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Research Article

Robust H_∞ Fault Detection for Networked Markov Jump Systems with Random Time-Delay

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This paper investigates the problem of robust H_∞ fault detection for networked Markov jump systems with random time-delay which is introduced by the network. The random time-delay is modeled as a Markov process, and the networked Markov jump systems are modeled as control systems containing two Markov chains. The delay-dependent fault detection filter is constructed. Furthermore, the sufficient and necessary conditions which make the closed-loop system stochastically stable and achieve prescribed H_∞ performance are derived. The method of calculating controller, fault detection filter gain matrices, and the minimal H_∞ attenuation level is also obtained. Finally, one numerical example is used to illustrate the effectiveness of the proposed method.

1. Introduction

Feedback control systems wherein the control loop is closed through a real-time network are called networked control systems (NCSs) [1, 2]. The information is exchanged among control system components (sensor, controller, actuator, etc.). Due to the advantages such as simple installation, reduced wiring, increased system agility, and high reliability, NCSs have been widely used in broad areas, for example, unmanned aerial vehicles, mobile sensor networks, environment monitoring, and automated highway systems [3–5]. However, the introduction of communication networks also brings communication constraints to the control systems, for example, network-induced time-delays and packet dropouts [6–8]. Fault detection (FD) is very important for practical control systems, especially in safe-critical systems [9–11]. The theory of FD for NCSs is different from that of the traditional control systems due to the limitations induced by the network, such as time-delays and data packet dropouts which should be taken into consideration.

In recent years, many results of FD for NCSs have been reported. In [12], the problem of FD for a kind of nonlinear NCS with time-delays and data packet dropouts was investigated, and the sufficient conditions for the existence of FD filter were presented in terms of linear matrix inequalities (LMIs) using Lyapunov function in the continuous domain. In [13], by considering random time-delays, the NCSs were modeled as discrete-time, finite-dimensional Markov jump linear systems (MJLSs). The FD problem was formulated as a robust H_∞ FD filter design problem, and the sufficient condition to solve this problem was given in terms of LMIs. In [14], with the presence of stochastic packet dropouts in the network, the problem of FD filter design for NCSs was investigated. A design method for FD filter which made the residual generation system stable in the mean-square sense was proposed by the MJLSs theory. In [15], the problem of robust FD filter design and optimization was investigated for NCSs with random delays. The NCSs were modeled as Markov jump systems by assuming that the random delays obeyed the Markov characteristics. Based on the model, an observer-based residual generator was constructed and the corresponding FD problem was formulated as a filtering problem. A sufficient condition for the existence of the desired FD filter was derived in terms of LMIs. In [16], by employing the multirate sampling method and the augmented state matrix method, the NCSs with long random delays were modeled as MJLSs. Then based on the model, a filter was designed for detecting faults. In [17], two independent Markov chains were introduced to describe the transmission characterization of the data packet dropouts in both channels from sensors to controller and from controller to actuator, and a nonlinear Markov jump system model was established. By employing a mode-dependent FD filter as residual generator, the FD filter design problem of nonlinear NCSs was formulated as a nonlinear H_∞ filtering problem. In [18], by use of the augmented matrix approach, the FD error dynamic systems were transformed to the MJLSs. With the established model and using the bounded real lemma (BRL) for MJLSs, a H_∞ observer-based FD filter was established in terms of LMIs to guarantee that the error between the residual and the weighted faults was made as small as possible. In [19], the problem of FD was investigated for NCSs with signal quantization and random packet dropouts. A residual generator was constructed, and the corresponding FD problem was converted into a H_∞ filtering problem. In [20], the time-delays from sensor to controller and the time-delays from sensor to actuator are both considered which were described by two independent Markov chains. H_∞ FD problem for NCSs with time-delays on condition that the transition probabilities were partly unknown was investigated.

Markov jump systems are appropriate to model the systems whose structures are subject to the random changes which are widely used in the field of communications systems, power systems, and so on; thus, they have attracted much attention [21–24]. It is significant and necessary to investigate the FD problems for NCSs with the Markov jump controlled plants. However, the controlled plants in most of the existing literature were assumed to be the time-invariant systems (see [12–20]). To the best of the authors' knowledge, up to now, very limited efforts have been devoted to investigating the FD problem for NCSs with the Markov jump controlled plant, which motivates our investigation.

Compared to the previous relevant works, the main contribution of this paper is that, for the Markov jump NCSs, the sufficient and necessary conditions for the stochastically stability of the closed-loop system are derived, and the method of calculating the minimal H_∞ attenuation $\gamma_{\min}$ is obtained by constructing proper Lyapunov function candidate.

The rest of this paper is organized as follows. The FD filter is constructed and the closed-loop system model is obtained in Section 2. The sufficient and necessary conditions which make the closed-loop system stochastically stable and achieve prescribed H_∞ performance are derived in Section 3. Section 4 presents the simulation results to show the effectiveness of the proposed method. The conclusions are provided in Section 5.

2. Problem Formulation

Without loss of generality, we assume that the time-delay τ_k only exists between sensor and controller, and τ_k is modeled as a homogeneous Markov chin which takes value in the set $M \triangleq \{0, \ldots, \tau\}$, and the transition probability matrix is $\Lambda = [\lambda_{ij}]$. That is, τ_k jumps from mode i to j with probability λ_{ij}, which is defined by $\lambda_{ij} = \Pr(\tau_{k+1} = j \mid \tau_k = i)$, where $\lambda_{ij} \geq 0$ and $\sum_{j=0}^{\tau} \lambda_{ij} = 1$, for all $i, j \in M$.

