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A PROPOSED MECHANISM FOR CUMULONIMBUS PERSISTENCE IN THE PRESENCE OF STRONG VERTICAL SHEAR

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ABSTRACT

Based on observations that vigorous convective elements in the atmosphere act as obstacles to the environmental flow, it is shown that hydrodynamic pressures exist near the strong updraft. After utilizing this result, a simple model of a quasi-steady-state cumulonimbus is proposed. It is then found that vertical shear in unidirectional environmental flow leads to enhanced vertical motions on the perimeter of the updraft. These intensified vertical motions would allow the updraft to be less adversely affected by the shearing forces and may explain the persistence or growth of such storms.

1. INTRODUCTION

The persistence and growth of large cumulonimbus systems in the presence of strong vertical shear has been investigated over the past several years. Observations that these storms resist deformation in the presence of strong shear have been recorded (cf. Hitschfeld, 1960; Ludlam, 1966). Newton and Newton (1959) postulated that the environmental air is forced to diverge and flow around a well-developed cumulonimbus. This idea has been substantiated by observations by Fujita and Grandoso (1968) as well as by Shmeter (1966). Newton (1963) investigated the effects of this forced divergence in a manner similar to that presented here. After drawing upon results of experimental hydrodynamics, it can be shown that the diverging flow of environmental air requires induced hydrodynamic pressures near the boundaries of the strong updraft core. For a vertically oriented, steady-state storm imbedded in sheared environmental flow, vertical gradients of hydrodynamic pressure, leading to vertical motions around the updraft core, would cause the updraft to be shielded from the erosive effects of entrainment.

2. THE EXISTENCE OF HYDRODYNAMIC PRESSURE NEAR A STRONG UPDRAFT

From the observation that vigorous convective elements maintain their form in the presence of strong vertical shear, the only logical conclusion seems to be that ascending parcels conserve at least a part of their horizontal momentum during vertical displacement (Bates and Newton, 1965). When extending this reasoning, it follows that for any vertically oriented convective system existing in an environment exhibiting strong vertical shear, the environmental air must flow around the system. The alternative explanation (flow through the system) is an obvious contradiction to the observed lack

of entrainment for large convective elements (Malkus, 1960). Recent observational verification of flow around a strong updraft core is included in an article by Fujita and Grandoso (1968). Figure 1 is reproduced from that publication and illustrates flow relative to a storm.

Interpretation of the updraft region as a barrier to the environmental flow without assuming any specific form for the updraft allows an application of the Bernoulli equation at a given level which yields a pressure coefficient (C_p) defined by

$$C_p = 1 - \left(\frac{V_c}{V_e}\right)^2, \qquad (1)$$

where the environmental flow has been assumed to be horizontal, steady, and incompressible. In equation (1), V_c is the relative wind speed near the moving updraft core, and V_e is the relative speed in the undisturbed environment upstream from the updraft. Relative speed is defined here as the environmental velocity with respect to a fixed point minus the translational velocity of the updraft. Note that C_p is a function of the position on the
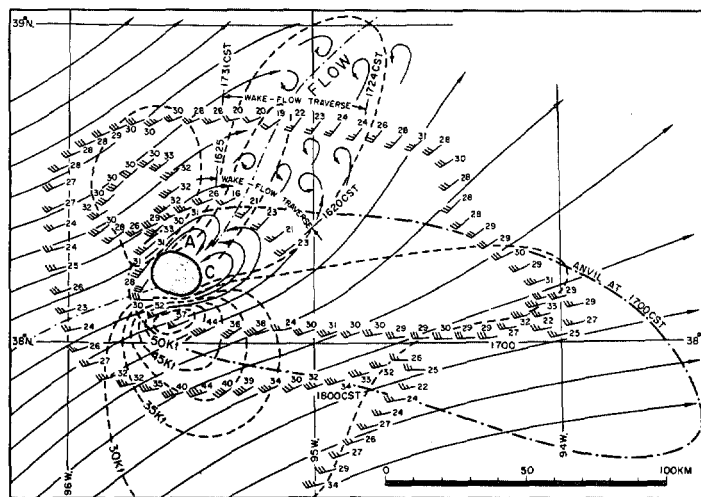

FIGURE 1.—Relative environmental winds at 500 mb in the vicinity of a cumulonimbus, after Fujita and Grandoso (1968).

barrier and varies from 1 at the stagnation point (where $V_c=0$) to negative values on the flanks (where $V_c>V_e$ as illustrated by fig. 1).

