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Research Article

Pricing Decisions of a Dual-Channel Supply Chain considering Supply Disruption Risk

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Supply disruption may cause strong complaints of customers, which is a cost loss for the firms in the supply chain. Obviously, if realizing that there is the disruption risk, the members in a supply chain will adjust their decisions. For analyzing the influence, we consider a popular supply chain mode with dual channels, where one manufacturer has its direct sales channel and one traditional retailer channel. The manufacturer may suffer a supply disruption so that all ordered products by the retailer or the direct retail channel will be lost, and the members in supply chain will bear the corresponding disruption penalty from the customers. By considering four structures with different market power relations, the closed-form optimal price decisions of the four models are given. We found that the disruption factor improves the sales prices for any member structure as compared to the supply chain without the disruption. And the direct retail prices in the different modes are the same as each other, but the price of the traditional channel is influenced by the market share. And the sorts of the sales prices under different structures are given. We also conduct some extensive numerical analysis and compare the results under different structures. We observe that the expected optimal profits of considering the external penalty are smaller than those of no external penalty, and we give a sort of the optimal expected profits. And we also provide the effects of some parameters on the optimal decisions and the optimal expected profits.

1. Introduction

With the rapid development of the Internet, customers are becoming more and more accustomed to online shopping. By means of an online channel, firms can deal with customers' order and get the logistic information timely. Therefore, those firms, such as Amazon, Dell, Jingdong, Alibaba, and Taobao, can be closer to customers and have a better understanding of customers' preference [1]. Thus, many manufacturers face the decision whether to add one direct channel in addition to the existing traditional channel. In order to improve competitiveness, many traditional manufacturers have begun or are beginning to set up their online direct sales channels. Therefore, the phenomenon that the traditional channel and direct sales channel coexisting is more common. Generally, it is called the dual-channel supply chain, which refers to the fact that the supply chain has the online (direct sales) and off-line (traditional retail) channels. In a dual-channel supply chain, the unreasonable pricing decisions may lead to the conflict between two channels, so one of the key issues in the dual-channel supply chain is the pricing problem.

With the help of the Internet, the direct sales channel in the dual-channel supply chain can realize the interaction of one to one between the firm and the customer, which is a good marketing channel. However, the information spread on Internet is faster; any change in supply or demand side may lead to a serious mismatch between supply and demand corresponding to a traditional channel. In 2017, the southern black sesame group (a Chinese company) obtained a lot of market concerns by a new IP (Intellectual Property) marketing and a favorable brand spokesperson, which directly promotes sales growth to exceed the firm's expectations. However, the firm's production line did not switch in time, so a serious stock-out happened for two channels [2]. And traditional disruption events, such as machine breakdown, raw material shortage, and workers strike, also can lead to one disruption. At August 16, 2017, all of CR Vanguard, WALMART, RT-MART, and other chain supermarkets expressed that Moutai (Chinese spirits brand) of 53 degrees is out of stock, and they do not know when to replenish it. At the same time, Moutai's own E-commerce platform also announced that there is shortage of products [3]. Similar situations also occurred in the sales process of Apple SE (2016, [4]). Therefore, the pricing problem of the dual-channel supply chain with disruption is important.

In this paper, we consider a dual-channel supply chain considering disruption risk, where one manufacturer has its direct sales channel and one traditional retailer channel. If the manufacturer suffered a supply disruption, then all ordered products by the retailer or the direct retail channel will be lost. Therefore, all members in supply chain will suffer the corresponding disruption penalties. In order to compare the effect of the members' power structure on pricing decision, we consider four structures with different market power relations, and the closed-form optimal price decisions are given in the four models. We found that the disruption factor improves the sales prices for any member structure as compared to the supply chain without the disruption. And the direct retail prices in the different modes are the same as each other, but the price of the traditional channel is influenced by the market share. And the sorts of the sales prices under different structures are given. We also conduct some extensive numerical analysis and compare the results under different structures. We observe that the expected optimal profits of considering the external penalty are smaller than those of no external penalty, and we give a sort of the optimal expected profits. And we also provide the effects of some parameters on the optimal decisions and the optimal expected profits.

The rest of the paper is organized as follows. Section 2 gives the literature review and our contribution. Section 3 gives the description and formulation of the problem. Section 4 gives the models and the closed-form optimal decisions. Section 5 provides some theoretical comparative analysis for optimal decisions, and some further numerical analyses are given in Section 6. The managerial insights and summary of this paper are shown in Section 7.

2. Literature Review

First, our research belongs to the pricing of the dual-channel supply chain. It refers to the fact that the supply chain has the online (direct sales) and off-line (traditional retail) channels. Second, it is related to the field of the dual-channel supply chain considering disruption risk.

The pricing of dual-channel supply chain had obtained much attention. Chiang et al. [5] considered the pricing game between a manufacturer with a direct channel and its traditional retailer and analyzed the gains and losses of the manufacturer's pricing strategies. Yao and Liu [6] study the competitive pricing policies between retail and e-tail distribution channels by the Bertrand and the Stackelberg price competition models and obtained the corresponding equilibrium pricing policies. They also compared the profit under different competitions and proposed the appropriate strategy for the manufacturer. Cattani et al. [7] analyzed the Stackelberg game where a manufacturer with a direct channel is competing with its traditional retailer. They determined the effect of pricing strategies on prices and profits of the supply chain members. Chen et al. [8] also considered the pricing problem in a dual-channel supply chain, where the manufacturer is the Stackelberg leader, and provided the coordination schemes. Cao et al. [9] considered the Bertrand game between the manufacturer and the retailer in a dual-channel supply chain, where they considered the asymmetric cost information and full information. Chen et al. [10] consider the pricing policies in a supply chain with one traditional retailer and one manufacturer. The manufacturer sells its product to the retailer or directly to consumers through an Internet channel and makes decision on the direct sales price. The retailer sells the product from the manufacturer and also sells the other substitute product from other manufacturer and decides two retail prices of two substitute products. They considers the settings of Nash and Stackelberg games. Chen et al. [11] studied a retailer Stackelberg supply chain; as compared to a single retail channel supply chain, a dual-channel supply chain can increase the profit of the manufacturer and the supply chain, and the retailer will also benefit from the direct channel when the maximum sales in the retail channel are high. They further propose a retailer's margin contract which can coordinate the dual-channel supply chain, and both the retailer and the manufacturer will be more profitable. In order to find the joint optimal prices including the wholesale price, the retail price in the traditional channel, and the selling price in the direct channel, Ding et al. [12] considered a hierarchical pricing mode and provided the corresponding criteria to identify and compare different operational strategies.

Some papers considered prices and other factors in a dual-channel supply. Dan et al. [13] examined the optimal retail services and prices in a dual-channel supply and evaluated the effects of retail service and the customer loyalty degree on the members' pricing. Ryan et al. [14] considered the coordination problem in a dual-channel supply chain, where each channel is modeled as a news-vendor problem, and the price and order quantity are decision variables. They proposed two contract schemes. Chen et al. [15] investigated the price and quality decisions in a dual-channel supply chain and showed that the quality can be improved when a new channel is introduced. Further, they employed two themes to characterize the impacts of channel structures. On similar topics, Hua et al. [16] investigated price and lead time decisions, and Wang et al. [17] investigated price and service decisions in dual-channel supply chains.

Corresponding to the above viewpoint, there are few researches considering the disruption factor under dual-channel supply chain framework. Sometimes some disruptive events in a supply chain will significantly affect the performance of the supply chain, such as machine breakdown, raw material shortage, and workers strike. One disruption may cause serious direct or indirect losses. It is reported that publicly trade firms experiencing disruptive events received negative stock market reactions with the magnitude of decline in market capitalization being as large as 10% [18, 19]. Generally, suppliers have a realization about the disruption probability as they have the information of inventory, resources, and production equipment. However, the downstream retailers are often unaware of disruptions until their orders cannot be fulfilled. The unaware causes the operation distortion and the performance loss of the supply chain, especially for the downstream retailers.

About the supply chain with disruption, we refer the reader to Snyder et al. [20]. We limit our discussion in the field of the dual-channel supply chain with the disruption risk. The ex ante disruption management is a risk management. Some papers considered the members' risk awareness. Xu et al. [21] investigated the coordination effect of a dual-channel supply chain when supply chain members have the risk awareness, where the pricing decisions are considered. They analyzed the effect of risk tolerance on the members' pricing and proposed a two-way revenue sharing contract. Li et al. [22] considered a dual-channel supply chain consisting of a risk-neutral supplier and a risk-averse retailer and examined how to adjust the prices and the production plan so that the potential maximal profit is obtained, where the CVaR criterion in the Stackelberg model is used. Liu et al. [23] investigated the effect of risk aversion on the optimal policies of a dual-channel supply chain under complete information and asymmetric information cases. Under the assumption that the optimal value added only depends on the value-added cost, the optimal prices under a risk-averse case are lower than those in a risk-neutral case. The information asymmetry increases the wholesale and retail prices but reduces direct sales price and tends to engender inefficiency.

