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Decuplet baryons in a hot medium

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Abstract The thermal properties of the light decuplet baryons are investigated in the framework of the thermal QCD sum rules. In particular, the behavior of the mass and residue of the Δ, Σ^*, Ξ^*, and Ω baryons with respect to temperature are analyzed taking into account the additional operators appearing in the Wilson expansion at finite temperature. It is found that the mass and residue of these particles remain overall unaffected up to $T \simeq 150$ MeV but, beyond this point, they start to diminish considerably.

1 Introduction

Over the past decade, the in-medium investigation of fundamental parameters of hadrons such as their mass, decay constant, widths and strong coupling constants among various hadrons has become one of the hot topics in hadron physics. Such investigations can help us not only to better analyze the hot and dense QCD matter produced by heavy ion collision experiments, but they also can help us get valuable knowledge on the perturbative and nonperturbative natures of QCD. Investigations of the parameters of different members of light baryons can also provide us with useful information on the structure of dense astrophysical objects like neutron stars, since at the center of the neutron stars there can be intensively produced the strange members of octet baryons as well as the decuplet baryons beside the nucleons.

The theoretical studies on the hadronic parameters at finite temperature have mainly been devoted to mesons. There are a few studies dedicated to the analysis of the light baryons in hot medium. It is well known that the investigation of parameters of both the decuplet and the octet baryons at finite temperature is very important in understanding of the SU(3) flavor chiral symmetry breaking. In Ref. [1], the temperature dependence of the pole mass of the octet and decuplet baryons is investigated using the chiral perturbation the-

ory. In Ref. [2], the baryon masses at finite temperature has been investigated in both the octet and the decuplet representations for the Nambu–Jona-Lasinio (NJL) and Polyakov–Nambu–Jona-Lasinio (PNJL) models. It has been found that the baryon masses decrease by increasing the temperature and there is a strong dependence on the melting or deconfinement temperature determined by the flavor content of the baryons. In Ref. [3], the mass of the decuplet baryons at finite temperature are investigated using thermal QCD sum rules. In this paper, the authors find that below $T \leq 0.11$ GeV the masses show very little dependence on the temperature, but decrease with increasing temperature above this point. The self energy of the Δ-baryon is also investigated at finite temperature and density using the real time formalism of thermal field theory in Ref. [4].

The aim of this article is to extend our previous studies on the thermal properties of nucleon and hyperons [5,6] and evaluate the behavior of the mass and residue of the decuplet Δ, Σ^*, Ξ^*, and Ω^- baryons with respect to temperature employing the thermal QCD sum rule method. This method is the extended version of the vacuum QCD sum rule [7,8] to finite temperature and first introduced by Bochkarev and Shaposhnikov [9]. This method has two new aspects compared to the case of zero temperature: the Lorentz invariance is broken at finite temperature with the choice of reference frames, for the restoration of which the four-velocity vector of the medium is introduced; and some new operators appear in the operator product expansion (OPE) at finite temperature and the vacuum condensates are replaced by their thermal expectation values. In our calculations, we use the thermal quark propagators containing new nonperturbative contributions appearing in the Wilson expansion at finite temperature. We use the expressions of the temperature-dependent quark and gluon condensates as well as the temperature-dependent fermionic and gluonic parts of the energy density calculated via lattice QCD. We find the temperature-dependent expression of the continuum threshold in decuplet channel using the obtained sum rules for the masses and residues.

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Table 1 The values of N and the quark flavors q_1, q_2, and q_3 for Δ^0, Σ^{*0}, Ξ^{*0}, and Ω^- decuplet baryons

	N	q_1	q_2	q_3
Δ^0	$\sqrt{1/3}$	d	d	u
Σ^{*0}	$\sqrt{2/3}$	u	d	s
Ξ^{*0}	$\sqrt{1/3}$	s	s	u
Ω^-	$1/3$	s	s	s

This study is organized as follows. In Sect. 2, the thermal QCD sum rules for the mass and residue of Δ, Σ^*, Ξ^*, and Ω^- decuplet baryons are derived. In Sect. 3, the numerical analyses of the obtained sum rules for the decuplet baryons as well as comparison of the results with those existing in the literature are presented.

2 Thermal QCD sum rules for the mass and residue of decuplet baryons

In this section, thermal QCD sum rules for the mass and residue of Δ, Σ^*, Ξ^*, and Ω decuplet baryons are derived. For this purpose, we choose the following two-point thermal correlation function:

$$\Pi_{\mu\nu}(p, T) = i \int \mathrm{d}^4x e^{ip\cdot x} \, Tr\{\rho \mathcal{T}[J_\mu^D(x)\bar{J}_\nu^D(0)]\}, \qquad (1)$$

where $J_\mu^D(x)$ is the interpolating current of **D** decuplet baryon, $\rho = e^{-\beta H}/Tre^{-\beta H}$ is the thermal density matrix of QCD with H being the QCD Hamiltonian, $T = 1/\beta$ is temperature and $\mathcal{T}$ indicates the time ordering operator.

For $J_\mu^D(x)$ interpolation current, we use the following expression in a compact form (see for instance [10–12]):

$$\begin{aligned} J_\mu^{\mathbf{D}}(x) &= N\epsilon_{abc}[(q_1^{T,a}(x)C\gamma_\mu q_2^b(x))q_3^c(x) \\ &\quad + (q_2^{T,a}(x)C\gamma_\mu q_3^b(x))q_1^c(x) \\ &\quad + (q_3^{T,a}(x)C\gamma_\mu q_1^b(x))q_2^c(x)], \end{aligned} \qquad (2)$$

where a, b, c are color indices and C denotes the charge conjugation operator. The values of normalization constant N and the q_1, q_2, and q_3 quarks for the considered light decuplet baryons are given in Table 1.

To construct the thermal sum rules for the considered decuplet baryons, the aforesaid correlation function is computed both in hadronic and OPE representations. Matching then the coefficients of sufficient structures from the two representations, through a dispersion relation, the sum rules for the mass and residue of decuplet baryons are obtained. In order to suppress the contributions of the higher resonances and continuum, a Borel transformation as well as continuum subtraction are performed.

