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Multi-valued (ψ, ϕ, ε, λ)-contraction in probabilistic metric space

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Abstract

In this article, we present a new definition of a class of contraction for multi-valued
case. Also we prove some fixed point theorems for multivalued (ψ, ϕ, ε, λ)-
contraction mappings in probabilistic metric space.

Keywords: probabilistic metric space, (ψ, φ, ε, λ)-contraction, fixed point

1 Introduction

The class of (ε, λ)-contraction as a subclass of B-contraction in probabilistic metric
space was introduced by Mihet [1]. He and other researchers achieved to some inter-
esting results about existence of fixed point in probabilistic and fuzzy metric spaces
[2-4]. Mihet defined the class of (ψ, ϕ, ε, λ)-contraction in fuzzy metric spaces [4]. On
the other hand, Hadzic et al. extended the concept of contraction to the multi valued
case [5]. They introduced multi valued (ψ - C)-contraction [6] and obtained fixed
point theorem for multi valued contraction [7]. Also Žikić generalized multi valued
case of Hick's contraction [8]. We extended (ϕ - k) - B contraction which introduced
by Mihet [9] to multi valued case [10]. Now, we will define the class of (ψ, ϕ, ε, λ)-
contraction in the sense of multi valued and obtain fixed point theorem.

The structure of article is as follows: Section 2 recalls some notions and known
results in probabilistic metric spaces and probabilistic contractions. In Section 3, we
will prove three theorems for multi valued (ψ, ϕ, ε, λ)- contraction.

2 Preliminaries

We recall some concepts from the books [11-13].

Definition 2.1. A mapping $T : [0, 1] \times [0, 1] \to [0, 1]$ is called a triangular norm (a
t-norm) if the following conditions are satisfied:

(1) $T(a, 1) = a$ for every $a \in [0, 1]$;

(2) $T(a, b) = T(b, a)$ for every $a, b \in [0, 1]$;

(3) $a \geq b, c \geq d \Rightarrow T(a, c) \geq T(b, d)$ $a, b, c, d \in [0, 1]$;

(4) $T(T(a, b), c) = T(a, T(b, c))$, $a, b, c \in [0, 1]$.

Basic examples are, $T_L(a, b) = \max\{a + b - 1, 0\}$, $T_P(a, b) = ab$ and $T_M(a, b) = \min\{a, b\}$.

Definition 2.2. If T is a t-norm and $(x_1, x_2, \ldots, x_n) \in [0, 1]^n$ $(n \geq 1)$, $\top_{i=1}^{\infty} x_i$ is
defined recurrently by $\top_{i=1}^{1} x_i = x_1$ and $\top_{i=1}^{n} x_i = T\left(\top_{i=1}^{n-1} x_i, x_n\right)$ for all $n \geq 2$. T can be

extended to a countable infinitary operation by defining $\top_{i=1}^{\infty} x_i$ for any sequence $(x_i)_{i\in N*}$ as $\lim_{n\to\infty} \top_{i=1}^{n} x_i$.

Definition 2.3. Let Δ_+ be the class of all distribution of functions $F : [0, \infty] \to [0, 1]$ such that:

(1) $F(0) = 0$,

(2) F is a non-decreasing,

(3) F is left continuous mapping on $[0, \infty]$.

D_+ is the subset of Δ_+ which $\lim_{x\to\infty} F(x) = 1$.

Definition 2.4. The ordered pair (S, F) is said to be a probabilistic metric space if S is a nonempty set and $F : S \times S \to D_+$ ($F(p, q)$ written by F_{pq} for every $(p, q) \in S \times S$) satisfies the following conditions:

(1) $F_{uv}(x) = 1$ for every $x > 0 \Rightarrow u = v$ ($u, v \in S$),

(2) $F_{uv} = F_{vu}$ for every $u, v \in S$,

(3) $F_{uv}(x) = 1$ and $F_{vw}(y) = 1 \Rightarrow F_{u,w}(x + y) = 1$ for every $u, v, w \in S$, and every $x, y \in R^+$.