Some papers considered the disruption in a dual-channel framework. Cao [24] studied the coordination of a dual-channel supply chain when the demands of two channels are simultaneously disrupted, where the pricing and production decisions are considered. They compared the profits under normal and disrupted cases and proposed an revenue sharing contract to coordinate the dual-channel supply chain. Huang et al. [25] developed a two-period pricing and production decision model in a dual-channel supply chain that experiences a demand disruption during the planning horizon. Their results indicate that the optimal production quantity has some robustness under a demand disruption in both centralized and decentralized dual-channel supply chains. Zhang et al. [26] study the coordinations in a dual-channel supply chain when demand disruption occurs. Yan et al. [27] conducted a dual-channel supply chains using centralized and decentralized decision-making models to making comparison analysis of the decisions before and after demand disruption. They find that the optimal decision is only influenced by demand disruption in the centralized decision-making model. The stability of the sales volume of the two models is related to the market share and the demand disruption. And there are some papers which considered the production cost disruption. Huang [1] studied a pricing and production problem in a dual-channel supply chain when the production costs are disrupted. When a production cost disruption occurs, the related deviation costs occur. They consider the problem in the centralized and decentralized dual-channel supply chain, respectively. The optimal prices and production quantity under the production cost disruption are derived, and they find that the decision-maker changes the production quantity only when the production cost disruption exceeds some thresholds. Soleimani et al. [28] studied the optimal decisions in a dual-channel supply under simultaneous demand and production cost disruptions. A game theoretical method is proposed to drive the optimal decisions. Then they find that the optimal prices are affected by sharing demands to the direct channel in both centralized and decentralized modes.

There are few papers about the pricing decisions of the dual-channel supply chain to consider the supply risk, in addition to Xiao and Shi [29]. Xiao and Shi [29] studied the pricing and channel priority strategies by using game theoretic models, where the dual-channel supply chain may result in the supply shortage caused by random yield. They studied the pricing and channel priority strategies under the supply random yield but only considered the Nash game. We consider the supply disruption risk, and our main goal is to compare the optimal decisions and system performances under different power structures. Although some papers considered different power structures, such as Yao and Liu [6], Chen et al. [10], and Chen et al. [15], they only considered pricing decisions of the dual-channel supply chain. In this paper, we aim to analyze the effect of disruption on the supplier, the retailers, and the whole supply chain, where the supplier shares its realization of the disruption probability. With a potential disruption risk, a manufacturer with dual channels is typically faced with the prices adjusting issue given two channel strategies, and the similar cases also will happen to the retailer. Therefore, our paper mainly focuses on how the prices which include the whole price and the sales prices are affected by the disruption risk; we consider different power structures including one centralized framework and three decentralized frameworks. To the best of our knowledge, no research in the literature has considered the problem. We provided the literature review in Table 1.

Our main contributions are given as follows.

(1) We consider the effects of the supply disruption factor on the pricing decisions and profits of a dual-channel supply chain under four frameworks, including one centralized framework and three decentralized frameworks, where three decentralized models are (a) manufacturer-leader Stackelberg game with disruption (MSPD), (b) retailer-leader Stackelberg game with disruption (RSPD), and (c) Nash game with disruption (NGPD).

(2) We introduce the internal and external penalty parameters into our models and analyze the effects of these parameters on pricing decisions and profits.

(3) We give the optimal pricing decisions under four decision frameworks and compare the effects of the disruption factor on the optimal decisions and profits. The results show that (a) the sales prices of considering the disruption factor in any member structure are larger than those of no disruption factor, (b) the optimal sale prices in the MSPD model and RSPD model are the same as each other, but the order of the wholesale prices varies as the disruption probability, (c) the order of the sales prices of traditional retail channels under different structures varies as the market basic size of the traditional retail channel; however, the order of the direct sale prices under different structures does not vary, and (d)

Table 1: Dual-channel supply chain considering pricing or disruption.

	Dual-channel	Static game	Stackelberg game	Disruption type	Risk attitude	Structural Contrast
Chiang et al. [5]	Yes	No	Manufacturer-leader	No	No	No
Cattani et al. [7]	Yes	No	Manufacturer-leader	No	No	No
Yao and Liu [6]	Yes	Bertrand	Manufacturer-leader	No	No	Yes
Dan et al. [13]	Yes	No	Manufacturer-leader	No	No	No
Chen et al. [8]	Yes	No	Manufacturer-leader	No	No	No
Cao et al. [9]	Yes	Bertrand	No	No	No	No
Ryan et al. [14]	Yes	No	Manufacturer-leader	No	No	No
Chen et al. [10]	Yes	Nash	Manufacturer-leader	No	No	Yes
Li et al. [22]	Yes	Nash	No	No	Consider	No
Liu et al. [23]	Yes	No	Manufacturer-leader	No	Consider	No
Chen et al. [11]	Yes	Nash	Retailer-leader	No	No	Yes
Chen et al. [15]	Yes	No	Manufacturer-leader	No	No	No
Hua et al. [16]	Yes	No	Manufacturer-leader	No	No	No
Wang et al. [17]	Yes	No	Manufacturer-leader	No	No	No
Xiao and Shi [29]	Yes	Nash	No	Random Yield	No	No
Cao [24]	Yes	No	Manufacturer-leader	Demand	No	No
Xu et al. [21]	Yes	No	Manufacturer-leader	No	Consider	No
Huang et al. [25]	Yes	No	Manufacturer-leader	Demand	No	No
Huang et al. [1]	Yes	No	Manufacturer-leader	Cost	No	No
Zhang et al. [26]	Yes	No	Manufacturer-leader	Cost/demand	No	No
Yan et al. [27]	Yes	No	Manufacturer-leader	No	No	No
Soleimani et al. [28]	Yes	No	Manufacturer-leader	Cost/demand	No	No
Ding et al. [12]	Yes	No	Retailer-leader	No	No	No

the disruption probability affects the market division of two channels, and we give the order of the influence degrees.

3. Problem Description and Formulation

We consider a two-echelon dual-channel supply chain consisting of one manufacturer and one retailer. The manufacturer sells a product to customers through its direct sale channel or one traditional retailer channel, so he not only needs to determine the wholesale price w to the traditional retailer but also needs to determine the direct sale price p_2 to the customers of the direct sales channel. Facing with the market demand, the retailer only needs to make decision on the its sale price p_1 to customers.

From Yao and Liu [6] and Yue and Liu [30], we consider the demand function to be the linear function with respect to the sales prices. The demand function in the traditional retail channel is expressed as

$$d_1(p_1, p_2) = \theta\alpha - \beta_p p_1 + r_p(p_2 - p_1), \quad (1)$$

and the demand function in the direct sale channel is expressed as

$$d_2(p_1, p_2) = (1-\theta)\alpha - \beta_p p_2 + r_p(p_1 - p_2). \quad (2)$$

In the above formulas, the subscripts 1 and 2 denote the traditional retail channel and the direct sale channel, respectively. Parameter α represents the total potential market size of the product. Parameter θ represents the percentage of the potential market demand α to the traditional retail channel, and $(1-\theta)$ represents the percentage of the potential market demand to the direct sales channel. β_p represents the coefficient of the demand price elasticity. r_p represents the coefficient of the cross-price elasticities of demand. The dual-channel structure is given as Figure 1.

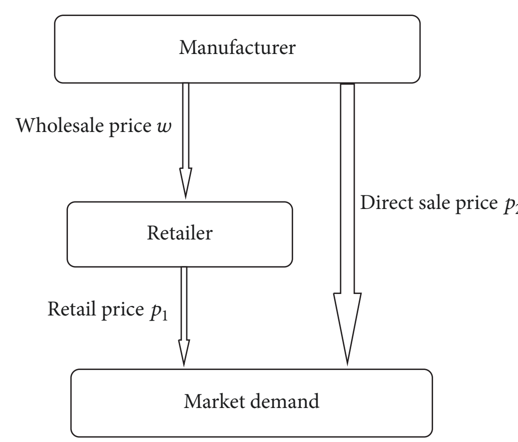


Figure 1: Dual-channel supply chain structure.

We assume that parameters $\theta, \alpha, \beta_p, r_p > 0$. For convenience, we set $\Lambda_p = \beta_p + r_p$. And we also set $\beta_p > r_p$. The assumption guarantees that the demand is decreasing as the increasing of the retail price; otherwise it is unreasonable [10, 31, 32].

Generally, some production disruption events in a supply chain are inevitable and have significantly affect the performance of the supply chain, such as machine breakdown and raw material shortage. When the production disruption occurs (such as machines break down), the manufacturer will not be able to satisfy any order, and he will suffer a penalty. Because the manufacturer needs to face the traditional retailer and the customers of direct sale channel, there are two parts of penalties. We call the penalty from the traditional retailer as the internal penalty, and the cost of per unit shortage is denoted by q. And we call the penalty from the direct sale channel as the external penalty; the cost of per unit shortage is denoted by q_r.