The hadronic side of the correlation function is obtained by inserting a complete set of intermediate state with spin

s into Eq. (1). After performing the spacetime integral, we get

$$\Pi_{\mu\nu}^{Had}(p, T) = -\frac{\langle 0|J_\mu^D(0)|D(p,s)\rangle_T \langle D(p,s)|J_\nu^{D\dagger}(0)|0\rangle_T}{p^2 - m_D^2(T)} \\ + \dots, \qquad (3)$$

where $\dots$ denotes the contributions of the higher states and continuum and $m_D(T)$ is the temperature-dependent mass of decuplet baryons. The matrix element $\langle 0|J_\mu^D(0)|D(p,s)\rangle_T$ is defined as

$$\langle 0|J_\mu^D(0)|D(p,s)\rangle_T = \lambda_D(T)u_\mu(p,s), \qquad (4)$$

where $u_\mu(p,s)$ is the Rarita–Schwinger spinor and $\lambda_D(T)$ is the temperature-dependent residue of D baryon. We shall remark that the J_μ^D current couples not only to the spin-3/2 but also to the spin-1/2 states. To get only the contributions of the decuplet baryons, we need to remove the unwanted contribution coming from the spin-1/2 states. To this end we employ the following procedures. The matrix element of J_μ^D between the spin-1/2 and vacuum states can be written as

$$\langle 0|J_\mu^D|\frac{1}{2}(p)\rangle_T = (Ap_\mu + B\gamma_\mu)u(p), \qquad (5)$$

where A and B are constants. By multiplication of both sides of Eq. (5) with γ_μ, and taking into account the condition $J_\mu^D\gamma^\mu = 0$, one immediately finds A in terms of B, i.e.

$$\left\langle 0|J_\mu^D(0)|\frac{1}{2}(p)\right\rangle_T = B\left(-\frac{4}{m_{\frac{1}{2}}}p_\mu + \gamma_\mu\right)u(p). \qquad (6)$$

From this equation we notice that the unwanted contributions coming from the spin-1/2 states are proportional to either p_μ or γ_μ. To eliminate these contributions, we order the Dirac matrices as $\gamma_\mu \not{p}\gamma_\nu$ and set to zero the terms with γ_μ in the beginning and γ_ν at the end and those proportional to p_μ and p_ν.

Finally, by inserting Eq. (4) into Eq. (3) and summing over the spin-3/2 states, the correlation function in hadronic side in the Borel scheme is obtained as

$$\hat{B}\Pi_{\mu\nu}^{Had}(p, T) = -\left[\lambda_D^2(T)e^{-m_D^2(T)/M^2}\right]\not{p}g_{\mu\nu} \\ -\left[\lambda_D^2(T)m_D(T)e^{-m_D^2(T)/M^2}\right]g_{\mu\nu} + \text{other structures}$$
including the four velocity vector of the medium, (7)

where M^2 is the Borel parameter to be fixed in next section.

On the other hand, the OPE side of the thermal correlation function is calculated in terms of the quark–gluon degrees of freedom in deep Euclidean region. By inserting the explicit forms of the interpolating currents into the correlation function given in Eq. (1) and contracting out all quark pairs via Wick's theorem, we obtain the

OPE side of thermal correlation function for the considered decuplet baryons in terms of the thermal light-quark propagators:

$$
\Pi_{\mu\nu}^{\text{OPE},\Delta}(p,T) = \frac{i}{3}\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x\, e^{ipx}
$$

$$
\times \Bigg\langle \Bigg\{ 2S_d^{ca'}(x)\gamma_\nu S_d'^{ab'}(x)\gamma_\mu S_u^{bc'}(x)
$$

$$
- 2S_d^{cb'}(x)\gamma_\nu S_d'^{aa'}(x)\gamma_\mu S_u^{bc'}(x)
$$

$$
+ 4S_d^{cb'}(x)\gamma_\nu S_u'^{ba'}(x)\gamma_\mu S_d^{ac'}(x)
$$

$$
+ 2S_u^{ca'}(x)\gamma_\nu S_d'^{ab'}(x)\gamma_\mu S_d^{bc'}(x)
$$

$$
- 2S_u^{ca'}(x)\gamma_\nu S_d'^{bb'}(x)\gamma_\mu S_d^{ac'}(x)
$$

$$
- S_u^{cc'}(x)Tr\Big[S_d^{ba'}(x)\gamma_\nu S_d'^{ab'}(x)\gamma_\mu\Big]
$$

$$
+ S_u^{cc'}(x)Tr\Big[S_d^{bb'}(x)\gamma_\nu S_d'^{aa'}(x)\gamma_\mu\Big]
$$

$$
- 4S_d^{cc'}(x)Tr\Big[S_u^{ba'}(x)\gamma_\nu S_d'^{ab'}(x)\gamma_\mu\Big]\Bigg\}\Bigg\rangle_T,
$$
(8)

$$
\Pi_{\mu\nu}^{\text{OPE},\Sigma^*}(p,T) = -\frac{2i}{3}\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x\, e^{ipx}
$$

$$
\times \Bigg\langle \Bigg\{ S_d^{ca'}(x)\gamma_\nu S_u'^{bb'}(x)\gamma_\mu S_s^{ac'}(x)
$$

$$
+ S_d^{cb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu S_u^{bc'}(x)
$$

$$
+ S_s^{ca'}(x)\gamma_\nu S_d'^{bb'}(x)\gamma_\mu S_u^{ac'}(x)
$$

$$
+ S_s^{cb'}(x)\gamma_\nu S_u'^{aa'}(x)\gamma_\mu S_d^{bc'}(x)
$$

$$
+ S_u^{ca'}(x)\gamma_\nu S_s'^{bb'}(x)\gamma_\mu S_d^{ac'}(x)
$$

$$
+ S_u^{cb'}(x)\gamma_\nu S_d'^{aa'}(x)\gamma_\mu S_s^{bc'}(x)
$$

$$
+ S_s^{cc'}(x)Tr\Big[S_d^{ba'}(x)\gamma_\nu S_u'^{ab'}(x)\gamma_\mu\Big]
$$

$$
+ S_u^{cc'}(x)Tr\Big[S_s^{ba'}(x)\gamma_\nu S_d'^{ab'}(x)\gamma_\mu\Big]
$$

$$
+ S_d^{cc'}(x)Tr\Big[S_u^{ba'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu\Big]\Bigg\}\Bigg\rangle_T,
$$
(9)

$$
\Pi_{\mu\nu}^{\text{OPE},\Xi^*}(p,T) = \frac{i}{3}\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x\, e^{ipx}
$$

$$
\times \Bigg\langle \Bigg\{ 2S_s^{ca'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu S_u^{bc'}(x)
$$

$$
- 2S_s^{cb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu S_u^{bc'}(x)
$$

$$
+ 4S_s^{cb'}(x)\gamma_\nu S_u'^{ba'}(x)\gamma_\mu S_s^{ac'}(x)
$$

$$
+ 2S_u^{ca'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu S_s^{bc'}(x)
$$