In this paper, the following Markov jump controlled plant is considered:

$$\begin{aligned} x_{k+1} &= A_{\theta_k} x_k + B_{u\theta_k} u_k + B_{d\theta_k} d_k + B_{f\theta_k} f_k, \\ y_k &= C_{\theta_k} x_k, \end{aligned} \tag{1}$$

where $x_k \in R^n$ is the state vector, $u_k \in R^m$ is the input vector, $y_k \in R^r$ is the measured output vector, $d_k \in R^d$ is the external disturbance noise belonging to $l_2 \in [0, \infty)$, and $f_k \in R^f$ is the fault to be detected. A_{θ_k}, $B_{u\theta_k}$, $B_{d\theta_k}$, $B_{f\theta_k}$, and C_{θ_k} are all known real constant matrices with appropriate dimensions. $\{\theta_k, k \geq 0\}$ is a discrete-time homogeneous Markov chain, which takes values in a finite set $G \triangleq \{1, \ldots, N\}$ with a transition probability matrix $\Pi = [\pi_{pq}]$; namely, for $\theta_k = p$, $\theta_{k+1} = q$, one has $\pi_{ij} = \Pr(\theta_{k+1} = q \mid \theta_k = p)$, where $\pi_{pq} \geq 0$ and $\sum_{q=1}^{N} \pi_{pq} = 1$, for all $p, q \in G$.

It is noticed that the information of θ_k is not available for the controller at the time instant k duo to the time-delay τ_k; however, the information of τ_k is known to the controller. Consequently, the controller gain can be designed depending on τ_k; that is,

$$u_k = K_{\tau_k} x_{k-\tau_k}. \tag{2}$$

Construct a full-order FD filter at the side of controller as follows:

$$\begin{aligned} \hat{x}_{k+1} &= A_{\theta_k} \hat{x}_k + B_{u\theta_k} K_{\tau_k} x_{k-\tau_k} + L_{\tau_k} \left(y_{k-\tau_k} - \hat{y}_{k-\tau_k} \right), \\ \hat{y}_k &= C_{\theta_k} \hat{x}_k, \\ r_k &= V \left(y_{k-\tau_k} - \hat{y}_{k-\tau_k} \right), \end{aligned} \tag{3}$$

where $\hat{x}_k \in R^n$ is the filter state vector, $r_k \in R^g$ is the residual vector which is sensitive to the fault, L_{τ_k} is the filter gain matrix to be determined, and V is the gain matrix of the residual r_k.

Define the state estimation error and residual error as follows:

$$\begin{aligned} e_k &= x_k - \hat{x}_k, \\ r_{ek} &= r_k - f_k. \end{aligned} \tag{4}$$

The closed-loop systems can be obtained as

$$\begin{aligned} \xi_{k+1} &= \widetilde{A}_{\theta_k} \xi_k + \left(\widetilde{B}_{u\theta_k} K_{\tau_k} I_1 - I_2 L_{\tau_k} \widetilde{C}_{\theta_k} \right) \xi_{k-\tau_k} + B_{\omega\theta_k} \omega_k, \\ r_{ek} &= V \widetilde{C}_{\theta_k} \xi_{k-\tau_k} - I_3 \omega_k, \\ \xi_k &= \eta_k, \quad k \in \{-\tau, \ldots, 0\}, \end{aligned} \tag{5}$$

where

$$\widetilde{A}_{\theta_k} = \begin{bmatrix} A_{\theta_k} & 0 \\ 0 & A_{\theta_k} \end{bmatrix},$$

$$\widetilde{B}_{u\theta_k} = \begin{bmatrix} B_{u\theta_k} \\ 0 \end{bmatrix},$$

$$\widetilde{C}_{\theta_k} = \begin{bmatrix} 0 & C_{\theta_k} \end{bmatrix},$$

$$\widetilde{B}_{\omega\theta_k} = \begin{bmatrix} B_{d\theta_k} & B_{f\theta_k} \\ B_{d\theta_k} & B_{f\theta_k} \end{bmatrix},$$
$$I_1 = \begin{bmatrix} I & 0 \end{bmatrix},$$
$$I_2 = \begin{bmatrix} 0 \\ I \end{bmatrix},$$
$$I_3 = \begin{bmatrix} 0 & I \end{bmatrix},$$
$$\xi_k^T = \begin{bmatrix} x_k^T & e_k^T \end{bmatrix},$$
$$\omega_k^T = \begin{bmatrix} d_k^T & f_k^T \end{bmatrix}. \tag{6}$$

Definition 1 (see [25]). System (5) is stochastically stable if for $\omega_k = 0$ and every initial mode $\tau_0 \in M$, $\theta_0 \in G$, there exists a finite matrix $W > 0$ such that

$$E\left\{\sum_{k=0}^{\infty} \|\xi_k\|^2 \mid \xi_0, \tau_0, \theta_0\right\} < \xi_0^T W \xi_0. \tag{7}$$

In this paper, our objective is to design controller (2) and the FD filter (3), such that one has the following:

(a) The closed-loop system (5) is stochastically stable for $\omega_k = 0$.

(b) Under the zero-initial conditions, the residual error r_{ek} satisfies the following H_∞ noise attenuation performance:

$$E\left\{\sum_{k=0}^{\infty} r_{ek}^T r_{ek}\right\} < \gamma^2 E\left\{\sum_{k=0}^{\infty} \omega_k^T \omega_k\right\}, \tag{8}$$

where $\gamma > 0$ is the attenuation level.

