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Properties of third-order half-linear dynamic equations with an unbounded neutral coefficient

Tongxing Li and Chenghui Zhang[*]

[*]Correspondence:
zchui@sdu.edu.cn
School of Control Science and
Engineering, Shandong University,
Jinan, Shandong 250061, P.R. China

Abstract

We study oscillation and asymptotic behavior of a class of third-order half-linear functional dynamic equations with an unbounded neutral coefficient. Several comparison theorems are presented that are essentially new.

MSC: 34K11; 34N05; 39A10

Keywords: asymptotic behavior; oscillation; neutral dynamic equation; third-order equation; time scale

1 Introduction

Neutral differential equations appear in modeling of the networks containing lossless transmission lines, in the study of vibrating masses attached to an elastic bar, as the Euler equation in some variational problems, in the theory of automatic control and in neuro-mechanical systems in which inertia plays an important role; see Hale [1].

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the theories of differential and of difference equations. Not only does the new theory of the so-called dynamic equations unify the theories of differential equations and difference equations, but also extends these classical cases to cases 'in between,' *e.g.*, to the so-called q-difference equations, when $\mathbb{T} = q^{\mathbb{N}_0} := \{q^t : t \in \mathbb{N}_0 \text{ for } q > 1\}$ (which has important applications in quantum theory (see [2])).

In this paper, we restrict our attention to oscillation and asymptotic behavior of the third-order half-linear neutral dynamic equation

$$\left[r\left(z^{\Delta\Delta}\right)^{\gamma}\right]^{\Delta}(t) + q(t)x^{\gamma}\left(\delta(t)\right) = 0, \tag{1.1}$$

where $t \in [t_0, \infty)_{\mathbb{T}} := [t_0, \infty) \cap \mathbb{T}$, $z := x + p \cdot x \circ \tau$, and we assume that the following conditions are satisfied:

(H$_1$)　$\gamma \leq 1$ is a quotient of odd positive integers;

(H$_2$)　$p \in C_{\mathrm{rd}}([t_0, \infty)_{\mathbb{T}}, [0, \infty))$ and $q \in C_{\mathrm{rd}}([t_0, \infty)_{\mathbb{T}}, (0, \infty))$;

(H$_3$)　$r \in C^1_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$, $\tau, \delta, \delta^{-1} \in C^1_{\mathrm{rd}}(\mathbb{T}, \mathbb{T})$, $r(t) > 0$, and $\lim_{t\to\infty} \tau(t) = \lim_{t\to\infty} \delta(t) = \infty$, where δ^{-1} denotes the inverse function of δ;

(H$_4$) $\tau([t_0, \infty)_{\mathbb{T}}) = [\tau(t_0), \infty)_{\mathbb{T}}$, $\delta^{-1}([t_0, \infty)_{\mathbb{T}}) = [\delta^{-1}(t_0), \infty)_{\mathbb{T}}$, $\tau^\Delta > 0$, and $(\delta^{-1})^\Delta > 0$.

We consider only those solutions x of (1.1) which satisfy $\sup\{|x(t)| : t \in [T, \infty)_{\mathbb{T}}\} > 0$ for all $T \in [t_0, \infty)_{\mathbb{T}}$ and assume that (1.1) possesses such solutions. As usual, a solution of (1.1) is called oscillatory if it has arbitrarily large generalized zeros on $[t_0, \infty)_{\mathbb{T}}$; otherwise, it is termed nonoscillatory.