$$
- 2S_u^{ca'}(x)\gamma_\nu S_s'^{bb'}(x)\gamma_\mu S_s^{ac'}(x)
$$

$$
- S_u^{cc'}(x)Tr\Big[S_s^{ba'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu\Big]
$$

$$
+ S_u^{cc'}(x)Tr\Big[S_s^{bb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu\Big]
$$

$$
- 4S_s^{cc'}(x)Tr\Big[S_u^{ba'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu\Big]\Bigg\}\Bigg\rangle_T,
$$
(10)

$$
\Pi_{\mu\nu}^{\text{OPE},\Omega^-}(p,T) = \epsilon_{abc}\epsilon_{a'b'c'}\int d^4x\, e^{ipx}
$$

$$
\times \Bigg\langle \Bigg\{ S_s^{ca'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu S_s^{bc'}(x)
$$

$$
- S_s^{ca'}(x)\gamma_\nu S_s'^{bb'}(x)\gamma_\mu S_s^{ac'}(x)
$$

$$
- S_s^{cb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu S_s^{bc'}(x)
$$

$$
+ S_s^{cb'}(x)\gamma_\nu S_s'^{ba'}(x)\gamma_\mu S_s^{ac'}(x)
$$

$$
- S_s^{cc'}(x)Tr\Big[S_s^{ba'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu\Big]
$$

$$
+ S_s^{cc'}(x)Tr\Big[S_s^{bb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu\Big]\Bigg\}\Bigg\rangle_T,
$$
(11)

where $S' = C S^T C$.

Note that the same structures as the hadronic side enter to the OPE side. The unwanted contributions are removed with the same procedures as the hadronic side of the correlation function.

To proceed, we need the thermal light-quark propagator, whose expression (expanded in terms of different operators having different mass dimensions) in x space is given as (see also [6,13])

$$
S_q^{ij}(x) = i\frac{\not{x}}{2\pi^2 x^4}\delta_{ij} - \frac{m_q}{4\pi^2 x^2}\delta_{ij}
$$

$$
- \frac{\langle \bar{q}q\rangle}{12}\delta_{ij} - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\Big[1 - i\frac{m_q}{6}\not{x}\Big]\delta_{ij}
$$

$$
+ \frac{i}{3}\Big[\not{x}\Big(\frac{m_q}{16}\langle\bar{q}q\rangle - \frac{1}{12}\langle u\Theta^f u\rangle\Big)
$$

$$
+ \frac{1}{3}\Big(u\cdot x\,\not{u}\langle u\Theta^f u\rangle\Big)\Big]\delta_{ij}
$$

$$
- \frac{ig_s\lambda_A^{ij}}{32\pi^2 x^2}G_{\mu\nu}^A(\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\not{x}),
$$
(12)

where m_q denotes the light-quark mass, $\langle\bar{q}q\rangle$ is the temperature-dependent light-quark condensate, $G_{\mu\nu}^A$ is the temperature-dependent external gluon field, $\Theta_{\mu\nu}^f$ is the fermionic part of the energy momentum tensor and λ_A^{ij} are the standard Gell–Mann matrices. As is seen, the temperature-dependent condensates are introduced instead of the vacuum saturated condensates (for details see for instance Refs. [14–16]). As also previously mentioned, the four-velocity vector of the medium u^μ is also introduced to restore the Lorentz invariance at finite temperature broken with the choice of the thermal rest frame. In the rest frame of the heat bath, the four-

velocity vector of the medium is written as $u^\mu = (1, 0, 0, 0)$, which leads to $u^2 = 1$ and $p \cdot u = p_0$. In Eq. (12), the terms containing the four-quark operators as well as terms with logarithms are neglected, since they give small contributions to the results.

By using the above thermal light-quark propagator in the coordinate space and performing the Fourier integral to go to momentum space and applying the Borel transformations as well as the continuum subtraction, after lengthy calculations, for the OPE side of the correlation function in the Borel scheme we obtain

$$\hat{B}\Pi_{\mu\nu}^{\mathrm{OPE}}(p_0, T) = \Pi_1^{\mathrm{OPE}}(p_0, T)\, \not{p} g_{\mu\nu}$$
$$+ \Pi_2^{\mathrm{OPE}}(p_0, T) g_{\mu\nu}$$
$$+ \text{other structures},\tag{13}$$

where Π_1^{OPE} and Π_2^{OPE} are functions of QCD degrees of freedom as well as the new operators. As examples, we present the explicit forms of these functions for Ξ^* particle in the appendix. Not that we have used the following relation to relate the two-gluon condensate to the gluonic part of the energy-momentum tensor $\Theta_{\lambda\sigma}^g$:

$$\langle Tr^c G_{\alpha\beta} G_{\mu\nu} \rangle = \frac{1}{24}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})\langle G_{\lambda\sigma}^a G^{a\lambda\sigma} \rangle$$
$$+ \frac{1}{6}[g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu} - 2(u_\alpha u_\mu g_{\beta\nu}$$
$$- u_\alpha u_\nu g_{\beta\mu} - u_\beta u_\mu g_{\alpha\nu} + u_\beta u_\nu g_{\alpha\mu})]$$
$$\times \langle u^\lambda \Theta_{\lambda\sigma}^g u^\sigma \rangle.\tag{14}$$

After calculation of both the hadronic and the OPE sides of the thermal correlation function, we match the coefficients of the structures $\not{p} g_{\mu\nu}$ and $g_{\mu\nu}$ to obtain the sum rules

$$-\lambda_D^2(T)e^{-m_D^2(T)/M^2} = \Pi_{1,D}^{\mathrm{OPE}}(p_0, T)\tag{15}$$

and

$$-\lambda_D^2(T)m_D(T)e^{-m_D^2(T)/M^2} = \Pi_{2,D}^{\mathrm{OPE}}(p_0, T)\tag{16}$$

by simultaneous calculations of which we get the temperature-dependent mass and residue of the particles under consideration.

