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Effects of Technological Change on Non-renewable Resource Extraction and Exploration

Eiji Sawada · Shunsuke Managi

Abstract This paper provides a non-renewable resource extraction model with both technological change and resource exploration. Especially, we consider two types of technology, extraction technology and exploration technology. We show how these technologies affect efficient non-renewable resource extraction differently. Then, progress in extraction technology drops marginal revenue of extraction and resource price by changing the structure of those dynamics, while progress in exploration technology drops marginal revenue of extraction and resource price remaining the structure of those dynamics. Finally, we illustrate the difference becomes significant when innovative technologies are developed using numerical examples.

1 Introduction

As a means to secure scarce resources, technologies play a crucial role in both resource extraction and resource exploration. From the perspective of resource economics, fewer reserves not only fail to meet a certain demand, but also they may make another unit of extraction more costly. Accordingly, it is necessary to expend excessive resources on explorative activities to make extraction more economic. However, explorative activities by themselves cannot keep extraction costs low because explo-

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ration itself may become more costly, based on the accumulation of the new findings. Thus, some technological breakthrough is required to resolve this situation.

Many countries consider the improvement of two technologies to be important policy measures.[1] One is the improvement of extraction efficiency technology, which lowers the given extraction cost for the remaining reserves, and the other is the improvement of exploration efficiency technology, which increases the exploration efficiency given the cost that is already determined by that time.[2] Although the need to improve these two technologies is emphasized, there is little discussion regarding the difference between their effects on the resource extraction schedule. Given the circumstances surrounding scarce resources, how do we identify the appropriate technology for attaining efficient and stable resource use?

In the resource economics literature, earlier studies explain the effects of resource exploration and technological change on resource extraction to bridge the gap between the real resource price path and the theoretical resource price path derived by the Hotelling rule (Hotelling 1931). The observed real resource price does not always continue to increase according to the Hotelling rule; rather, it sometimes follows a flat path or even begins to decrease. Stewart (1979), Arrow and Cheng (1982) and Pindyck (1978) succeed in obtaining various non-renewable resource price paths by incorporating resource exploration into the traditional Hotelling model.[3] Additionally, Slade (1982) explains similar results due to technological changes in extraction. Theoretically, any price path could be feasible if we could choose an arbitrary speed of technological change over time. In recent studies, Lin and Wagner (2007) explain why the price path of many non-renewable resources empirically becomes almost constant by estimating the supply and demand function using the Slade (1982) framework.

These earlier studies explain the effect of exploration and technological change on the use of non-renewable resources. However, to the best of our knowledge, no existing study considers resource exploration and technological change in the same model while focusing on exploration technology. As previously mentioned, it is important to incorporate both resource exploration and technological change into the same model when considering the efficient use of resources with fewer reserves.

This paper provides a theory for examining the efficient extraction of non-renewable resources that incorporates both resource exploration and technological change. We show how two types of technological change differently affect the efficient extraction of a non-renewable resource. In Sect. 2, we expand the economic model used by Pindyck (1978) by incorporating the two types of technological change. We consider a profit maximizing monopolistic producer that exploits reserves with incremental technological progress. In Sect. 3, we examine the difference between the two technologies in terms of their effects on the dynamics of the resource

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[1]The strategies of major countries are summarized in Critical materials strategy 2011 issued by U.S. Department of Energy (2011). In addition to extraction and exploration, substitution is also important to secure resources. Im et al. (2006), Chakravorty (2008) and Chakravorty et al. (2011) focus on substitution among multiple resources.

[2]Examples of the extraction and exploration technologies are bio leaching technology and remote sensing technology.

[3]Cairns (1990) is a good survey of this literature.

price, and we show that the two technologies affect the price path differently. Extraction technology decreases the marginal revenue of extraction and resource price by changing the structure of those dynamics, while exploration technology decreases the marginal revenue of extraction and the resource price by maintaining the structure of those dynamics. In Sect. 4, we present some numerical examples, and we show that the difference between the effects of the two technologies becomes significant when technology changes intermittently rather than smoothly. In Sect. 5, we present our conclusions.

