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Research Article

Thermal Radiation and Buoyancy Effects on Heat and Mass Transfer over a Semi-Infinite Stretching Surface with Suction and Blowing

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This study sought to investigate thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. Appropriate transformations were employed to transform the governing differential equations to nonsimilar form. The transformed equations were solved numerically by an efficient implicit, iterative finite-difference scheme. A parametric study illustrating the influence of wall suction or injection, radiation, Schmidt number and Grashof number on the fluid velocity, temperature and concentration is conducted. We conclude from the study that the flow is appreciably influenced by thermal radiation, Schmidt number, as well as fluid injection or suction.

1. Introduction

There are many transport processes which occur naturally and artificially in which flow is modified or driven by density differences caused by temperature, chemical composition differences and gradients, and material or phase constitution. Boundary layer flow and heat transfer over a continuously stretched surface has received considerable attention in recent years. This is because of the various possible engineering and metallurgical applications such as hot rolling, wire drawing, metal and plastic extrusion, continuous casting, glass fibre production, crystal growing, and paper production.

Gebhart and Pera [1] investigated flows resulting from buoyancy forces which arise from a combination of temperature and species concentration effects of comparable magnitude. This circumstance arises often, especially in the natural environment. Pera and Gebhart [2] analyzed the flow induced by the combined buoyancy effects due to thermal and chemical species diffusion adjacent to horizontal surfaces having uniform surface conditions with buoyancy effect primarily away from the surface.

Bestman [3] studied free convection boundary layer flow with simultaneous heat and mass transfer in a porous medium when the boundary wall moves in its own plane with suction. Hossain [4] investigated the effect of uniform transpiration rate on the heat and mass transfer characteristics in mixed convection flow of a viscous incompressible fluid along a vertical permeable plate. Acharya et al. [5] analyzed heat and mass transfer occurring in laminar boundary layer on a linearly accelerating surface with temperature-dependent heat source subject to suction or blowing.

Hossain et al. [6] determined the effect of radiation on natural convection flow of an optically thick viscous incompressible flow past a heated vertical porous plate with a uniform surface temperature and a uniform rate of suction where radiation is included by assuming the Rosseland diffusion approximation. Rahman and Mulolani [7] examined natural convection flow over a semi-infinite vertical plate at constant species concentration.

Hussain and Hossain [8] considered the problem of natural convection boundary layer flow, induced by the combined buoyancy forces from mass and thermal diffusion from a permeable vertical flat surface with non uniform surface temperature and concentration but a uniform rate of suction of fluid through the permeable surface.

Chamkha [9] considered the problem of steady, hydromagnetic boundary layer flow over an accelerating semi infinite porous surface in the presence of thermal radiation, buoyancy and heat generation or absorption. Hossain et al. [10] numerically investigated the effect of thermal radiation on natural convection flow along a uniformly heated vertical porous plate with variable viscosity and uniform suction velocity.

Abel et al. [11] investigated numerically natural convective flows, heat and mass transfer due to the combined effect of thermal and species diffusion in viscoelastic fluid. Devi and Kandasamy [12] analyzed the effects of a chemical reaction, heat and mass transfer on an accelerating surface with a heat source and thermal stratification in the presence of suction and injection.

Chamkha and Quadri [13] considered simultaneous heat and mass transfer by natural convection from a vertical semi-infinite plate embedded in a fluid saturated porous medium in the presence of wall suction or injection, heat generation or absorption effects, porous medium inertial and thermal dispersion effects. In general, the porous medium thermal dispersion effects increase the temperature of the fluid causing higher flow rates along the surface. However, this seems not to be the case in their study, as the peak values of the temperature and velocity profiles were lowered as porous medium thermal dispersion parameter increases.

Saha and Hossain [14] numerically studied the problem of laminar doubly diffusive free convection flows adjacent to a vertical surface in a stable thermally stratified medium. Abel et al. [15] analyzed the effect of the buoyancy force and thermal radiation in MHD boundary layer viscoelastic fluid flow over a continuously moving stretching surface.

Azizi et al. [16] investigated numerically the effects of thermal and buoyancy forces on both upward flow and downward flow of air in a vertical parallel-plates channel. Shateyi et al. [17] studied magnetohydrodynamic flow past a vertical plate with radiative heat transfer.

