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The effective field theory of nonsingular cosmology: II

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Abstract Based on the effective field theory (EFT) of cosmological perturbations, we explicitly clarify the pathology in nonsingular cubic Galileon models and show how to cure it in EFT with new insights into this issue. With the least set of EFT operators that are capable to avoid instabilities in nonsingular cosmologies, we construct a nonsingular model dubbed the Genesis-inflation model, in which a slowly expanding phase (namely, Genesis) with increasing energy density is followed by slow-roll inflation. The spectrum of the primordial perturbation may be simulated numerically, which shows itself a large-scale cutoff, as the large-scale anomalies in CMB might be a hint for.

1 Introduction

Inflation is still being eulogized for its simplicity and also criticized for its past-incompleteness [1,2]. A complete description of the early universe requires physics other than only implementing inflation.

To the best of current knowledge, the inflation scenario will be past-complete, only if it happens after a nonsingular bounce which is preceded by a contraction [3–7], or a slow expansion phase (namely, the Genesis phase) with increasing energy density [8–10]. These two possibilities will be called bounce-inflation and Genesis-inflation, respectively. Besides being past-complete, a bounce-inflation or Genesis-inflation scenario may explain the probable large-scale anomalies in cosmological microwave background (CMB) [6,11]. The nonsingular Quintom bounce [12–14] (see also [15]), the ekpyrotic universe [16,17], the Genesis scenario [18–21], and the slow expansion scenario [22–25] have acquired intensive attention. In classical nonsingular (past-complete) cosmologies, the null energy condition (NEC) must be violated for a period.

The ghost-free bounce models [26–30] have been obtained in cubic Galileon [31] and full Horndeski theory [32–34]. However, recently, it has been proved in Ref. [35] that there exists a "no-go" theorem for the cubic Galileon, i.e., the gradient instability ($c_s^2 < 0$ with c_s being the sound speed of the scalar perturbations) is inevitable in the corresponding models. See also [36] for the extension to the full Horndeski theory, and [37] for the attempts to avoid the "no-go" in Horndeski theory. Relevant studies can also be found in [38–40].

Recently, in Ref. [41] (see also [42]), we dealt with this issue in the framework of the effective field theory (EFT) [43–46], which has proved to be a powerful tool. In EFT, the quadratic action of the scalar perturbation could always be written in the form (see [41] for detailed derivations)

$$S_\zeta^{(2)} = \int d^4x a^3 c_1 \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right]; \tag{1}$$

where we have neglected higher-order spatial derivatives of the scalar perturbation ζ, the sound speed squared of scalar perturbation

$$c_s^2 = \left(\frac{\dot{c}_3}{a} - c_2\right) \Big/ c_1, \tag{2}$$

with the coefficients c_1, c_2 and c_3 being time dependent parameters in general, and $c_1 > 0$ is needed to avoid the ghost instability. The condition for avoiding the gradient instability is $c_s^2 \sim \dot{c}_3/a - c_2 > 0$, which is usually integrated as

$$c_3\Big|_{t_f} - c_3\Big|_{t_i} > \int_{t_i}^{t_f} a c_2 dt. \tag{3}$$

The condition of satisfying the inequality is to have c_3 cross 0, which is hardly possible in models based on the cubic

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Galileon [35,36]. However, we found that it can easily be satisfied by applying the EFT operator $R^{(3)}\delta g^{00}$ (with $R^{(3)}$ being the 3-dimensional Ricci scalar on the spacelike hypersurface and $\delta g^{00} = g^{00} + 1$), so that the gradient instability can be cured.

Though the integral approach (3) is simple and efficient, some details of curing the pathology might actually be missed. In this paper, based on the EFT, using a "non-integral approach", we revisit the nonsingular cosmologies. We begin straightly with (2), and we clarify the origin of pathology and show how to cure it in EFT with new insights into what is happening (Sect. 2). To have practice in this clarification, we build a stable model of the Genesis-inflation scenario by using the $R^{(3)}\delta g^{00}$ operator (Sect. 3). As a supplementary remark, we discuss a dilemma in the Genesis scenario (Sect. 4).

2 Re-proof of the "no-go" and its avoidance in EFT

The EFT is briefly introduced in Appendix A. In the unitary gauge, the quadratic action of tensor perturbation γ_{ij} is (see [41] for the derivation of Eqs. (4)–(8))

$$S_\gamma^{(2)} = \frac{M_p^2}{8}\int d^4x a^3 Q_T \left[\dot\gamma_{ij}^2 - c_T^2\frac{(\partial_k\gamma_{ij})^2}{a^2}\right], \tag{4}$$

where $Q_T = f + \frac{2m_4^2}{M_p^2} > 0$, $c_T^2 = f/Q_T > 0$, f and m_4^2 are coefficients defined in the EFT action (A4).

The quadratic action of the scalar perturbation ζ is given by Eq. (1) with

$$\begin{aligned} c_1 = &\frac{Q_T}{4\gamma^2 M_p^2}[2M_p^4 Q_T \dot f H - 2M_p^2 Q_T \\ &\times(2fM_p^2\dot H + \ddot f M_p^2 - 4M_2^4) \\ &-6\dot f M_p^2 m_3^3 + 3\dot f^2 M_p^4 + 3m_3^6], \end{aligned} \tag{5}$$

$$c_2 = fM_p^2, \tag{6}$$

$$c_3 = \frac{aM_p^2}{\gamma}Q_T Q_{\tilde m_4}, \tag{7}$$

$$\gamma = HQ_T - \frac{m_3^3}{2M_p^2} + \frac{1}{2}\dot f, \quad Q_{\tilde m_4} = f + \frac{2\tilde m_4^2}{M_p^2}, \tag{8}$$

where M_2^4, m_3^3 and $\tilde m_4^2$ are coefficients defined in the EFT action (A4), and they could be time dependent in general.

Only if $c_1 > 0$ and $c_s^2 > 0$, the model is free from ghost and gradient instabilities, respectively. In nonsingular cosmological models based on the cubic Galileon [28–30], $c_1 > 0$ is not hard to obtain, as can be seen from Eq. (5), since the cubic Galileon contributes the $\frac{m_3^3(t)}{2}\delta K\delta g^{00}$ operator in EFT. However, since c_3 is also affected by $\frac{m_3^3(t)}{2}\delta K\delta g^{00}$ through γ, $c_s^2 < 0$ is actually inevitable, as will be demonstrated in the following.