2 Resource Extraction and Exploration with Technological Change

We generalize the Pindyck (1978) monopolistic non-renewable resource extraction and exploration model by incorporating two types of technological change. A monopolistic producer chooses a level of production q_t from a resource stock R_t given an inverse demand function $p_t(q_t)$ at time t. To focus on the observation that how two technologies affect differently his or her optimal extraction and exploration, we assume that the monopolistic producer has perfect foresight on the technological progress. Time is discrete and runs through the interval $t \in [0, \infty]$. The average extraction cost is given by $C^1(R_t, z_t^1)$, where z_t^1 is an index of the state of extraction technology at time t. Following Farzin (1995), we do not assume investment for technological change, but we do assume $z_t^1 - z_{t-1}^1 > 0$, which implies an incremental improvement over time. The average extraction cost increases as the resource becomes depleted and decreases as the extraction technology is improved, and thus the average cost function satisfies $C_{R_t}^1 < 0$ and $C_{z_t^1}^1 < 0$. Throughout this paper, subscripts other than t denote partial derivatives.

Moreover, we assume that increases in the existing resource occur in response to the producer's explorative efforts denoted by w_t. We assume that the cost of the explorative effort is linear, as in Stewart (1979), but we also assume depletion of exploration. That is, the impact of one unit of explorative effort decreases based on the cumulative discoveries up to that time. The cost of exploration is expressed by $k w_t$ for a constant $k > 0$ and the total increase in resources is expressed by the discovery function $f(w_t, X_t, z_t^2)$, where X_t is the cumulative discoveries up to time t and z_t^2 indicates the state of exploration technology at time t. From our assumptions, the discovery function satisfies $f_{w_t} > 0$, $f_{X_t} < 0$. We further assume that exploration technology increases discoveries and improves incrementally over time, i.e., $f_{z_t^2} > 0$ and $z_t^2 - z_{t-1}^2 > 0$.

Furthermore, we put the usual assumptions on the second derivatives that $C_{R_t R_t} > 0$, $f_{w_t w_t} < 0$, $f_{X_t X_t} < 0$ and $f_{w_t X_t} < 0$.[4] Finally, we assume technological progress also decreases the depletion effect and increases the marginal discoveries, that is, $C_{R_t z_t^1} < 0$ and $f_{w_t z_t^2} > 0$.

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[4]These assumptions are required to satisfy the second order conditions of the profit maximization problem.

A monopolistic producer maximizes the sum of the present discounted value of the net profit:

$$\max_{q_t, w_t} \sum_{t=0}^{\infty} \rho^t \left[p_t(q_t)q_t - C^1(R_t, z_t^1)q_t - kw_t \right] \tag{1}$$

subject to

$$R_{t+1} - R_t = f(w_t, X_t, z_t^2) - q_t, \tag{2}$$

$$X_{t+1} - X_t = f(w_t, X_t, z_t^2), \tag{3}$$

where $\rho > 0$ is the discount factor derived from the constant market interest rate δ by $\rho := \frac{1}{1+\delta}$. The Lagrangian of this problem is

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \rho^t \big\{ & p_t(q_t)q_t - C^1(R_t, z_t^1)q_t - kw_t \\
& + \rho\lambda_{t+1}^1 \left(R_t + f(w_t, X_t, z_t^2) - q_t - R_{t+1} \right) \\
& + \rho\lambda_{t+1}^2 \left(X_t + f(w_t, X_t, z_t^2) - X_{t+1} \right) \big\}.
\end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial q_t} = \rho^t \left(MR_t - C^1(R_t, z_t^1) - \rho\lambda_{t+1}^1 \right) = 0, \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial w_t} = \rho^t \left(-k + \rho(\lambda_{t+1}^1 + \lambda_{t+1}^2) f_{w_t} \right) = 0, \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial R_t} = \rho^t \left(-C_{R_t}^1 q_t + \rho\lambda_{t+1}^1 - \lambda_t^1 \right) = 0, \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial X_t} = \rho^t \left(\rho\lambda_{t+1}^2 (1 + f_{X_t}) - \lambda_t^2 + \rho\lambda_{t+1}^1 f_{X_t} \right) = 0, \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{t+1}^1} = \rho^t \left(R_t + f(w_t, X_t, z_t^2) - q_t - R_{t+1} \right) = 0, \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{t+1}^2} = \rho^t \left(X_t + f(w_t, X_t, z_t^2) - X_{t+1} \right) = 0, \tag{9}$$

where $MR_t := p_t' q_t + p_t(q_t)$, i.e., MR_t is the marginal revenue by resource extraction at time t. Finally, the transversality conditions for the dynamics of extraction and explorative efforts are

$$\lim_{t \to \infty} \lambda_t^1 R_t = 0, \tag{10}$$

$$\lim_{t \to \infty} \lambda_t^2 = 0. \tag{11}$$

Equation (10) holds with complementary slackness (Farzin 1995). Equation (11) means that there are no additional costs associated with the cumulative discoveries