One disruption occurs at a certain probability. Let ε represent the occurrence probability of one disruption. If there is no disruption, the manufacturer's profit is

$$\begin{aligned}\Pi_{M_1}(w, p_2) &= (p_2 - c)\, d_2(p_1, p_2) \\ &\quad + (w - c)\, d_1(p_1, p_2),\end{aligned} \tag{3}$$

where c represents the unit production cost. If the disruption occurs, the manufacturer's profit function is

$$\Pi_{M_2}(w, p_2) = -q d_1(p_1, p_2) - q_r d_2(p_1, p_2). \tag{4}$$

Therefore, the manufacturer's expected profit function of considering the production disruption is given as follows:

$$\begin{aligned}\Pi_M(w, p_2) &= (1-\varepsilon)\, \Pi_{M_1}(w, p_2) + \varepsilon \Pi_{M_2}(w, p_2) \\ &= (1-\varepsilon)\left[(p_2 - c)\, d_2(p_1, p_2) + (w - c)\, d_1(p_1, p_2)\right] \\ &\quad - \varepsilon q d_1(p_1, p_2) - \varepsilon q_r d_2(p_1, p_2).\end{aligned} \tag{5}$$

In the following, we formulate the retailer's profit function. If there is no disruption, the retailer's profit function is

$$\Pi_{R_1}(p_1) = (p_1 - w)\, d_1(p_1, p_2). \tag{6}$$

When one disruption occurs, the retailer will get a compensation from the manufacturer. In addition, the retailer will also suffer a penalty from the external of the supply chain and obtain a compensation from the manufacturer. So if a disruption occurs, the retailer's profit function is

$$\Pi_{R_2}(p_1) = (q - q_r)\, d_1(p_1, p_2). \tag{7}$$

Therefore, the retailer's expected profit function is given as follows:

$$\begin{aligned}\Pi_R(p_1) &= (1-\varepsilon)\, \Pi_{R_1}(p_1) + \varepsilon \Pi_{R_2}(p_1) \\ &= (1-\varepsilon)(p_1 - w)\, d_1(p_1, p_2) \\ &\quad + \varepsilon (q - q_r)\, d_1(p_1, p_2).\end{aligned} \tag{8}$$

In order to analyze the effects of the disruption factor on the channel members' decisions, we will consider different market power structures in the following section. We further assume that all of members have complete information.

4. Models and Analysis

In this section, we consider four structures with different market powers relationship, which includes the centralized pricing model with disruption (CPD model), manufacturer Stackelberg leader with disruption (MSPD model), retailer Stackelberg leader with disruption (RSPD model), and Nash game with disruption (NGPD model).

4.1. Centralized Pricing with Disruption (CPD Model). From (5) and (8), we have the total supply chain profit as follows:

$$\begin{aligned}&\Pi_{SC} \\ &\quad = (1-\varepsilon)\left[(p_1 - c)\, d_1(p_1, p_2) + (p_2 - c)\, d_2(p_1, p_2)\right] \\ &\qquad - \varepsilon q_r \left(d_1(p_1, p_2) + d_2(p_1, p_2)\right).\end{aligned} \tag{9}$$

Theorem 1. *The total supply chain profit function Π_{SC} is concave in p_1 and p_2.*

Proof. Take the following derivatives:

$$\begin{aligned}\frac{\partial^2 \Pi_{sc}}{\partial p_1^2} &= \frac{\partial^2 \Pi_{sc}}{\partial p_2^2} = -2\Lambda_p (1-\varepsilon), \\ \frac{\partial^2 \Pi_{sc}}{\partial p_1 \partial p_2} &= \frac{\partial^2 \Pi_{sc}}{\partial p_2 \partial p_1} = 2 r_p (1-\varepsilon).\end{aligned} \tag{10}$$

Now we give the Hessian matrix of Π_{sc} with p_1 and p_2:

$$\begin{aligned}H_{sc} &= \begin{pmatrix} \dfrac{\partial^2 \Pi_{sc}}{\partial p_1^2} & \dfrac{\partial^2 \Pi_{sc}}{\partial p_1 \partial p_2} \\ \dfrac{\partial^2 \Pi_{sc}}{\partial p_2 \partial p_1} & \dfrac{\partial^2 \Pi_{sc}}{\partial p_2^2} \end{pmatrix} \\ &= \begin{pmatrix} -2\Lambda_p (1-\varepsilon) & 2 r_p (1-\varepsilon) \\ 2 r_p (1-\varepsilon) & -2\Lambda_p (1-\varepsilon) \end{pmatrix}.\end{aligned} \tag{11}$$

So, we have $\det H_{sc} = 4(1-\varepsilon)^2(\Lambda_p^2 - r_p^2) = 4(1-\varepsilon)^2 \beta_p (\Lambda_p + r_p) > 0$. The total supply chain profit function Π_{SC} is concave with respect to p_1 and p_2. □

Theorem 2. *In the CPD model, the optimal sales prices (denoted as p_{c1}^* and p_{c2}^*) are given as follows:*

$$p_{c1}^* = \frac{\alpha (r_p + \theta \beta_p)}{2\beta_p (\Lambda_p + r_p)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon} \frac{q_r}{2}, \tag{12}$$

$$p_{c2}^* = \frac{\alpha (\Lambda_p - \theta \beta_p)}{2\beta_p (\Lambda_p + r_p)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon} \frac{q_r}{2}. \tag{13}$$

Proof. From (9), we have

$$\frac{\partial \Pi sc}{\partial p_1} = (1-\varepsilon)\left[d_1(p_1,p_2) + (p_1-c)\frac{\partial d_1(p_1,p_2)}{\partial p_1} + (p_2-c)\frac{\partial d_2(p_1,p_2)}{\partial p_1}\right] + \varepsilon\beta_p q_r = (1-\varepsilon)\left(\theta\alpha - 2\Lambda_p p_1 + 2r_p p_2 + \beta_p c\right) + \varepsilon\beta_p q_r = (1-\varepsilon) \cdot \left[-2\Lambda_p p_1 + 2r_p p_2 + \theta\alpha + \beta_p c + \frac{\varepsilon}{1-\varepsilon}\beta_p q_r\right],$$
$$\frac{\partial \Pi sc}{\partial p_2} = (1-\varepsilon)\left[(p_1-c)\frac{\partial d_1(p_1,p_2)}{\partial p_2} + d_2(p_1,p_2) + (p_2-c)\frac{\partial d_2(p_1,p_2)}{\partial p_2}\right] + \varepsilon\beta_p q_r = (1-\varepsilon) \cdot \left[(1-\theta)\alpha - 2\Lambda_p p_2 + 2r_p p_1 + \beta_p c\right] + \varepsilon\beta_p q_r = (1-\varepsilon)\left[-2\Lambda_p p_2 + 2r_p p_1 + (1-\theta)\alpha + \beta_p c + \frac{\varepsilon}{1-\varepsilon}\beta_p q_r\right], \quad (14)$$

so the first-order condition equations are as follows:

$$(1-\varepsilon)\left(\theta\alpha - 2\Lambda_p p_1 + 2r_p p_2 + \beta_p c\right) + \varepsilon\beta_p q_r = 0,$$
$$(1-\varepsilon)\left((1-\theta)\alpha - 2\Lambda_p p_2 + 2r_p p_1 + \beta_p c\right) + \varepsilon\beta_p q_r = 0. \quad (15)$$

Therefore, we have

$$p_{c1}^* = \frac{\alpha(r_p + \theta\beta_p)}{2\beta_p(\Lambda_p + r_p)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon}\frac{q_r}{2},$$
$$p_{c2}^* = \frac{\alpha(\Lambda_p - \theta\beta_p)}{2\beta_p(\Lambda_p + r_p)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon}\frac{q_r}{2}. \quad (16)$$

The theorem holds. □

From Theorem 2, we easily know the following proposition.

Proposition 3. *The optimal sales prices of considering the disruption factor are larger than those of not considering the factor. And they are increasing in ε and q_r.*

When $\varepsilon = 0$, there is no disruption case, so the optimal sales prices when not considering the production disruption are as follows:

$$\widehat{p}_{c1}^* = \frac{\alpha(r_p + \theta\beta_p)}{2\beta_p(\Lambda_p + r_p)} + \frac{c}{2},$$
$$\widehat{p}_{c2}^* = \frac{\alpha(\Lambda_p - \theta\beta_p)}{2\beta_p(\Lambda_p + r_p)} + \frac{c}{2}. \quad (17)$$

Therefore, Proposition 3 is obvious.

From the proposition, we know that the customers will suffer a higher purchasing cost because of the increasing of the sale prices. The increasing is coming from the external penalty. From (12) and (13), the increased part on the optimal sale prices when considering the disruption factor is $(\varepsilon/(1-\varepsilon))(q_r/2)$, if the market does not give a penalty to the disruption, that is, $q_r = 0$, so the increased part $(\varepsilon/(1-\varepsilon))(q_r/2) = 0$. It means that the manufacturer will not suffer an unexpected disruption loss, so it does not need to improve the sales prices. However, there are some customers who have the negative impression on the disruption, so the penalty should exist in many cases. Once the manufacturer formed the perception, it will force customers to share the probable disruption loss. And the risk probability is larger, the customers' sharing loss is more.

Proposition 4. *The difference between the channel prices is not affected by the production disruption probability ε or the external penalty q_r.*

From (12), (13), and (17), the proposition holds. It means when the manufacturer improves the same size price difference when considering the disruption risk, the market size will not change. So the corresponding operation can be dealt with in proportion, such as inventory policy.