In the last decade, a considerable number of studies have been made on oscillation and nonoscillation of solutions to various types of dynamic equations on time scales. We refer the reader to [3–30] and the references cited therein. For oscillation of dynamic equations, the authors in [7, 9, 16, 17, 22, 29] studied the first-order delay dynamic equation

$$x^\Delta(t) + p(t)x(\tau(t)) = 0.$$

Agarwal *et al.* [4] considered the second-order delay dynamic equation

$$x^{\Delta\Delta}(t) + p(t)x(\tau(t)) = 0.$$

See also Braverman and Karpuz [10]. Agarwal *et al.* [6] and Saker [23] investigated the second-order half-linear neutral delay dynamic equation

$$\left[r(z^\Delta)^\gamma\right]^\Delta(t) + q(t)x^\gamma(t - \delta) = 0,$$

where $z(t) := x(t) + p(t)x(t - \tau)$ and $0 \leq p(t) < 1$. Regarding oscillation and asymptotic behavior of third-order dynamic equations, Erbe *et al.* [12] studied the equation

$$x^{\Delta^3}(t) + p(t)x(t) = 0.$$

Agarwal *et al.* [3] extended the results of [12] to the third-order delay dynamic equation

$$\left(r_1(r_2 x^\Delta)^\Delta\right)^\Delta(t) + q(t)x(\tau(t)) = 0.$$

Agarwal *et al.* [5], Hassan [14], and Li *et al.* [20, 21] examined equation (1.1) in the case where $p(t) = 0$. Assuming $0 \leq p(t) \leq 1$ or $-1 \leq p(t) \leq 0$, Grace *et al.* [13], Saker and Graef [24], Yang [27, 28], and Zhang *et al.* [30] obtained some oscillation results for (1.1). The analogue for (1.1) in the case $\mathbb{T} = \mathbb{Z}$ has been studied in the recent paper by Thandapani and Kavitha [31].

So far, there are very few results for oscillation and asymptotic properties of (1.1) in the case

$$\lim_{t \to \infty} p(t) = \infty. \tag{1.2}$$

Therefore, we use a comparison method to study (1.1) under the assumption that (1.2) is satisfied. In the sequel, all inequalities are assumed to hold eventually, that is, for all t large enough.

2 Main results

In what follows, τ^{-1} denotes the inverse function of τ,

$$Q(t) := \min\{q(\delta^{-1}(t)), q(\delta^{-1}(\tau(t)))\} \quad \text{and}$$

$$H(t) := \max\left\{\frac{1}{(\delta^{-1})^{\Delta}(t)}, \frac{p^{\gamma}(t)}{(\delta^{-1})^{\Delta}(\tau(t))\tau^{\Delta}(t)}\right\}.$$

Before stating the main results, we begin with the following lemma.

Remark 2.1 It follows from assumptions (H_3), (H_4), and [8, Theorem 1.93] that

$$(y(\delta^{-1}(t)))^{\Delta} = y^{\Delta}(\delta^{-1}(t))(\delta^{-1})^{\Delta}(t), \qquad (y(\delta^{-1}(\tau(t))))^{\Delta} = y^{\Delta}(\delta^{-1}(\tau(t)))(\delta^{-1}(\tau(t)))^{\Delta},$$

and

$$(\delta^{-1}(\tau(t)))^{\Delta} = (\delta^{-1})^{\Delta}(\tau(t))\tau^{\Delta}(t),$$

where y^{Δ} exists for $t \in \mathbb{T}^{k}$.

Lemma 2.1 (See [29]) *Assume* $p(t) \geq 0$, $\tau(t) \leq t$, *and* $\lim_{t\to\infty} \tau(t) = \infty$. *If*

$$\limsup_{t\to\infty} \sup_{\lambda \in E}\{\lambda e_{-\lambda p}(t, \tau(t))\} < 1, \quad \text{where } E := \{\lambda | \lambda > 0, 1 - \lambda p(t)\mu(t) > 0\},$$

then the delay dynamic inequality

$$x^{\Delta}(t) + p(t)x(\tau(t)) \leq 0 \tag{2.1}$$

has no eventually positive solutions.