A Menger space is a triple (S, F, T) where (S, F) is a probabilistic metric space, T is a triangular norm (abbreviated t-norm) and the following inequality holds $F_{uv}(x + y) \geq T$ $(F_{uw}(x), F_{wv}(y))$ for every $u, v, w \in S$, and every $x, y \in R^+$.

Definition 2.5. Let $\phi : (0, 1) \to (0, 1)$ be a mapping, we say that the t-norm T is ϕ-convergent if

$$\forall \delta \in (0, 1) \forall \lambda \in (0, 1) \exists s = s(\delta, \lambda) \in N \top_{i=1}^{n}(1 - \varphi^{s+i}(\delta)) > 1 - \lambda, \forall n \geq 1.$$

Definition 2.6. A sequence $(x_n)_{n\in N}$ is called a convergent sequence to $x \in S$ if for every $\varepsilon > 0$ and $\lambda \in (0, 1)$ there exists $N = N(\varepsilon, \lambda) \in N$ such that $F_{x_n x}(\varepsilon) > 1 - \lambda, \forall n \geq N$.

Definition 2.7. A sequence $(x_n)_{n\in N}$ is called a Cauchy sequence if for every $\varepsilon > 0$ and $\lambda \in (0, 1)$ there exists $N = N(\varepsilon, \lambda) \in N$ such that $F_{x_n x_{n+m}}(\varepsilon) > 1 - \lambda, \forall n \geq N \forall m \in \mathbb{N}$.

We also have

$$x_n \to^F x \Leftrightarrow F_{x_n x}(t) \to 1 \ \forall t > 0.$$

A probabilistic metric space (S, F, T) is called sequentially complete if every Cauchy sequence is convergent.

In the following, 2^S denotes the class of all nonempty subsets of the set S and $C(S)$ is the class of all nonempty closed (in the F-topology) subsets of S.

Definition 2.8 [14]. Let F be a probabilistic distance on S and $M \in 2^S$. A mapping f: $S \to 2^S$ is called continuous if for every $\varepsilon > 0$ there exists $\delta > 0$, such that

$$F_{uv}(\delta) > 1 - \delta \Rightarrow \forall x \in fu \exists y \in fv : F_{xy}(\varepsilon) > 1 - \varepsilon.$$

Theorem 2.1 [14]. Let (S, F, T) be a complete Menger space, $\sup_{0 \leq t < 1} T(t, t) = 1$ and $f : S \to C(S)$ be a continuous mapping. If there exist a sequence $(t_n)_{n\in N} \subseteq (0, \infty)$ with $\sum_1^{\infty} t_n < \infty$ and a sequence $(x_n)_{n\in N} \subseteq S$ with the properties:

$$x_{n+1} \in fx_n \text{ for all } n \text{ and } \lim_{n\to\infty} \top_{i=1}^{\infty} g_{n+i-1} = 1,$$

Where $g_n := F_{x_n x_{n+1}}(t_n)$, then f has a fixed point.

The concept of $(\psi, \phi, \varepsilon, \lambda)$ - B contraction has been introduced by Mihet [15]. We will consider comparison functions from the class φ of all mapping $\phi : (0, 1) \to (0, 1)$ with the properties:

(1) ϕ is an increasing bijection;

(2) $\phi(\lambda) < \lambda \ \forall \lambda \in (0, 1)$.

Since every such a comparison mapping is continuous, it is easy to see that if $\phi \in \varphi$, then $\lim_{n\to\infty}\phi^n(\lambda) = 0 \ \forall \lambda \in (0, 1)$.

Definition 2.9[15]. Let $(X, M, *)$ be a fuzzy Metric space. ψ be a map from $(0, \infty)$ to $(0, \infty)$ and ϕ be a map from $(0, 1)$ to $(0, 1)$. A mapping f: $X \to X$ is called $(\psi, \phi, \varepsilon, \lambda)$-contraction if for any $x, y \in X$, $\varepsilon > 0$ and $\lambda \in (0, 1)$.

$$M(x, y, \varepsilon) > 1 - \lambda \Rightarrow M(f(x), f(y), \psi(\varepsilon)) > 1 - \varphi(\lambda).$$

If ψ is of the form of $\psi(\varepsilon) = k\varepsilon \ (k \in (0, 1))$, one obtains the contractive mapping considered in [3].