For the purpose of FD, an evaluation function and a threshold should be provided, and in this paper the evaluation function J_k and a threshold J_{th} are selected as

$$J_k = E\left\{\sum_{\rho=l_0}^{l_0+k} \sqrt{r_\rho^T r_\rho}\right\},$$
$$J_{\text{th}} = \sup_{\omega_k \in L_2, f_k = 0} E\left\{\sum_{\rho=l_0}^{l_0+L_0} \sqrt{r_\rho^T r_\rho}\right\}, \tag{9}$$

where l_0 is the initial evaluation time instant and L_0 is the evaluation step length. The occurrence of fault can be detected by comparing J_k and J_{th} with the following rule:

$$J_k \le J_{\text{th}} \Longrightarrow \text{normal},$$
$$J_k > J_{\text{th}} \Longrightarrow \text{fault}. \tag{10}$$

Remark 2. It should be pointed out that if time-delay also exists between controller and actuator which is written as ν_k, the control input of the controlled plant (1) should be $K_{\tau_k} x_{k-\tau_k-\nu_k}$ which is different from the control input of the FD filter which is $K_{\tau_k} x_{k-\tau_k}$.

Remark 3. If there is no time-delay in system (5), the FD filter (3) can still detect the fault effectively.

Remark 4. In almost all the existing literatures related to the FD for NCSs, the standard infinite impulse response (IIR) filter (3) is commonly used. However, the researches about FD for NCSs using finite impulse response (FIR) filter including deadbeat dissipative FIR filtering, hybrid particle FIR filtering, and composite particle FIR filtering have not been reported, which is a completely new research area.

3. Main Results

In this section, the sufficient and necessary conditions which make system (5) stochastically stable will be derived. Further, we will present the calculation method of the controller gain matrix K_{τ_k}, the FD filter gain matrix L_{τ_k}, and the minimal H_∞ attenuation $\gamma_{\min}$ in terms of matrix inequalities. To proceed, we will need the following lemma.

Lemma 5 (see [26]). *For any positive-definite matrix R, scalars δ, δ_0 satisfying $\delta \ge \delta_0 \ge 1$, and vector function υ_l, one always has $(\sum_{l=\delta_0}^{\delta} \upsilon_l)^T R \sum_{l=\delta_0}^{\delta} \upsilon_l \le (\delta - \delta_0 + 1) \sum_{l=\delta_0}^{\delta} \upsilon_l^T R \upsilon_l$.*

Theorem 6. *When $\omega_k = 0$, the closed-loop system (5) is stochastically stable if and only if there exist positive-definite matrices $P_{i,p} > 0$, $P_{j,q} > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z_1 > 0$ and matrices K_i, L_i such that the inequality*

$$\Omega \triangleq \begin{bmatrix} \Omega_{11} & * & * \\ \Omega_{21} & \Omega_{22} & * \\ 0 & Z_1 & -Q_1 - Z_1 \end{bmatrix} < 0, \tag{11}$$

where

$$\Omega_{11} = \widetilde{A}_p^T \overline{P}_{j,q} \widetilde{A}_p + \tau^2 \widetilde{A}_p^T Z_1 \widetilde{A}_p + Q_1 + (\tau + 1) Q_2 - Z_1 - P_{i,p},$$
$$\Omega_{21} = \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right)^T \overline{P}_{j,q} \widetilde{A}_p + \tau^2 \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right)^T Z_1 \widetilde{A}_p + Z_1,$$
$$\Omega_{22} = \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right)^T \overline{P}_{j,q} \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right) + \tau^2 \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right)^T Z_1 \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right) - Q_2 - 2Z_1,$$
$$\overline{P}_{j,q} = \sum_{j=0}^{\tau} \sum_{q=1}^{N} \lambda_{ij} \pi_{pq} P_{j,q}, \tag{12}$$

holds for all i, $j \in M$ and $p, q \in G$.

Proof.

Sufficiency. Choose the Lyapunov function candidate as

$$V\left(\xi_k,\tau_k,\theta_k\right) \triangleq \xi_k^T \Psi_{\tau_k,\theta_k} \xi_k = \sum_{\mu=1}^{4} V_\mu\left(\xi_k,\tau_k,\theta_k\right), \tag{13}$$

where

$$\begin{aligned}
V_1\left(\xi_k,\tau_k,\theta_k\right) &= \xi_k^T P_{\tau_k,\theta_k}\xi_k, \\
V_2\left(\xi_k,\tau_k,\theta_k\right) &= \sum_{m=k-\tau}^{k-1} \xi_m^T Q_1 \xi_m, \\
V_3\left(\xi_k,\tau_k,\theta_k\right) &= \sum_{m=k-\tau_k}^{k-1} \xi_m^T Q_2 \xi_m + \sum_{n=-\tau+1}^{0}\sum_{m=k+n}^{k-1} \xi_m^T Q_2 \xi_m, \\
V_4\left(\xi_k,\tau_k,\theta_k\right) &= \sum_{n=-\tau+1}^{0}\sum_{m=k+n}^{k-1} \tau \varphi_m^T Z_1 \varphi_m, \\
\varphi_m &= \xi_{m+1} - \xi_m.
\end{aligned} \tag{14}$$

Apparently, we have $\Psi_{\tau_k,\theta_k} > 0$.

Along the solution of system (5), we have

$$\begin{aligned}
E\left\{\Delta V_1\right\} &= E\left\{\xi_{k+1}^T P_{\tau_{k+1},\theta_{k+1}} \xi_{k+1} \mid \tau_k = i,\ \theta_k = p\right\} \\
&\quad - \xi_k^T P_{\tau_k,\theta_k}\xi_k \\
&= \left(\widetilde{A}_p \xi_k + \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right)\xi_{k-\tau_k}\right)^T \\
&\quad \cdot \sum_{j=0}^{\tau}\sum_{q=1}^{N} \lambda_{ij}\pi_{pq} P_{j,q} \\
&\quad \cdot \left(\widetilde{A}_p \xi_k + \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right)\xi_{k-\tau_k}\right) - \xi_k^T P_{i,p}\xi_k, \\
E\left\{\Delta V_2\right\} &= \xi_k^T Q_1 \xi_k - \xi_{k-\tau}^T Q_1 \xi_{k-\tau}, \\
E\left\{\Delta V_3\right\} &= \xi_k^T Q_2 \xi_k - \xi_{k-i}^T Q_2 \xi_{k-i} + \sum_{l=k+1-\tau_{k+1}}^{k-1} \xi_l^T Q_2 \xi_l \\
&\quad - \sum_{l=k+1-\tau_k}^{k-1} \xi_l^T Q_2 \xi_l + \tau \xi_k^T Q_2 \xi_k - \sum_{l=k+1-\tau}^{k} \xi_l^T Q_2 \xi_l.
\end{aligned} \tag{15}$$