Motivated by the above referenced work and the vast possible industrial applications, it is of paramount interest in this study to consider effects of thermal radiation, buoyancy and suction/blowing on natural convection heat and mass transfer over a semi-infinite stretching surface. The essential difference between the current work and Chamkha [9] and other related work arises from the absence of an electrically conducting fluid. The inclusion of mass transfer, as well as suction and blowing and the exclusion of an electrically conducting fluid in the current work makes it different from Shateyi et al. [17]. The inclusion of thermal radiation differentiates this current work from other similar free convection heat and mass transfer studies.

2. Mathematical formulation

We consider a steady two-dimensional laminar boundary layer flow of an incompressible viscous fluid over a semi-infinite porous stretching surface. Heat and the concentration are supplied from the plate to the fluid at uniform rates. The chemical species diffuses into the nearby fluid inducing a buoyancy force. A change in the temperature of the fluid near the plate surface also results in additional buoyancy.

Introducing a Cartesian coordinate system, x-axis is chosen along the plate in the direction of flow and y-axis normal to it. The plate is maintained at a constant temperature T_w and the concentration is maintained at a constant value C_w. The ambient temperature of the flow is T_∞ and the concentration of uniform flow is C_∞. The concentration of diffusing species is very small in comparison to other chemical species and hence the thermal diffusion and diffusing thermal energy effects are neglected. Viscous dissipation in the energy is negligible. Variations in fluid properties are limited only to those density variations which affect the buoyancy terms and the radiative heat flux in the x-direction is considered negligible in comparison with that in the y-direction. The concentration is assumed to be nonreactive.

Under the usual Boussinesq approximation, the conservation equations for the steady, laminar, two-dimensional boundary layer flow problem under consideration can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_t(T - T_\infty) + g\beta_c(C - C_\infty), \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_t\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{2.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}. \tag{2.4}$$

The boundary conditions are

$$\begin{gathered} u(x,0) = ax, \qquad v(x,0) = V_w, \\ T(x,0) = T_w, \qquad C(x,0) = C_w, \\ u(x,\infty) = 0, \qquad T(x,\infty) = T_\infty, \qquad C(x,\infty) = C_\infty, \end{gathered} \tag{2.5}$$

where u, v are velocity components along x-axis and y-axis, respectively, g is the acceleration due to gravity, T is temperature, T_w is the wall temperature, T_∞ is the temperature of the uniform flow, α_t is thermal conductivity, c_p is the specific heat at constant pressure, ρ is density of the ambient fluid, and q_r is the component of radiative heat flux. C is the concentration of species, C_w is the wall concentration, C_∞ is the concentration of the uniform flow, D is the molecular diffusivity, V_w is suction/injection velocity, ν is the kinematic viscosity, β_t is the volumetric coefficient of thermal expansion, β_c is the volumetric coefficient of thermal expansion with concentration, and a is a stretching constant.

The radiative heat flux q_r is described by the Rosseland approximation such that

$$q_r = -\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y}, \tag{2.6}$$

where σ^* and K are the Stefan-Boltzman constant and the Roseland mean absorption coefficient, respectively. Following Chamkha [18] and others, we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This results in the following approximation:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{2.7}$$

Using (2.6) and (2.7) in the last term of (2.3), we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3K}\frac{\partial^2 T}{\partial y^2}. \tag{2.8}$$

We then nondimensionalize (2.1)–(2.4) using the following transformations:

$$\begin{gathered} x = L\xi, \qquad y = LGr^{-1/4}\eta, \qquad u = \frac{\nu}{L}Gr^{1/2}F, \qquad v = \frac{\nu}{L}Gr^{1/4}H, \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \end{gathered} \tag{2.9}$$

with L being the characteristic length, and the two Grashof numbers are given by $Gr = g\beta_t(T_w - T_\infty)L^3/\nu^2$ and $Gr_c = g\beta_c(C_w - C_\infty)L^3/\nu^2$.

Using these transformations, the governing equations become

$$\begin{gathered} \frac{\partial F}{\partial \xi} + \frac{\partial H}{\partial \eta} = 0, \\ F\frac{\partial F}{\partial \xi} + H\frac{\partial F}{\partial \eta} = \theta + N\phi + \frac{\partial^2 F}{\partial \eta^2}, \\ F\frac{\partial \theta}{\partial \xi} + H\frac{\partial \theta}{\partial \eta} = \frac{1}{Pr}(1+R)\frac{\partial^2 \theta}{\partial \eta^2}, \\ F\frac{\partial \phi}{\partial \xi} + H\frac{\partial \phi}{\partial \eta} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial \eta^2}, \end{gathered} \tag{2.10}$$

where $N = Gr_c/Gr$ is the buoyancy ratio, Pr is the Prandtl number, Sc is the Schmidt number, and $R = 16\sigma T_\infty^3/3K\lambda_c$ is the dimensionless thermal radiation coefficient.