Since $c_1 > 0$, the requirement of $c_s^2 > 0$ equals

$$\left(H\gamma + \frac{\dot Q_T}{Q_T}\gamma + \frac{\dot Q_{\tilde m_4}}{Q_{\tilde m_4}}\gamma - c_T^2\frac{\gamma^2}{Q_{\tilde m_4}} - \dot\gamma\right)\frac{Q_T Q_{\tilde m_4}}{\gamma^2} > 0. \tag{9}$$

Here, $Q_T \neq Q_{\tilde m_4}$ is required, which cannot be embodied by the Horndeski theory [32–34]. Thus whether $c_s^2 > 0$ or not is controlled by the parameter set $(H, \gamma, Q_T, c_T^2, Q_{\tilde m_4})$.

In the following, with condition (9), we will re-prove the "no-go" theorem for the cubic Galileon, and clarify how to cure it in EFT. Different from the proof in [35,41,42], the re-proof is directly based on the derivative inequality instead of integrating it, which we called "non-integral approach". We assume that after the beginning of the hot "big bang" or inflation, $\gamma = H > 0$, $\dot\gamma < 0$ and $Q_{\tilde m_4} = 1$.

2.1 Case I: initially $\gamma < 0$

Since initially $\gamma < 0$, γ has to cross 0 from $\gamma < 0$ to $\gamma > 0$ at t_γ. The analysis below is also applicable for all cases with γ crossing 0 from $\gamma < 0$ to $\gamma > 0$.

In the ekpyrotic and bounce models, initially $\gamma = H < 0$. In the Genesis model [18] and the slow expansion model [23], $H > 0$ during the Genesis, but actually $\gamma = H - \frac{m_3^3}{2M_p^2} < 0$, as discussed in Sect. 4. Both belong to Case I.

In the cubic Galileon case, $f = Q_T = Q_{\tilde m_4} = 1$. Around t_γ, condition (9) is

$$-\dot\gamma > 0. \tag{10}$$

We see that $c_s^2 < 0$ is inevitable around t_γ, since $\dot\gamma > 0$. Thus the nonsingular models based on the cubic Galileon is pathological, as first proved by LMR in [35].

In the EFT case, around t_γ, condition (9) requires

$$\left(\frac{\dot Q_T}{Q_T}\gamma + \frac{\dot Q_{\tilde m_4}}{Q_{\tilde m_4}}\gamma - \frac{c_T^2\gamma^2}{Q_{\tilde m_4}} - \dot\gamma\right)Q_{\tilde m_4} > 0. \tag{11}$$

We might have $c_s^2 > 0$, only if (considering only the case where only one of Q_T and $Q_{\tilde m_4}$ is modified while the unmodified one is unity) around $\gamma = 0$

$$\frac{\dot Q_T}{Q_T}\gamma > \dot\gamma, \tag{12}$$

$$\text{or}\quad Q_{\tilde m_4} < 0, \quad \text{or}\quad \frac{\dot Q_{\tilde m_4}}{Q_{\tilde m_4}}\gamma > \dot\gamma + \frac{c_T^2\gamma^2}{Q_{\tilde m_4}} \quad (\text{for } Q_{\tilde m_4} \geqslant 0). \tag{13}$$

In solution (12), at t_γ, $\gamma = 0$ suggests $Q_T = 0$. Here, since $\gamma = 0$ at t_γ, $c_1 \sim Q_T/\gamma^2$ diverges. One possibility of removing this divergence is that $\gamma \sim (t - t_\gamma)^p$ and $Q_T \sim (t - t_\gamma)^n$ around t_γ, with $n \geqslant 2p$ and p, n being constants. In Ijjas and Steinhardt's model [37], $\gamma \sim t - t_\gamma$, while $Q_T \sim (t - t_\gamma)^2$, which belongs to this case.

In the bounce model based on the cubic Galileon, Eq. (8) gives $\gamma = H - \frac{m_3^3}{2M_p^2} \neq H$. Generally, the NEC is violated when $\dot{H} > 0$, while the period of $c_s^2 < 0$ corresponds to the phase with $\gamma \simeq 0$ and $\dot{\gamma} > 0$. These two phases do not necessarily coincide, see Eq. (8). As pointed out by Ijjas and Steinhardt [37], it is the sign's change of γ that causes the pathology. Here, we reconfirmed this point.

In solution (13), if $Q_{\tilde{m}_4} > 0$, at t_γ, $\gamma = 0$ suggests $Q_{\tilde{m}_4} = 0$; while if $Q_{\tilde{m}_4} < 0$, since $Q_{\tilde{m}_4} = 1$ eventually, $Q_{\tilde{m}_4}$ must cross 0 at $t_{\tilde{m}_4}$ (generally $t_{\tilde{m}_4} \neq t_\gamma$), at which $\dot{Q}_{\tilde{m}_4}\gamma > c_T^2\gamma^2$ must be satisfied. In both cases, $Q_{\tilde{m}_4} = 0$ is required, as proposed by Cai et al. [41] and Creminelli et al. [42].

We see again the details of $Q_{\tilde{m}_4}$ crossing 0. In both the Genesis model and the bounce model, initially $Q_{\tilde{m}_4} = 1$, so if $Q_{\tilde{m}_4} < 0$ around t_γ, $Q_{\tilde{m}_4}$ must cross 0 twice. Thus it seems that $\dot{Q}_{\tilde{m}_4}\gamma > c_T^2\gamma^2$ is hard to implement. However, with (2) and (7), one always could solve $Q_{\tilde{m}_4}$ for any given c_s^2,

$$Q_{\tilde{m}_4} = \frac{\gamma}{aM_p^2}\int a\left(c_1 c_s^2 + c_2\right)\mathrm{d}t, \tag{14}$$

where $Q_T = 1$.

2.2 Case II: $\gamma > 0$ throughout

Since $\gamma > 0$ throughout, we must have $\dot{\gamma} \geqslant 0$ during some period initially,[1] otherwise γ will diverge in the infinite past.

In the cubic Galileon case, condition (9) is

$$H\gamma - \gamma^2 - \dot{\gamma} > 0. \tag{15}$$

In the bounce model, $H < 0$ in the contracting phase, and in the Genesis model, $H \sim 0$ in the Genesis phase, both suggest $H\gamma - \gamma^2 - \dot{\gamma} < 0$.[2] Thus $c_s^2 < 0$ is inevitable in the corresponding phases, so the nonsingular models based on the cubic Galileon is pathological.