Note that

$$\begin{aligned}
\sum_{l=k+1-\tau_{k+1}}^{k-1} \xi_l^T Q_2 \xi_l &= \sum_{l=k+1-\tau_k}^{k-1} \xi_l^T Q_2 \xi_l + \sum_{l=k+1-\tau_{k+1}}^{k-\tau_k} \xi_l^T Q_2 \xi_l \\
&\le \sum_{l=k+1-\tau_k}^{k-1} \xi_l^T Q_2 \xi_l + \sum_{l=k+1-\tau}^{k} \xi_l^T Q_2 \xi_l.
\end{aligned} \tag{16}$$

Hence, we can obtain

$$\begin{aligned}
E\left\{\Delta V_3\right\} &\le \xi_k^T Q_2 \xi_k - \xi_{k-i}^T Q_2 \xi_{k-i} + \tau \xi_k^T Q_2 \xi_k, \\
E\left\{\Delta V_4\right\} &= \tau^2 \varphi_k^T Z_1 \varphi_k - \sum_{l=k-\tau}^{k-1} \tau \varphi_l^T Z_1 \varphi_l \\
&= \tau^2 \varphi_k^T Z_1 \varphi_k - \sum_{l=k-i}^{k-1} \tau \varphi_l^T Z_1 \varphi_l - \sum_{l=k-\tau}^{k-i-1} \tau \varphi_l^T Z_1 \varphi_l \\
&\le \tau^2 \varphi_k^T Z_1 \varphi_k - \sum_{l=k-i}^{k-1} i \varphi_l^T Z_1 \varphi_l \\
&\quad - \sum_{l=k-\tau}^{k-i-1} (\tau - i)\, \varphi_l^T Z_1 \varphi_l.
\end{aligned} \tag{17}$$

By Lemma 5, one can obtain

$$\begin{aligned}
&- \sum_{l=k-i}^{k-1} i \varphi_l^T Z_1 \varphi_l - \sum_{l=k-\tau}^{k-i-1} (\tau - i)\, \varphi_l^T Z_1 \varphi_l \\
&\quad \le - \left[\xi_k - \xi_{k-i}\right]^T Z_1 \left[\xi_k - \xi_{k-i}\right] \\
&\qquad - \left[\xi_{k-i} - \xi_{k-\tau}\right]^T Z_1 \left[\xi_{k-i} - \xi_{k-\tau}\right].
\end{aligned} \tag{18}$$

From (15)–(18), we have

$$\begin{aligned}
E\left\{\Delta V\left(\xi_k,\tau_k,\theta_k\right) \mid \tau_k = i,\ \theta_k = p\right\} &\le \zeta_k^T \Omega \zeta_k \\
&\le -\lambda_{\min}(-\Omega)\, \zeta_k^T \zeta_k \\
&= -\lambda_{\min}(-\Omega)\left(\xi_k^T \xi_k + \xi_{k-i}^T \xi_{k-i} + \xi_{k-\tau}^T \xi_{k-\tau}\right) \\
&\le -\alpha \left\|\xi_k\right\|^2,
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
\zeta_k^T &= \left[\xi_k^T \quad \xi_{k-i}^T \quad \xi_{k-\tau}^T\right], \\
\alpha &= \inf\left\{-\lambda_{\min}(-\Omega)\right\} > 0.
\end{aligned} \tag{20}$$

From (19), we can see that for any $T \ge 1$

$$\begin{aligned}
E\left\{\sum_{k=0}^{\infty} \left\|\xi_k\right\|^2\right\} &\le \frac{1}{\alpha} E\left\{V\left(\xi_0,\tau_0,\theta_0\right)\right\} \\
&\quad - \frac{1}{\alpha} E\left\{V\left(\xi_{T+1},\tau_{T+1},\theta_{T+1}\right)\right\} \\
&\le \frac{1}{\alpha} E\left\{V\left(\xi_0,\tau_0,\theta_0\right)\right\} = \xi_0^T \Psi_{\tau_0,\theta_0} \xi_0.
\end{aligned} \tag{21}$$

That is, the closed-loop system (5) is stochastically stable according to Definition 1.

Necessity. Assume that the closed-loop system (5) is stochastically stable. Thus, we have

$$E\left\{\sum_{k=0}^{\infty} \left\|\xi_k\right\|^2 \mid \xi_0, \tau_0\right\} < \xi_0^T W \xi_0. \tag{22}$$

Let

$$\xi_k^T \widehat{\Psi}_{\tau_k,\theta_k} \xi_k = E\left\{\sum_{t=k}^{T} \xi_t^T H_{\tau_t,\theta_t} \xi_t\right\}, \tag{23}$$

where $H_{\tau_t,\theta_t} > 0$.

