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Common fixed point of a power graphic contraction pair in partial metric spaces endowed with a graph

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PakistanFull list of author information is
available at the end of the article**Abstract**

In this paper, we initiate a study of fixed point results in the setup of partial metric spaces endowed with a graph. The concept of a power graphic contraction pair of two mappings is introduced. Common fixed point results for such maps without appealing to any form of commutativity conditions defined on a partial metric space endowed with a directed graph are obtained. These results unify, generalize and complement various known comparable results from the current literature.

MSC: 47H10; 54H25; 54E50**Keywords:** partial metric space; common fixed point; directed graph; power graphic contraction pair

1 Introduction and preliminaries

Consistent with Jachymski [1], let X be a nonempty set and d be a metric on X . A set $\{(x, x) : x \in X\}$ is called a diagonal of $X \times X$ and is denoted by Δ . Let G be a directed graph such that the set $V(G)$ of its vertices coincides with X and $E(G)$ is the set of the edges of the graph with $\Delta \subseteq E(G)$. Also assume that the graph G has no parallel edges. One can identify a graph G with the pair $(V(G), E(G))$. Throughout this paper, the letters \mathbb{R} , \mathbb{R}^+ , ω and \mathbb{N} will denote the set of real numbers, the set of nonnegative real numbers, the set of nonnegative integers and the set of positive integers, respectively.

Definition 1.1 [1] A mapping $f : X \rightarrow X$ is called a Banach G -contraction or simply G -contraction if

- (a₁) for each $x, y \in X$ with $(x, y) \in E(G)$, we have $(f(x), f(y)) \in E(G)$,
(a₂) there exists $\alpha \in (0, 1)$ such that for all $x, y \in X$ with $(x, y) \in E(G)$ implies that
$$d(f(x), f(y)) \leq \alpha d(x, y).$$

Let $X^f := \{x \in X : (x, f(x)) \in E(G) \text{ or } (f(x), x) \in E(G)\}$.

Recall that if $f : X \rightarrow X$, then a set $\{x \in X : x = f(x)\}$ of all fixed points of f is denoted by $F(f)$. A self-mapping f on X is said to be

- (1) a Picard operator if $F(f) = \{x^*\}$ and $f^n(x) \rightarrow x^*$ as $n \rightarrow \infty$ for all $x \in X$;
- (2) a weakly Picard operator if $F(f) \neq \emptyset$ and for each $x \in X$, we have $f^n(x) \rightarrow x^* \in F(f)$ as $n \rightarrow \infty$;

(3) orbitally continuous if for all $x, a \in X$, we have

$$\lim_{k \rightarrow \infty} f^{nk}(x) = a \quad \text{implies} \quad \lim_{i \rightarrow \infty} f(f^{nk}(x)) = f(a).$$

The following definition is due to Chifu and Petrusel [2].

Definition 1.2 An operator $f : X \rightarrow X$ is called a Banach G -graphic contraction if

- (b₁) for each $x, y \in X$ with $(x, y) \in E(G)$, we have $(f(x), f(y)) \in E(G)$,
- (b₂) there exists $\alpha \in [0, 1)$ such that

$$d(f(x), f^2(x)) \leq \alpha d(x, f(x)) \quad \text{for all } x \in X^f.$$

If x and y are vertices of G , then a path in G from x to y of length $k \in \mathbb{N}$ is a finite sequence $\{x_n\}$, $n \in \{0, 1, 2, \dots, k\}$ of vertices such that $x_0 = x$, $x_k = y$ and $(x_{i-1}, x_i) \in E(G)$ for $i \in \{1, 2, \dots, k\}$.

Notice that a graph G is connected if there is a path between any two vertices and it is weakly connected if \tilde{G} is connected, where \tilde{G} denotes the undirected graph obtained from G by ignoring the direction of edges. Denote by G^{-1} the graph obtained from G by reversing the direction of edges. Thus,

$$E(G^{-1}) = \{(x, y) \in X \times X : (y, x) \in E(G)\}.$$

Since it is more convenient to treat \tilde{G} as a directed graph for which the set of its edges is symmetric, under this convention, we have that

$$E(\tilde{G}) = E(G) \cup E(G^{-1}).$$

If G is such that $E(G)$ is symmetric, then for $x \in V(G)$, the symbol $[x]_G$ denotes the equivalence class of the relation R defined on $V(G)$ by the rule:

yRz if there is a path in G from y to z .