Defining the hydrodynamic pressure (P_d) by

$$P_d=C_p(\tfrac{1}{2}\rho_e V_e^2), \qquad (2)$$

where the quantity in parentheses is the dynamic head (or kinetic energy per unit volume) of the undisturbed flow, permits the pressure near the updraft core (P_c) to be written as

$$P_c=P_e+P_d, \qquad (3)$$

where P_e is the pressure in the undisturbed flow. From equations (1) and (2) it is seen that unless $C_p=0$ (i.e., unless $V_c=V_e$), hydrodynamic pressure must exist near the updraft core. These equations also show that measurement of the horizontal velocity distribution around such a convective element would permit the calculation of the hydrodynamic pressure distribution at any given level.

In the absence of detailed measurements, it is convenient to obtain values of C_p from experimental hydrodynamics. Since, for frictionless flow, the dividing streamline may be replaced by a solid body without altering the external flow, choosing a representative equivalent body shape for the flow is feasible. Newton (1963) chose to represent the dividing streamline by a circular cylinder. However, from figure 1 it appears that the dividing streamline would be better modeled as an elliptical cylinder with the major axis in the direction of the undisturbed flow. The importance of the difference in models is illustrated by the fact that the magnitude of C_p may be three times as large on the flanks of a circular cylinder as at corresponding positions on a slender elliptical cylinder (Schlichting, 1960).

This investigation is restricted to the influence of hydrodynamic pressures on the upstream and flank regions of the barrier. In these regions, the agreement between theoretical and experimental hydrodynamics is quite good (Schlichting, 1960).

3. VERTICAL SHEAR AND VERTICAL GRADIENTS OF HYDRODYNAMIC PRESSURE

The vertical equation of motion for a unit mass of environmental air near the boundary of the updraft core is

$$\frac{dw}{dt}=-\frac{1}{\rho_e}\frac{\partial P}{\partial z}-g, \qquad (4)$$

neglecting viscous and Coriolis effects. Assuming that the undisturbed environmental flow is hydrostatic (i.e., $P_e=P_h$), equation (3) becomes

$$P_c=P_h+P_d. \qquad (5)$$

Thus, for regions near the strong updraft core where

$P=P_c$, equation (4) becomes

$$\frac{dw}{dt}=-\frac{1}{\rho_e}\frac{\partial}{\partial z}(P_h+P_d)-g. \qquad (6)$$

Since $\partial P_h/\partial z=-g\rho_e$, equation (6) becomes

$$\frac{dw}{dt}=-\frac{1}{\rho_e}\frac{\partial P_d}{\partial z}. \qquad (7)$$

Substituting (2) into (7) yields

$$\frac{dw}{dt}=-\left[C_pV_e\frac{\partial V_e}{\partial z}+\frac{C_pV_e^2}{2}\frac{\partial(\ln\rho_e)}{\partial z}+\frac{V_e^2}{2}\frac{\partial C_p}{\partial z}\right]. \qquad (8)$$

This equation shows that measurements of the vertical distributions of relative undisturbed environmental velocity, relative environmental velocity near the updraft core, and environmental density would allow the computation of the vertical accelerations near the core. That is, the storm could be split up into layers, and values of vertical acceleration in the layers could be computed. The first two terms in equation (8) represent the influence of vertical shear of the kinetic energy, while the third may be interpreted as the influence of veering environmental flow.

4. THE CUMULONIMBUS MODEL

Although the preceding developments have application to any obstacle to the environmental flow, even small-scale transitory convection with weak updrafts, a steady-state obstacle will now be assumed. The following results will be applicable to a large, quasi-steady-state cumulonimbus. The cumulonimbus is visualized as an organized convective region, invariant in spatial dimensions for a finite increment of time. Due to the vertical transport of horizontal momentum accomplished by the convection, the environmental air is forced to flow around the updraft core as previously discussed. The updraft core is assumed to have a vertical orientation that results in a dividing streamline (or equivalent body shape) that is not displaced horizontally with increasing height.

For many wind profiles in the vicinity of severe convective activity, only relatively small amounts of veering with height are found in the environmental flow above 2 to 3 km. Therefore, an environmental flow of constant direction is assumed. The role of the veering low-level flow has a much more important role in cumulonimbus dynamics than any contribution to hydrodynamic pressure gradients in those levels. Newton and Fankhauser (1964) have shown that the primary function of the low-level flow is to continually replenish the moist, potentially unstable air upon which the storm feeds.