Lemma 2.2 (See [17]) *Assume* $p(t) \geq 0$, $\tau(t) \leq t$ *and is nondecreasing with*

$$\lim_{t\to\infty} \tau(t) = \infty.$$

If there exists a $\lambda \in [0,1]$ *such that*

$$\liminf_{t\to\infty} \int_{\tau(t)}^{t} p(s)\Delta s > \lambda \quad \text{and} \quad \limsup_{t\to\infty} \int_{\tau(t)}^{\sigma(t)} p(s)\Delta s > 1 - (1 - \sqrt{1-\lambda})^2,$$

then the delay dynamic inequality (2.1) *has no eventually positive solutions.*

Lemma 2.3 *Assume* (1.2) *and let*

$$\int_{t_0}^{\infty} \frac{\Delta t}{r^{1/\gamma}(t)} = \infty. \tag{2.2}$$

If x *is a positive solution of* (1.1) *satisfying* $\lim_{t\to\infty} x(t) \neq 0$, *then* z *satisfies*

$$z > 0, \qquad z^{\Delta} > 0, \qquad z^{\Delta\Delta} > 0, \qquad (r(z^{\Delta\Delta})^{\gamma})^{\Delta} < 0, \tag{2.3}$$

eventually.

Proof Similar as in the proof of [30, Lemma 2.3], one obtains by (2.2) that either (2.3) holds or

$$z > 0, \qquad z^{\Delta} < 0, \qquad z^{\Delta\Delta} > 0, \qquad \left(r\left(z^{\Delta\Delta}\right)^{\gamma}\right)^{\Delta} < 0.$$

Since $\lim_{t\to\infty} p(t) = \infty$ and $\limsup_{t\to\infty} x(t) > 0$ (which implies that $\limsup_{t\to\infty} x(\tau(t)) > 0$ by (H$_4$)), it follows from $z(t) \geq p(t)x(\tau(t))$ that $\lim_{t\to\infty} z(t) = \infty$. Thus, the latter case cannot occur. The proof is complete. □

Lemma 2.4 *Assume that* (2.3) *is satisfied. Then*

$$z(t) \geq \left(r(t)\left(z^{\Delta\Delta}(t)\right)^{\gamma}\right)^{1/\gamma} \int_{t_1}^{t} \frac{t - \sigma(s)}{r^{1/\gamma}(s)} \Delta s. \tag{2.4}$$

Proof Since $r(z^{\Delta\Delta})^{\gamma}$ is decreasing, we obtain

$$z^{\Delta}(t) \geq z^{\Delta}(t) - z^{\Delta}(t_1) = \int_{t_1}^{t} \frac{(r(s)(z^{\Delta\Delta}(s))^{\gamma})^{1/\gamma}}{r^{1/\gamma}(s)} \Delta s \geq \left(r(t)\left(z^{\Delta\Delta}(t)\right)^{\gamma}\right)^{1/\gamma} \int_{t_1}^{t} \frac{\Delta s}{r^{1/\gamma}(s)}.$$

Thus

$$z(t) \geq \left(r(t)\left(z^{\Delta\Delta}(t)\right)^{\gamma}\right)^{1/\gamma} \int_{t_1}^{t} \int_{t_1}^{u} \frac{1}{r^{1/\gamma}(s)} \Delta s \Delta u.$$

By virtue of [15, Lemma 1], we have

$$\int_{t_1}^{t} \int_{t_1}^{u} \frac{1}{r^{1/\gamma}(s)} \Delta s \Delta u = \int_{t_1}^{t} \int_{\sigma(s)}^{t} \frac{1}{r^{1/\gamma}(s)} \Delta u \Delta s = \int_{t_1}^{t} \frac{t - \sigma(s)}{r^{1/\gamma}(s)} \Delta s.$$

Therefore, one has (2.4). This completes the proof. □

Below, we assume that $t_1 \in [t_0, \infty)_{\mathbb{T}}$ is large enough.