A graph G is said to satisfy the property (A) (see also [2]) if for any sequence $\{x_n\}$ in $V(G)$ with $x_n \rightarrow x$ as $n \rightarrow \infty$ and $(x_n, x_{n+1}) \in E(G)$ for $n \in \mathbb{N}$ implies that $(x_n, x) \in E(G)$.

Jachymski [1] obtained the following fixed point result for a mapping satisfying the Banach G -contraction condition in metric spaces endowed with a graph.

Theorem 1.3 [1] *Let (X, d) be a complete metric space and G be a directed graph and let the triple (X, d, G) have a property (A). Let $f : X \rightarrow X$ be a G -contraction. Then the following statements hold:*

1. $F_f \neq \emptyset$ if and only if $X_f \neq \emptyset$;
2. if $X_f \neq \emptyset$ and G is weakly connected, then f is a Picard operator;
3. for any $x \in X_f$ we have that $f|_{[x]_{\tilde{G}}}$ is a Picard operator;
4. if $f \subseteq E(G)$, then f is a weakly Picard operator.

Gwozdź-Lukawska and Jachymski [3] developed the Hutchinson-Barnsley theory for finite families of mappings on a metric space endowed with a directed graph. Bojor [4] obtained a fixed point of a φ -contraction in metric spaces endowed with a graph (see also [5]). For more results in this direction, we refer to [2, 6, 7].

On the other hand, Mathews [8] introduced the concept of a partial metric to obtain appropriate mathematical models in the theory of computation and, in particular, to give a modified version of the Banach contraction principle more suitable in this context. For examples, related definitions and work carried out in this direction, we refer to [9–19] and the references mentioned therein. Abbas *et al.* [20] proved some common fixed points in partially ordered metric spaces (see also [21]). Gu and He [22] proved some common fixed point results for self-maps with twice power type Φ -contractive condition. Recently, Gu and Zhang [23] obtained some common fixed point theorems for six self-mappings with twice power type contraction condition.

Throughout this paper, we assume that a nonempty set $X = V(G)$ is equipped with a partial metric p , a directed graph G has no parallel edge and G is a weighted graph in the sense that each vertex x is assigned the weight $p(x, x)$ and each edge (x, y) is assigned the weight $p(x, y)$. As p is a partial metric on X , the weight assigned to each vertex x need not be zero and whenever a zero weight is assigned to some edge (x, y) , it reduces to a loop (x, x) .

Also, the subset $W(G)$ of $V(G)$ is said to be complete if for every $x, y \in W(G)$, we have $(x, y) \in E(G)$.

Definition 1.4 Self-mappings f and g on X are said to form a power graphic contraction pair if

- (a) for every vertex v in G , (v, fv) and $(v, gv) \in E(G)$,
- (b) there exists $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ an upper semi-continuous and nondecreasing function with $\phi(t) < t$ for each $t > 0$ such that

$$p^\delta(fx, gy) \leq \phi(p^\alpha(x, y)p^\beta(x, fx)p^\gamma(y, gy)) \tag{1.1}$$

for all $(x, y) \in E(G)$ holds, where $\alpha, \beta, \gamma \geq 0$ with $\delta = \alpha + \beta + \gamma \in (0, \infty)$.

If we take $f = g$, then the mapping f is called a power graphic contraction.

The aim of this paper is to investigate the existence of common fixed points of a power graphic contraction pair in the framework of complete partial metric spaces endowed with a graph. Our results extend and strengthen various known results [8, 12, 13, 24].

2 Common fixed point results

We start with the following result.

Theorem 2.1 *Let (X, p) be a complete partial metric space endowed with a directed graph G . If $f, g : X \rightarrow X$ form a power graphic contraction pair, then the following hold:*

- (i) $F(f) \neq \emptyset$ or $F(g) \neq \emptyset$ if and only if $F(f) \cap F(g) \neq \emptyset$.
- (ii) If $u \in F(f) \cap F(g)$, then the weight assigned to the vertex u is 0.
- (iii) $F(f) \cap F(g) \neq \emptyset$ provided that G satisfies the property (A).
- (iv) $F(f) \cap F(g)$ is complete if and only if $F(f) \cap F(g)$ is a singleton.