3 Numerical computations and conclusions

To numerically analyze the sum rules for the masses and residues of Δ, Σ^*, Ξ^*, and Ω^- decuplet baryons at finite temperature, we use input parameters such as the quark masses as well as the light-quark and gluon condensates in vacuum. They are collected in Table 2. Beside these parameters, we need the temperature-dependent quark and gluon condensates as well as the temperature-dependent energy density. For the quark condensate, we

Table 2 Input parameters used in calculations [17–20]

Parameters	Values
p_0^Δ	1.231 GeV
$p_0^{\Sigma^*}$	1.383 GeV
$p_0^{\Xi^*}$	1.531 GeV
$p_0^{\Omega^-}$	1.672 GeV
m_u	$(2.3^{+0.7}_{-0.5})$ MeV
m_d	$(4.8^{+0.5}_{-0.3})$ MeV
m_s	(95 ± 5) MeV
m_0^2	(0.8 ± 0.2) GeV^2
$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle$	$-(0.24 \pm 0.01)^3$ GeV^3
$\langle 0|\bar{s}s|0\rangle$	$-0.8(0.24 \pm 0.01)^3$ GeV^3
$\left\langle 0 \mid \frac{1}{\pi}\alpha_s G^2 \mid 0 \right\rangle$	(0.012 ± 0.004) GeV^4

use the following parametrization which reproduces the lattice QCD and QCD sum rules results presented in [21–23]:

$$\langle \bar{q}q \rangle = \frac{\langle 0|\bar{q}q|0\rangle}{1 + e^{18.10042\left(1.84692\left[\frac{1}{\mathrm{GeV}^2}\right]T^2 + 4.99216\left[\frac{1}{\mathrm{GeV}}\right]T - 1\right)}},\tag{17}$$

where $\langle 0|\bar{q}q|0\rangle$ is the vacuum light-quark condensate and this function is valid up to a critical temperature $T_c = 197$ MeV.

For the gluonic and fermionic parts of the energy density, we get the parametrization

$$\langle \Theta_{00}^g \rangle = \langle \Theta_{00}^f \rangle = T^4 e^{\left(113.867\left[\frac{1}{\mathrm{GeV}^2}\right]T^2 - 12.190\left[\frac{1}{\mathrm{GeV}}\right]T\right)}$$
$$- 10.141\left[\frac{1}{\mathrm{GeV}}\right]T^5,\tag{18}$$

obtained using lattice QCD graphics given in [24] and valid for $T \geq 130$ MeV.

The temperature-dependent gluon condensate obtained using the QCD sum rules predictions and lattice QCD data in [25] is employed as

$$\langle G^2 \rangle = \langle 0|G^2|0\rangle\left[1 - 1.65\left(\frac{T}{T_c}\right)^{8.735} + 0.04967\left(\frac{T}{T_c}\right)^{0.7211}\right],\tag{19}$$

where $\langle 0|G^2|0\rangle$ is the gluon condensate in vacuum.

The temperature-dependent continuum threshold for the decuplet baryons is one of auxiliary parameters that should also be determined. For this aim, we use the obtained sum rules for the mass and residue in Eqs. (15) and (16) as well as an extra equation obtained from Eq. (15) by applying a derivative with respect to $-\frac{1}{M^2}$ to both sides. By eliminating the mass and residue from these equations, we get the continuum threshold in terms of the temperature, i.e.

Table 3 The working regions of M^2 for Δ, Σ^*, Ξ^* and Ω^- decuplet baryons

	M^2
Δ	$1.5\ \text{GeV}^2 \leqslant M^2 \leqslant 3.0\ \text{GeV}^2$
Σ^*	$1.7\ \text{GeV}^2 \leqslant M^2 \leqslant 3.5\ \text{GeV}^2$
Ξ^*	$2.0\ \text{GeV}^2 \leqslant M^2 \leqslant 3.8\ \text{GeV}^2$
Ω^-	$2.2\ \text{GeV}^2 \leqslant M^2 \leqslant 4.0\ \text{GeV}^2$

$$s_0(T) = s_0 \left[1 - 0.93 \left(\frac{T}{T_c} \right)^{12} \right], \tag{20}$$

where s_0 is the continuum threshold at $T = 0$. This parameter is not totally arbitrary but it depends on the energy of the first excited state with the same quantum numbers as the chosen interpolating currents for the decuplet baryons under consideration. The T-dependent continuum threshold extrapolates this condition to all temperatures. For s_0, we take the interval $[m_D(0)+0.4]^2\ \text{GeV}^2 \leq s_0 \leq [m_D(0)+0.6]^2\ \text{GeV}^2$ in which the physical quantities show relatively weak dependence on

it. By comparison of Eq. (20) with the expression of the temperature-dependent continuum threshold of the hyperons [6] we see that there is an extra 0.93 coefficient in the decuplet case.

Finally, we determine the working regions for the Borel mass parameter M^2. To this aim, we require that not only the contributions of the higher states and continuum are adequately suppressed but also the portion of perturbative part exceeds the nonperturbative contributions and the series of the OPE converge. This leads to the working windows of the Borel mass parameter for different decuplet baryons as presented in Table 3.