The boundary conditions become

$$F(\xi,0)=1,\qquad H(\xi,0)=V_0,\qquad \theta(\xi,0)=\phi(\xi,0)=1,$$
$$F(\xi,\infty)=H(\xi,\infty)=0,\qquad \theta(\xi,\infty)=\phi(\xi,\infty)=0, \tag{2.11}$$

where $V_0 = LV_wGr^{-1/4}/\nu$ is the dimensionless wall normal velocity such that $V_0 > 0$ indicates injection and $V_0 < 0$ indicates suction at the surface.

3. Method of solution

Equations (2.10) are coupled with nonlinear partial differential ones which possess no similarity or closed-form solutions. Therefore a numerical solution of the problem under consideration is needed (see, e.g., Abel et al. [11], Abel et al. [15], Chamkha and Khaled [19], among others). In general, closed-form or similarity solutions are very useful in validating numerical methods. Since 1970, when Blottner first discussed it, the implicit finite-difference method has proven to be adequate and accurate for equations similar to (2.10). For this reason, the implicit finite-difference method discussed by Blottner [20] is employed in the present work. Also since the finite-difference method is more accurate and more flexible in setting the limiting condition far from the surface than most numerical methods such as Runge-kutta methods, it is one of the reasons why it is adopted in the present work. Finite-difference procedure leads to a system which is triadiagonal and therefore speedy to solve and also economical of memory space to store the coefficients. Although the shooting methods can be used for solving problems presented in this study, they often present problems of instability. Finite-difference methods have better stability characteristics, though they generally require more work to obtain a specified accuracy.

Owing to the nonlinear nature of the equations, we employed an iterative procedure with 10^{-6} as the maximum absolute error between two successive iterations. The computational domain consisted of more than 300 nodal points of nonuniform distribution employed to accommodate steep changes in the velocity, temperature, and chemical species in the immediate vicinity of the wall. After many numerical experiments were performed to assess grid independence and accuracy of the results, the choice of an initial step size of $\nabla\eta_1$ of 0.001 and a growth factor k of 1.02, such that $\nabla\eta_{i+1} = k\nabla\eta_i$, was made. We chose the relative difference between the current and the previous iterations to be the convergence criterion. When this difference reached 10^{-6}, the solution was assumed converged and the iteration process was terminated.

4. Results and discussion

A graphical representation of the numerical results is illustrated in Figure 1 through Figure 7 to show the influence of the wall suction or blowing, Schmidt number, radiation parameter, and Grashof number. Figure 1 depicts the influence of the suction/injection parameter V_0 on the flow velocity in the boundary layer. It is now known that imposition of wall fluid injection increases the hydrodynamic boundary layer which indicates an increase in the fluid velocity. However, the exact opposite behaviour is produced by imposition of wall fluid suction. These behaviours are clear from Figures 1(a) and 1(b). As is clearly depicted in Figure 1, the velocity profiles rise from an initial velocity 1 up to respective maximum values before asymptotically tend to zero as we move away from the moving wall. In Figure 1(b), we can see that as the