X_t as $t \to \infty$. Efficient extraction and exploration for the monopolistic producer are characterized by Eqs. (4)–(11).

3 The Difference Between Extraction and Exploration Technologies

First, we examine the effect of the extraction technology. Define α_t by

$$\alpha_t := MR_t - C^1(R_t, z_t^1). \tag{12}$$

α_t is the extraction rent for a monopolistic producer at time t. Then, by Eqs. (4), (6), and (8), we have the following modified Hotelling rule for resource extraction:

$$\frac{\alpha_t - \alpha_{t-1}}{\alpha_{t-1}} = \delta + \frac{C_{R_t}^1 (R_t - R_{t+1})}{\alpha_{t-1}} + \frac{C_{R_t}^1 f(w_t, X_t, z_t^2)}{\alpha_{t-1}}. \tag{13}$$

The LHS of Eq. (13) is the rate of extraction rent change and the RHS is the sum of the interest rate, reserve dependent cost effects and exploration effects, respectively. By the exploration effect, the monopolistic producer extracts their reserves in such a way that the extraction rent rises at less than the interest rate minus the reserve dependent cost effect (Pindyck 1978). To identify the effect of technological change of extraction, we rearrange Eq. (13) by a linear approximation of the difference in average extraction cost:

$$MR_t - MR_{t-1} = \delta\alpha_{t-1} + C_{R_t}^1 \left(R_t + f(w_t, X_t, z_t^2) - R_{t+1} \right)$$
$$+ C_{R_t}^1 (R_t - R_{t-1}) + C_{z_t^1}^1 (z_t^1 - z_{t-1}^1). \tag{14}$$

Equation (14) characterizes the dynamics of marginal revenue of extraction. The last term on the RHS of Eq. (14) is multiplied by the technological change of extraction. Thus the extraction technology changes the structure of the dynamics of the marginal revenue of extraction. Because by our assumption we have $C_{z_t^1}^1 (z_t^1 - z_{t-1}^1) < 0$, the marginal revenue of extraction rises more slowly as extraction technology advances. The level of marginal revenue also decreases because planned reserves would increase with technological progress. The same thing can be said about resource price (we show numerical examples later).

Next, we examine the effect of exploration technology. Define β_t by

$$\beta_t := \left(MR_t - C^1(R_t, z_t^1) \right) - \frac{k}{f_{w_t}}; \tag{15}$$

$(MR_t - C^1(R_t, z_t^1))$ is the increasing revenue from one unit of reserves found by explorative efforts and $\frac{k}{f_{w_t}}$ is the cost to find one unit of reserves. Thus, β_t is the exploration rent for a monopolistic producer at time t. By rearranging Eqs. (4), (5), and (7), noticing the definition of ρ, we have the following condition for efficient

resource exploration:[5]

$$\frac{\beta_t - \beta_{t-1}}{\beta_{t-1}} = \delta + \frac{k}{\beta_{t-1}} \frac{f_{X_t}}{f_{w_t}}. \tag{16}$$

This expression is very similar to the Hotelling rule. The second term of the RHS of Eq. (16) is the accumulation dependent effect. If the accumulated discoveries do not affect the increase in resources, the second term on the RHS of Eq. (16) vanishes. Then, under efficient exploration by a monopolistic producer, the exploration rent increases according to the interest rate.