According to the philosophy of the method used, the physical quantities should be practically independent the auxiliary parameters M^2 and s_0. To see how this condition is satisfied for the decuplet baryons, we plot their mass and residue versus M^2 for different fixed values of the continuum threshold s_0 at $T = 0$ in Figs. 1 and 2. From these figures, we see that these quantities depend on both M^2 and s_0 very weakly in their working intervals.
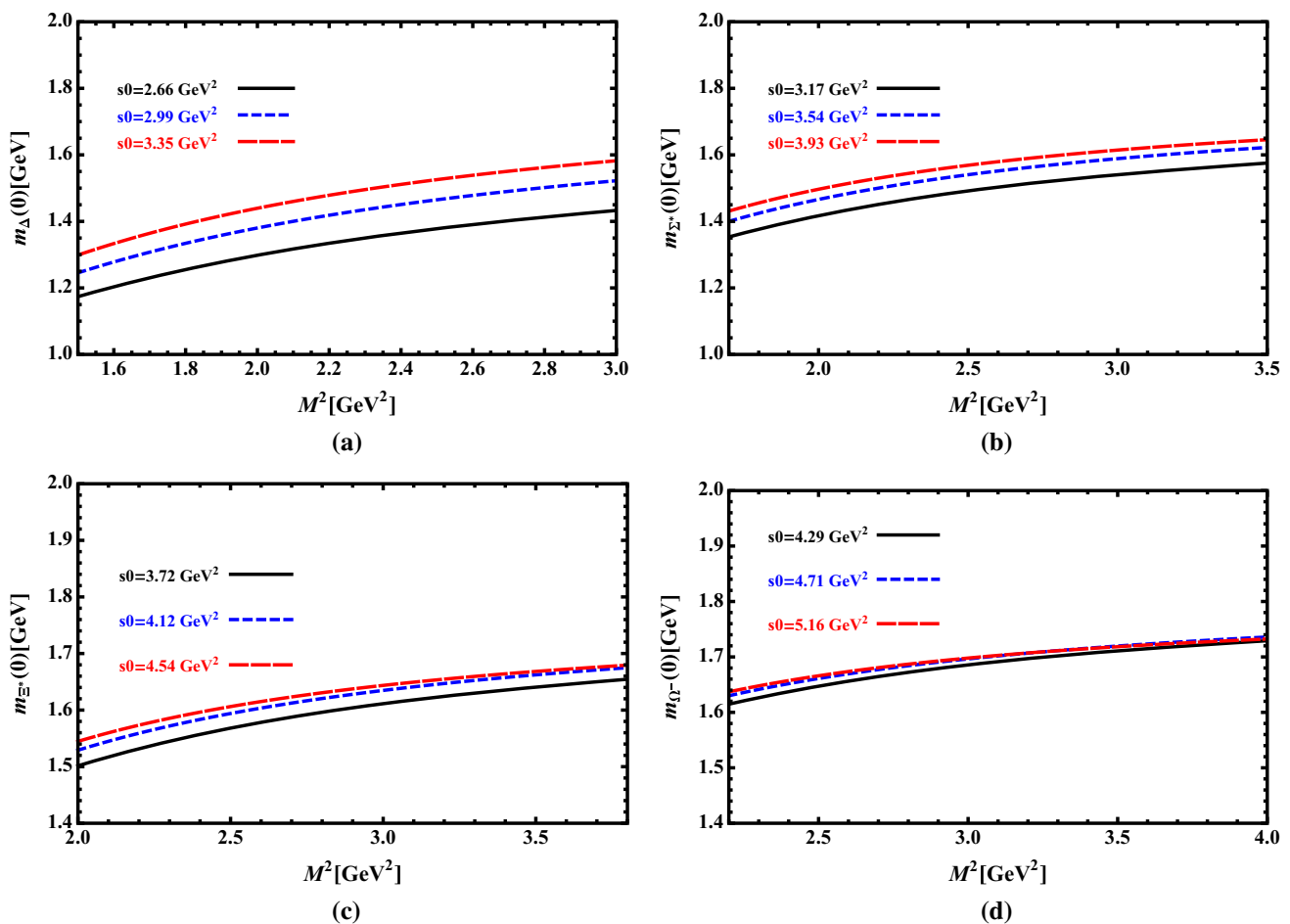
Fig. 1 **a** The mass of the Δ baryon as a function of M^2 for different fixed values of s_0 at $T = 0$. **b** The same as **a** but for Σ^* baryon. **c** The same as **a** but for Ξ^* baryon. **d** The same as **a** but for the Ω^- baryon
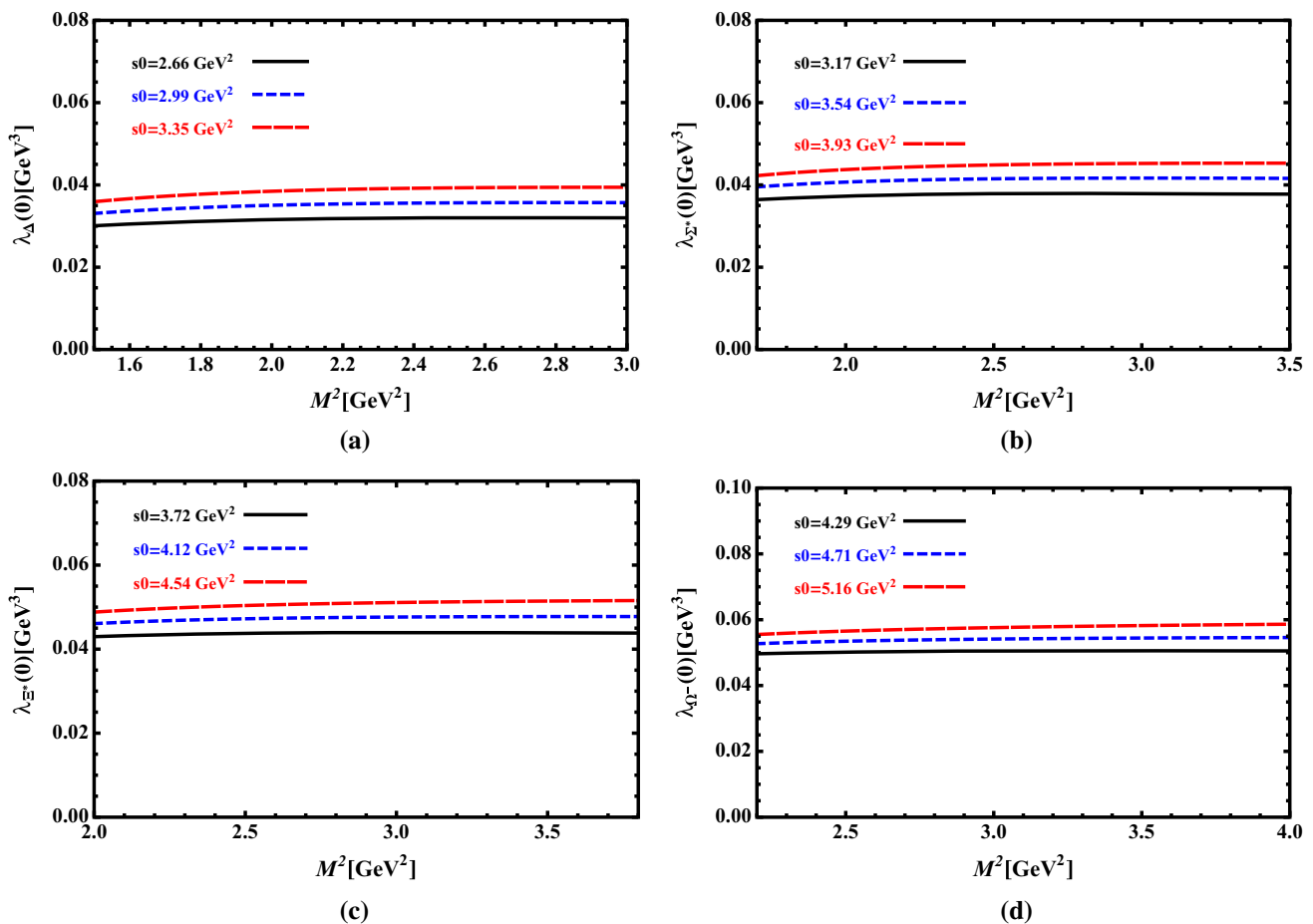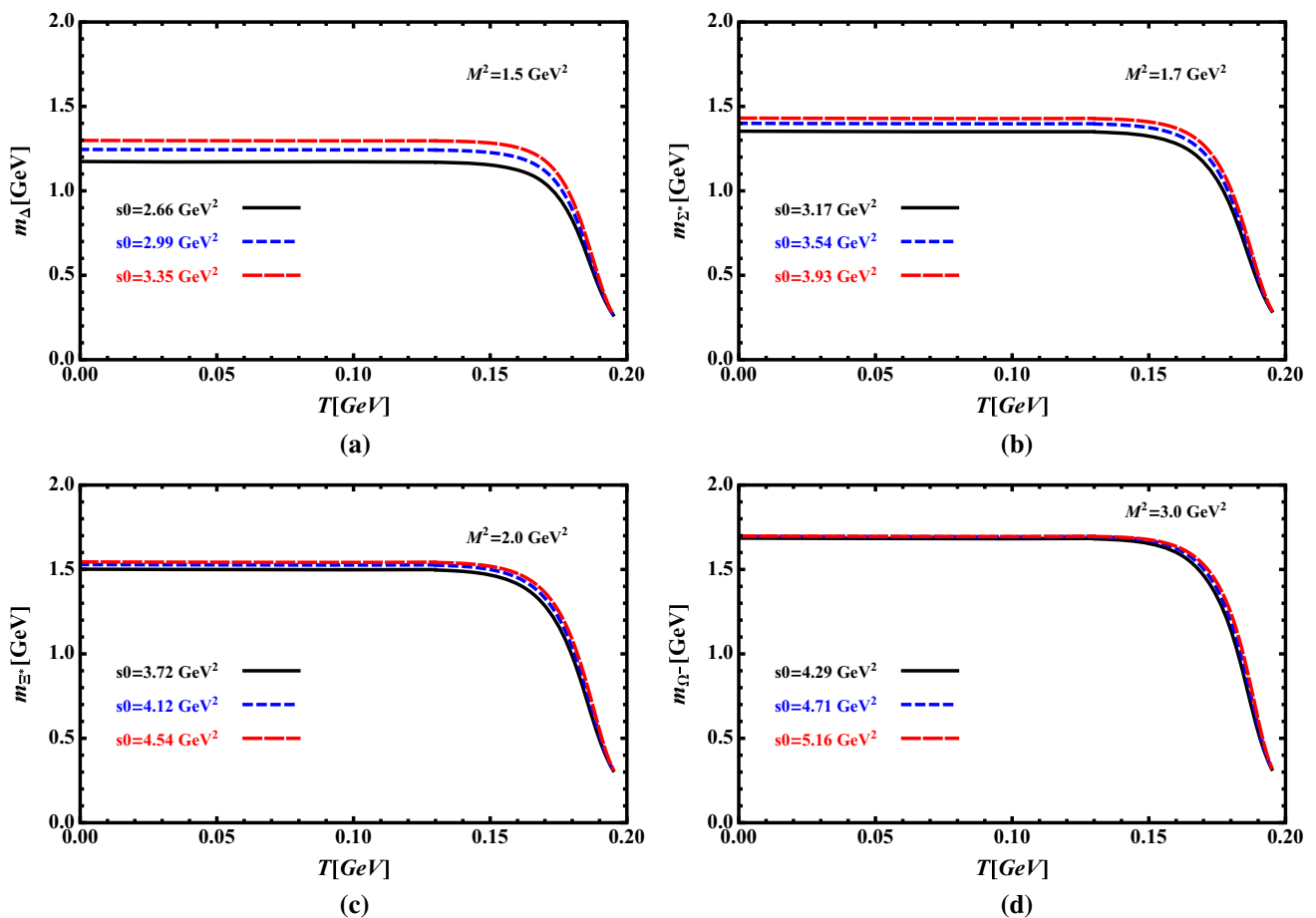
Fig. 2 **a** The residue of the Δ baryon as a function of M^2 for different fixed values of s_0 at $T = 0$. **b** The same as **a** but for Σ^* baryon. **c** The same as **a** but for Ξ^* baryon. **d** The same as **a** but for the Ω^- baryon