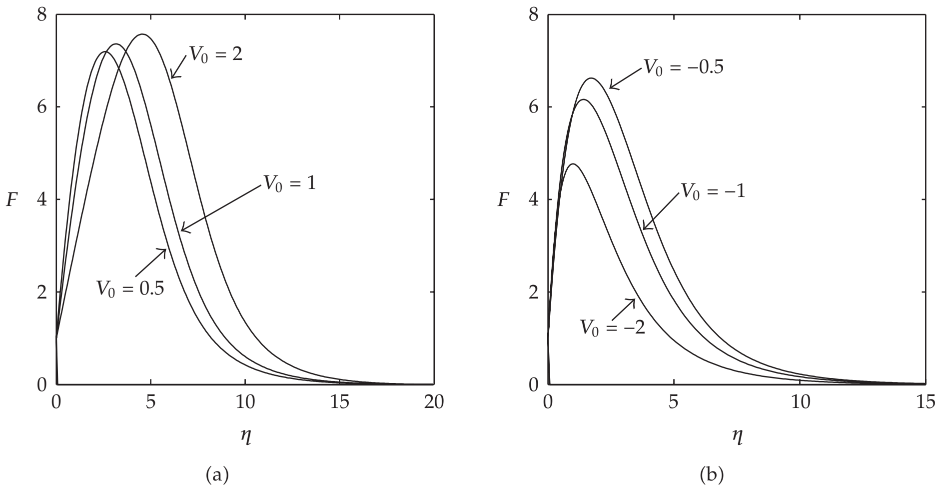


Figure 1: The variation of the velocity distribution with increasing (a) injection parameter and (b) suction parameter with $\xi = 5,\ Gr = G_c = 1,\ Pr = 0.72,\ Sc = 1,\ R = 1$.

suction parameter increases, the maximum fluid velocity decreases. This can be physically interpreted by the fact that suction is to take away the warm solute on the vertical plate thereby decreasing the velocity with a reduction in the intensity of the natural convection rate.

Figure 2 shows the effect of the injection parameter on the temperature and concentration profiles. As injection rate increases, more warm fluid is added and thus the thermal and concentration boundary layer thicknesses increase. In Figure 3, it is shown that as suction parameter value increases, both the temperature and concentration profiles decrease. This is because as the suction rate is increased, more warm fluid is taken away from the boundary layer.

Diffusing chemical species of most interest in air has Schmidt numbers in the range from 0.1 to 10 [8]. In the present investigation, we consider hydrogen ($Sc = 0.22$), water vapour ($Sc = 0.66$), and carbon dioxide ($Sc = 0.94$). The effect of these chemical species on the velocity and concentration distribution is shown in Figure 4. It can be seen that the presence of a heavier species (lager Schmidt number) is to decrease both the fluid velocity and the concentration in the boundary layer. This is due to the thinning of the momentum and concentration boundary layer with the introduction of a heavier species diffusion.

The effects of thermal radiation parameter R on the velocity and temperature profiles in the boundary layer are illustrated in Figures 5(a) and 5(b), respectively. Increasing the thermal radiation parameter R produces an increase in the thermal condition of the fluid and its thermal boundary layer. More flow is induced in the boundary layer by the increase in the fluid temperature thereby causing the velocity of the fluid to increase as well.

Figure 6(a) shows that the velocity rises steeply near the vertical wall as the Grashof number is increased. Moving away from the wall, a cross flow in the velocity is induced as the velocity profiles turn to zero at slower rates for small Grashof numbers. The thermal boundary layer and the concentration boundary layer reduce as the Grashof number increases causing the fluid temperature to reduce at every point other than the wall. It is observed that the effect of the Grashof number is to reduce the concentration distribution as concentration species is dispersed away. This is clearly depicted in Figures 6(b) and 7(a). In

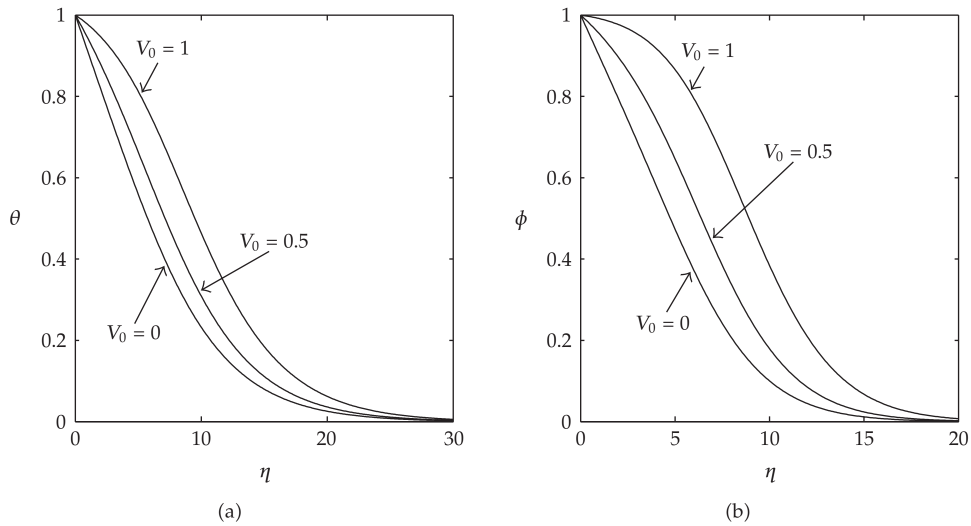


Figure 2: The variation of (a) the temperature distribution and (b) the concentration profiles with increasing injection parameter numbers with $\xi = 5$, $Gr = Gr_c = 1$, $Pr = 0.72$, $Sc = 1$, $R = 1$.