To see the effect of the technological change of exploration, rearranging Eq. (16) by linear approximation of the difference of marginal discoveries by explorative efforts gives

$$\alpha_t - \alpha_{t-1} = \delta \beta_{t-1} - k \frac{f_{X_t}}{f_{w_t}} - \frac{k}{f_{w_t} f_{w_{t-1}}} \left(f_{w_t X_t} (X_t - X_{t-1}) + f_{w_t z_t^2} (z_t^2 - z_{t-1}^2) \right). \tag{17}$$

By substituting Eq. (17) into Eq. (13) and after some manipulations,[6]

$$
\begin{aligned}
& C_{R_t}^1 \left(R_t + f\left(w_t, X_t, z_t^2 \right) - R_{t+1} \right) \\
& = -\frac{k}{f_{w_t} f_{w_{t-1}}} \left[\delta f_{w_t} + f_{w_{t-1}} f_{X_t} + f_{w_t} f_{X_t} (X_t - X_{t-1}) \right] \\
& \quad - \left[\frac{k f_{w_t z_t^2} (z_t^2 - z_{t-1}^2)}{f_{w_t} f_{w_{t-1}}} \right].
\end{aligned}
\tag{18}
$$

We can substitute Eq. (18) into Eq. (14) to find an expression for $MR_t - MR_{t-1}$ depending on $z_t^2 - z_{t-1}^2$. However, the structure of the dynamics remains as in Eq. (14). Thus, technological change of exploration does not change the structure of the dynamics of the marginal revenue of extraction. This point is crucially different from the case for extraction technology.

The second term of RHS of Eq. (18) implies how much technological progress decreases the cost to find one unit of reserves, $\frac{k}{f_w}$. This cost reduction mitigates increasing of average extraction cost due to decreasing reserves, $C_{R_t}^1(\cdot)$. This is the reason that progress in exploration technology drops the marginal revenue (and resource price). We summarize the above discussion in the following proposition.

Proposition *The conditions for an efficient extraction and exploration schedule for a monopolistic producer with incremental technologies are characterized by Eqs. (13) and (16). Furthermore, extraction and exploration technologies affect the efficient extraction differently. Progress in extraction technology drops the marginal revenue of extraction and resource prices by changing the structure of the dynamics, whereas progress in exploration technology drops the marginal revenue of extraction and resource prices by maintaining the structure of the dynamics.*

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[5]See Appendix A.1 for a derivation in detail.

[6]See Appendix A.2 for a derivation in detail.

4 Numerical Examples

In the previous section, we found that extraction and exploration technology affect the efficient extraction differently. We argue that a technology choice is significant in actual policies if this difference brings about a substantial change to resource price, extraction, and exploration schedules. However, it is hard to observe the changes on schedules in detail in an analytical way. In this section, we therefore examine further properties of the two technologies with a numerical approach.

We illustrate some numerical examples in three scenarios (no progress, extraction progress, exploration progress) using the specified model.[7] For simplicity, we only consider the time interval $t \in [0, 29]$. First of all, we consider the case where technology changes once every 15 years or once every 10 years, i.e., some innovative technologies are applied to resource extraction or resource exploration. Secondly, we assume technology changes every year at a constant speed. In the no progress scenario, the two technologies are sustained on a constant level. Under the extraction progress and exploration progress scenarios, only one of the technologies will improve at $t = 15$ or at $t = 10$ and $t = 20$ or at every period. Following Pindyck (1978), we specify the demand function, the average extraction cost function and the discovery function as

$$q_t = a - bp_t, \tag{19}$$

$$C^1\left(R_t, z_t^1\right) = \frac{A}{R_t z_t^1}, \tag{20}$$

$$f\left(w_t, X_t, z_t^2\right) = \alpha w_t^\beta \exp\left(-\frac{\gamma}{z_t^2} X_t\right), \tag{21}$$

where $a, b, A, \alpha, \beta, \gamma$ are all positive constant.

Figure 1(a) illustrates the time paths for the resource price, Fig. 1(b) illustrates the time paths for the marginal revenue of extraction and Fig. 1(c) illustrates the time paths for the explorative efforts when technology changes only at $t = 15$. The no progress scenario shows the typical path for the Hotelling rule, where the resource price and marginal revenue rise over time. Under the extraction progress scenario, the paths for the resource price and marginal revenue change, starting from a lower level and rising more slowly after the technological progress. Conversely, under the exploration progress scenario, explorative efforts increase drastically with technological progress. However, the paths for the resource price and the marginal revenue of extraction shift slightly downward.