Theorem 2.1 *Assume* (1.2) *and* (2.2). *If the first-order neutral dynamic inequality*

$$\left(y\left(\delta^{-1}(t)\right) + y\left(\delta^{-1}\left(\tau(t)\right)\right)\right)^{\Delta} + \frac{Q(t)}{H(t)} \left(\int_{t_1}^{t} \frac{t - \sigma(s)}{r^{1/\gamma}(s)} \Delta s\right)^{\gamma} y(t) \leq 0 \tag{2.5}$$

has no positive solutions, then every solution of (1.1) *is oscillatory or tends to zero as* $t \to \infty$.

Proof Let x be a nonoscillatory solution of (1.1) and $\lim_{t\to\infty} x(t) \neq 0$. Without loss of generality, we may assume $x > 0$ eventually. Then we have (2.3) due to Lemma 2.3. It follows from (1.1) and [8, Theorem 1.93] that for all sufficiently large t,

$$\frac{(r(\delta^{-1}(t))(z^{\Delta\Delta}(\delta^{-1}(t)))^{\gamma})^{\Delta}}{(\delta^{-1})^{\Delta}(t)} + p^{\gamma}(t) \frac{(r(\delta^{-1}(\tau(t)))(z^{\Delta\Delta}(\delta^{-1}(\tau(t))))^{\gamma})^{\Delta}}{(\delta^{-1}(\tau(t)))^{\Delta}}$$

$$+ q\left(\delta^{-1}(t)\right)x^{\gamma}(t) + p^{\gamma}(t)q\left(\delta^{-1}\left(\tau(t)\right)\right)x^{\gamma}\left(\tau(t)\right) = 0. \tag{2.6}$$

By virtue of [32, Lemma 2] and the definition of z, we obtain

$$
\begin{aligned}
q\big(\delta^{-1}(t)\big)x^{\gamma}(t) + p^{\gamma}(t)q\big(\delta^{-1}\big(\tau(t)\big)\big)x^{\gamma}\big(\tau(t)\big) &\geq Q(t)\big[x^{\gamma}(t) + p^{\gamma}(t)x^{\gamma}\big(\tau(t)\big)\big] \\
&\geq Q(t)\big[x(t) + p(t)x\big(\tau(t)\big)\big]^{\gamma} \\
&= Q(t)z^{\gamma}(t).
\end{aligned}
\tag{2.7}
$$

Applications of (2.6) and (2.7) yield

$$
\begin{aligned}
&\frac{(r(\delta^{-1}(t))(z^{\Delta\Delta}(\delta^{-1}(t)))^{\gamma})^{\Delta}}{(\delta^{-1})^{\Delta}(t)} + p^{\gamma}(t)\frac{(r(\delta^{-1}(\tau(t)))(z^{\Delta\Delta}(\delta^{-1}(\tau(t))))^{\gamma})^{\Delta}}{(\delta^{-1}(\tau(t)))^{\Delta}} \\
&\quad + Q(t)z^{\gamma}(t) \leq 0.
\end{aligned}
\tag{2.8}
$$

Therefore, we get by (2.8) and the definition of H that

$$
\big(r\big(\delta^{-1}(t)\big)\big(z^{\Delta\Delta}\big(\delta^{-1}(t)\big)\big)^{\gamma} + r\big(\delta^{-1}\big(\tau(t)\big)\big)\big(z^{\Delta\Delta}\big(\delta^{-1}\big(\tau(t)\big)\big)\big)^{\gamma}\big)^{\Delta} + \frac{Q(t)}{H(t)}z^{\gamma}(t) \leq 0,
$$

which implies by (2.4) that

$$
\begin{aligned}
&\big(r\big(\delta^{-1}(t)\big)\big(z^{\Delta\Delta}\big(\delta^{-1}(t)\big)\big)^{\gamma} + r\big(\delta^{-1}\big(\tau(t)\big)\big)\big(z^{\Delta\Delta}\big(\delta^{-1}\big(\tau(t)\big)\big)\big)^{\gamma}\big)^{\Delta} \\
&\quad + \frac{Q(t)}{H(t)}r(t)\big(z^{\Delta\Delta}(t)\big)^{\gamma}\left(\int_{t_1}^{t}\frac{t-\sigma(s)}{r^{1/\gamma}(s)}\Delta s\right)^{\gamma} \leq 0.
\end{aligned}
\tag{2.9}
$$