3 Main results

In this section we will generalize the Definition 2.9 to multi valued case in probabilistic metric spaces.

Definition 3.1. Let S be a nonempty set, $\phi \in \varphi$, ψ be a map from $(0, \infty)$ to $(0, \infty)$ and F be a probabilistic distance on S. A mapping $f : S \to 2^S$ is called a multi-valued $(\psi, \phi, \varepsilon, \lambda)$-contraction if for every $x, y \in S$, $\varepsilon > 0$ and for all $\lambda \in (0, 1)$ the following implication holds:

$$F_{xy}(\varepsilon) > 1 - \lambda \Rightarrow \forall p \in fx \exists q \in fy : F_{pq}(\psi(\varepsilon)) > 1 - \varphi(\lambda).$$

Now, we need to define some conditions on the t-norm T or on the contraction mapping in order to be able to prove fixed point theorem. These two conditions are parallel. If one of them holds, Theorem 3.1 will obtain.

Definition 3.2[11]. Let (S, F) be a probabilistic metric space, M a nonempty subset of S and $f : M \to 2^S$ - $\{\varnothing\}$, a mapping f is weakly demicompact if for every sequence $(p_n)_{n\in \mathbb{N}}$ from M such that $p_{n+1}\in fp_n$, for every $n \in \mathbb{N}$ and $\lim F_{p_{n+1},p_n}(\varepsilon) = 1$, for every $\varepsilon > 0$, there exists a convergent subsequence $(p_{n_j})_{j\in\mathbb{N}}$.

The other condition is mentioned in the Theorem 3.1.

Theorem 3.1. Let (S, F, T) be a complete Menger space with $\sup_{0 \le a < 1} T(a, a) = 1$, $M \in C(S)$ and $f : M \to C(M)$ be a multi-valued $(\psi, \phi, \varepsilon, \lambda)$-contraction, where the series $\Sigma\psi^n(\varepsilon)$ is convergent for every $\varepsilon > 0$ and $\phi \in \varphi$. Let there exists $x_0 \in M$ and $x_1 \in fx_0$ such that $F_{x_0x_1} \in D_+$. If f is weakly demicompact or

$$\lim_{n\to\infty} \top_{i=1}^{\infty}(1 - \varphi^{n+i-1}(\varepsilon)) = 1 \text{ for every } \varepsilon > 0 \tag{1}$$

then there exists at least one element $x \in M$ such that $x \in fx$.

Proof. Since there exists $x_0 \in M$ and $x_1 \in fx_0$ such that $F_{x_0x_1} \in D_+$, hence for every $\lambda \in (0, 1)$ there exists $\varepsilon > 0$ such that $F_{x_0x_1} > 1 - \lambda$. The mapping f is a $(\psi, \phi, \varepsilon, \lambda)$-contraction and therefore there exists $x_2 \in fx_1$ such that

$$F_{x_2x_1}(\psi(\varepsilon)) > 1 - \varphi(\lambda)$$

Continuing in this way we obtain a sequence $(x_n)_{n \in N}$ from M such that for every $n \geq 2$, $x_n \in fx_{n-1}$ and

$$F_{x_n, x_{n-1}}(\psi^{n-1}(\varepsilon)) > 1 - \varphi^{n-1}(\lambda). \tag{2}$$

Since the series $\Sigma \psi^n(\varepsilon)$ is convergent we have $\lim_{n \to \infty} \psi^n(\varepsilon) = 0$ and by assumption $\phi \in \varphi$, so $\lim_{n \to \infty} \phi^n(\lambda) = 0$. We infer for every $\varepsilon_0 > 0$ that

$$\lim_{n \to \infty} F_{x_n x_{n-1}}(\varepsilon_0) = 1. \tag{3}$$

Indeed, if $\varepsilon_0 > 0$ and $\lambda_0 \in (0, 1)$ are given, and $n_0 = n_0(\varepsilon_0, \lambda_0)$ is enough large such that for every $n \geq n_0$, $\psi^n(\varepsilon) \leq \varepsilon_0$ and $\phi^n(\lambda) \leq \lambda_0$ then

$$F_{x_{n+1} x_n}(\varepsilon_0) \geq F_{x_{n+1} x_n}(\psi^n(\varepsilon)) > 1 - \varphi^n(\lambda) > 1 - \lambda_0 \text{ for every } n \geq n_0.$$

If f is weakly demicompact (3) implies that there exists a convergent subsequence $(x_{n_k})_{k \in N}$.