Figures 2(a), (b), and (c) illustrate the time paths when technology changes twice. The time paths for the resource price and the marginal revenue of extraction change with every improvement in extraction technology. By contrast, those paths again just shift downward under the exploration technology scenario. As our economic model shows in the previous section, technological change of exploration does not affect the structure of the dynamics of the resource price or the marginal revenue of extraction.

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[7]All solutions of the numerical calculations are indicated in Appendix B.
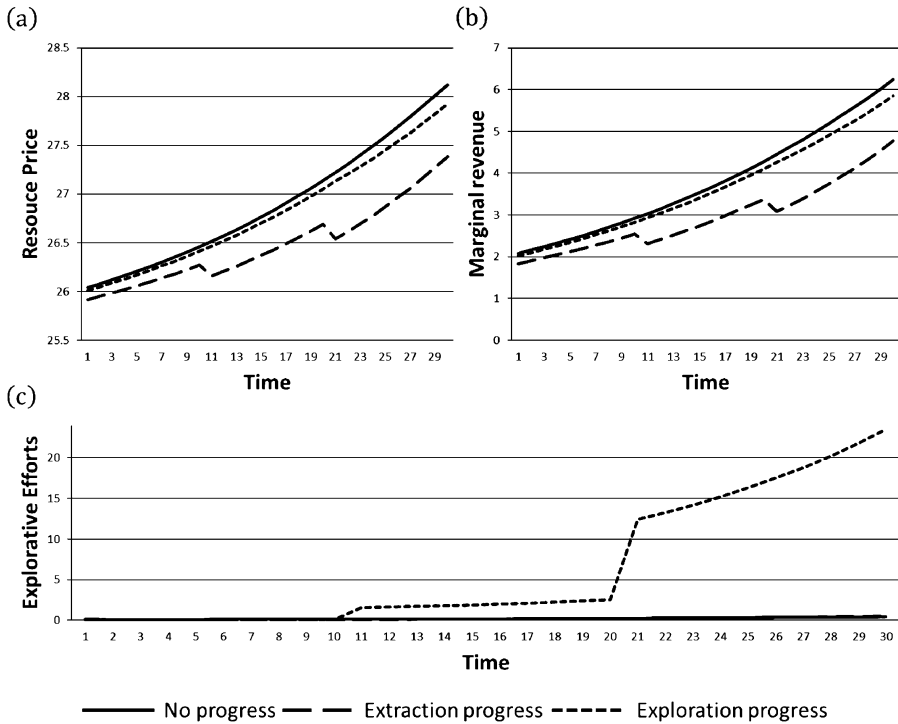
(a)

(b)

(c)

——— No progress — — Extraction progress ----- Exploration progress
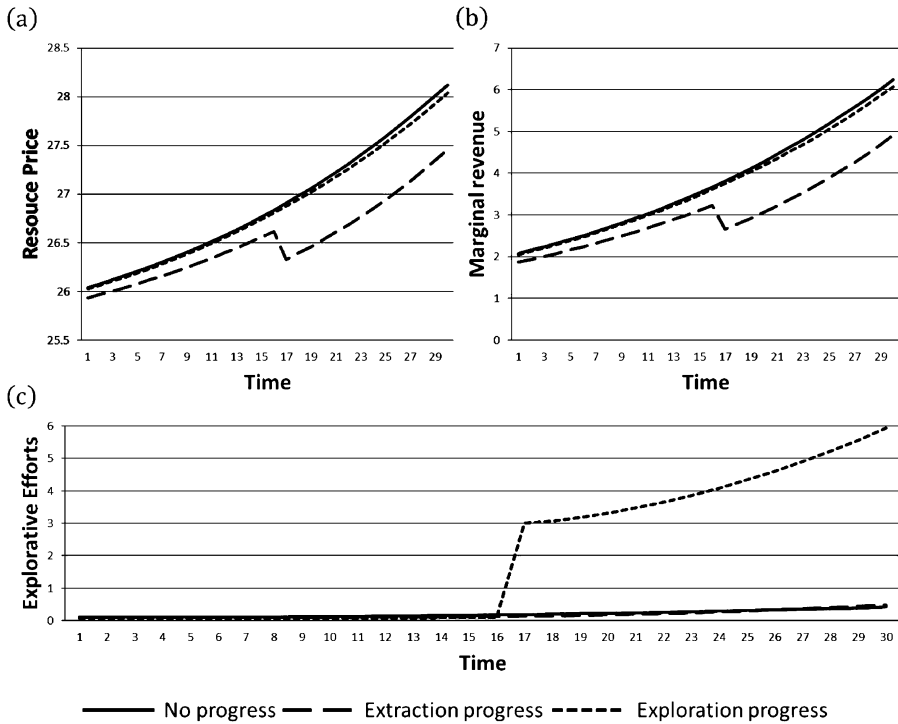
Fig. 1 Efficient schedules when technologies progress once. Note that $\delta = 0.05$, $\lambda_{30}^1 = 5$, $k = 0.5$, $a = 25$, $b = 0.5$, $A = 250$, $\alpha = 2$, $\beta = 0.5$, $\gamma = 0.5$ for all scenarios. **a** Time paths for the resource price when technology changes once every 15 years. In the *no progress scenario*, $z_t^1 = z_t^2 = 1$ for $t \in [0, 29]$. In the *extraction progress scenario*, $z_t^1 = 1$ for $t \in [0, 14]$, $z_t^1 = 10$ for $t \in [15, 29]$ and $z_t^2 = 1$ for $t \in [0, 29]$. In the *exploration progress scenario*, $z_t^1 = 1$ for $t \in [0, 29]$, $z_t^2 = 1$ for $t \in [0, 14]$ and $z_1^2 = 10$ for $t \in [15, 29]$. **b** Time paths for the marginal revenue of resource extraction for the same parameters as used in **a**. **c** Time paths for explorative efforts for the same parameters as used in **a**