We see the Genesis model in the cubic Galileon version again in detail. During the slow expansion (Genesis phase), $H \sim 1/(-t)^n$ with the constant $n > 1$. Thus

$$\frac{\dot{\gamma}}{H\gamma} \simeq \frac{\dot{H}}{H^2} \sim (-t)^{n-1} \gg 1, \tag{16}$$

which implies $H\gamma \ll \dot{\gamma}$. Thus with (15), we see that $c_s^2 < 0$ is inevitable in the slow expansion phase. It seems that if $n = 1$, $H\gamma \ll \dot{\gamma}$ might be avoided. However, when $n = 1$, we have $H = p/(-t)$ and $a \sim 1/(-t)^p$ with constant p, thus $a \to 0$ in the infinite past. From (16), we see that $c_s^2 > 0$ requires $p = H^2/\dot{H} > 1$. Therefore, the universe is singular, or from another point of view, it is geodesically incomplete since the affine parameter of the graviton geodesics $\int_{t_i}^{t_f} a\mathrm{d}t$ is finite for $p > 1$ when $t_i \to -\infty$.

In the EFT case, condition (9) requires

$$\left(\frac{\dot{Q}_T}{Q_T}\gamma + \frac{\dot{Q}_{\tilde{m}_4}}{Q_{\tilde{m}_4}}\gamma - \frac{c_T^2\gamma^2}{Q_{\tilde{m}_4}} + H\gamma - \dot{\gamma}\right)Q_{\tilde{m}_4} > 0. \tag{17}$$

We might have $c_s^2 > 0$, only if (considering only the case where either Q_T or $Q_{\tilde{m}_4}$ is modified)

$$\frac{\dot{Q}_T}{Q_T} > c_T^2\gamma - H + \frac{\dot{\gamma}}{\gamma}, \tag{18}$$

$$\text{or}\quad \frac{\dot{Q}_{\tilde{m}_4}}{Q_{\tilde{m}_4}} < \frac{c_T^2\gamma}{Q_{\tilde{m}_4}} - H + \frac{\dot{\gamma}}{\gamma} \quad (\text{initially } Q_{\tilde{m}_4} < 0). \tag{19}$$

Generally, $-H\gamma + \dot{\gamma} > 0$, as in the Genesis model and the bounce model. Thus the solution (18) suggests $\dot{Q}_T > 0$, so that we will have $Q_T = 0$ in the infinite past. Thus based on (12) and (18), it seems that though the pathology can be cured by applying Q_T, $Q_T = 0$ is inevitable. A model with (18) has been proposed by Kobayashi [36] ($Q_T \sim \frac{1}{(-t)^p}$, $p > n > 1$). During the Genesis $\gamma \sim H \sim 1/(-t)^n$, $n > 1$, (17) is

$$\left(\dot{Q}_T/Q_T\right)^{-1}\frac{\dot{\gamma}}{\gamma} = n/p < 1. \tag{20}$$

Initially, $Q_T \sim \frac{1}{(-t)^p} = 0$.

In solution (19), $Q_{\tilde{m}_4}$ must cross 0 at $t_{\tilde{m}_4}$ to $Q_{\tilde{m}_4} > 0$, as pointed out by Cai et al. [41] and Creminelli et al. [42]. Around $t_{\tilde{m}_4}$, $\dot{Q}_{\tilde{m}_4} > c_T^2\gamma$ must be satisfied.

In (17), if $Q_{\tilde{m}_4} > 0$ throughout,

$$\frac{\dot{Q}_{\tilde{m}_4}}{Q_{\tilde{m}_4}} > \frac{c_T^2\gamma}{Q_{\tilde{m}_4}} - H + \frac{\dot{\gamma}}{\gamma} \tag{21}$$

is obtained. Thus, similar to (18), we have $Q_{\tilde{m}_4} = 0$ (which definitely requires $\gamma = 0$) in the infinite past. In the Genesis model, $Q_{\tilde{m}_4} \sim 1/(-t)^p$ and $\gamma \sim 1/(-t)^n$ with $p > n$, since $\dot{Q}_{\tilde{m}_4}/Q_{\tilde{m}_4} > \dot{\gamma}/\gamma$. However, $p > n$ indicates $\dot{Q}_{\tilde{m}_4} < \gamma$ in the infinite past ($Q_{\tilde{m}_4} = 0$), which violates the inequality (21). Thus $Q_{\tilde{m}_4} > 0$ throughout seems unworkable (Table 1).

[1] Of course, in Case II, we could also have $\dot{\gamma} < 0$ during some period, but what we focus on is the period (i.e., $\dot{\gamma} \geqslant 0$) where pathologies appear.

[2] In the case where γ grows from 0 initially, (15) is also obeyed no more.

Table 1 Pathology in nonsingular cubic Galileon cosmological models and its cure in EFT by either Q_T or $Q_{\tilde{m}_4}$

	Initially $\gamma < 0$	$\gamma > 0$ throughout
Nonsingular cubic Galileon models		
Crossing 0 for γ?	✓	×
$c_s^2 < 0$ is inevitable ("no-go")?	✓	✓
Phase with $c_s^2 < 0$ (Pathological phase)	$\dot{\gamma} > 0$ around $\gamma \simeq 0$	$H\gamma - \dot{\gamma} < \gamma^2$
Curing pathology in EFT		
Conditions of $c_s^2 > 0$	(11)	(17)
Applying Q_T	(12)	(18)
Applying $Q_{\tilde{m}_4}$	(13)	(19)

3 Application to Genesis-inflation

In this section, we will build a nonsingular model with the solution (19), in which the slow-roll inflation is preceded by a Genesis phase. A Genesis phase is a slowly expanding phase originating from the Minkowski vacuum with a drastic violation of NEC, i.e., $\epsilon \ll -1$; thus the energy density is increasing with the expansion of the universe and hence is free from the initial singularity [18,19] (see also [22]). As will be shown below, our model cannot only get rid of the pathology of instability, but also give rise to a flat spectrum with interesting features at large scales.

3.1 The setup of the model

The action of the model is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + M_p^2 g_1(\phi)X + g_2(\phi)X\Box\phi + g_3(\phi)X^2 - M_p^4 V(\phi) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00}\right], \tag{22}$$

where $X = -\nabla_\mu\phi\nabla^\mu\phi/2, \Box\phi = \nabla_\mu\nabla^\mu\phi$, and ϕ is a dimensionless scalar field, so dimensionless are $g_1(\phi), g_2(\phi), g_3(\phi)$ and $V(\phi)$.