Thus, using $r(z^{\Delta\Delta})^{\gamma} := y$ in (2.9), one can see that y is a positive solution of (2.5). This contradicts our assumptions and the proof is complete. □

Applying additional conditions on the arguments of (2.5), one can deduce from Theorem 2.1 various criteria for (1.1).

Theorem 2.2 *Assume* (1.2), (2.2), *and* $\tau(t) \leq t$. *If the first-order dynamic inequality*

$$
w^{\Delta}(t) + \frac{Q(t)}{2H(t)}\left(\int_{t_1}^{t}\frac{t-\sigma(s)}{r^{1/\gamma}(s)}\Delta s\right)^{\gamma}w\big(\tau^{-1}\big(\delta(t)\big)\big) \leq 0
\tag{2.10}
$$

has no positive solutions, then every solution of (1.1) *is oscillatory or tends to zero as* $t \to \infty$.

Proof We assume that x is a positive solution of (1.1) and $\lim_{t\to\infty} x(t) \neq 0$. As in the proof of Theorem 2.1, $y := r(z^{\Delta\Delta})^{\gamma} > 0$ is decreasing and satisfies (2.5). Let us denote

$$
w(t) := y\big(\delta^{-1}(t)\big) + y\big(\delta^{-1}\big(\tau(t)\big)\big).
\tag{2.11}
$$

It follows from $\tau(t) \leq t$ that

$$
w(t) \leq 2y\big(\delta^{-1}\big(\tau(t)\big)\big).
$$

Substituting this into (2.5), we get that w is a positive solution of (2.10). This contradiction completes the proof. □

Corollary 2.1 *Assume* (1.2), (2.2), *and* $\tau(t) \leq t$. *If* $\delta(t) \leq \tau(t)$ *and*

$$\limsup_{t \to \infty} \sup_{\lambda \in E} \left\{ \lambda e_{-\lambda Q_1}\left(t, \tau^{-1}\left(\delta(t)\right)\right) \right\} < 1, \tag{2.12}$$

where

$$E := \left\{ \lambda \mid \lambda > 0, 1 - \lambda Q_1(t)\mu(t) > 0 \right\} \quad and \quad Q_1(t) := \frac{Q(t)}{2H(t)}\left(\int_{t_1}^{t} \frac{t - \sigma(s)}{r^{1/\gamma}(s)} \Delta s \right)^{\gamma},$$

then every solution of (1.1) *is oscillatory or tends to zero as* $t \to \infty$.

Proof According to Lemma 2.1, condition (2.12) guarantees that (2.10) has no positive solutions. Application of Theorem 2.2 completes the proof. □

Corollary 2.2 *Assume* (1.2), (2.2), $\tau(t) \leq t$, *and* $\delta(t) \leq \tau(t)$. *If there exists a* $\lambda \in [0, 1]$ *such that*

$$\liminf_{t \to \infty} \int_{\tau^{-1}(\delta(t))}^{t} Q_1(s)\Delta s > \lambda \quad and \quad \limsup_{t \to \infty} \int_{\tau^{-1}(\delta(t))}^{\sigma(t)} Q_1(s)\Delta s > 1 - (1 - \sqrt{1 - \lambda})^2, \tag{2.13}$$

where Q_1 *is defined as in Corollary* 2.1, *then every solution of* (1.1) *is oscillatory or tends to zero as* $t \to \infty$.