Suppose that (1) holds and prove that $(x_n)_{n \in N}$ is a Cauchy sequence. This means that for every $\varepsilon_1 > 0$ and every $\lambda_1 \in (0, 1)$ there exists $n_1(\varepsilon_1, \lambda_1) \in N$ such that

$$F_{x_{n+p} x_n}(\varepsilon_1) > 1 - \lambda_1 \tag{4}$$

for every $n_1 \geq n_1(\varepsilon_1, \lambda_1)$ and every $p \in N$.

Let $n_2(\varepsilon_1) \in N$ such that $\sum_{n \geq n_2(\varepsilon_1)} \psi^n(\varepsilon) < \varepsilon_1$. Since $\sum_{n=1}^{\infty} \psi^n(\varepsilon)$ is convergent series such a natural number $n_2(\varepsilon_1)$ exists. Hence for every $p \in N$ and every $n \geq n_2(\varepsilon_1)$ we have that

$$F_{x_{n+p+1}, x_n}(\varepsilon_1) \geq \top_{i=1}^{p+1} F_{x_{n+i}, x_{n+i-1}}(\psi^{n+i-1}(\varepsilon)),$$

and (2) implies that

$$F_{x_{n+p+1}, x_n}(\varepsilon_1) \geq \top_{i=1}^{p+1}(1 - \varphi^{n+i-1}(\lambda))$$

for every $n \geq n_2(\varepsilon_1)$ and every $p \in N$.
For every $p \in N$ and $n \geq n_2(\varepsilon_1)$

$$\top_{i=1}^{p+1}(1 - \varphi^{n+i-1}(\lambda)) \geq \top_{i=1}^{\infty}(1 - \varphi^{n+i-1}(\lambda))$$

and therefore for every $p \in N$ and $n \geq n_2(\varepsilon_1)$,

$$F_{x_{n+p+1}, x_n}(\varepsilon_1) \geq \top_{i=1}^{\infty}(1 - \varphi^{n+i-1}(\lambda)). \tag{5}$$

From (1) it follows that there exists $n_3(\lambda_1) \in N$ such that

$$\top_{i=1}^{\infty}(1 - \varphi^{n+i-1}(\lambda)) > 1 - \lambda_1 \tag{6}$$

for every $n \geq n_3(\lambda_1)$. The conditions (5) and (6) imply that (4) holds for $n_1(\varepsilon_1, \lambda_1) = \max(n_2(\varepsilon_1), n_3(\lambda_1))$ and every $p \in N$. This means that $(x_n)_{n \in N}$ is a Cauchy sequence and since S is complete there exists $\lim_{n \to \infty} x_n$. Hence in both cases there exists $(x_{n_k})_{k \in N}$ such that

$$\lim_{k \to \infty} x_{n_k} = x.$$

It remains to prove that $x \in fx$. Since $fx = \overline{fx}$ it is enough to prove that $x \in \overline{fx}$ i.e., for every $\varepsilon_2 > 0$ and $\lambda_2 \in (0, 1)$ there exists $b_{\varepsilon_2, \lambda_2} \in fx$ such that

$$F_{x, b_{\varepsilon_2, \lambda_2}}(\varepsilon_2) > 1 - \lambda_2. \tag{7}$$

Since $\sup_{x < 1} T(x, x) = 1$ for $\lambda_2 \in (0, 1)$ there exists $\delta(\lambda_2) \in (0, 1)$ such that $T(1 - \delta(\lambda_2), 1 - \delta(\lambda_2)) > 1 - \lambda_2$.