Proof To prove (i), let $u \in F(f)$. By the given assumption, $(u, gu) \in E(G)$. Assume that we assign a non-zero weight to the edge (u, gu) . As $(u, u) \in E(G)$ and f and g form a power graphic contraction, we have

$$\begin{aligned} p^\delta(u, gu) &= p^\delta(fu, gu) \\ &\leq \phi(p^\alpha(u, u)p^\beta(u, fu)p^\gamma(u, gu)) \\ &= \phi(p^{\alpha+\beta}(u, u)p^\gamma(u, gu)) \\ &\leq \phi(p^{\alpha+\beta}(u, gu)p^\gamma(u, gu)) \\ &= \phi(p^\delta(u, gu)) \\ &< p^\delta(u, gu), \end{aligned}$$

a contradiction. Hence, the weight assigned to the edge (u, gu) is zero and so $u = gu$. Therefore, $u \in F(f) \cap F(g) \neq \emptyset$. Similarly, if $u \in F(g)$, then we have $u \in F(f)$. The converse is straightforward.

Now, let $u \in F(f) \cap F(g)$. Assume that the weight assigned to the vertex u is not zero, then from (1.1), we have

$$\begin{aligned} p^\delta(u, u) &= p^\delta(fu, gu) \\ &\leq \phi(p^\alpha(u, u)p^\beta(u, fu)p^\gamma(u, gu)) \\ &= \phi(p^{\alpha+\beta+\gamma}(u, u)) \\ &= \phi(p^\delta(u, u)) \\ &< p^\delta(u, u), \end{aligned}$$

a contradiction. Hence, (ii) is proved.

To prove (iii), we will first show that there exists a sequence $\{x_n\}$ in X with $fx_{2n} = x_{2n+1}$ and $gx_{2n+1} = x_{2n+2}$ for all $n \in \mathbb{N}$ with $(x_n, x_{n+1}) \in E(G)$, and $\lim_{n \rightarrow \infty} p(x_n, x_{n+1}) = 0$.

Let x_0 be an arbitrary point of X . If $fx_0 = x_0$, then the proof is finished, so we assume that $fx_0 \neq x_0$. As $(x_0, fx_0) \in E(G)$, so $(x_0, x_1) \in E(G)$. Also, $(x_1, gx_1) \in E(G)$ gives $(x_1, x_2) \in E(G)$. Continuing this way, we define a sequence $\{x_n\}$ in X such that $(x_n, x_{n+1}) \in E(G)$ with $fx_{2n} = x_{2n+1}$ and $gx_{2n+1} = x_{2n+2}$ for $n \in \mathbb{N}$.

We may assume that the weight assigned to each edge (x_{2n}, x_{2n+1}) is non-zero for all $n \in \mathbb{N}$. If not, then $x_{2k} = x_{2k+1}$ for some k , so $fx_{2k} = x_{2k+1} = x_{2k}$, and thus $x_{2k} \in F(f)$. Hence, $x_{2k} \in F(f) \cap F(g)$ by (i). Now, since $(x_{2n}, x_{2n+1}) \in E(G)$, so from (1.1), we have

$$\begin{aligned} p^\delta(x_{2n+1}, x_{2n+2}) &= p^\delta(fx_{2n}, gx_{2n+1}) \\ &\leq \phi(p^\alpha(x_{2n}, x_{2n+1})p^\beta(x_{2n}, fx_{2n})p^\gamma(x_{2n+1}, gx_{2n+1})) \\ &= \phi(p^\alpha(x_{2n}, x_{2n+1})p^\beta(x_{2n}, x_{2n+1})p^\gamma(x_{2n+1}, x_{2n+2})) \\ &= \phi(p^{\alpha+\beta}(x_{2n}, x_{2n+1})p^\gamma(x_{2n+1}, x_{2n+2})) \\ &< p^{\alpha+\beta}(x_{2n}, x_{2n+1})p^\gamma(x_{2n+1}, x_{2n+2}), \end{aligned}$$

which implies that

$$p^{\alpha+\beta}(x_{2n+1}, x_{2n+2}) < p^{\alpha+\beta}(x_{2n}, x_{2n+1}),$$

a contradiction if $\alpha + \beta = 0$. So, take $\alpha + \beta > 0$, and we have

$$p(x_{2n+1}, x_{2n+2}) < p(x_{2n}, x_{2n+1})$$

for all $n \in \mathbb{N}$. Again from (1.1), we have

$$\begin{aligned} p^\delta(x_{2n+2}, x_{2n+3}) &= p^\delta(gx_{2n+1}, fx_{2n+2}) \\ &= p^\delta(fx_{2n+2}, gx_{2n+1}) \\ &\leq \phi(p^\alpha(x_{2n+2}, x_{2n+1})p^\beta(x_{2n+2}, fx_{2n+2})p^\gamma(x_{2n+1}, gx_{2n+1})) \\ &= \phi(p^\alpha(x_{2n+1}, x_{2n+2})p^\beta(x_{2n+2}, x_{2n+3})p^\gamma(x_{2n+1}, x_{2n+2})) \\ &= \phi(p^{\alpha+\gamma}(x_{2n+1}, x_{2n+2})p^\beta(x_{2n+2}, x_{2n+3})) \\ &< p^{\alpha+\gamma}(x_{2n+1}, x_{2n+2})p^\beta(x_{2n+2}, x_{2n+3}), \end{aligned}$$

which implies that

$$p^{\alpha+\gamma}(x_{2n+2}, x_{2n+3}) < p^{\alpha+\gamma}(x_{2n+1}, x_{2n+2}).$$