The dividing streamline is assumed to occur within the visible cloud boundary to allow for entrainment effects that exist at the cloud-clear air boundary. Assuming the visible boundary of a cumulonimbus to be representative of the dividing streamline would imply no exchange of

momentum at the boundary which is obviously undesirable for a cloud model moving with a velocity different from that of its environment.

The environmental flow is assumed to be horizontal, steady, horizontally uniform, incompressible, and to exhibit vertical shear. The translational velocity of the updraft (V_u) is assumed constant and in the direction of the undisturbed environmental flow. This assumption may seem undesirable since the observed trajectories of most well-developed cumulonimbi tend to be to the right of the undisturbed flow. However, following the suggestion of Newton and Fankhauser (1964), it is postulated that the deviation to the right for a nonrotating cumulonimbus is primarily a result of new growth on the side of the storm where low-level moisture is continually replenished along with resultant dissipation on the opposite side. With this concept, it is plausible to assume that at every instant the existing cloud matter moves in a downstream direction and that the observed deflection is due to preferential growth.

Incorporating equation (8) into the model leads to the expression for vertical acceleration due to vertical gradients of hydrodynamic pressure:

$$\frac{dw}{dt} = -C_p V_e \left[\frac{V_e}{2} \frac{\partial (\ln \rho_e)}{\partial z} + \frac{\partial V_e}{\partial z} \right] \qquad (9)$$

since, for the model, $\partial C_p / \partial z = 0$.

Analysis of several soundings in the vicinity of well-organized, severe convection in the Great Plains illustrated that the vertical distribution of environmental density may be approximated by the relation

$$\rho_e = \rho_0 \exp[-Kz] \qquad (10)$$

where $K = 0.1$ km^{-1}. Figure 2 shows a comparison between (10) and values of ρ_e computed from such soundings assuming an ideal gas. Substituting (10) into (9) yields

$$\frac{dw}{dt} = C_p V_e \left[\frac{K V_e}{2} - \frac{\partial V_e}{\partial z} \right]. \qquad (11)$$

Recall that for the model the relative velocity (V_e) is related to the environmental velocity with respect to a fixed point (U_e) by $V_e = U_e - V_u$, where V_u is the translational velocity of the updraft. Thus, $\partial V_e / \partial z = \partial U_e / \partial z$ since $\partial V_u / \partial z = 0$ by assumption. Evaluation of equation (11) requires the knowledge of only three parameters, namely: a) C_p, which is only a function of the shape of the obstacle to the flow and the position relative to that obstacle; b) the horizontal environmental velocity (U_e) and its variation with height; and c) the translational velocity of the obstacle. Thus, assuming an equivalent body shape for the dividing steamline and having a vertical profile of horizontal environmental winds and a translational velocity of the system, one may compute the contribution to vertical motion due to vertical gradients of hydrodynamic pressure.

In choosing a representative equivalent body shape for the dividing streamline, it may be assumed that the up-
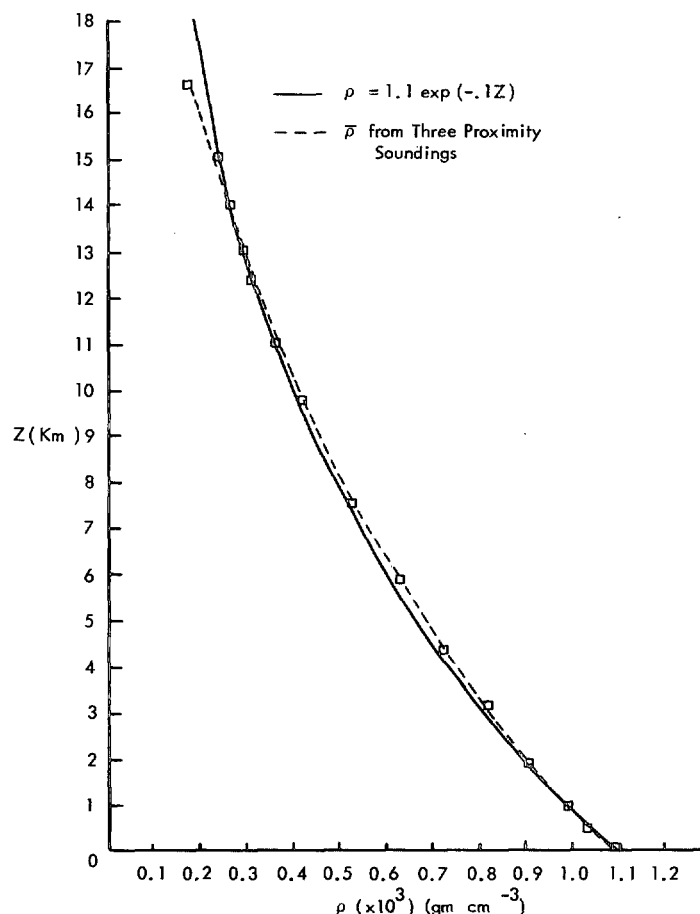