Mapped into the EFT action (A4), (22) corresponds to

$$f = 1, \tag{23}$$

$$\Lambda(t) = M_p^4 V - \frac{1}{2}g_2\dot{\phi}^2(3H\dot{\phi} + \ddot{\phi}) + \frac{1}{4}g_3\dot{\phi}^4, \tag{24}$$

$$c(t) = \frac{M_p^2}{2}g_1\dot{\phi}^2 - \frac{1}{2}g_2\dot{\phi}^2(3H\dot{\phi} - \ddot{\phi}) + \frac{1}{2}g_{2,\phi}\dot{\phi}^4 + \frac{1}{2}g_3\dot{\phi}^4, \tag{25}$$

$$M_2^4(t) = -\frac{1}{4}g_2\dot{\phi}^2(3H\dot{\phi} + \ddot{\phi}) + \frac{1}{4}g_{2,\phi}\dot{\phi}^4 + \frac{1}{2}g_3\dot{\phi}^4, \tag{26}$$

$$m_3^3(t) = -g_2\dot{\phi}^3, \tag{27}$$

$$m_4^2 = 0, \tag{28}$$

$$\tilde{m}_4^2 \neq 0. \tag{29}$$

We can get the background equations

$$3H^2M_p^2 = \frac{M_p^2}{2}g_1\dot{\phi}^2 - 3g_2H\dot{\phi}^3 + \frac{1}{2}g_{2,\phi}\dot{\phi}^4 + \frac{3}{4}g_3\dot{\phi}^4 + M_p^4 V, \tag{30}$$

$$\dot{H}M_p^2 = -\frac{M_p^2}{2}g_1\dot{\phi}^2 + \frac{3}{2}g_2H\dot{\phi}^3 - \frac{1}{2}g_2\dot{\phi}^2\ddot{\phi} - \frac{1}{2}g_{2,\phi}\dot{\phi}^4 - \frac{1}{2}g_3\dot{\phi}^4, \tag{31}$$

$$0 = g_1\ddot{\phi} + 3g_1H\dot{\phi} + \frac{1}{2}g_{1,\phi}\dot{\phi}^2 - \frac{9g_2H^2\dot{\phi}^2}{M_p^2} - \frac{3g_2\dot{H}\dot{\phi}^2}{M_p^2} - \frac{6g_2H\dot{\phi}\ddot{\phi}}{M_p^2} + \frac{2g_{2,\phi}\dot{\phi}^2\ddot{\phi}}{M_p^2} + \frac{g_{2,\phi\phi}\dot{\phi}^4}{2M_p^2} + \frac{3g_3H\dot{\phi}^3}{M_p^2} + \frac{3g_3\dot{\phi}^2\ddot{\phi}}{M_p^2} + \frac{3g_{3,\phi}\dot{\phi}^4}{4M_p^2} + M_p^2V_\phi, \tag{32}$$

where "$_{,\phi} = d/d\phi$" and "$_{,\phi\phi} = d^2/d\phi^2$".

Initially, the universe is slowly expanding (in the Genesis phase), $H \simeq 0$. We set $V = 0$, $g_1 = -f_1 e^{2\phi}$, $g_2 = f_2$ and $g_3 = f_3$, see e.g. Ref. [9], with $f_{1,2,3}$ being dimensionless constants. Thus with Eq. (30), we have $\frac{M_p^2}{2}g_1\dot{\phi}^2 + \frac{3}{4}g_3\dot{\phi}^4 = 0$, which suggests

$$e^{2\phi} = \frac{3f_3}{2M_p^2 f_1}\dot{\phi}^2. \tag{33}$$

The solution is

$$\dot{\phi} = \frac{1}{(-t)}, \quad t < 0. \tag{34}$$

Equation (31) reads $\dot{H}=\frac{f_3-2f_2}{4M_p^2}\dot{\phi}^4$. Thus we get

$$H=\frac{f_3-2f_2}{12M_p^2}\frac{1}{(-t)^3} \tag{35}$$

after the integration. In principle, there could be a constant, i.e., $H=\frac{f_3-2f_2}{12M_p^2}\frac{1}{(-t)^3}+\text{const}$, however, in that case we will have $H\approx\text{const}$ initially, which is geodesically incomplete (see also [47]).

Additionally, from Eq. (35), we have

$$a(t)=\mathrm{e}^{\int H\mathrm{d}t}=\exp\left(\frac{f_3-2f_2}{24M_p^2t^2}\right)\simeq 1+\left(\frac{f_3-2f_2}{24M_p^2t^2}\right), \tag{36}$$

while we set $a(-\infty)=1$.

During inflation, we set $g_1=1$ and $g_2=g_3=0$, since we require that the inflationary phase is controlled by a simple slow-roll field.[3]

3.2 The primordial perturbation and its spectrum

In the unitary gauge, the quadratic action of the scalar perturbation is presented in the form of Eq. (1). The coefficients c_i are (substituting Eqs. (23)–(29) into (5)–(7))

$$c_1=\frac{\dot{\phi}^2}{4M_p^2\gamma^2}[2\dot{\phi}^2M_p^2(g_{2,\phi}+2g_3) -2g_2M_p^2(3H\dot{\phi}+\ddot{\phi})+3g_2^2\dot{\phi}^4]-\frac{\dot{H}M_p^2}{\gamma^2}, \tag{37}$$

$$c_2=M_p^2, \tag{38}$$

$$c_3=\frac{aM_p^2}{\gamma}Q_{\tilde{m}_4}, \tag{39}$$

where

$$\gamma=H+\frac{g_2}{2M_p^2}\dot{\phi}^3,\quad Q_{\tilde{m}_4}=1+\frac{2\tilde{m}_4^2}{M_p^2}. \tag{40}$$

The sound speed squared c_s^2 of the scalar perturbation is defined in Eq. (2). Here, when $\tilde{m}_4^2\equiv 0$ or $Q_{\tilde{m}_4}=1$, the sound speed squared of the scalar perturbation is reduced to

$$c_{s0}^2=1+\frac{4\dot{\phi}^2[g_2M_p^2(\ddot{\phi}-H\dot{\phi})+g_2^2\dot{\phi}^4+\dot{\phi}^2M_p^2(g_{2,\phi}+g_3)]}{4\dot{H}M_p^4+\dot{\phi}^2[2g_2M_p^2(3H\dot{\phi}+\ddot{\phi})-3g_2^2\dot{\phi}^4-2\dot{\phi}^2M_p^2(g_{2,\phi}+2g_3)]}. \tag{41}$$

It is easy to see that $c_{s0}^2=1$ for inflation, since $g_2=g_3=0$, but not for Genesis. However, using the operator $\frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00}$, we could always set $c_s^2=1$ in the Genesis phase, which requires $\tilde{m}_4^2=-\frac{2M_p^2(f_2+f_3)}{4f_2+f_3}$. This suggests $Q_{\tilde{m}_4}=-\frac{3f_3}{4f_2+f_3}$ is a constant at $|-t|\gg 1$, which is consistent with the solution (19).