We remark that the difference between technologies does not immediately determine the superiority of a technology. While extraction technology can lead to a large change in resource prices, it may make the resource price unstable and increase the risk for the demand on the resources. Moreover, the difference of effects between technologies becomes smaller when the speed of progress is constant. Figures 3(a), (b) and (c) illustrate the time paths when technology changes every year at constant speed. Here, unlike Fig. 1 and Fig. 2, every path is just growing over time. Therefore, the type of technology becomes more important, especially for innovative technologies.

This has important policy implications on the decision of research and development investment in the non-renewable resource area. As long as the price of the scarce resources is stable, policy makers do not have to pay much attention on the choice of technologies. However, if policy makers suffer from widely fluctuating prices of
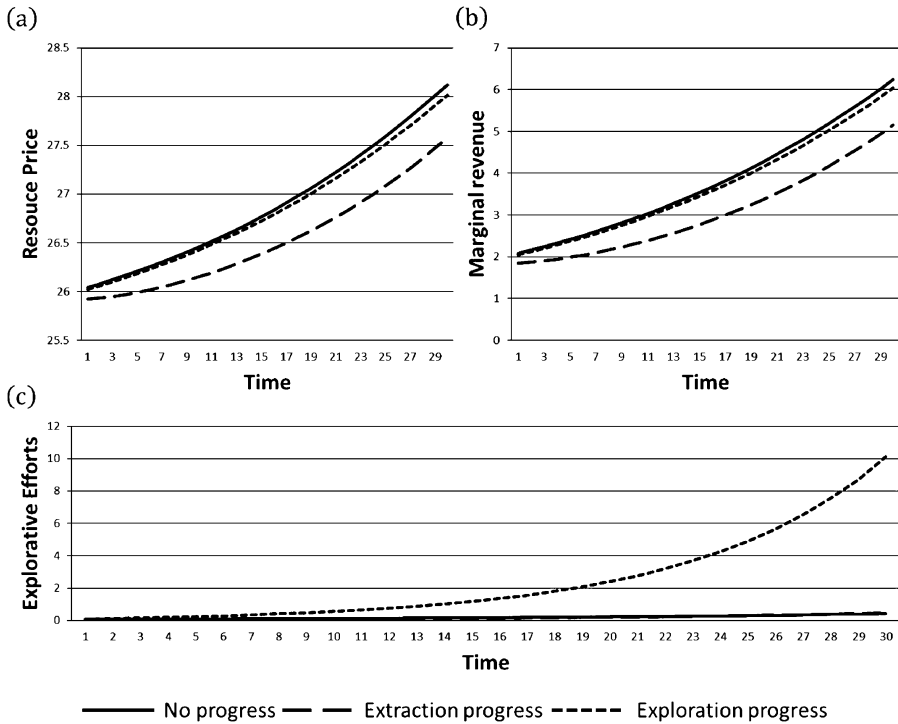
(a)