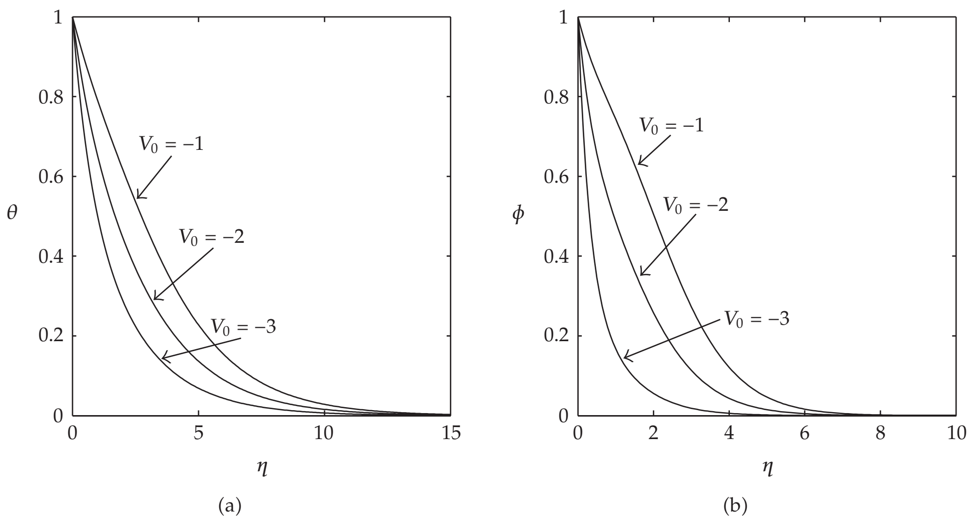
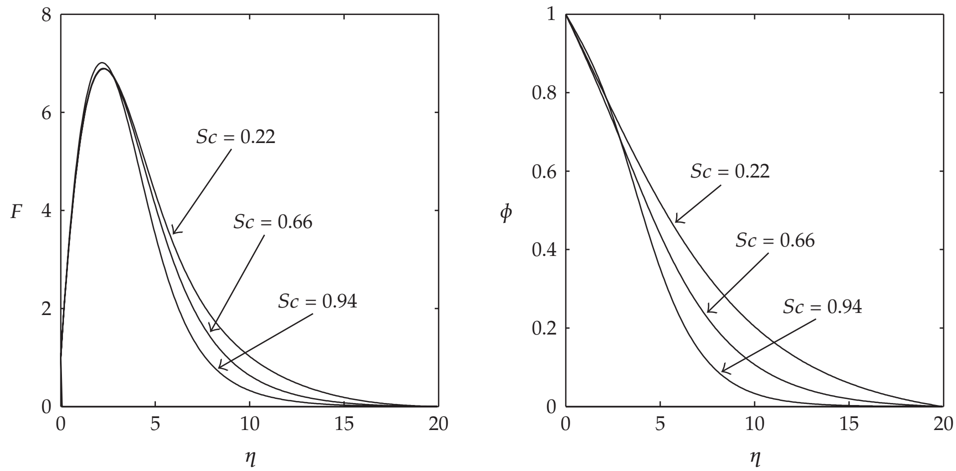


Figure 3: The variation of (a) the temperature distribution and (b) the concentration profiles with increasing injection parameter numbers with $\xi = 5$, $Gr = Gr_c = 1$, $Pr = 0.72$, $Sc = 1$, $R = 1$.

Figure 7(b) we see that radiation has no significant effect on the concentration composition of the flow. However, it can be seen that increasing radiation slightly reduces the concentration boundary layer.