FIGURE 2.—Comparison of a mean density profile in the vicinity of severe convective activity and the assumed density function.

draft core acts as a vertical line source imbedded in horizontally uniform flow. With this assumption, the upstream half of the body approaches an elliptic cylinder with the major axis in the direction of flow (Schlichting, 1960). The slenderness of the resultant elliptic cylinder is a function of the source strength and the environmental velocity at each level. That is, the slenderness of the obstacle is inversely proportional to the difference in horizontal momentum between the air within the updraft core and that in the environment. As the difference in horizontal momentum increases, the elliptic cylinder approaches the circular cylinder postulated by Newton (1963) and Goldman (1968).

For an elliptic cylinder of slenderness 4 (i.e., an elliptic cylinder with the ratio of the major axis to the minor axis equal to four), the pressure coefficient near the flanks is approximately equal to -0.5 (Schlichting, 1960) for Reynolds numbers in the subcritical range. Values of C_p decrease to -1.0 near the flanks as the body shape approaches a circular cylinder. Although $C_p = 1.0$ by definition at the stagnation point for any body of finite slenderness, values of C_p near the stagnation point used are equal in magnitude to those near the flanks. In this way the

results may be applied to a much broader region of the upstream side of the body.

Because it is desired not to overestimate the effects of vertical gradients of hydrodynamic pressure, values of C_p used in computations to follow are:

$$C_p = -0.4 \text{ near the flanks}$$

and

$$C_p = +0.4 \text{ near the stagnation point.}$$

Since, from equation (11), the contribution to vertical motion is directly proportional to the magnitude of C_p, effects claimed for this model are less than half as large as those that would result from assuming a cylindrical obstacle. However, as an existing updraft grows in the horizontal dimension normal to the environmental flow, the contribution to vertical motion increases since the magnitude of C_p increases. The values obtained in this paper are intended to represent a reasonable lower limit for a large storm.

Since little is known a priori about the increment of time a given air parcel would spend in a given vertical acceleration field, direct finite-difference integration of the vertical acceleration to obtain the vertical velocity is not possible. However, a reasonable approximation may be obtained by assuming the vertical acceleration and velocity in any given layer to be a linear function of z. One may write

$$\frac{dw}{dt} = \frac{1}{2}\frac{d(w^2)}{dz}. \tag{12}$$

Taking the mean value of dw/dt in the layer and integrating yields

$$w_2^2 = w_1^2 + 2\overline{\frac{dw}{dt}}\Delta z, \tag{13}$$

where Δz is the thickness of the layer considered. Writing $\overline{dw/dt} = 0.5(a_1 + a_2)$, where a_1 and a_2 denote the vertical acceleration at level z_1 and z_2, respectively, allows one to write

$$w_2^2 = w_1^2 + \Delta z(a_1 + a_2). \tag{14}$$

Equation (14) must be modified in order to consider both positive and negative velocities. The necessary modification is to replace w_n^2 by $|w_n|w_n$. This allows (14) to be written as

$$|w_2|w_2 = |w_1|w_1 + \Delta z(a_1 + a_2). \tag{15}$$

Equation (15) allows negative accelerations to yield negative velocities while (14) cannot be applied in this way. By using this result applied to successive layers beginning at the level where w_n is zero and computing w_{n+1} from previously determined values of a_n and a_{n+1}, the vertical velocity profile can be computed. In the actual computation, the following relation was used:

$$w_m = \pm |[\Delta z(a_n + a_m) + w_n|w_n|]|^{1/2}, \tag{16}$$

where $m = n + 1$ and the sign is chosen to agree with the sign of the term in brackets.
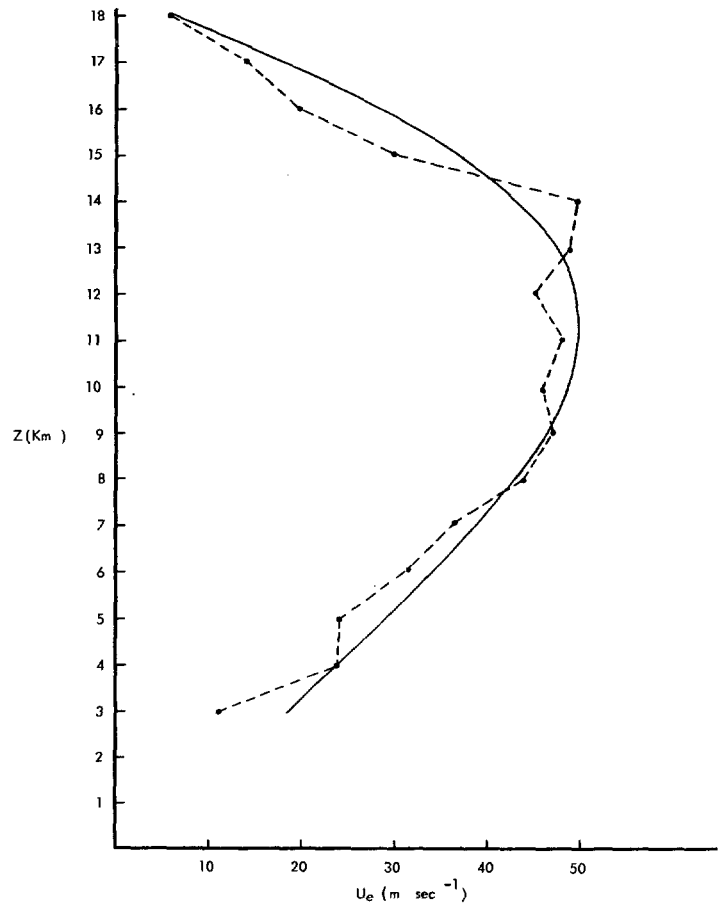

FIGURE 3.—Comparison of an observed and an assumed vertical profile of horizontal environmental velocity. (Observations from Carswell Air Force Base, Tex., Apr. 4, 1965.)

5. NUMERICAL RESULTS OF THE MODEL

Due to the manner in which upper air data are reported, discontinuities in the vertical shear are the rule rather than the exception. Equation (11) illustrates the extreme effects upon computed vertical accelerations due to these apparent discontinuities. To avoid the undeterminable shears appearing in real data, one can either smooth the sounding (i.e., approximate the shear) or assume that an analytic function is representative of the actual wind profile. The accuracy of (16) increases as Δz decreases; and since analytic functions have continuous derivatives at every point, the analytic function approximation offers obvious advantages.

The vertical profile of horizontal environmental velocity is assumed to be suitably represented by an analytic function of the form