We arrive at a contradiction in case $\alpha + \gamma = 0$. Therefore, we must take $\alpha + \gamma > 0$; consequently, we have

$$p(x_{2n+2}, x_{2n+3}) < p(x_{2n+1}, x_{2n+2})$$

for all $n \in \mathbb{N}$. Hence,

$$p^\delta(x_n, x_{n+1}) \leq \phi(p^\delta(x_{n-1}, x_n)) < p^\delta(x_{n-1}, x_n) \tag{2.1}$$

for all $n \in \mathbb{N}$. Therefore, the decreasing sequence of positive real numbers $\{p^\delta(x_n, x_{n+1})\}$ converges to some $c \geq 0$. If we assume that $c > 0$, then from (2.1) we deduce that

$$0 < c \leq \limsup_{n \rightarrow \infty} \phi(p^\delta(x_{n-1}, x_n)) \leq \phi(c) < c,$$

a contradiction. So, $c = 0$, that is, $\lim_{n \rightarrow \infty} p^\delta(x_n, x_{n+1}) = 0$ and so we have $\lim_{n \rightarrow \infty} p(x_n, x_{n+1}) = 0$. Also,

$$p^\delta(x_n, x_{n+1}) \leq \phi(p^\delta(x_{n-1}, x_n)) \leq \dots \leq \phi^n(p^\delta(x_0, x_1)). \tag{2.2}$$

Now, for $m, n \in \mathbb{N}$ with $m > n$,

$$\begin{aligned} p^\delta(x_n, x_m) &\leq p^\delta(x_n, x_{n+1}) + p^\delta(x_{n+1}, x_{n+2}) + \dots + p^\delta(x_{m-1}, x_m) \\ &\quad - p^\delta(x_{n+1}, x_{n+1}) - p^\delta(x_{n+2}, x_{n+2}) - \dots - p^\delta(x_{m-1}, x_{m-1}) \\ &\leq \phi^n(p^\delta(x_0, x_1)) + \phi^{n+1}(p^\delta(x_0, x_1)) + \dots + \phi^{m-1}(p^\delta(x_0, x_1)) \end{aligned}$$

implies that $p^\delta(x_n, x_m)$ converges to 0 as $n, m \rightarrow \infty$. That is, $\lim_{n, m \rightarrow \infty} p(x_n, x_m) = 0$. Since (X, p) is complete, following similar arguments to those given in Theorem 2.1 of [9], there exists a $u \in X$ such that $\lim_{n, m \rightarrow \infty} p(x_n, x_m) = \lim_{n \rightarrow \infty} p(x_n, u) = p(u, u) = 0$. By the given hypothesis, $(x_{2n}, u) \in E(G)$ for all $n \in \mathbb{N}$. We claim that the weight assigned to the edge (u, gu) is zero. If not, then as f and g form a power graphic contraction, so we have

$$\begin{aligned} p^\delta(x_{2n+1}, u) &= p^\delta(fx_{2n}, gu) \\ &\leq \phi(p^\alpha(x_{2n}, u)p^\beta(x_{2n}, fx_{2n})p^\gamma(u, gu)) \\ &= \phi(p^\alpha(x_{2n}, u)p^\beta(x_{2n}, x_{2n+1})p^\gamma(u, gu)). \end{aligned} \tag{2.3}$$

We deduce, by taking upper limit as $n \rightarrow \infty$ in (2.3), that

$$\begin{aligned} p^\delta(u, gu) &\leq \limsup_{n \rightarrow \infty} \phi(p^\alpha(x_{2n}, u)p^\beta(x_{2n}, x_{2n+1})p^\gamma(u, gu)) \\ &\leq \phi(p^\alpha(u, u)p^\beta(u, u)p^\gamma(u, gu)) \\ &\leq \phi(p^{\alpha+\beta+\gamma}(u, gu)) \\ &< p^\delta(u, gu), \end{aligned}$$

a contradiction. Hence, $u = gu$ and $u \in F(f) \cap F(g)$ by (i).