4.2. Pricing Model with Disruption in MS Game (MSPD). In this subsection, the manufacturer acts as the leader of Stackelberg game and the retailer acts as the follower. The manufacturer first decides the wholesale price for the traditional channel and the retail price for the direct sales channel; then the retailer decides the retail price according to the manufacturer's decisions. The MSPD model is given as follows:

$$\max_{w,p_2} \Pi_M = (1-\varepsilon)\left[(p_2-c)d_2(p_1,p_2) + (w-c)d_1(p_1,p_2)\right] - \varepsilon\left(q d_1(p_1,p_2) - q_r d_2(p_1,p_2)\right)$$
$$\text{where, } p_1 \text{ is the optimal solution of the following optimization problem} \quad (18)$$
$$\max_{p_1} \Pi_R = (1-\varepsilon)(p_1-w)d_1(p_1,p_2) + \varepsilon(q-q_r)d_1(p_1,p_2).$$

Theorem 5. *For any given w and p_2,*

(a) *the retailer's expected profit function Π_R is concave to p_1;*

(b) *the retailer's best response function (denoted as $p'_{m1}(p_2, w)$) is as follows:*

$$p'_{m1}(p_2, w) = \frac{\theta\alpha + r_p p_2}{2\Lambda_p} + \frac{w}{2} - \frac{\varepsilon}{1-\varepsilon}\frac{q - q_r}{2}. \quad (19)$$

Proof. The first-order derivative of $\Pi_R(p_1)$ is

$$\begin{aligned}\frac{\partial \Pi_R(p_1)}{\partial p_1} &= (1-\varepsilon)\left[d_1(p_1, p_2) + (p_1 - w)\frac{\partial d_1(p_1, p_2)}{\partial p_1}\right] + \varepsilon(q - q_r)\frac{\partial d_1(p_1, p_2)}{\partial p_1} = (1-\varepsilon)\left(\theta\alpha - 2\Lambda_p p_1 + r_p p_2 + w\Lambda_p\right) - \varepsilon(q - q_r)\Lambda_p \\ &= (1-\varepsilon)\left[\theta\alpha - 2\Lambda_p p_1 + r_p p_2 + w\Lambda_p - \frac{\varepsilon}{1-\varepsilon}(q - q_r)\Lambda_p\right].\end{aligned} \quad (20)$$

The second-order derivative of $\Pi_R(p_1)$ is

$$\frac{\partial^2 \Pi_R(p_1)}{\partial p_1^2} = -2\Lambda_p(1-\varepsilon) < 0. \quad (21)$$

So the retailer's expected profit function Π_R is concave to p_1. From the first-order condition $\partial \Pi_R(p_1)/\partial p_1 = 0$, we have

$$p'_{m1}(p_2, w) = \frac{\theta\alpha + r_p p_2}{2\Lambda_p} + \frac{w}{2} - \frac{\varepsilon}{1-\varepsilon}\frac{q - q_r}{2}. \quad (22)$$

In summary, the theorem holds. □

Proposition 6. *For any given wholesale price w and direct retail price p_2,*

(a) *if $q > q_r$, the retailer's sales price of considering the disruption factor is larger than that of not considering the factor and is decreasing in the disruption probability ε;*

(b) *otherwise, it is smaller than that of not considering the factor and is increasing in the disruption probability ε.*

When $\varepsilon = 0$, there is no disruption case, so the optimal retailer's reaction function is

$$\widehat{p}'_{m1}(p_2, w) = \frac{\theta\alpha + r_p p_2}{2\Lambda_p} + \frac{w}{2}. \quad (23)$$

Comparing (19) and (23), Proposition 6 is obvious. When $q > q_r$, the retailer can obtain extra profit from the disruption, so it does not need to improve the sales price for dealing with the disruption risk. And the risk is bigger; the retailer can obtain a greater subsidy from the manufacturer. And, the external penalty is smaller, the sales price is lower. The case seems to be the case that the manufacturer undertakes all disruption risk and gives an extra subsidy to the retailer. However, under the manufacturer acts the leader of Stackelberg game and the final result is uncertain. In most cases, the risk is shared by all members in the supply chain, the relation $q \le q_r$ is more suitable. The retailer will directly share some penalty. The risk is bigger; the retailer needs to share more risk costs so that the sales price is improved, which is consistent with the centralized model.

Theorem 7. *The manufacturer's expected profit function $\Pi_M(w, p_2, p'_{m1}(p_2, w))$ is concave with respect to p_2 and w.*

Proof. Take the first-order derivative as follows:

$$\begin{aligned}\frac{\partial \Pi_M(w, p_2, p'_{m1}(p_2, w))}{\partial p_2} &= (1-\varepsilon)\left[d_2(p'_{m1}, p_2) + (p_2 - c)\frac{\partial d_2(p'_{m1}, p_2)}{\partial p_2} + (w - c)\frac{\partial d_1(p'_{m1}, p_2)}{\partial p_2}\right] - \varepsilon q\frac{\partial d_1(p'_{m1}, p_2)}{\partial p_2} - \varepsilon q_r\frac{\partial d_2(p'_{m1}, p_2)}{\partial p_2}, \\ \frac{\partial \Pi_M(w, p_2, p'_{m1}(p_2, w))}{\partial w} &= (1-\varepsilon)\cdot\left[(p_2 - c)\frac{\partial d_2(p'_{m1}, p_2)}{\partial w} + d_1(p'_{m1}, p_2) + (w - c)\frac{\partial d_1(p'_{m1}, p_2)}{\partial w}\right] - \varepsilon q\frac{\partial d_1(p'_{m1}, p_2)}{\partial w} - \varepsilon q_r\frac{\partial d_2(p'_{m1}, p_2)}{\partial w},\end{aligned} \quad (24)$$

where

$$\begin{aligned}\frac{\partial d_1(p'_{m1}, p_2)}{\partial p_2} &= r_p - \Lambda_p\frac{\partial p'_{m1}(p_2, w)}{\partial p_2} = \frac{1}{2}r_p, \\ \frac{\partial d_2(p'_{m1}, p_2)}{\partial p_2} &= -\Lambda_p + r_p\frac{\partial p'_{m1}(p_2, w)}{\partial p_2} = \frac{r_p^2 - 2\Lambda_p^2}{2\Lambda_p}, \\ \frac{\partial d_1(p'_{m1}, p_2)}{\partial w} &= -\Lambda_p\frac{\partial p'_{m1}(p_2, w)}{\partial w} = -\frac{1}{2}\Lambda_p, \\ \frac{\partial d_2(p'_{m1}, p_2)}{\partial w} &= r_p\frac{\partial p'_{m1}(p_2, w)}{\partial w} = \frac{1}{2}r_p.\end{aligned} \quad (25)$$

The second-order derivative is as follows:

$$\frac{\partial^2\Pi_M}{\partial p_2^2} = \frac{(1-\varepsilon)\left(r_p^2 - 2\Lambda_p^2\right)}{\Lambda_p},$$
$$\frac{\partial^2\Pi_M}{\partial w\partial p_2} = \frac{\partial^2\Pi_M}{\partial p_2\partial w} = r_p(1-\varepsilon), \tag{26}$$
$$\frac{\partial^2\Pi_M}{\partial w^2} = -\Lambda_p(1-\varepsilon).$$

So, we have

$$\det H_M = \begin{pmatrix} \frac{\partial^2\Pi_M}{\partial p_2^2} & \frac{\partial^2\Pi_M}{\partial p_2\partial w} \\ \frac{\partial^2\Pi_M}{\partial w\partial p_2} & \frac{\partial^2\Pi_M}{\partial w^2} \end{pmatrix} = \det\begin{pmatrix} \frac{(1-\varepsilon)\left(r_p^2-2\Lambda_p^2\right)}{\Lambda_p} & r_p(1-\varepsilon) \\ r_p(1-\varepsilon) & -\Lambda_p(1-\varepsilon) \end{pmatrix} = 2(1-\varepsilon)^2\beta_p\left(\Lambda_p + r_p\right) > 0. \tag{27}$$

The manufacturer's expected profit function $\Pi_M(w, p_2, p_1'(p_2, w))$ is concave with respect to p_2 and w. □

Theorem 8. *In the MSPD model, the manufacturer's optimal wholesale price (denoted as w_m^*) and the optimal retail price for the direct market channel (denoted as p_{m2}^*) are*

$$w_m^* = p_{c1}^* + \frac{\varepsilon(q - q_r)}{1-\varepsilon} = \frac{\alpha\left(r_p + \theta\beta_p\right)}{2\beta_p\left(\Lambda_p + r_p\right)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon}\left(q - \frac{q_r}{2}\right), \tag{28}$$

$$p_{m2}^* = \frac{\alpha\left(\Lambda_p - \theta\beta_p\right)}{2\beta_p\left(\Lambda_p + r_p\right)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon}\frac{q_r}{2}. \tag{29}$$

