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Heat Transfer Measurements on a Rotating Disk

G. CARDONE*, T. ASTARITA and G.M. CARLOMAGNO

University of Naples, DETEC, P.le Tecchio 80, 80125 Naples, Italy

Heat transfer to a rotating disk is measured for a wide range of Reynolds number values in the laminar, transitional and turbulent flow regimes. Measurements are performed by making use of the *heated-thin-foil* technique and by gauging temperature maps with an infrared scanning radiometer. The use of the IR radiometer is advantageous on account of its relatively good spatial resolution and thermal sensitivity and because it allows one to perform measurements down to very low local Reynolds numbers. Data is obtained on three disks, having an external diameter varying from 150*mm* to 450*mm*; the smallest disk is used only to measure the adiabatic wall temperature and can rotate up to 21,000*rpm*. Heat transfer results are presented in terms of Nusselt and Reynolds numbers based on the local radius and show a substantial agreement with previous experimental and theoretical analyses. Transition to turbulent flow is found at about $Re = 250{,}000$. A discussion about the role played by the adiabatic wall temperature is also included.

Keywords: Rotating disk, convective heat transfer, IR thermography, adiabatic wall temperature

INTRODUCTION

The laminar flow due to an infinite flat disk rotating in still air is one of the few exact solutions of the three-dimensional Navier-Stokes equations. This type of flow was first theoretically investigated with an approximate method by von Kàrmàn [1921] who found that it resembles a boundary layer flow but with a boundary layer thickness independent of the radial distance. The tangential component of the shear stress at the disk surface imparts a circumferential velocity to the adjacent fluid layer which in turn, due to the centrifugal forces, also moves radially outwards. Rogers and Lance [1960] calculated accurate solutions by means of a numerical integration of the governing equations.

Wagner [1948] first evaluated the convective heat transfer coefficient by finding an approximate solution in the laminar regime based on the von Kàrmàn velocity distribution. The Wagner relation between the local Nusselt and Reynolds numbers is:

$$Nu = a \cdot \sqrt{Re} \tag{1}$$

where a is a constant equal to 0.335 for $Pr = 0.74$.

Millsaps and Pohlhausen [1952] solved, still in the laminar regime, the exact equation of the thermal field by reducing the system of partial differential

*Corresponding author.

equations to an ordinary one by means of a similarity solution method. They imposed a boundary condition of constant temperature all over the disk and included the viscous dissipation effects. When the latter can be neglected and $Pr = 0.71$, a is equal to 0.326.

Cobb and Saunders [1956], by testing a disk rotating from 30 to 2,500*rpm*, performed an experimental investigation on the mean heat transfer coefficient for a range of conditions from entirely laminar to mixed laminar-turbulent flow. In the laminar regime, they pointed out that eq. (1) is still valid when the average $\overline{Nu}$ and $\overline{Re}$ numbers are used. The lowest tested Reynolds number based on the disk radius is about 100,000. Although their data show a dependence of Nu from Re with an exponent which seems lower than 0.5, they affirmed that, in the laminar range, experimental results fit eq. (1) with a coefficient a equal to 0.36. Moreover at the lowest tested $\overline{Re}$, their findings are much larger than those predicted by eq. (1). This may be due to the influence of the natural convection around the disk which becomes important at the lowest rotational speeds they tested. Cobb and Saunders detected the onset of transition to turbulent flow at about $\overline{Re} = 240{,}000$. They also give the following correlation between the local Nusselt and Reynolds numbers in the turbulent regime:

$$Nu = 0.0193 \cdot Re^{0.8} \tag{2}$$

which was obtained by means of the classic Reynolds analogy and the friction moment coefficient data of Theodorsen and Regier [1944].

Kreith *et al.* [1959] experimentally evaluated mass transfer rates from a rotating disk of naphthalene under laminar and turbulent flow conditions and related their results to the heat transfer coefficients by means of an analogy method. They found a good agreement with theoretical data in the laminar range ($a = 0.34$ for $Pr = 0.71$) and located the onset of transition in the range $200{,}000 < Re < 250{,}000$.

Popiel and Boguslawski [1975] measured the heat transfer coefficient at a certain location over a disk rotating at different angular speeds and found the coefficient a of eq. (1) to be equal to 0.33 for laminar flow conditions. Also in their data the Nusselt number seems to depend on the Reynolds number with an exponent much lower than 0.5. Furthermore, at the lowest tested Re ($\cong 7{,}000$), Popiel and Boguslawski, who also took into account the effects of natural convection, measured a Nusselt number which is about 35% larger than the theoretical prediction. They found the onset of transition at about $Re = 195{,}000$. In the transitional and turbulent regimes their experimental data fit respectively the relationships:

$$Nu = 10 \cdot 10^{-20} \cdot Re^{4} \tag{3}$$

for Re ranging from 195,000 to 250,000 and:

$$Nu = 0.0188 \cdot Re^{0.8} \tag{4}$$

for Re greater than 250,000.

It has to be pointed out that all the works mentioned before deal with a boundary condition of isothermal disk which, in the laminar flow regime [see eq. (1)], coincides with the boundary condition of constant heat flux. For turbulent flow and for the boundary condition of a power law temperature difference ΔT, between disk and surrounding fluid, Northrop and Owen [1988], according to the solution obtained by Dorfman [1963], presented the following correlation for the local Nusselt number:

$$Nu = 0.0197\,(n + 2.6)^{+0.2}\,Pr^{0.6}\,Re^{0.8} \tag{5}$$

where n is the exponent of the power-law temperature difference profile:

$$\Delta T = c\,r^{n} \tag{6}$$

and c is a constant.