(b)

(c)

Fig. 2 Efficient schedules when technologies progress twice. Note that $\delta = 0.05$, $\lambda_{30}^1 = 5$, $k = 0.5$, $a = 25$, $b = 0.5$, $A = 250$, $\alpha = 2$, $\beta = 0.5$, $\gamma = 0.5$ for all scenarios. **a** Time paths for the resource price when technology changes once every 10 years. In the *no progress scenario*, $z_t^1 = z_t^2 = 1$ for $t \in [0, 29]$. In the *extraction progress scenario*, $z_t^1 = 1$ for $t \in [0, 9]$, $z_t^1 = 2$ for $t \in [10, 19]$, $z_t^1 = 250$ for $t \in [20, 29]$ and $z_t^2 = 1$ for $t \in [0, 29]$. In the *exploration progress scenario*, $z_t^1 = 1$ for $t \in [0, 29]$, $z_t^2 = 1$ for $t \in [0, 9]$, $z_t^2 = 10$ for $t \in [10, 19]$ and $z_1^2 = 50$ for $t \in [20, 29]$. **b** Time paths for the marginal revenue of resource extraction for the same parameters as used in **a**. **c** Time paths for explorative efforts for the same parameters as used in **a**

those resources, then they need to shift the innovation development grants from extraction to exploration.[8]

5 Conclusion

We have examined the dynamics of non-renewable resource extraction for a monopolistic producer using resource exploration and two types of technological change. Our analysis indicates that extraction and exploration technologies have different effects on efficient resource extraction. Extraction technology changes the structure of

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[8]In reality, the widely fluctuating prices of scarce resources often result in extra costs to policy makers and individual firms. Researches in financial economics have considered that the price path of the scarce resources is unpredictable because the price can easily be controlled by strategic or speculative activities (Radetzki 1989; Sari et al. 2010).

(a)

(b)

(c)

—— No progress — — Extraction progress ----- Exploration progress

Fig. 3 Efficient schedules when technologies progress at a constant speed. Note that $\delta = 0.05$, $\lambda_{30}^1 = 5$, $k = 0.5$, $a = 25$, $b = 0.5$, $A = 250$, $\alpha = 2$, $\beta = 0.5$, $\gamma = 0.5$ for all scenarios. **a** Time paths for the resource price when technology changes every year. In the *no progress scenario*, $z_t^1 = z_t^2 = 1$ for $t \in [0, 29]$. In the *extraction progress scenario*, $z_t^1 = 1 + t\Delta_1$, $\Delta_1 = 0.1$ for $t \in [0, 29]$ and $z_t^2 = 1$ for $t \in [0, 29]$. In the *exploration progress scenario*, $z_t^1 =$ for $t \in [0, 29]$ and $z_t^2 = 1 + t\Delta_2$, $\Delta_2 = 0.5$ for $t \in [0, 29]$. **b** Time paths for the marginal revenue of resource extraction for the same parameters as used in **a**. **c** Time paths for explorative efforts for the same parameters as used in **a**

the dynamics of the resource price, whereas exploration technology only changes the value of the resource price. Furthermore, we show that the difference becomes significant for innovative technologies for a specified model.

Thus far, the discussion of the effect of technology has proceeded without distinguishing between the types of technology in both theory and policymaking. However, the difference between the two technologies is expected to affect a number of issues related to efficient resource use. Accordingly, our findings will be applicable to other studies of non-renewable resource use.

Finally, this paper has two limitations. First, following the assumption in Farzin (1995), our study also assumed that an economic agent has perfect foresight on the technological progress. However, as noted in Farzin (1995), uncertainty can have significant implications for the dynamics of resource use. Moreover, uncertainty may also play an important role in terms of resource exploration. Pindyck (1980) and Cairns (1990) noted that the insight provided by the exploration process in a context of certainty is limited. Second, we assumed that marginal extraction cost is constant within a given period. This assumption ruled out the effect of technological progress

on the marginal change of the extraction cost. For example, Farzin (1995) has shown that the technology which decreases the marginal extraction cost and the one which decreases the depletion effects affect differently the paths of the marginal extraction cost, scarcity rent, and resource price using a general cost function. Therefore, the application of the general cost function might provide further classification of the technology type and may provide further insight into the technology choice problem. These interesting analyses are left for future studies.