$$U_e = A \sin\left(\frac{\pi z}{B}\right)\exp[z/C] + U_0, \tag{17}$$

where U_0 is the velocity at anemometer height. Values of A, B, C, and U_0 may be varied in such a way that reason-
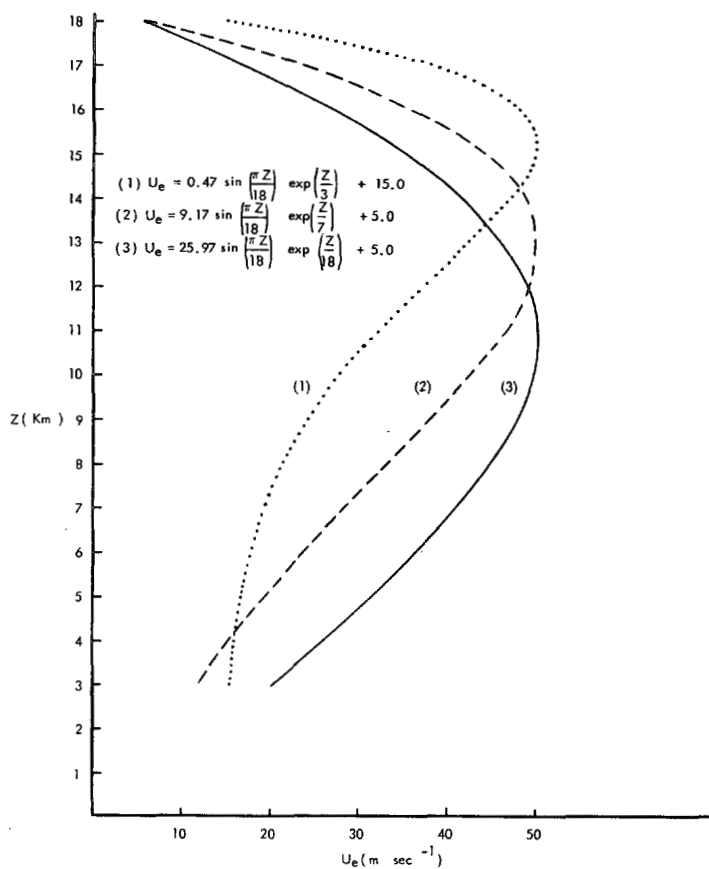
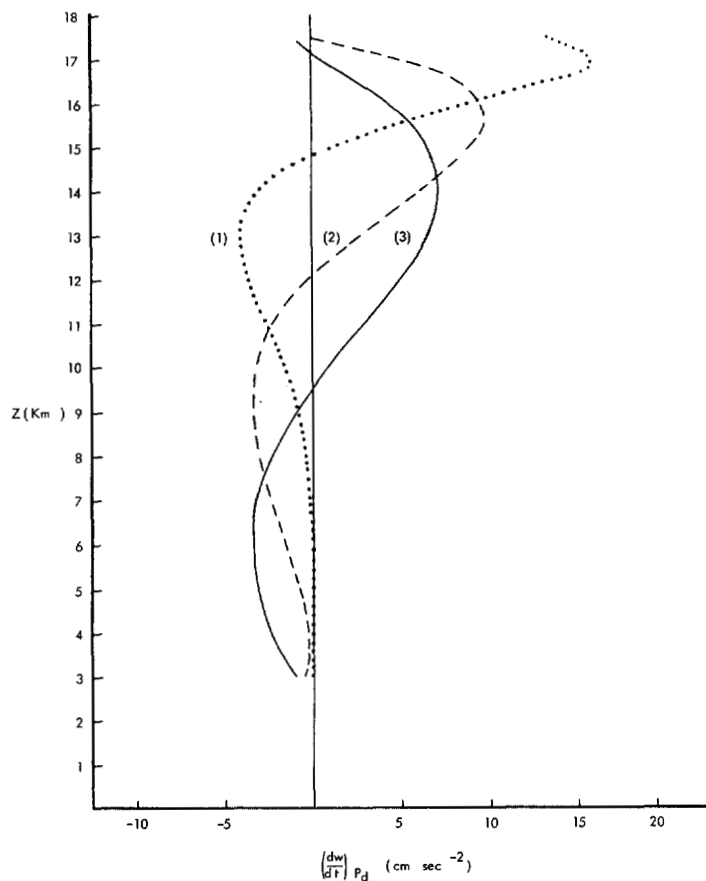FIGURE 4.—Vertical profiles of horizontal environmental velocity used in the model.

FIGURE 5.—Vertical accelerations near the upstream side of the cumulonimbus model for the profiles shown in figure 4.

able agreement between reported wind profiles and the representation is obtained. Figure 3 illustrates one such comparison. Figure 4 illustrates three of the possible profiles obtainable by varying the parameters A, B, C, and U_0. Note that for each of the profiles it was assumed that the maximum horizontal velocity was 50 m sec^{-1}.

Equations (11) and (17) were used to compute the acceleration profiles shown in figures 5 and 6. It was assumed that $V_u = 15$ m sec^{-1}. Equation (16) was used to compute the profiles of vertical velocity shown in figures 7 and 8, using a height increment of 10 m. The number on each curve in figures 5 through 8 identifies the applicable family of parameters (and hence the vertical profile of environmental winds) as given in figure 4.

Simultaneous consideration of figures 4, 5, and 6 illustrates the fluctuating dominance of the density gradient term and the shear term in equation (11). That is, for regions of weak shear and high relative velocity, the density gradient term dominates (11). Conversely, for levels of strong shear or low relative velocity, the shear term dominates. Figure 7 is the vertical velocity profile near the trailing (or stagnation) side of the storm model. The level at which the sum of the shear term and density gradient term is a minimum was found, and integration using equation (16) was carried out in both directions

from that level. In figure 8, the lowest level of positive (upward) acceleration was chosen as the level for beginning the integration.

Although the vertical acceleration profiles for the model (figs. 5 and 6) are not excessive, figures 7 and 8 illustrate the large vertical velocities that may evolve for neutrally buoyant air parcels. The latter two figures are based upon the assumption that the air parcels considered remain in their respective acceleration fields throughout their vertical displacement. Since the assumption was made that these parcels remain neutrally buoyant, the accelerations indicated would have to be added to any buoyancy existing.