Assume $\xi_k \neq 0$, from (23), it can be easily inferred that $\widehat{\Psi}_{\tau_k,\theta_k}$ is bounded, and the following limit exists:

$$\begin{aligned} \xi_k^T \Psi_{\tau_k,\theta_k} \xi_k &\triangleq \lim_{T\to\infty} \xi_k^T \widehat{\Psi}_{\tau_k,\theta_k} \xi_k \\ &= \lim_{T\to\infty} E\left\{\sum_{t=k}^{T} \xi_t^T H_{\tau_t,\theta_t} \xi_t\right\}. \end{aligned} \tag{24}$$

Since (24) holds for any ξ_k, we have $\Psi_{\tau_k,\theta_k} = \lim_{T\to\infty} \widehat{\Psi}_{\tau_k,\theta_k}$. Since $H_{\tau_t,\theta_t} > 0$, it can be seen that $\Psi_{\tau_k,\theta_k} > 0$ from (24).

Let us consider

$$\begin{aligned} &E\left\{\xi_k^T \widehat{\Psi}_{\tau_k,\theta_k} \xi_k - \xi_{k+1}^T \widehat{\Psi}_{\tau_{k+1},\theta_{k+1}} \xi_{k+1} \mid \tau_k = i,\ \theta_k = p\right\} \\ &\quad = \xi_k^T H_{\tau_k,\theta_k} \xi_k > 0. \end{aligned} \tag{25}$$

Letting $T \to \infty$, we have

$$\begin{aligned} &E\left\{\xi_k^T \Psi_{\tau_k,\theta_k} \xi_k - \xi_{k+1}^T \Psi_{\tau_{k+1},\theta_{k+1}} \xi_{k+1} \mid \tau_k = i,\ \theta_k = p\right\} \\ &\quad = -E\left\{\Delta V\left(\xi_k, \tau_k, \theta_k\right) \mid \tau_k = i,\ \theta_k = p\right\} \geq -\zeta_k^T \Omega \zeta_k \\ &\quad > 0, \end{aligned} \tag{26}$$

which completes the proof. □

Corollary 7. *When $\omega_k \neq 0$, consider the closed-loop system (5) and let $\gamma > 0$ be a given real scalar. If there exist $P_{i,p} > 0$, $P_{j,q} > 0$, $F_{j,q} > 0$, $Q_1 > 0$, $Q_2 > 0$, $Y_1 > 0$, $Z_1 > 0$ and matrices K_i, L_i such that*

$$\begin{bmatrix} \Xi_{11} & * & * \\ \Xi_{21} & \Xi_{22} & * \\ \Xi_{31} & 0 & \Xi_{33} \end{bmatrix} < 0, \tag{27}$$

$$\begin{aligned} P_{j,q}^{-1} F_{j,q} &= I, \\ Z_1^{-1} Y_1 &= I, \end{aligned} \tag{28}$$

where

$$\begin{aligned} \Xi_{11} &= \begin{bmatrix} Q_1 + (\tau+1) Q_2 - Z_1 - P_{i,p} & * & * \\ Z_1 & -Q_2 - 2Z_1 & * \\ 0 & 0 & -\gamma^2 I \end{bmatrix}, \\ \Xi_{21} &= \begin{bmatrix} 0 & Z_1 & 0 \\ \widetilde{A}_p - I & \widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p & \widetilde{B}_{\omega p} \end{bmatrix}, \\ \Xi_{22} &= \begin{bmatrix} Q_1 - Z_1 & * \\ 0 & -Y_1 \end{bmatrix}, \\ \Xi_{31} &= \begin{bmatrix} \sqrt{\lambda_{i0}\pi_{p1}} \widetilde{A}_p & \sqrt{\lambda_{i0}\pi_{p1}} \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right) & \sqrt{\lambda_{i0}\pi_{p1}} \widetilde{B}_{\omega p} \\ \vdots & \vdots & \vdots \\ \sqrt{\lambda_{i\tau}\pi_{pN}} \widetilde{A}_p & \sqrt{\lambda_{i\tau}\pi_{pN}} \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right) & \sqrt{\lambda_{i\tau}\pi_{pN}} \widetilde{B}_{\omega p} \end{bmatrix}, \\ \Xi_{33} &= \operatorname{diag}\left\{-F_{0,1}, \ldots, -F_{\tau,N}\right\}, \end{aligned} \tag{29}$$

holds for all $i, j \in M$, $p, q \in G$, system (5) is stochastically stable with H_∞ performance index γ.

Proof. From (19), we can obtain

$$\begin{aligned} &E\left\{\Delta V\left(\xi_k, \tau_k, \theta_k\right) \mid \tau_k = i,\ \theta_k = p\right\} + E\left\{r_{ek}^T r_{ek}\right\} \\ &\quad - \gamma^2 E\left\{\omega_k^T \omega_k\right\} \leq \varsigma_k^T \overline{\Omega} \varsigma_k, \end{aligned} \tag{30}$$

where

$$\overline{\Omega} = \begin{bmatrix} \Omega_{11} & * & * & * \\ \Omega_{21} & \overline{\Omega}_{22} & * & * \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & * \\ 0 & Z_1 & 0 & -Q_1 - Z_1 \end{bmatrix},$$

$$\begin{aligned} \overline{\Omega}_{22} &= \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right)^T \overline{P}_{j,q} \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right) \\ &\quad + \tau^2 \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right)^T Z_1 \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right) \\ &\quad - Q_2 - 2Z_1 + \left(V\widetilde{C}_p\right)^T V\widetilde{C}_p, \end{aligned} \tag{31}$$

$$\begin{aligned} \Omega_{31} &= \widetilde{B}_p^T \overline{P}_{j,q} \widetilde{A}_p, \\ \Omega_{32} &= \widetilde{B}_p^T \overline{P}_{j,q} \left(\widetilde{B}_{up} K_i I_1 - I_2 L_i \widetilde{C}_p\right) - I_3^T V \widetilde{C}_p, \\ \Omega_{33} &= \widetilde{B}_p^T \overline{P}_{j,q} \widetilde{B}_p + I_3^T I_3 - \gamma^2 I, \\ \varsigma_k^T &= \begin{bmatrix} \xi_k^T & \xi_{k-i}^T & \omega_k^T & \xi_{k-\tau}^T \end{bmatrix}. \end{aligned}$$

If $\overline{\Omega} < 0$, from (30) and under zero-initial condition, we have $E\{\sum_{k=0}^{\infty} r_{ek}^T r_{ek}\} < \gamma^2 E\{\sum_{k=0}^{\infty} \omega_k^T \omega_k\}$.