The final task is to investigate the variations of the mass and residue of Δ, Σ^*, Ξ^*, and Ω^- baryons with respect to temperature and compare the obtained results on the behavior of these quantities with respect temperature with our previous work for nucleon and hyperons [5,6] as well as existing predictions in the literature [1–3]. For this purpose, we plot these quantities as a function of temperature in Figs. 3 and 4. These figures indicate that the mass and residue of Δ, Σ^*, Ξ^*, and Ω^- decuplet baryons remain approximately unchanged up to $T \cong 0.15$ GeV, however, after this point, they start to diminish rapidly by increasing the temperature. The main reason behind such behavior is the variation of the T-dependent continuum threshold with respect to temperature presented in Eq. (20) and calculated, for the first time in the present study, in the decuplet channel. Our analyses show that this quantity does not change up to $T \cong 0.15$ GeV, beyond which it drastically falls. At the critical or deconfinement temperature, the continuum threshold falls with amount of 93 %. As we already mentioned the continuum threshold separates the ground state from the higher states and continuum. It appears in the upper limits of many integrals in the

sum rules obtained for the masses and residues (see the OPE expressions in the appendix) where their lower limits are roughly zero because of the light-quark masses. When temperature is increased the region of integration gets smaller. The temperature-dependent quark condensate that gives a higher contribution after the perturbative part also shows a similar behavior and kills many terms in the expressions of the sum rules as it approaches zero near the critical temperature. Such variations of the parameters of the hadrons under consideration near to the critical temperature can be considered as a sign of a transition to the quark–gluon plasma (QGP) as the new phase of the matter.