The equation of motion of ζ is

$$u''+\left(c_s^2k^2-\frac{z''}{z}\right)u=0, \tag{42}$$

with $u=z\zeta$, $z=\sqrt{2a^2c_1}$; the prime denotes the derivative with respect to the conformal time $\tau=\int\mathrm{d}t/a$. The initial state is the Minkowski vacuum, thus $u=\frac{1}{\sqrt{2c_sk}}\mathrm{e}^{-ic_sk\tau}$ for ζ modes deep inside the horizon. The power spectrum of ζ is

$$P_{\mathcal{R}}=\frac{k^3}{2\pi^2}\left|\frac{u}{z}\right|^2. \tag{43}$$

In the following, we will analytically estimate the spectrum of the scalar perturbation. We set $c_s^2=1$ throughout for simplicity, which could be implemented by using $Q_{\tilde{m}_4}(t)$, as will be illustrated by the numerical simulation.

In the Genesis phase, substituting Eqs. (34), (35) into (5), we have

$$c_1=\frac{108f_3M_p^4}{(4f_2+f_3)^2}(-t)^2. \tag{44}$$

Thus

$$z=\frac{6\sqrt{6f_3}M_p^2}{4f_2+f_3}(-t)\cdot\exp\left(\frac{f_3-2f_2}{24M_p^2t^2}\right). \tag{45}$$

Then it is straightforward to obtain $\frac{z''}{z}\approx\frac{(f_3-2f_2)^2}{72M_p^4\tau^6}\approx\frac{0}{\tau^2}$, where $\tau=\int\frac{1}{a}\mathrm{d}t\approx t$. Thus the solution of Eq. (42) is

$$u_1=\frac{\sqrt{-\pi\tau}}{2}[C_{11}\cdot H_{1/2}^{(1)}(-k\tau)+C_{12}\cdot H_{1/2}^{(2)}(-k\tau)], \tag{46}$$

where C_{11} and C_{12} are functions of k, $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are Hankel functions of the first and the second kind of νth order, respectively. The initial condition $u=\frac{1}{\sqrt{2k}}\mathrm{e}^{-ik\tau}$ indicates

$$C_{11}=i,\quad C_{12}=0. \tag{47}$$

In the inflation phase, $c_1=\epsilon M_p^2$, thus $z=\sqrt{2\epsilon a^2M_p^2}$. We set $\epsilon\ll 1$ as a constant during inflation. Then $z''/z\approx(2+3\epsilon)/\tau^2$. The solution of Eq. (42) is

[3] The behaviors of these g_i in the two phases can easily be matched together by making use of some shape functions [7,48].

$$u_2 = \frac{\sqrt{-\pi\tau}}{2}[C_{21} \cdot H^{(1)}_{\nu_2}(-k\tau) + C_{22} \cdot H^{(2)}_{\nu_2}(-k\tau)] \tag{48}$$

with $\nu_2 \approx 3/2 + \epsilon$.

We require that $u_1(\tau_m) = u_2(\tau_m)$ and $u_1'(\tau_m) = u_2'(\tau_m)$, with τ_m approximately corresponding to the beginning time of inflation phase, and we obtain

$$C_{21} = -\frac{i}{4}e^{-ik\tau_m}\sqrt{\frac{-\pi}{2k\tau_m}}[2k\tau_m H^{(2)}_{\nu_2-1}(-k\tau_m) \\ +(2\nu_2 - 1 - 2ik\tau_m)H^{(2)}_{\nu_2}(-k\tau_m)], \tag{49}$$

$$C_{22} = \frac{i}{4}e^{-ik\tau_m}\sqrt{\frac{-\pi}{2k\tau_m}}[2k\tau_m H^{(1)}_{\nu_2-1}(-k\tau_m) \\ +(2\nu_2 - 1 - 2ik\tau_m)H^{(1)}_{\nu_2}(-k\tau_m)]. \tag{50}$$

The power spectrum of ζ is given by

$$P_{\mathcal{R}} = P^{\rm inf}_{\mathcal{R}} \cdot |C_{21} - C_{22}|^2, \quad k \ll aH, \tag{51}$$

where $P^{\rm inf}_{\mathcal{R}} = \frac{H^2_{\rm inf}}{8\pi^2 M_p^2\epsilon} \cdot \left(\frac{k}{aH}\right)^{3-2\nu_2}$ is the power spectrum of the scalar perturbation modes that exit the horizon during inflation. We see that for the perturbation modes exiting the horizon in the Genesis phase, $-k\tau_m \ll 1$, $|C_{21} - C_{22}|^2 \simeq (-k\tau_m)^2$, thus $P_{\mathcal{R}} \sim k^2$ is strong blue-tilted, while for the perturbation modes exiting the horizon in the inflation phase, $-k\tau_m \gg 1$, $|C_{21} - C_{22}|^2 \simeq 1$, thus $P_{\mathcal{R}} \sim k^{3-2\nu_2} = k^{-2\epsilon}$ is flat with a slightly red tilt.

Tensor perturbation is unaffected by the $R^{(3)}\delta g^{00}$ operator. Its quadratic action is given in Eq. (4) with $Q_T = 1$ and $c_T^2 = 1$. The spectrum of primordial GWs can be calculated similarly; see also Ref. [8]. Since $z_T''/z_T = a''/a$, we have

$$P_T = P^{\rm inf}_T \cdot |C_{21} - C_{22}|^2, \quad k \ll aH, \tag{52}$$

where $P^{\rm inf}_T = \frac{2H^2_{\rm inf}}{\pi^2 M_p^2} \cdot \left(\frac{k}{aH}\right)^{3-2\nu_2}$ is the power spectrum of tensor perturbation modes that exit the horizon during inflation. Thus the spectrum of primordial GWs has a shape similar to that of the scalar perturbation.

3.3 Numerical simulation

In the numerical calculation, we set

$$g_1(\phi) = \frac{f_1 e^{2\phi}}{1 + f_1 e^{2\phi}} \tanh[q_1(\phi - \phi_0)], \tag{53}$$

$$g_{2,3}(\phi) = f_{2,3}\left(\frac{1 - \tanh[q_{2,3}(\phi - \phi_0)]}{2}\right), \tag{54}$$

$$V(\phi) = \frac{\lambda}{2}(\phi - \phi_1)^2 \left(\frac{1 + \tanh[q_4(\phi - \phi_2)]}{2}\right), \tag{55}$$

with $f_{1,2,3}$, $q_{1,2,3,4}$, $\phi_{0,1,2}$ and λ being dimensionless constants. When $\phi \ll \phi_0$, we have $g_1 = -f_1 e^{2\phi}$, $g_2 = f_2$ and $g_3 = f_3$, which brings a Genesis phase (36), while $\phi \gg \phi_0$, we have $g_1 = 1$ and $g_2 = g_3 = 0$, the slow-roll inflation will occur with $V(\phi) \sim \phi^2$. When $\phi \ll \phi_2$, $V(\phi) \approx 0$, while $\phi \gg \phi_2$, $V(\phi) \approx \frac{\lambda}{2}(\phi - \phi_1)^2$. We do not require $\phi_0 = \phi_2$ but $\phi_0 > \phi_2$.