Competing Interests

The authors declare that they have no competing interests.

Appendix A: Derivation of Key Equations

A.1 Derivation of Eq. (16)

By the definition of β_t and Eqs. (4) and (5),

$$\rho \lambda_{t+1}^1 = MR_t - C^1\left(R_t, z_t^1\right) = \beta_t + \frac{k}{f_{w_t}}, \tag{22}$$

$$\rho \lambda_{t+1}^2 = -\left(MR_t - C^1\left(R_t, z_t^1\right)\right) + \frac{k}{f_{w_t}} = -\beta_t, \tag{23}$$

$$\lambda_t^2 = -\frac{\beta_{t-1}}{\rho}. \tag{24}$$

By substituting Eqs. (22), (23), and (24) into Eq. (7), we have

$$\left(-\beta_t + \beta_t + \frac{k}{f_{w_t}}\right) f_{X_t} - \beta_t + \frac{\beta_{t-1}}{\rho} = 0. \tag{25}$$

By rearranging Eq. (25), noticing the definition of ρ, we have the following condition for efficient resource exploration:

$$\frac{\beta_t - \beta_{t-1}}{\beta_{t-1}} = \delta + \frac{k}{\beta_{t-1}} \frac{f_{X_t}}{f_{w_t}}.$$

A.2 Derivation of Eq. (18)

By Eq. (13),

$$\alpha_t - \alpha_{t-1} = \delta \alpha_{t-1} + C_{R_t}^1\left(R_t + f\left(w_t, X_t, z_t^2\right) - R_{t+1}\right). \tag{26}$$

Because $\alpha_{t-1} := MR_{t-1} - C^1(R_{t-1}, z_{t-1}^1)$,

$$\alpha_t - \alpha_{t-1} = \delta\left(MR_{t-1} - C^1\left(R_{t-1}, z_{t-1}^1\right)\right) + C_{R_t}^1\left(R_t + f\left(w_t, X_t, z_t^2\right) - R_{t+1}\right). \tag{27}$$

By substituting Eq. (17) into Eq. (27),

$$\delta\beta_{t-1} - k\frac{f_{X_t}}{f_{w_t}} - \frac{k}{f_{w_t}f_{w_{t-1}}}\big(f_{w_t X_t}(X_t - X_{t-1}) + f_{w_t z_t^2}\big(z_t^2 - z_{t-1}^2\big)\big)$$
$$= \delta\big(MR_{t-1} - C^1\big(R_{t-1}, z_{t-1}^1\big)\big) + C_{R_t}^1\big(R_t + f\big(w_t, X_t, z_t^2\big) - R_{t+1}\big). \quad (28)$$

Because $\beta_{t-1} := MR_{t-1} - C^1\big(R_{t-1}, z_{t-1}^1\big) - \frac{k}{f_{w_{t-1}}}$,

$$\delta\left(MR_{t-1} - C^1\big(R_{t-1}, z_{t-1}^1\big) - \frac{k}{f_{w_{t-1}}}\right)$$
$$- k\frac{f_{X_t}}{f_{w_t}} - \frac{k}{f_{w_t}f_{w_{t-1}}}\big(f_{w_t X_t}(X_t - X_{t-1}) + f_{w_t z_t^2}\big(z_t^2 - z_{t-1}^2\big)\big)$$
$$= \delta\big(MR_{t-1} - C^1\big(R_{t-1}, z_{t-1}^1\big)\big) + C_{R_t}^1\big(R_t + f\big(w_t, X_t, z_t^2\big) - R_{t+1}\big). \quad (29)$$

Rearranging Eq. (29), we have

$$C_{R_t}^1\big(R_t + f\big(w_t, X_t, z_t^2\big) - R_{t+1}\big)$$
$$= -\frac{k}{f_{w_t}f_{w_{t-1}}}\big[\delta f_{w_t} + f_{w_{t-1}}f_{X_t} + f_{w_t}f_{X_t}(X_t - X_{t-1})\big] - \left[\frac{kf_{w_t z_t^2}(z_t^2 - z_{t-1}^2)}{f_{w_t}f_{w_{t-1}}}\right].$$

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