If $\delta'(\lambda_2)$ is such that

$$T(1 - \delta'(\lambda_2), 1 - \delta'(\lambda_2)) > 1 - \delta(\lambda_2)$$

and $\delta''(\lambda_2) = \min(\delta(\lambda_2), \delta'(\lambda_2))$ we have that

$$\begin{aligned}
T(1 - \delta''(\lambda_2), T((1 - \delta''(\lambda_2), 1 - \delta''(\lambda_2)))) &\geq T(1 - \delta(\lambda_2), T((1 - \delta'(\lambda_2), 1 - \delta(\lambda_2)))) \\
&\geq T(1 - \delta(\lambda_2), 1 - \delta(\lambda_2)) \\
&> 1 - \lambda_2.
\end{aligned}$$

Since $\lim_{k \to \infty} x_{n_k} = x$ there exists $k_1 \in N$ such that $F_{x, x_{n_k}}\left(\dfrac{\varepsilon}{3}\right) > 1 - \delta''(\lambda_2)$ for every $k \geq k_1$. Let $k_2 \in N$ such that

$$F_{x_{n_k}, x_{n_k+1}}\left(\frac{\varepsilon_2}{3}\right) > 1 - \delta''(\lambda_2) \text{ for every } k \geq k_2.$$

The existence of such a k_2 follows by (3). Let $\varepsilon \in R_+$ be such that $\psi(\varepsilon) < \dfrac{\varepsilon_2}{3}$ and $k_3 \in N$ such that $F_{x_{n_k}, x}(\varepsilon) > 1 - \delta''(\lambda_2)$ for every $k \geq k_3$. Since f is a $(\psi, \phi, \varepsilon, \lambda)$-contraction there exists $b_{\varepsilon_2, \lambda_2, k} \in fx$ such that

$$F_{x_{n_k+1}, b_{\varepsilon_2, \lambda_2, k}}(\psi(\varepsilon)) > 1 - \varphi(\delta''(\lambda_2)) \text{ for every } k \geq k_3.$$

Therefore for every $k \geq k_3$

$$\begin{aligned}
F_{x_{n_k+1}, b_{\varepsilon_2, \lambda_2, k}}\left(\frac{\varepsilon_2}{2}\right) &\geq F_{x_{n_k+1}, b_{\varepsilon_2, \lambda_2, k}}(\psi(\varepsilon)) \\
&> 1 - \varphi(\delta''(\lambda_2)) \\
&> 1 - \delta''(\lambda_2)
\end{aligned}$$

If $k \geq \max(k_1, k_2, k_3)$ we have

$$\begin{aligned}
F_{x, b_{\varepsilon_2, \lambda_2, k}}(\varepsilon_2) &\geq T\left(F_{x, x_{n_k}}\left(\frac{\varepsilon_2}{3}\right), T\left(F_{x_{n_k}, x_{n_k+1}}\left(\frac{\varepsilon_2}{3}\right), F_{x_{n_k+1}, b_{\varepsilon_2, \lambda_2, k}}\left(\frac{\varepsilon_2}{3}\right)\right)\right) \\
&\quad T(1 - \delta''(\lambda_2), T(1 - \delta''(\lambda_2), 1 - \delta''(\lambda_2))) \\
&> 1 - \lambda_2
\end{aligned}$$

and (7) is proved for $b_{\varepsilon_2, \lambda_2} = b_{\varepsilon_2, \lambda_2, k}$, $k \geq \max(k_1, k_2, k_3)$. Hence $x \in \overline{fx} = fx$, which means x is a fixed point of the mapping f.

Now, suppose that instead of $\Sigma \psi^n(\varepsilon)$ be convergent series, ψ is increasing bijection.

Theorem 3.2. Let (S, F, T) be a complete Menger space with $\sup_{0 \leq a < 1} T(a, a) = 1$ and $f : S \to C(S)$ be a multi-valued $(\psi, \phi, \varepsilon, \lambda)$-contraction.

If there exist $p \in S$ and $q \in fp$ such that $F_{pq} \in D_+$, ψ is increasing bijection and $\lim_{n \to \infty} \top_{i=1}^{\infty}(1 - \varphi^{n+i-1}(\lambda)) = 1$, for every $\lambda \in (0, 1)$, then, f has a fixed point.