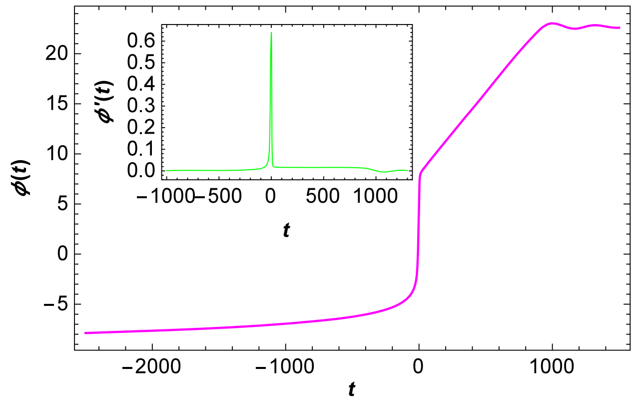


Fig. 1 The evolution of ϕ and $\dot\phi$, while we set $f_1 = 5$, $f_2 = -0.23$, $f_3 = -13f_2$, $q_1 = 1$, $q_2 = 0.2$, $q_3 = 0.2$, $q_4 = 2$, $\lambda = 4 \times 10^{-4}$, $\phi_0 = 7$, $\phi_1 = 22.7$ and $\phi_2 = 5.2$

We start the simulation at $t_i \ll -1$, and we set

$$\dot\phi(t_i) = \frac{1}{(-t_i)}, \quad \phi(t_i) = \frac{1}{2}\ln\left[\frac{3f_3}{2f_1M_p^2}\frac{1}{(-t_i)^2}\right], \tag{56}$$

and

$$a(t_i) = 1, \quad H(t_i) = \frac{f_3 - 2f_2}{12M_p^2}\frac{1}{(-t_i)^3}. \tag{57}$$

We show the evolution of ϕ and $\dot\phi$ in Fig. 1, and the evolution of a, H and ϵ in Fig. 2. In Fig. 3a, c_1 is plotted, and $c_1 > 0$ is satisfied. In Fig. 3b, we see that γ does not cross 0, which implies that, in the Genesis phase, $c^2_{s0} < 0$ (see Fig. 4a), as proved in Sect. 4. By including the operator $R^{(3)}\delta g^{00}$, we could have $c_s^2 > 0$ and so cure the gradient instability. The spectrum of the scalar perturbation can be simulated numerically, which is plotted in Fig. 5. The spectrum obtained has a cutoff at large scale $k < k_*$ and is nearly scale-invariant for $k > k_*$, as displayed in Eq. (51).

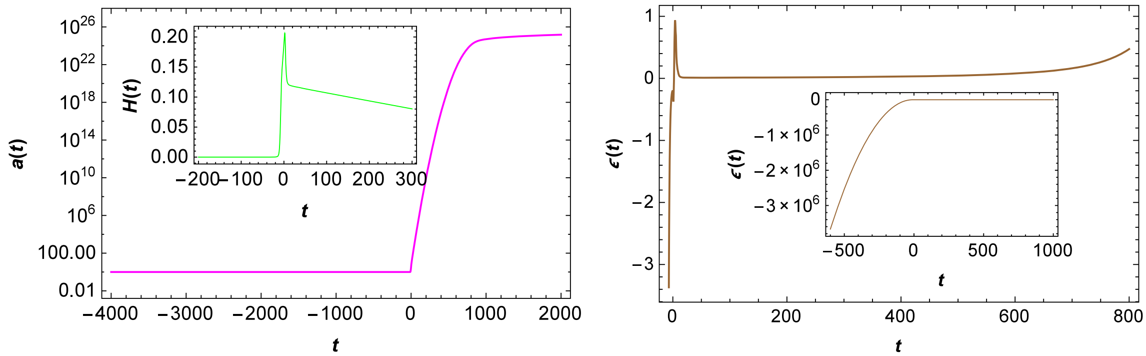
4 The dilemma of γ in the Genesis scenario

In the Genesis scenario based on the cubic Galileon, see [18] (see also [23]), we have

$$\gamma = H + \frac{f_2}{2M_p^2}\dot\phi^3 = \frac{f_3 + 4f_2}{12M_p^2}\dot\phi^3 \tag{58}$$

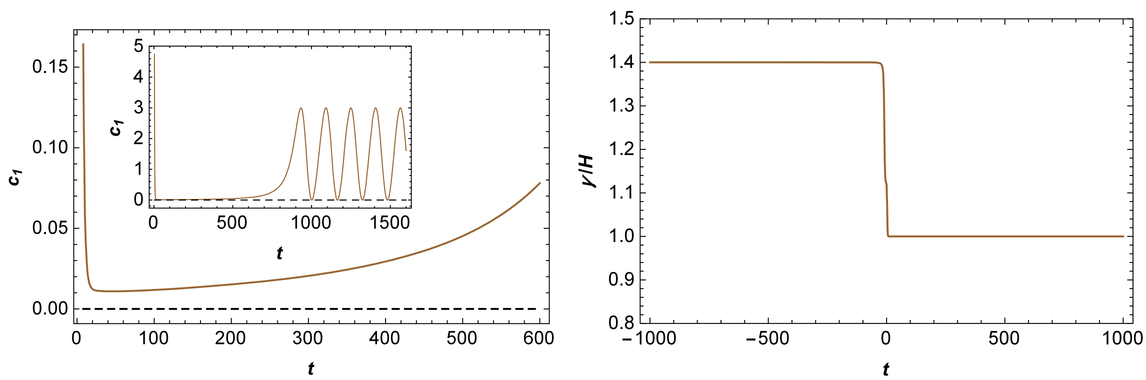
during the Genesis, where $f_2 < 0$.