The average results on the masses obtained in the limit $T \to 0$ are $m_\Delta(0) = 1.239 \pm 0.148$ GeV, $m_{\Sigma^*}(0) = 1.394 \pm 0.167$ GeV, $m_{\Xi^*}(0) = 1.525 \pm 0.183$ GeV and $m_{\Omega^-}(0) = 1.693 \pm 0.203$ GeV which, within the errors, are in good consistencies with the experimental data [18]. We also obtain the average values $\lambda_\Delta(0) = 0.038 \pm 0.010$ GeV^3, $\lambda_{\Sigma^*}(0) = 0.043 \pm 0.012$ GeV^3, $\lambda_{\Xi^*}(0) = 0.053 \pm 0.014$ GeV^3, and $\lambda_{\Omega^-}(0) = 0.068 \pm 0.019$ GeV^3 for the residues at zero temperature which are also consistent with the results of Ref.
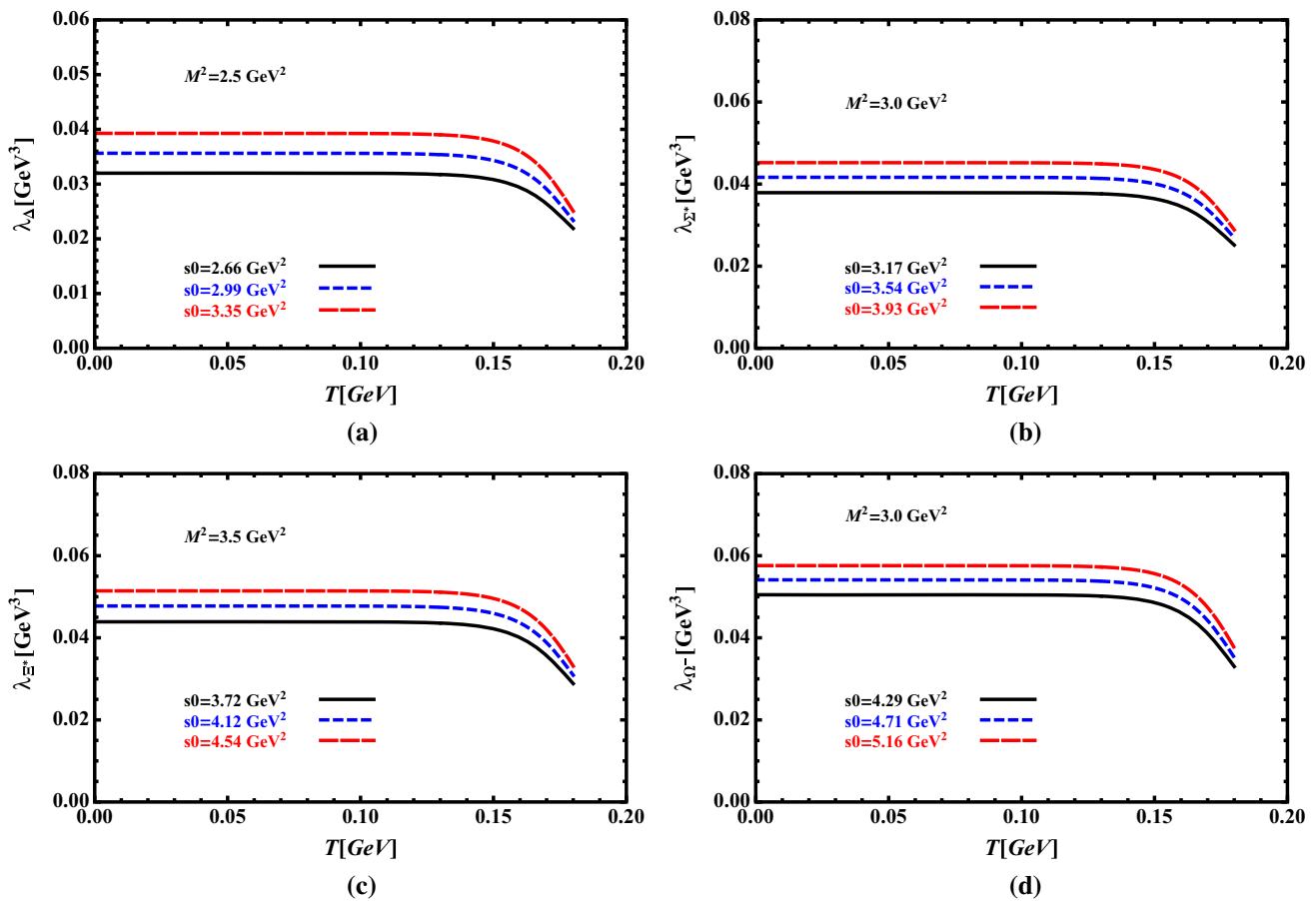
Fig. 3 **a** The mass of the Δ baryon as a function of temperature at fixed values of s_0 and M^2. **b** The same as **a** but for Σ^* baryon. **c** The same as **a** but for Ξ^* baryon. **d** The same as **a** but for the Ω^- baryon

[26]. Looking at the values of zero-temperature masses and residues we see that there is a considerably large SU(3) flavor symmetry breaking effect from Δ (consisting of up and down quarks) and Ω^- (consisting of only s quark). These violations are with amounts of 27 and 44 % for the mass and residue, respectively. However, when considering the thermal behavior of the mass and residue of decuplet baryons we cannot detect any SU(3) flavor violation effects and all members demonstrate roughly the same trend with respect to temperature. Near deconfinement or the critical temperature, the masses of Δ, Σ^*, Ξ^*, and Ω^- decuplet baryons fall with amount of roughly 80 %, while the residues overall reduce with amount of 35 % compared to their vacuum values. Though the zero-temperature values of the mass and residue of the decuplet baryons are very different from those of the octet baryons [5,6], the behaviors of the mass and residue of the decuplet baryons in terms of temperature obtained in the present work are similar to those of the nucleon and hyperons [5,6] and we do not see the effect of the spin on the thermal properties of baryons with the same quark contents. The variations of the masses versus temperature for

the decuplet baryons obtained in the present study are also in agreement with those of Ref. [2,3], i.e., in all of these studies the masses reduce rapidly with respect to T near to the critical temperature.

In conclusion, we investigated the mass and residue of Δ, Σ^*, Ξ^*, and Ω^- decuplet baryons at finite temperature in the framework of thermal QCD sum rules to acquire sum rules for these quantities in terms of different QCD degrees of freedom and the new operators appearing at finite temperature. We found the fit function for the temperature-dependent continuum threshold in terms of vacuum threshold. By fixing the auxiliary parameters, M^2 and s_0, and using the temperature-dependent quark and gluon condensates as well as the thermal average of the energy density obtained via lattice QCD and QCD sum rules, we analyzed the behaviors of the masses and residues versus temperature. We observed that the mass and residue of the decuplet baryons remain unchanged with the variations of temperature up to $T \cong 0.15$ GeV, after this point they decrease rapidly so that near the critical temperature the masses and residues reach roughly 20 and 65 % of their vacuum values, respectively. The melting of the baryons near

Fig. 4 **a** The residue of the Δ baryon as a function of temperature at fixed values of s_0 and M^2. **b** The same as **a** but for Σ^* baryon. **c** The same as **a** but for Ξ^* baryon. **d** The same as **a** but for the Ω^- baryon

the critical or deconfinement temperature may be considered as a sign of transition to the QGP phase that are searched for at different colliders. We also extracted the numerical values of the masses and residues at $T \to 0$ limit which are in good consistencies with the existing experimental data as well as other predictions in the literature.

Our results may be used in analyses of the results of the future heavy ion collision experiments. The T-dependent quantities considered in the present work, especially the temperature-dependent residues, can be used in the study of the strong couplings of the particles under consideration with other hadrons as well as their electromagnetic properties and radiative decays at finite temperature. The residue is the main input parameter in such studies.