Finally, to prove (iv), suppose the set $F(f) \cap F(g)$ is complete. We are to show that $F(f) \cap F(g)$ is a singleton. Assume on the contrary that there exist u and v such that $u, v \in F(f) \cap F(g)$ but $u \neq v$. As $(u, v) \in E(G)$ and f and g form a power graphic contraction, so

$$\begin{aligned} 0 &< p^\delta(u, v) = p^\delta(fu, fv) \\ &\leq \phi(p^\alpha(u, v)p^\beta(u, fu)p^\gamma(v, gv)) \\ &= \phi(p^\alpha(u, v)p^\beta(u, u)p^\gamma(v, v)) \\ &\leq \phi(p^\delta(u, v)), \end{aligned}$$

a contradiction. Hence, $u = v$. Conversely, if $F(f) \cap F(g)$ is a singleton, then it follows that $F(f) \cap F(g)$ is complete. □

Corollary 2.2 *Let (X, p) be a complete partial metric space endowed with a directed graph G . If we replace (1.1) by*

$$p^\delta(f^s x, g^t y) \leq \phi(p^\alpha(x, y)p^\beta(x, f^s x)p^\gamma(y, g^t y)), \tag{2.4}$$

where $\alpha, \beta, \gamma \geq 0$ with $\delta = \alpha + \beta + \gamma \in (0, \infty)$ and $s, t \in \mathbb{N}$, then the conclusions obtained in Theorem 2.1 remain true.

Proof It follows from Theorem 2.1, that $F(f^s) \cap F(g^t)$ is a singleton provided that $F(f^s) \cap F(g^t)$ is complete. Let $F(f^s) \cap F(g^t) = \{w\}$, then we have $f(w) = f(f^s(w)) = f^{s+1}(w) = f^s(f(w))$, and $g(w) = g(g^t(w)) = g^{t+1}(w) = g^t(g(w))$ implies that fw and gw are also in $F(f^s) \cap F(g^t)$. Since $F(f^s) \cap F(g^t)$ is a singleton, we deduce that $w = fw = gw$. Hence, $F(f) \cap F(g)$ is a singleton. □

The following remark shows that different choices of α , β and γ give a variety of power graphic contraction pairs of two mappings.

Remarks 2.3 Let (X, p) be a complete partial metric space endowed with a directed graph G .

(R1) We may replace (1.1) with the following:

$$p^3(fx, gy) \leq \phi(p(x, y)p(x, fx)p(y, gy)) \quad (2.5)$$

to obtain conclusions of Theorem 2.1. Indeed, taking $\alpha = \beta = \gamma = 1$ in Theorem 2.1, one obtains (2.5).

(R2) If we replace (1.1) by one of the following condition:

$$p^2(fx, gy) \leq \phi(p(x, y)p(x, fx)), \quad (2.6)$$

$$p^2(fx, gy) \leq \phi(p(x, y)p(y, gy)), \quad (2.7)$$

$$p^2(fx, gy) \leq \phi(p(x, fx)p(y, gy)), \quad (2.8)$$

then the conclusions obtained in Theorem 2.1 remain true. Note that

- (i) if we take $\alpha = \beta = 1$ and $\gamma = 0$ in (1.1), then we obtain (2.6),
- (ii) take $\alpha = \gamma = 1$, $\beta = 0$ in (1.1) to obtain (2.7),
- (iii) use $\beta = \gamma = 1$, $\alpha = 0$ in (1.1) and obtain (2.8).

(R3) Also, if we replace (1.1) by one of the following conditions:

$$p(fx, gy) \leq \phi(p(x, y)), \quad (2.9)$$

$$p(fx, gy) \leq \phi(p(x, fx)), \quad (2.10)$$

$$p(fx, gy) \leq \phi(p(y, gy)), \quad (2.11)$$

then the conclusions obtained in Theorem 2.1 remain true. Note that

- (iv) take $\alpha = 1$ and $\beta = \gamma = 0$ in (1.1) to obtain (2.9),
- (v) to obtain (2.10), take $\beta = 1$, $\alpha = \gamma = 0$ in (1.1),
- (vi) if one takes $\gamma = 1$, $\alpha = \beta = 0$ in (1.1), then we obtain (2.11).

Remark 2.4 If we take $f = g$ in a power graphic contraction pair, then we obtain fixed point results for a power graphic contraction.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors read and approved the final manuscript.

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