For other computations not illustrated, it was found that increasing the maximum environmental flow and simultaneously increasing the storm velocity led to similar results. However, for any level where a storm velocity greater than the environmental velocity occurs, a symmetric streamlined body shape must be assumed if any region other than the flanks of a storm are to be considered. Even for streamlined body shapes, the flow in the wake region may be highly turbulent and cannot be steady state for relative velocities greater than a few meters per second. Thus the contribution to vertical
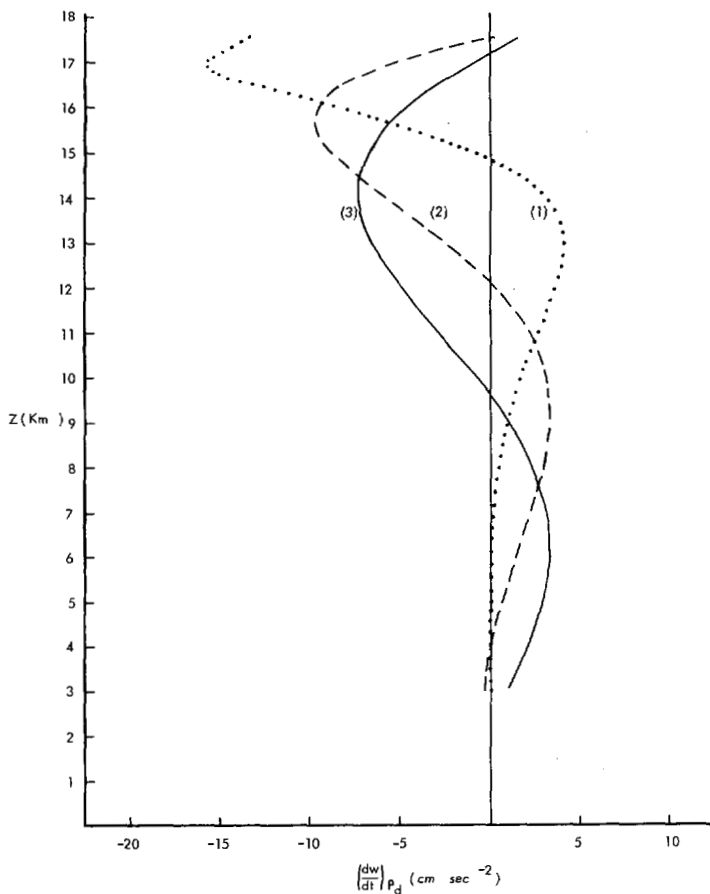
FIGURE 6.—Vertical accelerations near the flanks of the cumulo
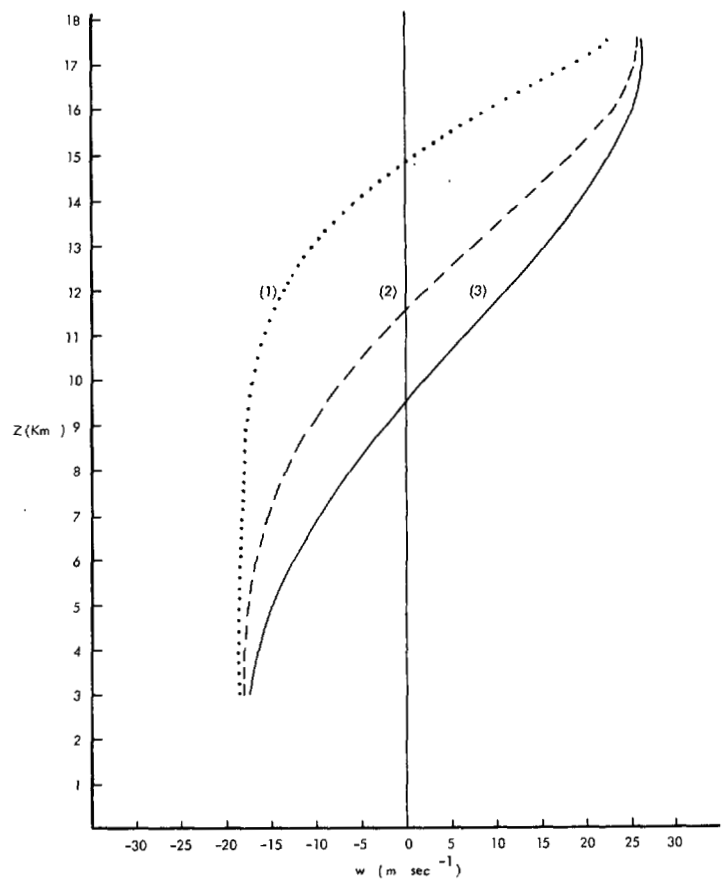nimbus model for the profiles shown in figure 4.