By Schur compliment, $\overline{\Omega} < 0$ is equivalent to

$$\begin{bmatrix} \Xi_{11} & * & * \\ \Xi_{21} & \overline{\Xi}_{22} & * \\ \Xi_{31} & 0 & \overline{\Xi}_{33} \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{aligned} \overline{\Xi}_{22} &= \begin{bmatrix} Q_1 - Z_1 & * \\ 0 & -Z_1^{-1} \end{bmatrix}, \\ \overline{\Xi}_{33} &= \operatorname{diag}\left\{-P_{0,1}^{-1}, \ldots, -P_{\tau,N}^{-1}\right\}. \end{aligned} \tag{33}$$

Letting $P_{j,q}^{-1} = F_{j,q}$, $j \in M$, $q \in G$, $Z_1^{-1} = Y_1$, (27) and (28) can be obtained, which completes the proof. □

In Corollary 7, the conditions are a set of LMIs with some inversion constraints. Though they are nonconvex which cannot be solved by using the existing convex optimization tool, we can use the cone complementarity linearization (CCL) algorithm [27] to transform this problem into the nonlinear minimization problem as follows:

$$\begin{aligned} \min \quad & \operatorname{tr}\left(\sum_{j=0}^{\tau}\sum_{q=1}^{N} P_{j,q}F_{j,q} + Z_1 Y_1\right) \\ \text{s.t.} \quad & (27), (35), \end{aligned} \tag{34}$$

$$\begin{gathered} \begin{bmatrix} P_{j,q} & I \\ I & F_{j,q} \end{bmatrix} > 0, \quad j \in M, \; q \in G, \\ \begin{bmatrix} Z_1 & I \\ I & Y_1 \end{bmatrix} > 0. \end{gathered} \tag{35}$$

Furthermore, the iterative algorithm which can be used to calculate the controller gain K_i, FD gain matrix L_i, and the minimal H_∞ attenuation $\gamma_{\min}$ is given bellow.

Algorithm 8.

Step 1. Let $\gamma = \gamma_0$ and set the maximum iterations number as $n_{\max}$.

Step 2. Find a feasible solution satisfying (27), (35) and set it as $(P_{j,s}^0, F_{j,s}^0, Z_1^0, Y_1^0, K_i^0, L_i^0)$. Let $k = 0$.

Step 3. Solve the following LMI optimization problem for variables $(P_{j,q}, F_{j,q}, Z_1, Y_1, K_i, L_i)$:

$$\begin{aligned} \min \quad & \operatorname{tr}\left(\sum_{j=0}^{\tau}\sum_{q=1}^{N}\left(P_{j,q}^k F_{j,q} + P_{j,q}F_{j,q}^k\right) + Z_1^k Y_1 + Z_1 Y_1^k\right), \\ \text{subject to} \quad & (27), (35); \end{aligned} \tag{36}$$

set $(P_{j,s}^k = P_{j,s}$, $F_{j,s}^k = F_{j,s}$, $Z_1^k = Z_1$, $Y_1^k = Y_1$, $K_i^k = K_i$, $L_i^k = L_i)$.

Step 4. If (27) and (28) are satisfied, let $\gamma = \gamma - \delta$, $\delta > 0$ and return to Step 3. If the number of iterations exceeds $n_{\max}$, the iteration is terminated.

Step 5. Check γ: if $\gamma = \gamma_0$, the optimization problem has no solutions within the maximum iterations number $n_{\max}$. Otherwise, $\gamma_{\min} = \gamma + \delta$.

Remark 9. In this paper, we assume that the transition probabilities of τ_k and θ_k are completely known. When transition probabilities of τ_k and θ_k are partly unknown, we can separate the unknown ones from the known ones; see [28].

4. Numerical Example

In this section, we present an example to demonstrate the effectiveness of the proposed method. Consider the controlled plant with the following parameter:

$$\begin{aligned} A_1 &= \begin{bmatrix} 1 & 0 \\ 0.25 & 0.15 \end{bmatrix}, \\ B_{u1} &= \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}, \\ B_{d1} &= \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\ B_{f1} &= \begin{bmatrix} 0.23 \\ 0.81 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0.65 & 0.05 \\ 0.1 & 0.3 \end{bmatrix}, \\ B_{u2} &= \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}, \\ B_{d2} &= \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \\ B_{f2} &= \begin{bmatrix} 0.37 \\ 0.32 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, \end{aligned} \tag{37}$$

the system mode $\theta_k \in \{1, 2\}$, and the transition probability matrix of θ_k is $\Omega = \left[\begin{smallmatrix} 0.2 & 0.8 \\ 0.9 & 0.1 \end{smallmatrix}\right]$. The random time-delay $\tau_k \in \{0, 1\}$ and the transition probability matrix is $\Lambda = \left[\begin{smallmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{smallmatrix}\right]$. Given $V = 0.1$, by Corollary 7, we can obtain the delay-dependent controller gain matrix K_i, filter gain matrix L_i, and $\gamma_{\min}$ as follows:

$$\begin{aligned} K_0 &= \begin{bmatrix} -0.0004 & 0.003 \end{bmatrix}, \\ L_0 &= \begin{bmatrix} 0.0481 \\ -0.1333 \end{bmatrix}, \\ K_1 &= \begin{bmatrix} 0.0036 & -0.0020 \end{bmatrix}, \\ L_1 &= \begin{bmatrix} -0.0342 \\ -0.0906 \end{bmatrix}, \\ \gamma_{\min} &= 1.6125. \end{aligned} \tag{38}$$