Proof By virtue of Lemma 2.2, condition (2.13) implies that (2.10) has no positive solutions. Application of Theorem 2.2 yields the result. □

Theorem 2.3 *Assume* (1.2), (2.2), *and* $\tau(t) \geq t$. *If the first-order dynamic inequality*

$$w^{\Delta}(t) + \frac{Q(t)}{2H(t)}\left(\int_{t_1}^{t} \frac{t - \sigma(s)}{r^{1/\gamma}(s)} \Delta s \right)^{\gamma} w\left(\delta(t)\right) \leq 0 \tag{2.14}$$

has no positive solutions, then every solution of (1.1) *is oscillatory or tends to zero as* $t \to \infty$.

Proof We assume that x is a positive solution of (1.1) and $\lim_{t \to \infty} x(t) \neq 0$. As in the proof of Theorem 2.1, $y := r(z^{\Delta\Delta})^{\gamma} > 0$ is decreasing and satisfies (2.5). We denote w by (2.11). In view of $\tau(t) \geq t$, we obtain

$$w(t) \leq 2y\left(\delta^{-1}(t)\right).$$

Substitution of this term into (2.5) implies that w is a positive solution of (2.14). This contradiction completes the proof. □

Corollary 2.3 *Assume* (1.2), (2.2), *and* $\tau(t) \geq t$. *If* $\delta(t) \leq t$ *and*

$$\limsup_{t \to \infty} \sup_{\lambda \in E} \left\{ \lambda e_{-\lambda Q_1}\left(t, \delta(t)\right) \right\} < 1, \tag{2.15}$$

where E *and* Q_1 *are defined as in Corollary* 2.1, *then every solution of* (1.1) *is oscillatory or tends to zero as* $t \to \infty$.

Proof By virtue of Lemma 2.1, condition (2.15) ensures that (2.14) has no positive solutions. Application of Theorem 2.3 yields the result. □

Corollary 2.4 *Assume* (1.2), (2.2), $\tau(t) \geq t$, *and* $\delta(t) \leq t$. *If there exists a* $\lambda \in [0, 1]$ *such that*

$$\liminf_{t \to \infty} \int_{\delta(t)}^{t} Q_1(s) \Delta s > \lambda \quad and \quad \limsup_{t \to \infty} \int_{\delta(t)}^{\sigma(t)} Q_1(s) \Delta s > 1 - (1 - \sqrt{1 - \lambda})^2, \tag{2.16}$$

where Q_1 *is defined as in Corollary* 2.1, *then every solution of* (1.1) *is oscillatory or tends to zero as* $t \to \infty$.

Proof By Lemma 2.2, condition (2.16) guarantees that (2.14) has no positive solutions. Application of Theorem 2.3 completes the proof. □

Remark 2.2 Note that oscillation results can be also obtained for $\gamma \geq 1$; in this case, one simply has to replace Q in [32, Lemma 1] with a function $Q/2^{\gamma-1}$ and proceed as above.

Example 2.1 For $t \geq 1$, consider the third-order neutral differential equation

$$\left[x(t) + tx(t-1)\right]''' + \left[e^{-2} + (t-3)e^{-1}\right]x(t-2) = 0. \tag{2.17}$$

It is not difficult to verify that $Q(t)/H(t) \geq 1$. Applications of Theorem 2.2 and [18, Theorem 2.1.1] imply that every solution of (2.17) is oscillatory or satisfies $\lim_{t \to \infty} x(t) = 0$. As a matter of fact, one such solution is $x(t) = e^{-t}$.

Remark 2.3 Some other examples may be given easily. For instance, we take $\tau(t) = t - \tau_0$ and $\delta(t) = t - \delta_0$ for $\mathbb{T} = \mathbb{Z}$, we put $\tau(t) = t + hk$ and $\delta(t) = t - hk$ for $\mathbb{T} = h\mathbb{Z} := \{hk : k \in \mathbb{Z}\}$, we set $\tau(t) = qt$ and $\delta(t) = t/q$ for $\mathbb{T} = q^{\mathbb{N}_0}$, etc.

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