And the retailer's optimal retail price (denoted as p_{m1}^*) for the traditional channel is

$$p_{m1}^* = \frac{\alpha\left[\beta_p\theta\left(3\Lambda_p + r_p\right) + 2\Lambda_p r_p\right]}{4\Lambda_p\beta_p\left(\Lambda_p + r_p\right)} + \frac{c\left(\Lambda_p + r_p\right)}{4\Lambda_p} + \frac{\varepsilon q_r}{1-\varepsilon}\frac{\Lambda_p + r_p}{4\Lambda_p}. \tag{30}$$

Proof. From Theorem 7, using the first-order conditions,

$$\frac{\partial\Pi_M\left(w, p_2, p_{m1}'(p_2, w)\right)}{\partial p_2} = 0,$$
$$\frac{\partial\Pi_M\left(w, p_2, p_{m1}'(p_2, w)\right)}{\partial w} = 0, \tag{31}$$

we can get (28) and (29). Substituting (28) and (29) into (19), we have (30). □

Proposition 9. *For the optimal decisions in the MSPD model, there are the following:*

(a) *The optimal sale prices of considering the disruption factor are larger than those of not considering the factor. And they are increasing in ε and q_r.*

(b) *The optimal wholesale price is larger than that of not considering the factor if $q > q_r/2$, and it is increasing in the disruption probability ε or the internal penalty q and is decreasing in the external penalty q_r; otherwise, it is smaller than that of not considering the factor and is decreasing in the disruption probability ε or the internal penalty q and is increasing in the external penalty q_r.*

(c) *The difference between the channel prices is affected by the production disruption ε and the external penalty q_r.*

From (29) and (30), we know that the optimal sales prices is larger than that of $\varepsilon = 0$. It means that both of two retail channels are to decease the disruption cost by improving the sales prices. And the bigger risk and penalty cost will lead to the bigger optimal sales prices because it can transfer more disruption cost to customers. Those are similar to the case in the centralized model.

However, the relation is not always. From Part (b) in Proposition 9, when the internal penalty is large enough, that is, $q > q_r/2$, the optimal wholesale price is larger. In fact, the retailer may be profitable from one disruption event because the internal penalty can be viewed as a subsidy for the retailer. But the manufacturer acts as the leader of Stackelberg game; he can transfer some parts of disruption risk to the retailer by improving the wholesale price. Therefore, when $q > q_r/2$, the retailer seems to be profitable, but, in fact, the profit is offset by the improved wholesale price. In other words, the bigger disruption risk or internal penalty cost will lead to the bigger wholesale price, or the smaller external penalty cost brings to a bigger wholesale price.

In decentralized framework, the direct channel and the traditional channel consider different decision factors, so the price variation from the disruption risk is different for two channels. Then, for the change of the disruption probability, it is difficult for keeping the price difference unchanged; that is, the market sizes change as the variation of the disruption probability. In most cases, $\beta_p \geq r_p$, so it is impossible for the case $(\Lambda_p + r_p)/4\Lambda_p = 1/2$.

4.3. Pricing Model with Disruption in RS Game (RSPD). In this subsection, the retailer acts as the Stackelberg leader and the manufacturer acts as the follower. The retailer first decides the retail price for the traditional channel; then the manufacturer decides the wholesale price and the retail price for the direct channel. The RSPD model is given as follows:

$$\max_{p_1} \Pi_R = (1-\varepsilon)(p_1 - w) d_1(p_1, p_2) + \varepsilon (q - q_r) d_1(p_1, p_2)$$

$$\text{where, } w \text{ and } p_2 \text{ are the optimal solution of the following optimization problem} \quad (32)$$

$$\max_{w,p_2} \Pi_M = (1-\varepsilon)\left[(p_2 - c) d_2(p_1, p_2) + (w - c) d_1(p_1, p_2)\right] - \varepsilon \left(q d_1(p_1, p_2) + q_r d_2(p_1, p_2)\right).$$

We first derive the manufacturer's decisions as follows. Without loss of generality, let m be the retailer margin of this product enjoyed by the retailer in the traditional market channel, that is,

$$p_1 = w + m, \quad (33)$$

where $m > 0$. We first solve the inner-optimization problem.

Theorem 10. *For any given p_1, there are*

(a) *the manufacturer's expected profit Π_M is concave with respect to the p_2 and w;*

(b) *the manufacturer's best wholesale response function (denoted as $w_r'(p_1)$) and the manufacturer's best retail response function (denoted as p_{r2}') are*

$$p_{r2}' = \frac{\alpha\left(\Lambda_p - \theta\beta_p\right)}{2\beta_p\left(\Lambda_p + r_p\right)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon}\frac{q_r}{2}, \quad (34)$$

$$w_r'(p_1) = \frac{\alpha\left(r_p + \theta\beta_p\right)}{\beta_p\left(\Lambda_p + r_p\right)} + c + \frac{\varepsilon q}{1-\varepsilon} - p_1. \quad (35)$$

Proof. The first-order derivatives of Π_M are

$$\begin{aligned} &\frac{\partial \Pi_M(w, p_2)}{\partial p_2} \\ &\quad = (1-\varepsilon)\left[(1-\theta)\alpha - 2\Lambda_p p_2 + r_p p_1 + c\beta_p + w r_p\right] \\ &\qquad - \varepsilon q r_p + \varepsilon q_r \Lambda_p, \\ &\frac{\partial \Pi_M(w, p_2)}{\partial w} \\ &\quad = (1-\varepsilon)\left[\theta\alpha - \Lambda_p p_1 + 2 r_p p_2 + c\beta_p - w\Lambda_p\right] \\ &\qquad + \varepsilon q \Lambda_p - \varepsilon q_r r_p. \end{aligned} \quad (36)$$

And then, their second-order derivatives are

$$\begin{aligned} \frac{\partial^2 \Pi_M}{\partial p_2^2} &= \frac{\partial^2 \Pi_M}{\partial w^2} = -2\Lambda_p(1-\varepsilon), \\ \frac{\partial^2 \Pi_M}{\partial w \partial p_2} &= \frac{\partial^2 \Pi_M}{\partial p_2 \partial w} = 2 r_p (1-\varepsilon). \end{aligned} \quad (37)$$

So, we have

$$\begin{aligned} \det H_M &= \begin{pmatrix} \dfrac{\partial^2 \Pi_M}{\partial p_2^2} & \dfrac{\partial^2 \Pi_M}{\partial p_2 \partial w} \\ \dfrac{\partial^2 \Pi_M}{\partial w \partial p_2} & \dfrac{\partial^2 \Pi_M}{\partial w^2} \end{pmatrix} \\ &= \det \begin{pmatrix} -2\Lambda_p(1-\varepsilon) & 2r_p(1-\varepsilon) \\ 2r_p(1-\varepsilon) & -2\Lambda_p(1-\varepsilon) \end{pmatrix} \\ &= 4(1-\varepsilon)^2 \beta_p\left(\Lambda_p + r_p\right) > 0. \end{aligned} \quad (38)$$

In summary, the manufacturer's expected profit function Π_M is concave in p_2 and w.

From the following first-order condition,

$$\begin{aligned} \frac{\partial \Pi_M(w, p_2)}{\partial p_2} &= 0, \\ \frac{\partial \Pi_M(w, p_2)}{\partial w} &= 0, \end{aligned} \quad (39)$$

we can easily get

$$\begin{aligned} p_{r2}' &= \frac{\alpha\left(\Lambda_p - \theta\beta_p\right)}{2\beta_p\left(\Lambda_p + r_p\right)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon}\frac{q_r}{2}, \\ w_r'(p_1) &= \frac{\alpha\left(r_p + \theta\beta_p\right)}{\beta_p\left(\Lambda_p + r_p\right)} + c + \frac{\varepsilon q}{1-\varepsilon} - p_1. \end{aligned} \quad (40)$$

The theorem holds. □

Proposition 11. *For any given p_1, there are*

(a) *the sale prices of the direct retail channel when considering the disruption factor are larger than those of not considering the factor. And they are increasing in ε and q_r;*

(b) *the manufacturer's wholesale price is smaller than that of not considering the factor, and it is increasing in the disruption probability ε or the internal penalty q.*

From (34), we know that the optimal sales prices are larger than that of $\varepsilon = 0$. It means that the manufacturer is to decrease the disruption cost by improving the sales prices. And the bigger risk and penalty cost will lead to the bigger optimal sales prices because it can transfer more disruption cost to customers. Those are similar to the case in the centralized model. From (35), we easily know that Part (b) in Proposition 11 holds. Faced with an internal penalty, it is reasonable that the manufacturer improves the sale price, the penalty or the disruption probability is bigger, and the sale price is higher.