The initial value $x_{-1} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $x_0 = \begin{bmatrix} -1.1 & 1 \end{bmatrix}^T$, $\hat{x}_{-1} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $\hat{x}_0 = \begin{bmatrix} 1 & -1.1 \end{bmatrix}^T$, $\tau_{-1} = \tau_0 = 0$, $\theta_{-1} = \theta_0 = 0$. Assume that the external disturbance d_k is uniformly distributed random signal on $[-0.15 \;\; 0.15]$; when there is no

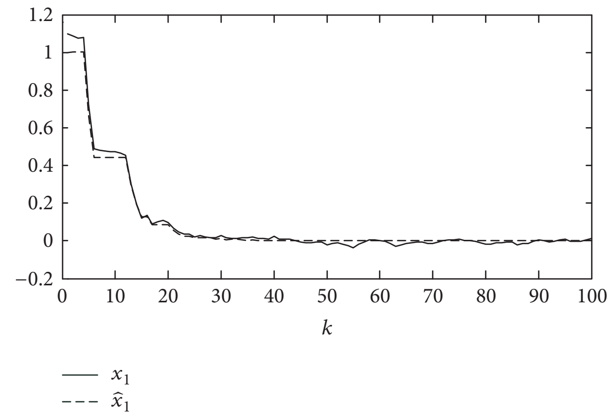

FIGURE 1: The state x_1 and its estimated value $\hat{x}_1$.

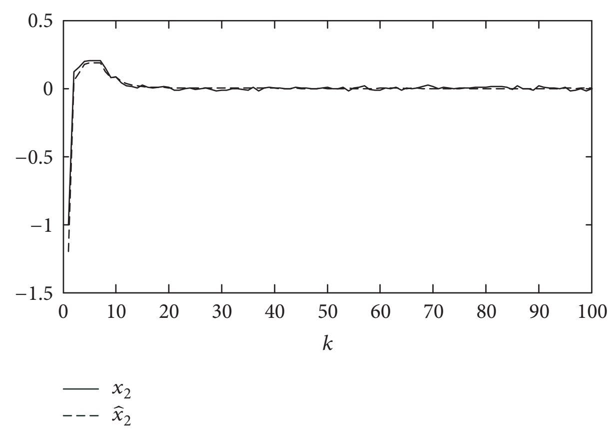

FIGURE 2: The state x_2 and its estimated value $\hat{x}_2$.

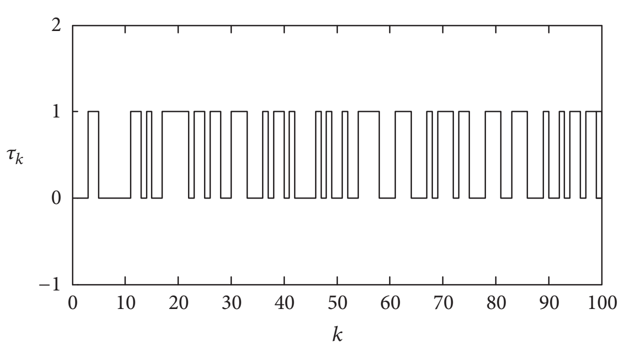

FIGURE 3: The time-delay τ_k.

fault, the trajectories of the closed-loop system's states and the corresponding estimated value are shown in Figures 1 and 2.

We can see that the filter can track the states of the system closely. Assume the fault signal is

$$f_k = \begin{cases} 0.5, & k = 10, \ldots, 20 \\ 0, & \text{others.} \end{cases} \tag{39}$$

The residual evaluation function is adopted as $J_k = E\{\sum_{\rho=0}^{k} \sqrt{r_\rho^T r_\rho}\}$, and the FD threshold can be obtained as $J_k = \sup_{\omega_k \in L_2, f_k=0} E\{\sum_{\rho=0}^{k} \sqrt{r_\rho^T r_\rho}\} = 0.0122$. Figures 3 and 4

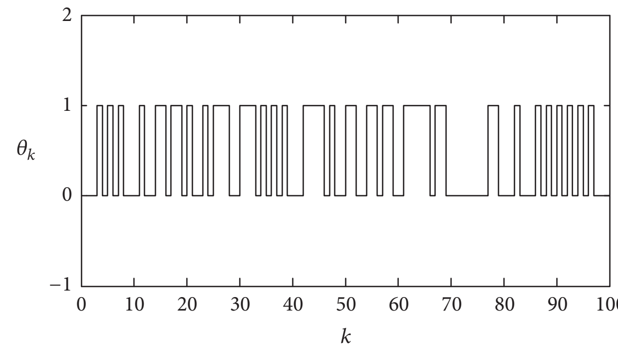

FIGURE 4: The system mode θ_k.

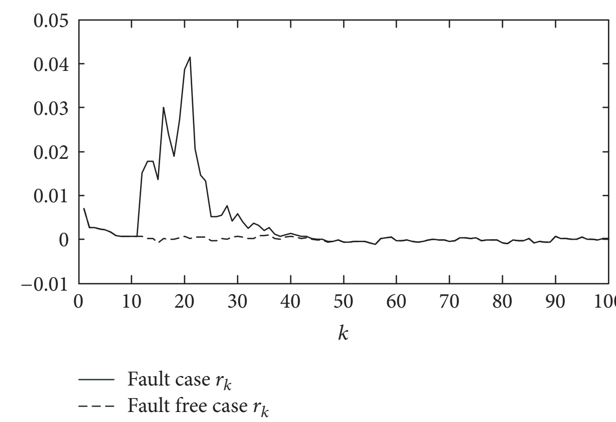

FIGURE 5: The residual signal r_k.