Proof. Let $\varepsilon > 0$ be given and $\delta \in (0, 1)$ be such that $\delta < \min\{\varepsilon, \psi^{-1}(\varepsilon)\}$ or $\psi(\delta) < \varepsilon$ since ψ is increasing bijection. If $F_{uv}(\delta) > 1-\delta$ then, due to $(\psi, \phi, \varepsilon, \lambda)$- contraction for each $x \in fu$ we can find $y \in fv$ such that $F_{xy}(\psi(\delta)) > 1 - \phi(\delta)$, from where we obtain that $F_{xy}(\varepsilon) > F_{xy}(\psi(\delta)) > 1 - \phi(\delta) > 1 - \delta > 1 - \varepsilon$. So f is continuous. Next, let $p_0 = p$ and $p_1 = q$ be in fp_0. Since $F_{pq} \in D_+$, hence for every $\lambda \in (0, 1)$ there exist $\varepsilon > 0$ such that $F_{pq}(\varepsilon) > 1 - \lambda$, namely $F_{p_0 p_1}(\varepsilon) > 1 - \lambda$.

Using the contraction relation we can find $p_2 \in fp_1$ such that $F_{p_1 p_2}(\psi(\varepsilon)) > 1 - \varphi(\lambda)$, and by induction, p_n such that $p_n \in fp_{n-1}$ and $F_{p_{n-1} p_n}(\psi^{n-1}(\varepsilon)) > 1 - \varphi^{n-1}(\lambda)$ for all $n \geq 1$. Defining $t_n = \psi^n(\varepsilon)$, we have $g_j = F_{p_j p_{j+1}}(t_j) \geq 1 - \varphi^j(\lambda), \forall j$, so $\lim_{n \to \infty} \top_{i=1}^\infty g_{n+i-1} \geq \lim_{n \to \infty} \top_{i=1}^\infty (1 - \varphi^{n+i-1}(\lambda)) = 1$.

On the other hand the sequence (p_n) is a Cauchy sequense, that is:

$$\forall \varepsilon > 0 \, \exists n_0 = n_0(\varepsilon) \in N : F_{p_n p_{n+m}}(\varepsilon) > 1 - \in, \forall n \geq n_0, \forall m \in N.$$

Suppose that $\varepsilon > 0$, then:

$$\lim_{n \to \infty} \top_{i=1}^\infty g_{n+i+1} = 1 \Rightarrow \exists n_1 = n_1(\varepsilon) \in N : \top_{i=1}^m g_{n+i-1} > 1 - \varepsilon, \quad \forall n \geq n_1, \quad \forall m \in N.$$

Since the series $\sum_{n=1}^\infty t_n$ is convergent, there exists $n_2(= n_2(\varepsilon))$ such that $\sum_{n=n_2}^\infty t_n < \varepsilon$.

Let $n_0 = \max\{n_1, n_2\}$, then for all $n \geq n_0$ and $m \in N$ we have:

$$F_{p_n p_{n+m}}(\varepsilon) \geq F_{p_n p_{n+m}}\left(\sum_{i=n}^{n+m-1} t_i\right) \geq \top_{i=1}^m F_{p_{n+i-1} p_{n+1}}(t_{n+i-1})$$

$$= \top_{i=1}^m g_{n+i-1} > 1 - \varepsilon,$$

as desired.

Now we can apply Theorem 2.1 to find a fixed point of f. The theorem is proved. □

When ψ is increasing bijection and $\lim_{n \to \infty} \psi^n(\lambda)$ be zero, by using demicompact contraction we have another theorem.

Theorem 3.3. Let (S, F, T) be a complete Menger space, T a t-norm such that $\sup_{0 \leq a < 1} T(a, a) = 1$, M a non-empty and closed subset of $S, f : M \to C(M)$ be a multi-valued $(\psi, \phi, \varepsilon, \lambda)$- contraction and also weakly demicompact. If there exist $x_0 \in M$ and $x_1 \in fx_0$ such that $F_{x_0 x_1} \in D_+$, ψ is increasing bijection and $\lim_{n \to \infty} \psi(\lambda) = 0$ then, f has a fixed point.