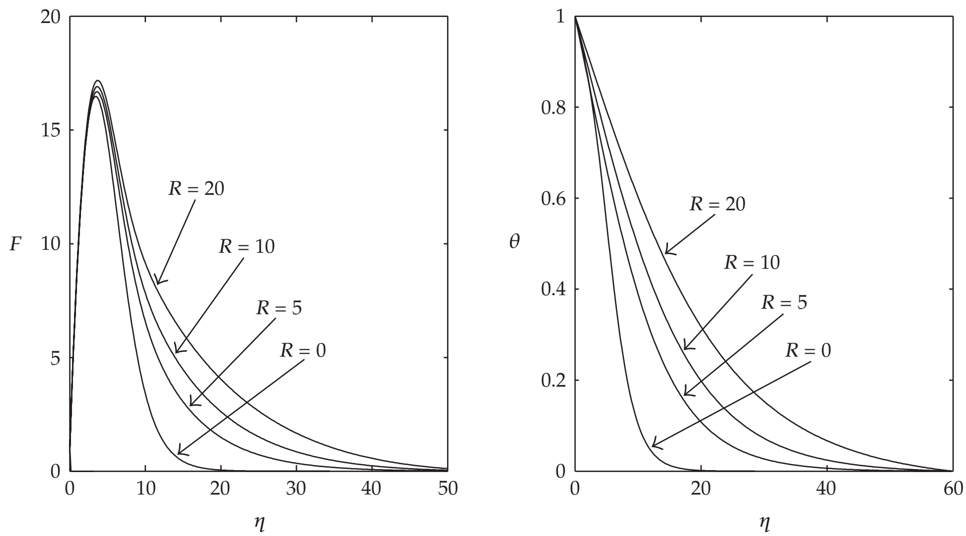
5. Conclusion

In this paper, investigations were made on the effects of thermal radiation, combined buoyancy and suction/blowing on natural convection heat and mass transfer over a semi-infinite stretching surface. Implicit finite difference method was employed and graphical



(a) (b)

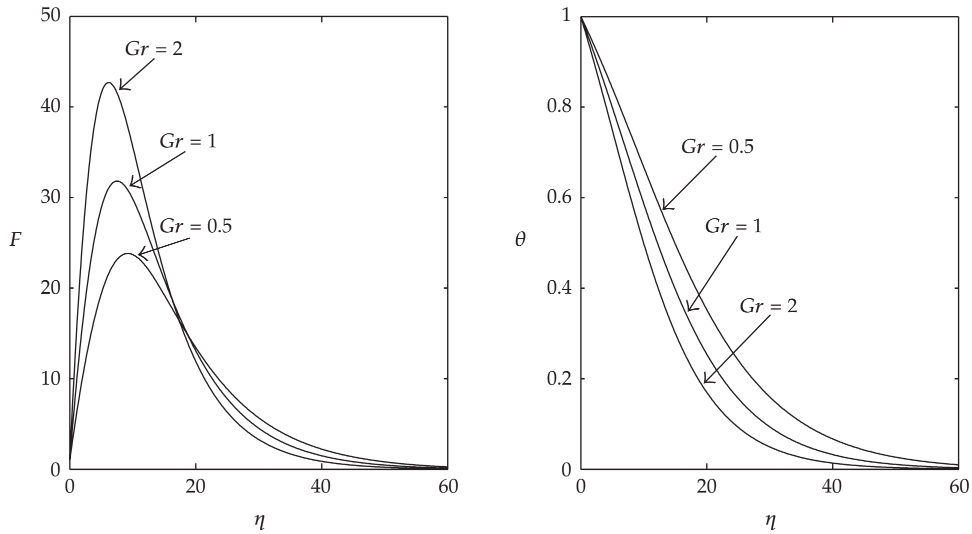
Figure 4: The variation of (a) the velocity and (b) the concentration profiles with different Schmidt numbers with $\xi = 5,\ Gr = Gr_c = 1,\ Pr = 0.72,\ V_0 = 0.1,\ R = 1$.



(a) (b)