In Ref. [18], $f_3 = -f_2$, which suggests $\gamma = \frac{f_2}{4M_p^2}\dot\phi^3 < 0$. Thus if a hot "big bang" or inflation ($\gamma = H > 0$) starts after



(a) a and H **(b)** ϵ

Fig. 2 The evolution of a, H and ϵ, while we set $f_1 = 5$, $f_2 = -0.23$, $f_3 = -13 f_2$, $q_1 = 1$, $q_2 = 0.2$, $q_3 = 0.2$, $q_4 = 2$, $\lambda = 4 \times 10^{-4}$, $\phi_0 = 7$, $\phi_1 = 22.7$ and $\phi_2 = 5.2$



(a) c_1 **(b)** γ/H

Fig. 3 The evolution of c_1, γ/H and ϵ, while we set $f_1 = 5$, $f_2 = -0.23$, $f_3 = -13 f_2$, $q_1 = 1$, $q_2 = 0.2$, $q_3 = 0.2$, $q_4 = 2$, $\lambda = 4 \times 10^{-4}$, $\phi_0 = 7$, $\phi_1 = 22.7$ and $\phi_2 = 5.2$

the Genesis phase, γ must cross 0 at t_γ ($c_{s0}^2 < 0$ around t_γ, which may be cured by applying $Q_{\tilde{m}_4}$). It is obvious that when $\gamma = 0$, c_1 in (1) will be divergent. Though this divergence might not be a problem, it will affect the numerical simulation for perturbations [49,50], unless Q_T/γ^2 is finite at t_γ, as in Ijjas and Steinhardt's model [37].

In the model of [9], the Genesis is followed by Galileon inflation [51]. Though $f_3 = -f_2$ and $\gamma = \frac{f_2}{4M_p^2}\dot{\phi}^3 < 0$ in the Genesis phase, one might also have $\gamma < 0$ for Galileon inflation, since $g_2 \neq 0$ in (22) during inflation. Thus it seems that γ might not necessarily cross 0. However, after inflation, γ crossing 0 is still inevitable.

In our model, the Genesis is followed by the slow-roll inflation, $\gamma = H > 0$ for inflation. To not cross 0, initially γ must satisfy $\gamma > 0$. In the Genesis phase, this suggests $f_3 > -4 f_2$. Thus we will have $\gamma > 0$ throughout. However, for the cubic Galileon model, the expense is

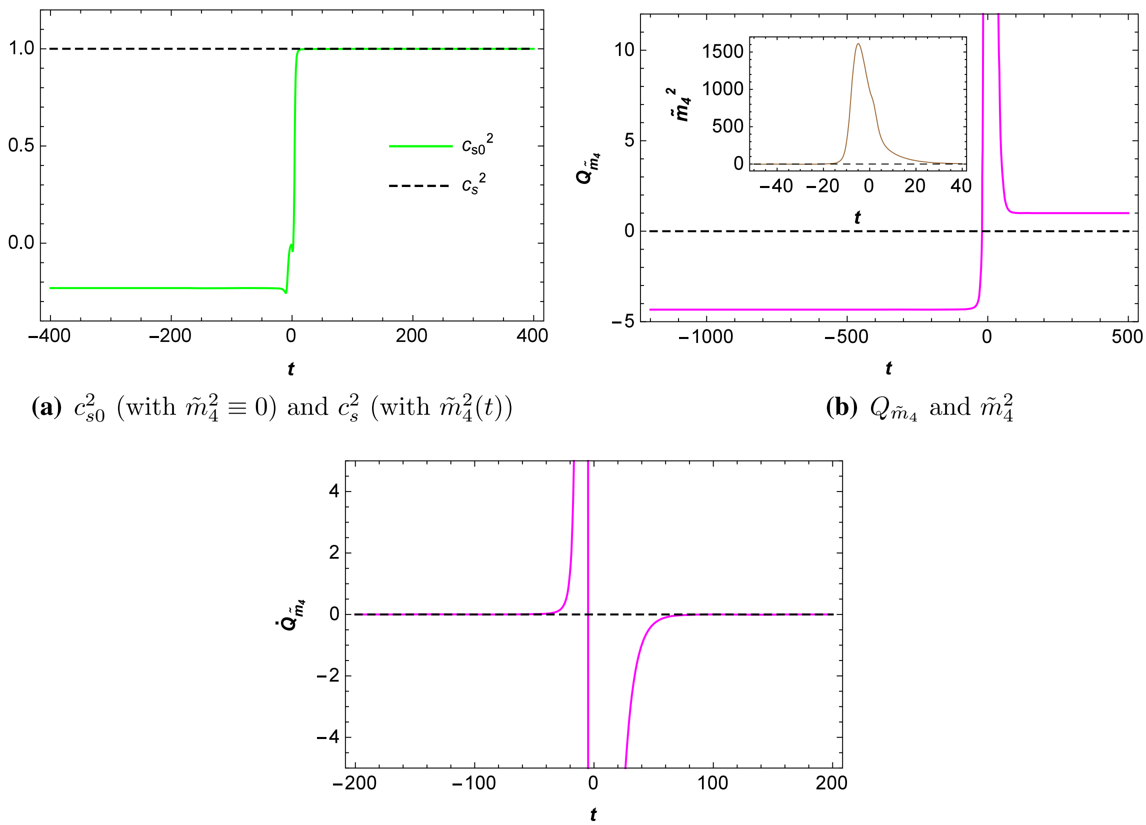
$$c_{s0}^2 = 1 - \frac{4 f_2 + 4 f_3}{3 f_3} < 0 \tag{59}$$

during the Genesis. Here, this pathology is cured in EFT by applying (19).

5 Conclusion

Based on the EFT of cosmological perturbations, we revisit the nonsingular cosmologies, using the "non-integral approach". By doing this, we could have a clearer understanding of the pathology in nonsingular Galileon models and its cure in EFT.

We clarify the application of the operator $\tilde{m}_4^2 R^{(3)} \delta g^{00}/2$ in EFT, which is significant for curing the gradient instability. We show that if $Q_{\tilde{m}_4} < 0$ around $\gamma = 0$ is adopted to cure the gradient instability, in solution (13) (with $\gamma < 0$ and $Q_{\tilde{m}_4} = 1$ initially), $Q_{\tilde{m}_4}$ must cross 0 twice; while in solution (19) (with $\gamma > 0$ throughout), initially $Q_{\tilde{m}_4} < 0$ must be satisfied, $Q_{\tilde{m}_4}$ will cross 0 to $Q_{\tilde{m}_4} > 0$ at $t_{\tilde{m}_4}$, and crosses 0 only once. Thus at a certain time, $Q_{\tilde{m}_4}$ meeting 0 is required, as pointed out first by Cai et al. [41], and also by Creminelli et al. [42].