Proof. We can construct a sequence $(p_n)_{n \in N}$ from M, such that $p_1 = x_1 \in fx_0$, $p_{n+1} \in fp_n$. Given $t > 0$ and $\lambda \in (0, 1)$, we will show that

$$\lim_{n \to \infty} F_{p_{n+1} p_n}(t) = 1. \tag{11}$$

Indeed, since $F_{x_0 x_1} \in D_+$, hence for every $\xi > 0$ there exist $\eta > 0$ such that $F_{x_0 x_1}(\eta) > 1 - \xi$, and by induction $F_{p_{n-1} p_n}(\psi^n(\eta)) > 1 - \varphi^n(\xi)$ for all $n \in N$. By choosing n such that $\psi^n(\eta) < t$ and $\phi^n(\xi) < \lambda$, we obtain

$$F_{p_{n+1} p_n}(t) > 1 - \lambda.$$

Since t and λ are arbitrary, the proof of (1) is complete.

By Definition 3.2, there exists a subsequence $(p_{n_j})_{j \in \mathbb{N}}$ such that $\lim_{j \to \infty} p_{n_j}$ exists. We shall prove that $x = \lim_{j \to \infty} p_{n_j}$ is a fixed point of f. Since fx is closed, $fx = \overline{fx}$, and therefore, it remains to prove that $x = \overline{fx}$, i.e., for every $\varepsilon > 0$ and $\lambda \in (0, 1)$, there exist $b(\varepsilon, \lambda) \in fx$, such that $F_{x,b(\varepsilon,\lambda)}(\varepsilon) > 1 - \lambda$. From the condition $\sup_{0 \leq a < 1} T(a, a) = 1$ it follows that there exists $\eta(\lambda) \in (0, 1)$ such that

$$u > 1 - \eta(\lambda) \Rightarrow T(u, u) > 1 - \lambda.$$

Let $j_1(\varepsilon, \lambda) \in \mathbb{N}$ be such that

$$F_{p_{n_j},x}\left(\psi^{-1}\left(\frac{\varepsilon}{2}\right)\right) > 1 - \frac{\eta(\lambda)}{2} \text{ for every } j \geq j_1(\varepsilon, \lambda).$$

Since $x = \lim_{j \to \infty} p_{n_j}$, such a number $j_1(\varepsilon, \lambda)$ exists. Since f is $(\psi, \phi, \varepsilon, \lambda)$-contraction and ψ is increasing bijection, for $p_{n_j+1} \in f p_{n_j}$ there exists $b_j(\varepsilon) \in fx$ such that

$$F_{p_{n_j+1},b_{j(\varepsilon)}}\left(\frac{\varepsilon}{2}\right) > 1 - \varphi\left(\frac{\eta(\lambda)}{2}\right) > 1 - \frac{\eta(\lambda)}{2} \text{ for every } j \geq j_1(\varepsilon, \lambda).$$

From (1) it follows that $\lim_{j \to \infty} p_{n_j+1} = x$ and therefore, there exists $j_2(\varepsilon, \lambda) \in \mathbb{N}$ such that $F_{x,p_{n_j+1}}\left(\frac{\varepsilon}{2}\right) > 1 - \frac{\eta(\lambda)}{2}$ for every $j \geq j_2(\varepsilon, \lambda)$. Let $j_3(\varepsilon, \lambda) = \max\{j_1(\varepsilon, \lambda), j_2(\varepsilon, \lambda)\}$. Then, for every $j \geq j_3(\varepsilon, \lambda)$ we have $F_{x,b_j(\varepsilon)}(\varepsilon) \geq T\left(F_{x,p_{n_j+1}}\left(\frac{\varepsilon}{2}\right), F_{p_{n_j+1},b_{j(\varepsilon)}}\left(\frac{\varepsilon}{2}\right)\right) > 1 - \lambda$. Hence, if $j > j_3(\varepsilon, \lambda)$, then, we can choose $b(\varepsilon, \lambda) = b_j(\varepsilon) \in fx$. The proof is complete. □

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Authors' contributions
PA defined the definitions and wrote the introduction, preliminaries and abstract. AB proved the theorems. AB has approved the final manuscript. Also PA has verified the final manuscript

Competing interests
The authors declare that they have no competing interests.

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