FIGURE 7.—Vertical velocities for a neutrally buoyant parcel near
the upstream side of the cumulonimbus model arising from the
acceleration profile of figure 5.

motion due to vertical gradients of hydrodynamic pressure
is highly questionable in this region.

6. CONCLUSIONS

The effect of vertical shear has been shown to produce
an enhancement of vertical motions near the strong
updraft core characteristic of severe storms. Upward
accelerations on the flanks of such a storm may add to
any buoyancy forces existing there, or may actually
induce buoyancy due to release of latent heat and precipi-
tation. Downward accelerations on the trailing edge of
the storm leading to the evaporatively cooled outflow or
"thunderstorm front" are also at least augmented by
vertical gradients of hydrodynamic pressure, and perhaps
are initiated by such gradients.

The results stated here are most applicable to those
levels called the "barrier flow layer" by Goldman (1968)
and the "throat" of the storm by Bates (1961). This
layer has been assumed to be quite deep in these com-
putations. However, a less deep layer does not invalidate
the results. The results indicate that, although strong
vertical shear might be expected to force the convective
column to lean in a down-shear direction, the strong shear
may simultaneously yield an effect enhancing the vertical
motion near a strong updraft core. This increased vertical

motion in air that is a combination of environmental and
cloud air serves to decrease the entrainment of drier
environmental air into the column and thus allow greater
vertical velocities within the column. The overall effect
can either be interpreted as increasing the source strength
of the barrier to the flow or as allowing less time for the
environmental air to impart its horizontal momentum to
the rising air parcels having relatively small horizontal
momentum. Of course, one must realize that the effect
discussed here is only one factor in an extremely complex
system. A persistent cumulonimbus is a result of a very
delicate balance between dynamical and thermodynamical
interactions. However, perhaps the most important
mechanism leading to the resistance to shearing forces
exhibited by many severe storms is the vertical gradient of
hydrodynamic pressure which is proportional to the shear
itself. This implies that the convective system containing
strong updrafts can be reinforced by the very mechanism
that would seem to destroy it—the vertical shear of the
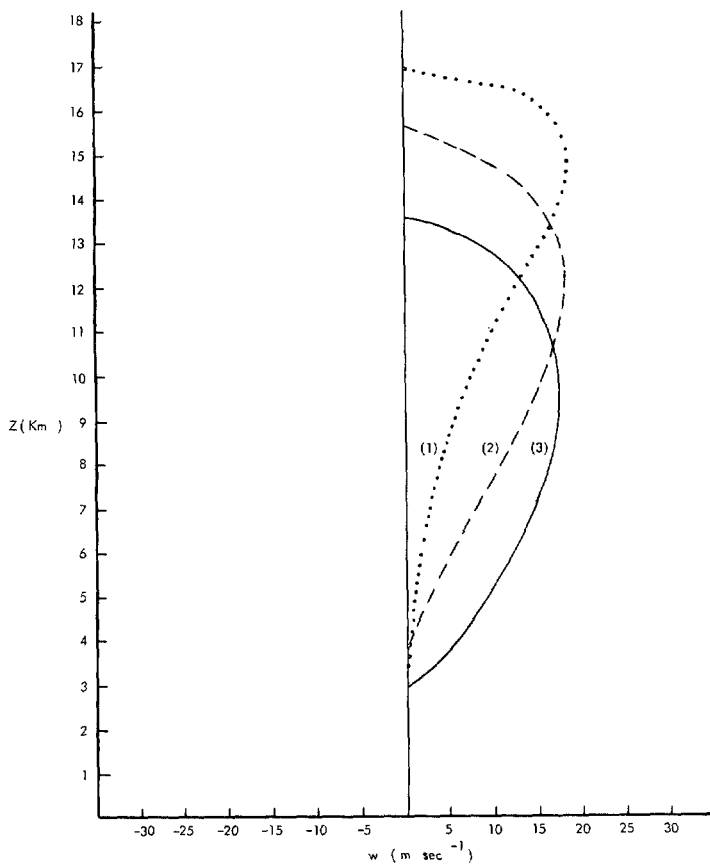environmental flow.

ACKNOWLEDGMENTS

FIGURE 8.—Vertical velocities for a neutrally buoyant parcel near the flanks of the cumulonimbus model arising from the acceleration profile of figure 6.

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