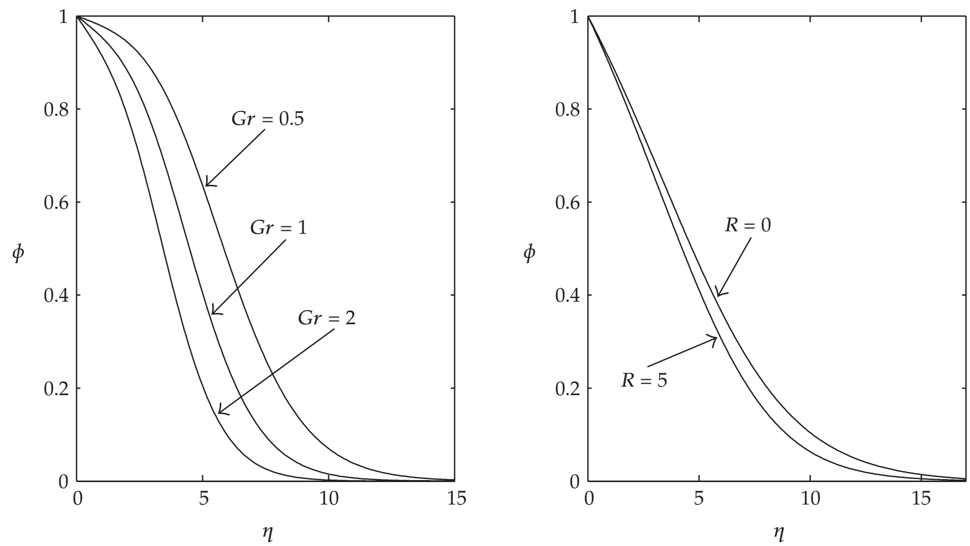
Figure 5: The variation of (a) velocity and (b) the temperature profiles with increasing radiation parameter numbers with $\xi = 5,\ Gr = Gr_c = 1,\ Pr = 0.72,\ V_0 = 0.1,\ Sc = 1$.

results were obtained to illustrate the details of flow characteristics and their dependence on some of the physical parameters. It was found that when the Grashof number increased, the fluid velocity increased. However, this same effect was found to decrease both thermal and concentration boundary layers. The present analysis has shown that the flow is appreciably influenced by thermal radiation. It was observed that increasing the thermal radiation parameter produces significant increases in the thermal conditions of the fluid temperature which consequently induces more fluid in the boundary layer through buoyancy effect,



(a) (b)

Figure 6: The variation of (a) velocity and (b) the temperature distribution with increasing Grashof number values with $\xi = 5$, $Pr = 0.72$, $Sc = 1$, $V_0 = 0.1$, $R = 1$.



(a) (b)

Figure 7: The variation of concentration distribution (a) with increasing buoyancy numbers with $\xi = 5$, $Pr = 0.72$, $Sc = 1$, $V_0 = 0.1$, $R = 1$ and (b) with increasing radiation parameter numbers with $\xi = 5$, $Gr = Gr_c = 1$, $Pr = 0.72$, $Sc = 1$, $V_0 = 0.1$.

causing the velocity in the fluid there to increase. The hydrodynamic boundary layer and thermal boundary layer thicknesses were observed to increase as a result of increasing radiation. However, the concentration boundary layer thickness was reduced as a result of increases in the thermal radiation parameter. It was also observed that increasing the Schmidt number caused reduction in the concentration distribution in the boundary layer.

The study noted that velocity, temperature, and concentration profiles decrease with increases in the suction effect and that injection has opposite effects on these profiles. It is hoped that the present work will serve as a tool for understanding more complex problems involving various physical effects investigated in this study.

Nomenclature

a: Stretching constant
C: Species concentration at any point in the flow field
C_w: Species concentration at the wall
c_p: Specific heat at constant pressure
C_∞: Species concentration at the free stream
D: Molecular diffusivity of the species concentration
F: Nondimensional streamwise velocity
H: Nondimensional normal velocity
g: Acceleration due to gravity
Gr_c: Concentration buoyancy parameter
Gr: Grashof number
K: Mean absorption coefficient
L: Typical length scale
N: Buoyancy ratio
Pr: Prandtl number
q_r: Rossel and approximation
R: Thermal radiation parameter
Sc: Schmidt number
T: Fluid temperature at any point
T_w: Fluid temperature at the wall
T_∞: Free stream temperature
u: Streamwise velocity
v: Normal velocity
V_0: Dimensionless wall normal velocity
V_w: Suction/injection velocity
x: Streamwise coordinate axis
y: Normal coordinate axis.

Greek Symbol

α_t: Thermal conductivity
μ: Dynamic viscosity
ν: Kinematic viscosity
β_c: Volumetric coefficient expansion with concentration
β_t: Volumetric coefficient of thermal expansion
ρ: Density of the fluid
σ^*: Stefan-Boltzman constant
ξ: Scaled streamwise variable
η: Scaled normal variable
θ: Nondimensional temperature
ϕ: Dimensionless concentration
λ_c: Fluid thermal conductivity.

Subscripts

∞: Far away from the wall surface
w: At the wall surface.

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