(a) c_{s0}^2 (with $\tilde{m}_4^2 \equiv 0$) and c_s^2 (with $\tilde{m}_4^2(t)$)

(b) $Q_{\tilde{m}_4}$ and $\tilde{m}_4^2$

(c) $\dot{Q}_{\tilde{m}_4}$

Fig. 4 The sound speed of the scalar perturbation with and without $\tilde{m}_4^2$, while we set $f_1 = 5$, $f_2 = -0.23$, $f_3 = -13 f_2$, $q_1 = 1$, $q_2 = 0.2$, $q_3 = 0.2$, $q_4 = 2$, $\lambda = 4 \times 10^{-4}$, $\phi_0 = 7$, $\phi_1 = 22.7$ and $\phi_2 = 5.2$

We also clarify that in the bounce model with $\gamma < 0$ initially, $c_s^2 < 0$ will occur in the phase with $\gamma \simeq 0$ and $\dot{\gamma} > 0$, while the NEC is violated when $\dot{H} > 0$ (bounce phase); these two phases do not necessarily coincide. As pointed out by Ijjas and Steinhardt [37], it is the sign's change of γ that causes $c_s^2 < 0$. Here, we verify this point. In the Genesis model, see [9,18], and also see [23], the case is similar, as discussed in Sect. 4.

The nonsingular model with the solution (19) ($\gamma > 0$ throughout) has not been studied before. In Sect. 3, we design such a model, in which a slow expansion phase (namely, the Genesis phase) is followed by slow-roll inflation. Under the unitary gauge, since $\dot{\gamma} > 0$ and $\gamma > 0$ (not crossing 0), the evolution of primordial perturbations can be simulated numerically. The simulation displays that the spectrum acquires a large-scale cutoff, as expected in Ref. [8].

We conclude that, based on EFT, not only a stable nonsingular cosmological scenario may be built without getting involved in unknown physics, but also the phenomenological possibilities of its implementation are far richer than expected (see also [52,53] for the higher spatial derivative operators).

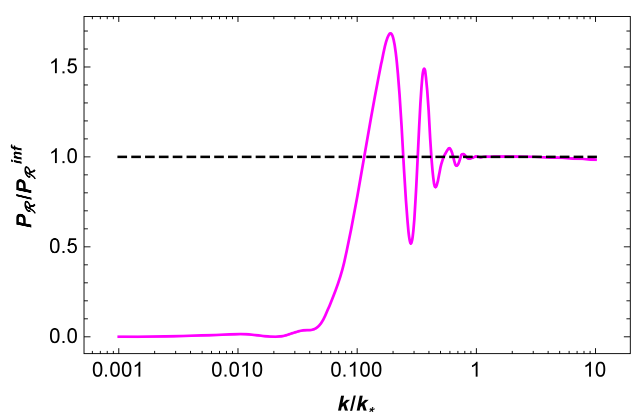


Fig. 5 The spectrum $P_{\mathcal{R}}$ of the scalar perturbation, while we set $f_1 = 5$, $f_2 = -0.23$, $f_3 = -13 f_2$, $q_1 = 1$, $q_2 = 0.2$, $q_3 = 0.2$, $q_4 = 2$, $\lambda = 4 \times 10^{-4}$, $\phi_0 = 7$, $\phi_1 = 22.7$, $\phi_2 = 5.2$ and k_* corresponds to the comoving wave number of the perturbation mode which exits the horizon around the beginning of inflation

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Appendix A: EFT of cosmological perturbations

With the ADM line element, we have

$$g_{\mu\nu} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix},$$
$$g^{\mu\nu} = \begin{pmatrix} -N^{-2} & \frac{N^j}{N^2} \\ \frac{N^i}{N^2} & h^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix}, \tag{A1}$$

and $\sqrt{-g} = N\sqrt{h}$, where $N_i = h_{ij}N^j$. The unit one-form tangent vector is defined as $n_\nu = n_0(\mathrm{d}t/\mathrm{d}x^\mu) = (-N, 0, 0, 0)$ and $n^\nu = g^{\mu\nu}n_\mu = (1/N, -N^i/N)$, which satisfies $n_\mu n^\mu = -1$. On the hypersurface, the induced 3-dimensional metric is $H_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$; thus

$$H_{\mu\nu} = \begin{pmatrix} N_k N^k & N_j \\ N_i & h_{ij} \end{pmatrix}, \quad H^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h^{ij} \end{pmatrix}. \tag{A2}$$

The extrinsic curvature is $K_{\mu\nu} \equiv \frac{1}{2}\mathcal{L}_n H_{\mu\nu}$, where $\mathcal{L}_n$ is the Lie derivative with respect to n^μ. The induced 3-dimensional Ricci scalar $R^{(3)}$ associated with $H_{\mu\nu}$ is

$$R^{(3)} = R + K^2 - K_{\mu\nu}K^{\mu\nu} - 2\nabla_\mu(Kn^\mu - n^\nu\nabla_\nu n^\mu). \tag{A3}$$

Without higher-order spatial derivatives, the EFT reads [41]

$$\begin{aligned} S = \int \mathrm{d}^4x\sqrt{-g}\Bigg[& \frac{M_p^2}{2} f(t)R - \Lambda(t) - c(t)g^{00} \\ & + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} \\ & - m_4^2(t)(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}) \\ & + \frac{\tilde{m}_4^2(t)}{2} R^{(3)}\delta g^{00}\Bigg] + S_m[g_{\mu\nu}, \psi_m], \end{aligned} \tag{A4}$$

where $\delta g^{00} = g^{00} + 1$, $\delta K_{\mu\nu} = K_{\mu\nu} - H_{\mu\nu}H$ with H being the Hubble parameter. The coefficient set $(f, c, \Lambda, M_2, m_3, m_4, \tilde{m}_4)$ specifies different theories and could be time dependent in general.[4] A particular subset ($m_4 = \tilde{m}_4$) of EFT (A4) is the Horndeski theory. $S_m[g_{\mu\nu}, \psi_m]$ is the matter part, which is minimally coupled to the metric $g_{\mu\nu}$.

To obtain the quadratic actions for scalar and tensor perturbations, we will work in the unitary gauge, thus we set

$$h_{ij} = a^2 e^{2\zeta}(e^\gamma)_{ij}, \quad \gamma_{ii} = 0 = \partial_i\gamma_{ij}. \tag{A5}$$

Then we follow the standard method first used by Maldacena [54], it is straightforward (though tedious) to obtain the quadratic actions of scalar perturbation ζ and tensor perturbation γ_{ij}, as exhibited in Eqs. (1) and (4), respectively (see [41] for detailed derivations).

[4] Different conventions of the nomenclatures of these coefficients were adopted during the development of the EFT of cosmological perturbations (see e.g., [43–46]). Here, we follow the convention used in Refs. [45,46].

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