Appendix: OPE expressions for Ξ^*

In this appendix, as examples, we present the functions $\Pi_1^{\text{OPE}}(p_0, T)$ and $\Pi_2^{\text{OPE}}(p_0, T)$ for Ξ^* baryon which are obtained as

$$\Pi_1^{\text{OPE}}(p_0, T)$$
$$= \frac{1}{160\pi^4} \int_{(2m_s+m_u)^2}^{s_0(T)} \mathrm{d}s \exp\left(-\frac{s}{M^2}\right) s^2$$
$$+ \frac{\langle \bar{q}q \rangle}{48\pi^2} \Bigg[m_0^2(8m_s - m_u) + (4m_u - 16m_s)$$
$$\times \int_{(2m_s+m_u)^2}^{s_0(T)} \mathrm{d}s \exp\left(-\frac{s}{M^2}\right) \Bigg]$$
$$+ \frac{\langle \bar{s}s \rangle}{24\pi^2} \Bigg[m_0^2(3m_s + 4m_u) - 4(m_s + 2m_u)$$

$$\times \int_{(2m_s+m_u)^2}^{s_0(T)} ds \, \exp\left(-\frac{s}{M^2}\right)\Bigg]$$

$$-\frac{\langle u\Theta^f u\rangle}{9\pi^2}\left[4p_0^2 - \int_{(2m_s+m_u)^2}^{s_0(T)} ds \, \exp\left(-\frac{s}{M^2}\right)\right]$$

$$+\frac{\alpha_s\langle u\Theta^g u\rangle}{9\pi^3}\int_{(2m_s+m_u)^2}^{s_0(T)} ds \, \exp\left(-\frac{s}{M^2}\right)$$

$$+\frac{5\langle\alpha_s G^2\rangle}{144\pi^3}\int_{(2m_s+m_u)^2}^{s_0(T)} ds \, \exp\left(-\frac{s}{M^2}\right) - \langle\bar{s}s\rangle^2$$

$$\times\left[\frac{(2m_0^2-4M^2)}{9M^2}\right] - \langle\bar{q}q\rangle\langle\bar{s}s\rangle\left[\frac{(4m_0^2-8M^2)}{9M^2}\right]$$

$$+\frac{\langle\bar{q}q\rangle\langle u\Theta^f u\rangle}{81M^6}\{2m_0^2 m_u(-7M^2-8p_0^2)+48M^2$$

$$\times[m_s M^2 + m_u(M^2+2p_0^2)]\}$$

$$+\frac{\langle\bar{s}s\rangle\langle u\Theta^f u\rangle}{81M^6}\{4m_0^2 m_s(-7M^2-8p_0^2)+48M^2$$

$$\times[3m_s M^2 + m_u M^2 + 4m_s p_0^2]\}$$

$$+\left[\frac{\alpha_s\langle\bar{q}q\rangle\langle u\Theta^g u\rangle}{108\pi M^4}m_u + \frac{\alpha_s\langle\bar{s}s\rangle\langle u\Theta^g u\rangle}{54\pi M^4}m_s\right]$$

$$\times(3m_0^2-8M^2)$$

$$-\left[\frac{\langle\bar{q}q\rangle\langle\alpha_s G^2\rangle}{432\pi M^4}m_u + \frac{\langle\bar{s}s\rangle\langle\alpha_s G^2\rangle}{216\pi M^4}m_s\right]$$

$$\times(3m_0^2-20M^2)+\frac{2\alpha_s\langle u\Theta^f u\rangle\langle u\Theta^g u\rangle}{27\pi M^4}(M^2+p_0^2)$$

$$-\frac{\langle\alpha_s G^2\rangle\langle u\Theta^f u\rangle}{27\pi M^4}(3M^2-4p_0^2)$$

$$-\frac{2\langle u\Theta^f u\rangle^2}{9M^2}(5+2t+5t^2), \tag{A.1}$$

$$\Pi_2^{OPE}(p_0,T) = \frac{(2m_s+m_u)}{64\pi^4}\int_{(2m_s+m_u)^2}^{s_0(T)} ds \, \exp\left(-\frac{s}{M^2}\right)s^2$$

$$+\frac{\langle\bar{q}q\rangle+2\langle\bar{s}s\rangle}{72\pi^2}$$

$$\times\left[\int_{(2m_s+m_u)^2}^{s_0(T)}(3m_0^2-8s)ds \, \exp\left(-\frac{s}{M^2}\right)\right]$$

$$+\frac{2\langle u\Theta^f u\rangle}{9\pi^2}(2m_s+m_u)$$

$$\times\left[p_0^2 + \int_{(2m_s+m_u)^2}^{s_0(T)} ds \, \exp\left(-\frac{s}{M^2}\right)\right]$$

$$+\frac{\alpha_s\langle u\Theta^g u\rangle}{36\pi^3}(2m_s+m_u)$$

$$\times\int_{(2m_s+m_u)^2}^{s_0(T)} ds \, \exp\left(-\frac{s}{M^2}\right)$$

$$+\frac{\langle\alpha_s G^2\rangle}{48\pi^3}(2m_s+m_u)$$

$$\times\int_{(2m_s+m_u)^2}^{s_0(T)} ds \, \exp\left(-\frac{s}{M^2}\right)$$

$$+\langle\bar{s}s\rangle^2\left[\frac{-5m_0^2 m_s+18m_u M^2}{27M^2}\right]+\langle\bar{q}q\rangle\langle\bar{s}s\rangle$$

$$\times\left[\frac{-5m_0^2(m_s+m_u)+36m_s M^2}{27M^2}\right]$$

$$+\frac{4\langle\bar{q}q\rangle\langle u\Theta^f u\rangle}{27M^4}$$

$$\times[-4M^2(2M^2+p_0^2)+m_0^2(3M^2+p_0^2)]$$

$$+\frac{8\langle\bar{s}s\rangle\langle u\Theta^f u\rangle}{27M^4}$$

$$\times[-4M^2(2M^2+p_0^2)+m_0^2(3M^2+p_0^2)]$$

$$-\frac{2\alpha_s\langle\bar{q}q\rangle\langle u\Theta^g u\rangle}{27\pi M^2}(m_0^2-2M^2)$$

$$-\frac{4\alpha_s\langle\bar{s}s\rangle\langle u\Theta^g u\rangle}{27\pi M^2}(m_0^2-2M^2)$$

$$+\frac{\langle\bar{q}q\rangle\langle\alpha_s G^2\rangle}{54\pi M^2}(m_0^2-4M^2)$$

$$+\frac{\langle\bar{s}s\rangle\langle\alpha_s G^2\rangle}{27\pi M^2}(m_0^2-4M^2)$$

$$+\frac{16\langle u\Theta^f u\rangle^2}{81M^4}(2m_s+m_u)(3M^2+14p_0^2). \tag{A.2}$$

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