Theorem 12. *The retailer's expected profit function Π_R is concave to p_1.*

Proof. Substituting (34) and (35) into the outer-optimization problem, and making the first-order and the second derivatives, there are

$$\begin{aligned}\frac{\partial \Pi_R\left(p_1 \mid p_{r2}', w_r'\right)}{\partial p_1} &= (1-\varepsilon)\left[2d_1\left(p_1, p_{r2}'\right) - \left(p_1 - w_r'\right)\Lambda_p\right] \\ &\quad - \varepsilon\left(q - q_r\right)\Lambda_p, \\ \frac{\partial^2 \Pi_R\left(p_1 \mid p_{r2}', w_r'\right)}{\partial p_1^2} &= -4\Lambda_p(1-\varepsilon) < 0,\end{aligned} \tag{41}$$

so the retailer's expected profit function Π_R is concave to p_1. □

Theorem 13. *In the RSPD model, the retailer's optimal retail price (denoted as p_{r1}^*) for the traditional channel is*

$$\begin{aligned}p_{r1}^* &= \frac{\alpha\left[\beta_p\theta\left(3\Lambda_p + r_p\right) + 2\Lambda_p r_p\right]}{4\Lambda_p\beta_p\left(\Lambda_p + r_p\right)} + \frac{c\left(\Lambda_p + r_p\right)}{4\Lambda_p} \\ &\quad + \frac{\varepsilon q_r}{1-\varepsilon}\frac{\Lambda_p + r_p}{4\Lambda_p}\end{aligned} \tag{42}$$

and the manufacturer's optimal wholesale (denoted as w_r^*) and the retail price (denoted as p_{r2}^*) are

$$p_{r2}^* = \frac{\alpha\left(\Lambda_p - \theta\beta_p\right)}{2\beta_p\left(\Lambda_p + r_p\right)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon}\frac{q_r}{2}, \tag{43}$$

$$\begin{aligned}w_r^* &= \frac{\alpha\left(2\Lambda_p r_p + \theta\beta_p^2\right)}{4\Lambda_p\beta_p\left(\Lambda_p + r_p\right)} + \frac{c\left(2\Lambda_p + \beta_p\right)}{4\Lambda_p} \\ &\quad + \frac{\varepsilon}{1-\varepsilon}\left(q - q_r\frac{\Lambda_p + r_p}{4\Lambda_p}\right).\end{aligned} \tag{44}$$

Proof. From $\partial\Pi_R(p_{r1} \mid p_{r2}', w_r')/\partial p_1 = 0$, we get

$$\begin{aligned}p_{r1}^* &= \frac{\alpha\left[\beta_p\theta\left(3\Lambda_p + r_p\right) + 2\Lambda_p r_p\right]}{4\Lambda_p\beta_p\left(\Lambda_p + r_p\right)} + \frac{c\left(\Lambda_p + r_p\right)}{4\Lambda_p} \\ &\quad + \frac{\varepsilon q_r}{1-\varepsilon}\frac{\Lambda_p + r_p}{4\Lambda_p}.\end{aligned} \tag{45}$$

Substituting it into (34) and (35), we can get (43) and (44). □

Proposition 14. *For the optimal decisions in the RSPD model, there are*

(a) *the optimal sale prices of considering the disruption factor are larger than those of not considering the factor. And they are increasing in ε and q_r;*

(b) *the optimal wholesale price is larger than that of not considering the factor if $q/q_r > (\Lambda_p + r_p)/4\Lambda_p$, and it is increasing in the disruption probability ε or the internal penalty q and is decreasing in the external penalty q_r; otherwise, it is smaller than that of not considering the factor and is decreasing in the disruption probability ε or the internal penalty q and is increasing in the external penalty q_r;*

(c) *the difference between the channel prices is affected by the production disruption ε and the external penalty q_r.*

Those in Proposition 14 are similar to Proposition 9, except for the condition in Part (b) in the above proposition.

4.4. Pricing Model with Disruption in Nash Game (NGPD). In this subsection, we consider that the manufacturer and the retailer have the same bargaining power. The model is given as follows:

$$\begin{aligned}&\max_{w,p_2} \Pi_M \\ &\quad = (1-\varepsilon)\left[\left(p_2 - c\right)d_2\left(p_1, p_2\right) + (w - c)\,d_1\left(p_1, p_2\right)\right] \\ &\qquad - \varepsilon q d_1\left(p_1, p_2\right) - \varepsilon q_r d_2\left(p_1, p_2\right), \\ &\max_{p_1} \Pi_R \\ &\quad = (1-\varepsilon)\left(p_1 - w\right)d_1\left(p_1, p_2\right) \\ &\qquad + \varepsilon\left(q - q_r\right)d_1\left(p_1, p_2\right).\end{aligned} \tag{46}$$

We need to simultaneously solve two optimization problems in the above model.

Theorem 15. *In the NGPD model, the retailer's optimal retail price (denoted as p_{n1}^*), the manufacturer's optimal wholesale price (denoted as w_n^*), and retail price (denoted as p_{n2}^*) are*

$$\begin{aligned}p_{n1}^* &= \frac{\alpha\left(r_p + \theta\beta_p\right)}{2\beta_p\left(\Lambda_p + r_p\right)} + \frac{\theta\alpha + c\left(2\Lambda_p + r_p\right)}{6\Lambda_p} \\ &\quad + \frac{\varepsilon q_r}{1-\varepsilon}\frac{r_p + 2\Lambda_p}{6\Lambda_p},\end{aligned} \tag{47}$$

$$p_{n2}^* = \frac{\alpha\left(\Lambda_p - \theta\beta_p\right)}{2\beta_p\left(\Lambda_p + r_p\right)} + \frac{c}{2} + \frac{\varepsilon}{1-\varepsilon}\frac{q_r}{2}, \tag{48}$$

$$\begin{aligned}w_n^* &= \frac{\alpha\left(r_p + \theta\beta_p\right)}{2\beta_p\left(\Lambda_p + r_p\right)} + \frac{c\left(3\Lambda_p + \beta_p\right) - \theta\alpha}{6\Lambda_p} \\ &\quad + \frac{\varepsilon}{1-\varepsilon}\left(q - q_r\frac{r_p + 2\Lambda_p}{6\Lambda_p}\right).\end{aligned} \tag{49}$$

Proof. From Part (a) in Theorems 5 and 10 and (20) and (36), we easily know the theorem. □

Proposition 16. *For the optimal decisions in the NGPD model, there are*

(a) *the optimal sale prices of considering the production disruption case are larger than those of no disruption. And they are increasing in ε and q_r;*

(b) *the optimal wholesale price is larger than that of not considering the factor if $q/q_r > (r_p + 2\Lambda_p)/6\Lambda_p$, and it is increasing in the disruption probability ε or the internal penalty q and is decreasing in the external penalty q_r; otherwise, it is smaller than that of not considering the factor and is decreasing in the disruption probability ε or the internal penalty q and is increasing in the external penalty q_r;*

(c) *the difference between the channel prices is affected by the production disruption ε and the external penalty q_r.*

5. Comparative Analysis for Optimal Decisions

In this section, in order to compare the difference of the disruption effects, we will compare the optimal decisions of the above four decision frameworks.

Proposition 17. *The sales prices of considering the disruption factor in any member structure are larger than those of no disruption factor and are increasing in ε and q_r.*

This is obvious from Part (a) in Propositions 9, 14, and 16. It means that the supply chain under any member structure will transfer their disruption cost to customers by improving the sales prices. And the risk probability is bigger; customers will bear the bigger risk cost. Those will not change as the changing of supply chain structures. In fact, one disruption does not necessarily occur during a sales season, so the supply chain does not necessarily pay for the disruption. However, the customers will have to pay a higher price to supply chain because of the disruption consideration.

Proposition 18. *For the MSPD mode and RSPD mode, the optimal sale prices satisfy*

$$p_{m1}^* = p_{r1}^*; \quad p_{m2}^* = p_{r2}^*. \tag{50}$$

However, for the wholesale prices, when $\varepsilon < \varepsilon'$, there is

$$w_m^* > w_r^*; \tag{51}$$

otherwise, $w_m^* < w_r^*$, where $\varepsilon' = (1 - c\beta_p^2)/(1 + q_r\beta_p^2 - c\beta_p^2)$.

Proof. Equations (50) are obvious from Theorems 8 and 13. However, we need to compare (28) and (44) for obtaining (51). □

The proposition shows that the optimal sale prices in the MSPD model and RSPD model are the same as each other. It means that the two modes bring the same extra costs to customers, and the total market sales are the same; however the members' profit will be different because of the different wholesale prices in two modes. The different wholesale prices denote the different splitting way. Equation $\varepsilon < (1 - c\beta_p^2)/(1 + q_r\beta_p^2 - c\beta_p^2)$ means the time when the disruption risk is little. Under the case, there is $w_m^* > w_r^*$; that is, the mode of the manufacturer as Stackelberg leader is relatively better for the manufacturer; otherwise, the mode of the manufacturer as Stackelberg leader is relatively better for the retailer.

Proposition 19. *For the sales prices of traditional channels, there are the following:*

(a) *If $\theta\alpha < \overline{\theta\alpha}$, then*

$$p_{c1}^* < p_{r1}^* < p_{n1}^*. \tag{52}$$

(b) *If $\theta\alpha > \overline{\theta\alpha}$, then*

$$p_{c1}^* > p_{r1}^* > p_{n1}^*, \tag{53}$$

where $\overline{\theta\alpha} = \beta_p(c + \varepsilon q_r/(1 - \varepsilon))$.