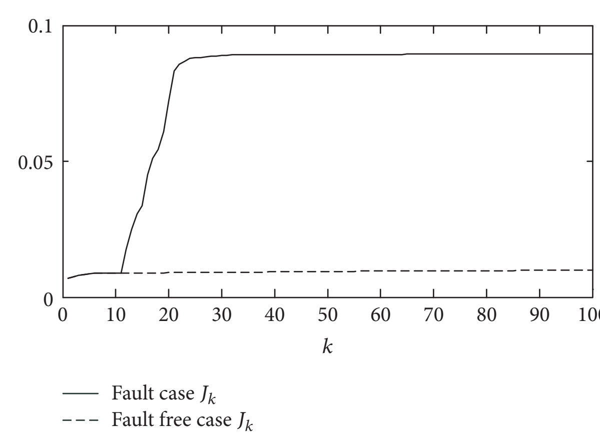

FIGURE 6: The residual evaluation function J_k and the threshold J_{th}.

show one simulation run of the time-delay and the system mode under the transition probability matrices Π and Λ, respectively.

Figures 5 and 6 show the residual signal r_k and the residual evaluation function J_k, respectively, from which we can see that when fault occurs, r_k and J_k change obviously. Moreover, it is noticed that $J_{11} = 0.0114 < J_{\text{th}} < J_{12} = 0.0170$. This means that the fault has been detected at the third time period after it occurs.

5. Conclusion

With the presence of random time-delay introduced by the network, the problem of robust H_∞ FD for networked Markov jump systems is investigated in this paper. By constructing delay-dependent FD filter, the closed-loop systems are established. The sufficient and necessary conditions which make the closed-loop system stochastically stable and achieve prescribed H_∞ performance are derived. The method of calculating controller, FD filter gain matrices, and the minimal H_∞ attenuation level is also obtained. The numerical example shows that the proposed FD filter is both sensitive to the fault and also robust to the exogenous disturbance.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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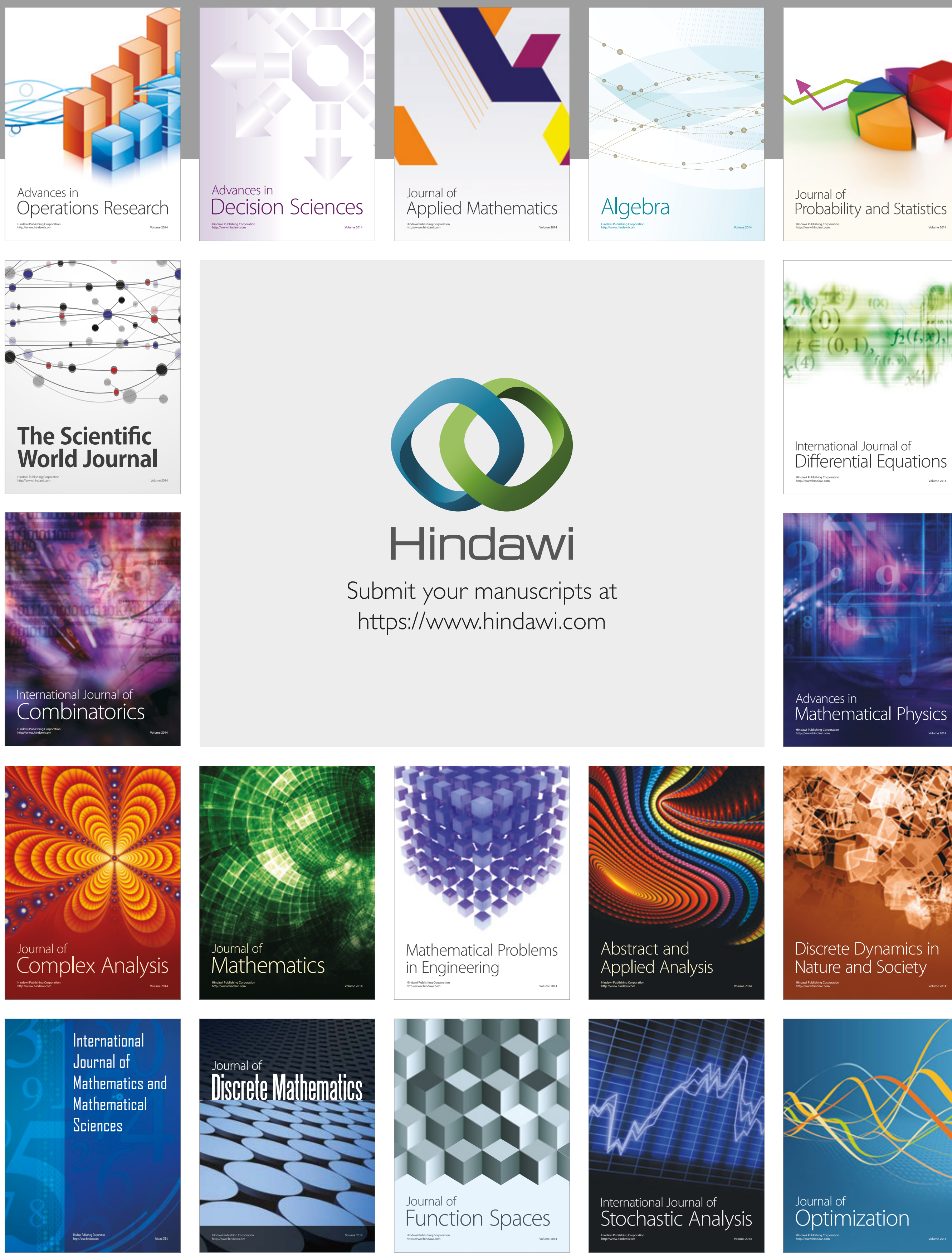
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