right)^2\left(\frac{\Omega}{\omega}\right)^2 B_{44}\right) + k^2\left(-K_2^2A_{88} + 2K_1K_2L_2 - K_1^2L_3\right), \\
\delta = k^6\left[L_1L_3L_4 - 4\left(\rho c^2\right)^2\left(\frac{\Omega}{\omega}\right)^2 L_4\right] - k^4L_1K_2^2, \\
L_1 = A_{11} - \rho c^2\Omega^*, \qquad L_2 = A_{12} + A_{78}, \\
L_3 = A_{77} - \rho c^2\Omega^*, \\
L_4 = B_{66} + \rho j c^2 + \frac{\chi}{k^2}, \qquad \Omega^* = 1 + \frac{\Omega^2}{\omega^2}.
\end{gathered} \tag{A.1}$$

The values of ζ_1, ζ_2, and ζ_3 and η_1, η_2, and η_3 are obtained as

$$\begin{aligned}
\zeta_i = i\Bigg[&\left(\frac{m_i}{k}\left(\frac{m_i}{k}\frac{A_{12}+A_{78}}{A_{88}} - \frac{2\rho c^2}{A_{88}}\frac{\Omega}{\omega}\right) - \frac{K_2}{K_1}\left(\frac{m_i^2}{k^2} - \frac{A_{11}-\rho c^2\Omega^*}{A_{88}}\right)\right) \\
&\times\left(\frac{K_2}{K_1}\left(\frac{m_i}{k}\frac{A_{12}+A_{78}}{A_{88}} + \frac{2\rho c^2}{A_{88}}\frac{\Omega}{\omega}\right) + \frac{m_i}{k}\frac{A_{22}}{A_{88}}\left(\frac{m_i^2}{k^2} - \frac{A_{77}-\rho c^2\Omega^*}{A_{22}}\right)\right)^{-1}\Bigg], \\
\frac{\eta_i}{k} = &\left(\left[\left(\frac{m_i^2}{k^2}\frac{\left(A_{12}+A_{78}\right)^2}{A_{22}A_{88}} - 4\frac{\left(\rho c^2\right)^2}{A_{22}A_{88}}\left(\frac{\Omega}{\omega}\right)^2\right) - \left(\frac{m_i^2}{k^2} - \frac{A_{11}-\rho c^2\Omega^*}{A_{88}}\right)\left(\frac{m_i^2}{k^2} - \frac{A_{77}-\rho c^2\Omega^*}{A_{22}}\right)\right]\right) \\
&\times\left(\left[\frac{K_2}{A_{88}}\left(\frac{m_i}{k}\frac{A_{12}+A_{78}}{A_{22}} + 2\frac{\rho c^2}{A_{22}}\frac{\Omega}{\omega}\right) + \frac{m_i}{k}\frac{K_1}{A_{88}}\left(\frac{m_i^2}{k^2} - \frac{A_{77}-\rho c^2\Omega^*}{A_{22}}\right)\right]\right)^{-1}.
\end{aligned} \tag{A.2}$$

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