Proof. Comparing (12) and (42) and comparing (42) and (47), we can obtain the proposition. □

Proposition 19 shows that when the market basic size $\theta\alpha$ of the traditional retail channel is small enough, the traditional retailer's sales price is smallest in the centralized mode and then in the Stackelberg game mode (including MSPD and RSPD) and it is larger in Nash game mode. However, when the market basic size $\theta\alpha$ of the traditional retail channel is large enough, the order is opposite.

However, for the optimal manufacturer's direct sale prices, the relation is different. From (13), (29), (43), and (48), we easily have the following proposition.

Proposition 20. *The optimal manufacturer's direct sale prices of different modes satisfy*

$$p_{c_2}^* = p_{m2}^* = p_{r2}^* = p_{n2}^*. \tag{54}$$

The proposition shows that the optimal manufacturer's direct sale prices in the different modes are the same as each other.

Proposition 21. *The disruption probability ε may affect the market division of two channels, and the same disruption variation will lead to different effects; the degree order is*

$$\text{NSPD} > \text{MSPD} = \text{RSDP} > \text{CPD}. \tag{55}$$

Proof. From the demand models in (1) and (2), the market division is affected by the sales prices. From Proposition 20, we only need to compare the traditional retailer's sales prices under different modes. Comparing the effect of the disruption probability ε, we only need to consider the coefficient of

Table 2: Optimal expected profits with $\varepsilon = 0.3$ and $\theta = 0.6$.

Scenario	$q_r \neq 0$			$q_r = 0$		
	Π_{sc}	Π_M	Π_R	Π_{sc}	Π_M	Π_R
CPD	5.0807×10^3			5.2325×10^3		
MSPD	4.4897×10^3	3.8988×10^3	590.94	4.6275×10^3	4.0225×10^3	605
RSPD	4.4897×10^3	3.3078×10^3	1181.88	4.6275×10^3	3.4175×10^3	1210
NGPD	4.8181×10^3	3.7675×10^3	1050.56	4.9636×10^3	3.8881×10^3	1075.56

ε. Because $(r_p+2\Lambda_p)/6\Lambda_p > (\Lambda_p+r_p)/4\Lambda_p$, we have NSPD > MSPD. From Proposition 16, there is MSPD = RSDP. From Part (c) in Proposition 4, the difference between the channel prices is not affected by the production disruption probability ε, so the disruption probability does not affect the market division. Therefore, the proposition holds. □

The proposition means that the centralized model is firstly considered for the dealing with the disruption. Then it is a leader-follower structure, and no matter who is the leader, the effect on the customers market is the same; that is, the whole supply chain has the same market competition. The structure under equivalent member abilities is the most susceptible mode for the disruption risk.

6. Numerical Experiment

In this section, we will further give some numerical analyses about equilibrium decisions and profits under disruption and give some conclusions. A selected basic set of parameters are $\alpha = 1000$, $c = 20$, $\beta_p = 8$, $r_p = 6$, $\Lambda_p = 14$, $\theta = 0.6$, $\varepsilon = 0.3$, $q = 4$, $q_r = 1.5$.

6.1. Optimal Expected Profit Analysis. From Table 2, we have the following results when $\varepsilon = 0.3$ and $\theta = 0.6$.

From the table, there are

(a) the biggest total supply chain expected profit is achieved in CPD model and then in NGPD model. The total supply chain expected profit in MSPD model is equal to the RSPD model, and the two models achieve the smallest total supply chain expected profits;

(b) the highest manufacturer's expected profit is achieved in MSPD model and then in NGPD model. And the lowest manufacturer's expected profit is achieved in RSPD model. Conversely, the highest retailer's expected profit is achieved in RSPD model and then in NGPD model. And the lowest manufacturer's expected profit is achieved in MSPD model;

(c) the expected optimal profits of considering the external penalty are smaller than those of no external penalty, which is reasonable in practice.

According to the above analysis, we can get some conclusions as follows: compared with the four decentralized supply chain models, the total supply chain realizes his biggest expected profit under the centralized model. Comparing among the three decentralized supply chain models, the total supply chain realizes his highest expected profit in Nash game scenario. However, If manufacturer is a leader in the supply chain, it will have the advantage to get the higher expected profit. And so it is retailer.

6.2. Sensitivity Analysis of Optimal Profit on Parameters ε, q, q_r. In this section, we analyze the changes of the optimal decisions with parameters ε, q, q_r.

From Figure 2, we give the change of the expected profits with parameter ε; we can see that the expected profits are all decreasing in ε.

From Figure 3, we give the change of the expected profits with parameter q, we find that the expected profits are all independent with q. It is an interesting result.

From Figure 4, we give the change of the expected profits with parameter q_r; we can see that the expected profits are all decreasing in q_r.

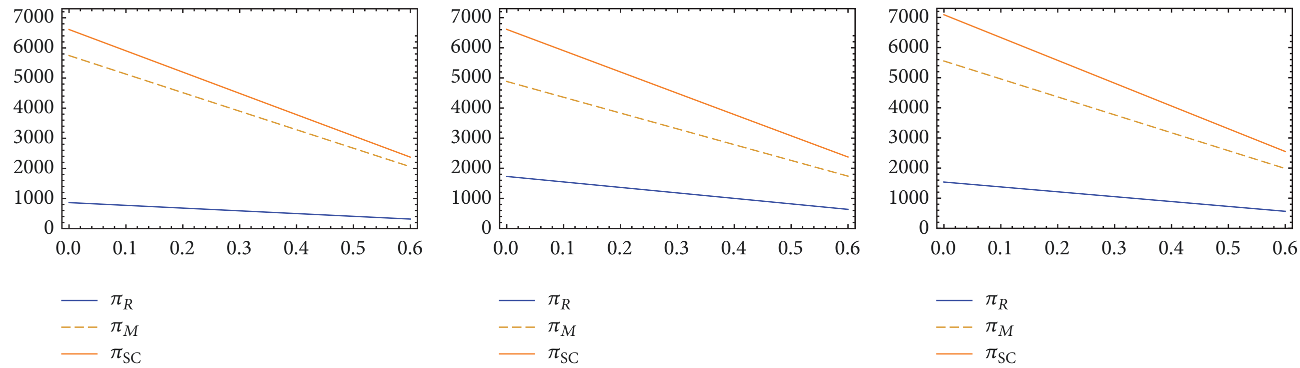
6.3. Sensitivity Analysis of Parameters θ. In this section, we analyze the changes of the optimal decisions and the expected profits with parameter θ. θ represents the percentage of the potential market size α that goes to the traditional channel.

In Figure 5, we give the changes of the optimal decisions with parameter θ (which are presented in the follow); we can see that the optimal retail price p_1 is decided by the retailer and the wholesale price w is decided by the manufacturer for the traditional market channel increase with the increases of θ. On the contrary, the retail price p_2 decided by the manufacturer for the direct market channel decreases with the increase of θ.

In Figure 6, we give the changes of the expected profits with parameter θ; we can see that the manufacturer's expected profit Π_M decreases with the increase of θ. However, the retailer's expected profit Π_R increases with the increase of θ. So, we see that the total supply chain profits Π_{SC} decrease and then increase with the increase of θ. In addition, we see that the retailer's profit Π_R increases fastest in the RSPD model, followed by in the NGPD model. The retailer's expected profit Π_M increases slowest in the MSPD model.

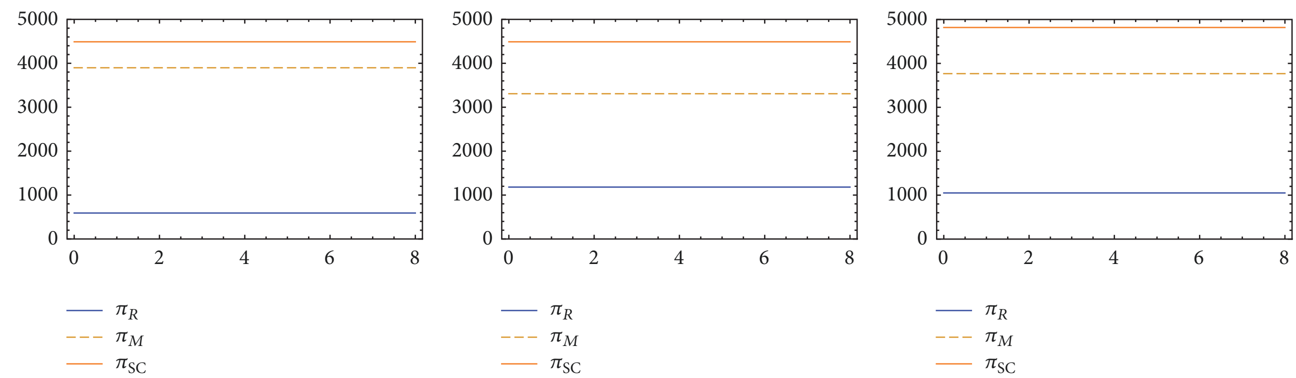
7. Managerial Insights and Summary

In this paper, we analyze pricing decisions of a dual-channel supply chain considering disruption risk. Firstly, we analyze the pricing decisions in centralized model; then we analyze the cases in three decentralized models (MSPD model, RSPD model, and NGPD model). The closed-form optimal price decisions of four models are given. And we provided the contrast analysis of the pricing decisions under the different cases. Furthermore, we make some numerical analysis.



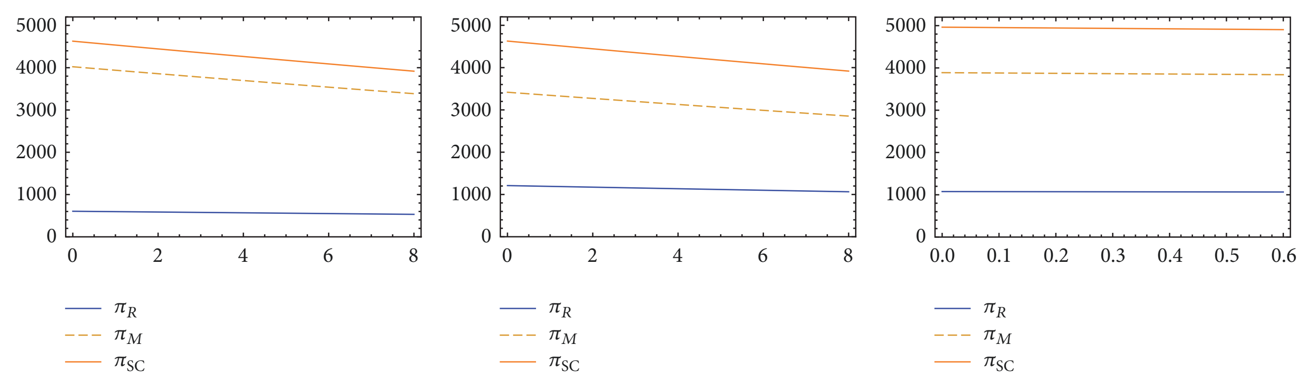
(a) In MSPD model (b) In RSPD model (c) In NGPD model

Figure 2: The change of the expected profits with the parameters ε.



(a) In MSPD model (b) In RSPD model (c) In NGPD model

Figure 3: The change of expected profits with the parameters q.

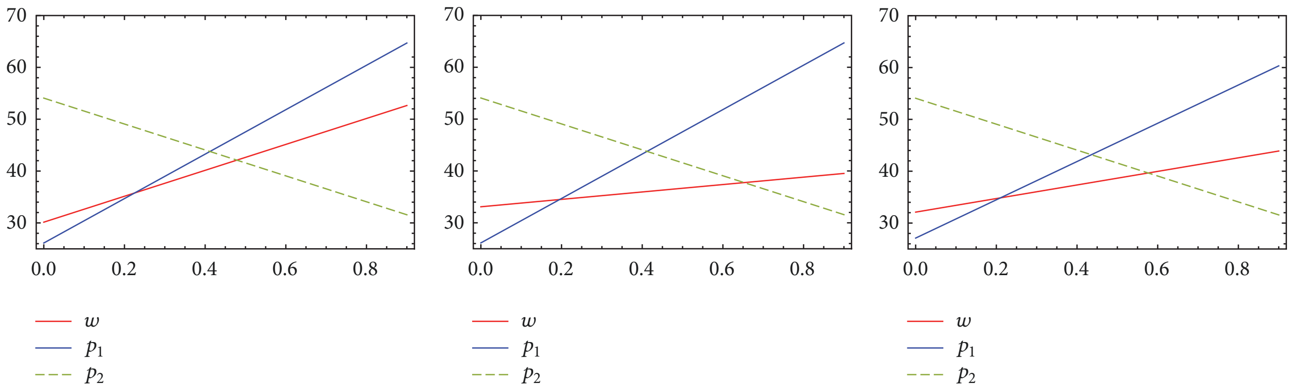


(a) In MSPD model (b) In RSPD model (c) In NGPD model

Figure 4: The change of expected profits with the parameters q_r.

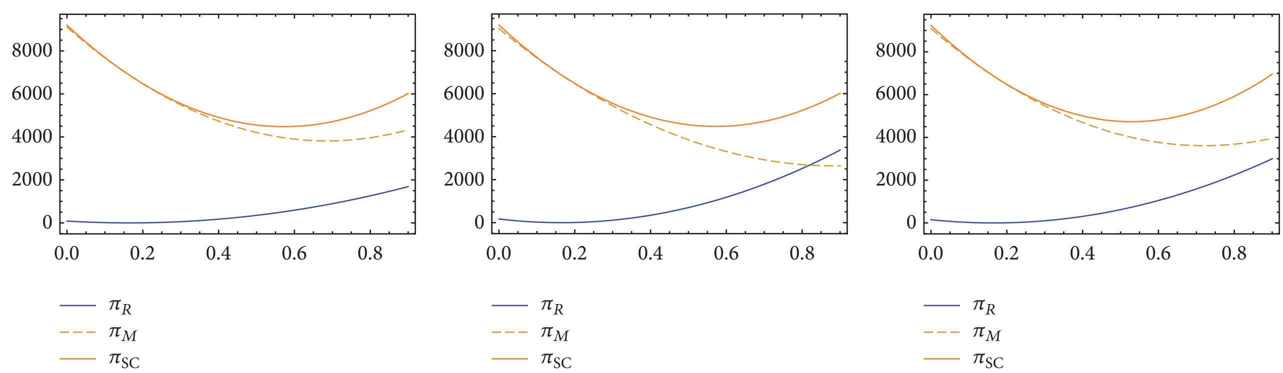
By the theoretical and numerical analysis, we have the following managerial insights:

(1) When considering the existence of a supply disruption risk, the manufacturer is willing to declare the information to the customer, and it is unfavourable to all customers. The sales prices of the direct sales and traditional retail channels in any power structure are improved when the members of supply chain consider the supply disruption risk, and the prices improvement is increasing as disruption probability and the shortage penalty degree. The supply disruption may result in complaint from the customers and affect the business reputation and sales quantity, which can be seen as a market punishment. Once the firms formed the perception, they will force customers to share the probable disruption loss. And the risk probability is larger, the customers' sharing is more. Therefore, when the supply chain may have a supply disruption and the market penalty may exist, the firms with short-term income goals have the motive to raise the sale



(a) In MSPD model (b) In RSPD model (c) In NGPD model

Figure 5: The change of optimal decisions profits with the parameters θ.



(a) In MSPD model (b) In RSPD model (c) In NGPD model

Figure 6: The change of expected profits with the parameters θ.

price to compensate for the loss of the future shortage. The customers will suffer a higher purchasing cost because of the increasing of the sale prices, and the reason of the prices increasing comes from the customer side, which means that all risk costs will be bore only by the customers. And, in order to obtain a probability revenue, a monopolistic manufacturer is willing to declare the information to the customer.

(2) The traditional retailer should select a low internal penalty for the manufacturer corresponding to the external penalty. If the internal penalty is large enough compared with external penalty, the wholesale prices in all decentralized decision structures are larger than that of not considering disruption risk. Otherwise, the wholesale prices are smaller. For the manufacturer, both of the customers of its direct sales channel and the traditional retailer are its downstream. If the penalty from its downstream is increased, the manufacturer's natural motivation is to raise the price to its downstream. When internal penalty is larger than the external penalty, raising price is the only way that the manufacturer responds to the probability loss, although it will hurt the retailer. However, if the internal penalty is smaller than the external penalty, the manufacturer can subsidize a part of the probability revenue from the direct sales channel to the traditional channel, so the wholesale price has a drop chance. It is obvious that a low internal penalty is more profitable for the retailer because the wholesale price is decreased and the sales price is improved, which also reflects the joint effort to deal with the risk.

(3) When the disruption risk is small enough, the mode of the manufacturer as Stackelberg leader is relatively better for the manufacturer; otherwise, the mode of the manufacturer as Stackelberg leader is relatively better for the retailer. From proposition, the optimal sale prices of the MSPD mode and RSPD mode are the same as each other. However, when the disruption risk is small enough, the wholesale price of the MSPD price is larger. Therefore when the disruption risk is small enough, the MSPD mode is profitable for the manufacturer. If the customers feel the supply disruption information to be higher, the supply chain members will more have motivations to raise their sales prices. Corresponding to the case with an enough small disruption risk, the manufacturer can gain the greater probability revenue under the case with large disruption risk, so he has an ability to subsidize traditional channel.

(4) When dealing with the disruption risk, the minimal damage model for customers is the centralized model. Then it is a leader-follower structure, and no matter who is the leader, the effect on the customers market is the same. The structure under equivalent member abilities is the most damaging

mode for customers. However, the Nash model is profitable in the decentralized models.

(5) Both of the disruption probability and the external penalty will increase the loss of supply chain; the decision makers should try to decrease the occurrence probability of the disruption and control user complaints for the disruption.

There are also many extensive researches in our paper. Firstly, the demand function is a single period setting, but, in the real word, the demand is dynamic. Secondly, we assume the manufacturer and the retailer have the symmetric information about demand and the products. So, we can extend the paper to consider the dual-channel with asymmetry information. Thirdly, in the paper, we consider that the manufacturer and the retailer are risk neutral; we can extend it to risk averse and risk appetite. Finally, we can also consider insurance contract to alleviate disruptions in dual-channel supply chain.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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