The aim of this paper is to produce new data, including some at very low local Reynolds number and to discuss critically the validity limits of the previous correlations presented in the literature. Data about the adiabatic wall temperature behavior is also indicated for the laminar and turbulent flow regimes and in particular in the laminar regime data is compared with an analytical solution reported in the appendix. A major peculiarity of the present work lies in the fact that an infrared (IR) imaging system (IR scanning radiometer or thermograph) is employed to measure the disk surface temperature, to evaluate the wall convective heat transfer coefficient. The advantage of having a non-contact temperature measuring device, such as the IR scanning radiometer, is well exploited to carry out heat transfer measurements in the vicinity of the disk axis of rotation, i.e. down to very low local Reynolds numbers.

EXPERIMENTAL APPARATUS AND PROCEDURE

A sketch of the experimental apparatus is shown in Fig 1. The disk section consists of a 300*mm* (or 150*mm* or 450*mm*) in diameter aluminium (or light-alloy steel) cup filled with a 20*mm* thick polyurethane foam on which a printed circuit board is glued. The circuit is used to generate, by Joule effect, an uniform heat flux on the disk surface, while the polyurethane foam thermally insulates the face of the disk not exposed to air. Electric power is supplied to the printed circuit by means of a mercury rotating contact. The smallest disk, which is only used to measure adiabatic wall temperatures, is not heated.

A pulley, that is connected by a transmission belt to an electric motor, is fixed on the transmission shaft supporting the disk. The rotating speed of the disk, which may be positioned either perpendicular or parallel to the gravitational force, can be varied in a continuous way within the range 100-4,500*rpm* for the two larger disks and up to 21,000*rpm* for the smallest one. The disk angular speed is continuously monitored by a tachometer. It has to be pointed out that, in order to measure heat transfer coefficients at low local Reynolds numbers, one can either have the disk rotating at a low angular speed, or measure the temperature very close to the disk rotating axis. In the present tests the possibility of implementing the second procedure makes it possible to keep relatively high rotating speed so as to diminish natural convection effects.

The printed circuit board is designed so as to achieve a constant heat flux over the disk surface and therefore the thickness and the width of its conducting tracks, having a spiral shape, are manufactured with very close tolerances. Tracks are 35μm thick, 2*mm* wide and placed at 2.5*mm* pitch; the overall thickness of the board is 0.2*mm*. Details about the printed circuit are reported in Cardone *et al.* [1993]. To enhance the emitted IR radiation detection, the measured board surface is coated with a thin layer of black paint which has an emissivity coefficient equal to 0.95 in the working IR window of the employed scanner.

The infrared thermographic system is based on AGEMA Thermovision 880. The field of view (which depends on the optic focal length and on the viewing distance) is scanned by the Hg-Cd-Te detector in the 8–12μm IR window. Nominal sensitivity, expressed in terms of noise equivalent temperature difference is $0.1°C$ when the scanned object is at ambient temperature. The scanning spatial resolution is 175 instantaneous fields of view per line at 50% slit response function. A $20° \times 20°$ lens is used during the tests. The thermal image is digitized in a frame of 140×140 *pixels* $\times$ 8 *bits*. In order to achieve an azimuthal spatial resolution the *line scan* facility of AGEMA 880 IR camera is also used to take temperature radial profiles (see Cardone *et al.* [1993]).

An application software is developed to correlate measured temperatures to heat transfer coefficients by

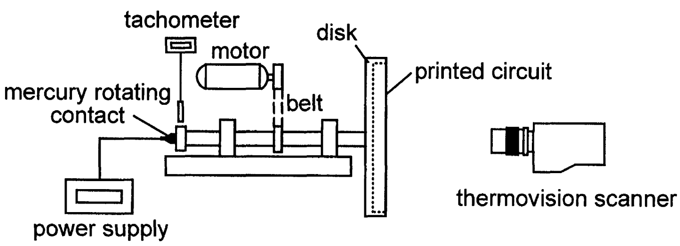

FIGURE 1 Experimental apparatus.

means of the so called *heated-thin-foil* technique (Carlomagno and de Luca [1989]):

$$h = \frac{q_j - q_{ra}}{T_w - T_{aw}} \tag{7}$$

where q_j is the Joule heating, q_{ra} is the radiative heat flux to ambient, T_w is the measured wall temperature and T_{aw} the adiabatic wall temperature of the flow. The radiative thermal losses q_{ra} are computed by using the measured T_w, while the conductive ones toward the inner polyurethane foam, are neglected. T_{aw} is measured by means of the same thermographic technique under the assumption that it coincides with the disk surface temperature when the Joule heating is suppressed; in effects a small correction of the order of few percent has to be made to take into account thermal radiation effects. The role played by the use of T_{aw} in eq. (7) in place of the ambient temperature T_a will be discussed later.

An error analysis, based on calibration accuracy of infrared system and repeatability of measurements indicates that heat transfer coefficients measurements are accurate to within about $\pm 3\%$.

RESULTS

In Fig 2 a thermographic image of the disk (D = 300*mm*), rotating at 576*rpm* and subject to a heat flux $q_j = 407 W/m^2$, is shown. Due to the relatively low angular speed, the flow is expected to be laminar everywhere over the disk. According to this, as shown by the thermogram, the disk surface exhibits a uniform wall temperature which in the present case is about 45°*C*. In fact, the laminar correlation of eq. (1) predicts the h coefficient to be constant over the disk; as a consequence, from eq. (7) it arises that, for a uniform heat flux boundary condition and uniform distribution of T_{aw} (in the low subsonic regime T_{aw} coincides with the ambient temperature), the wall temperature must be constant. Near the periphery of the disk a temperature decrease, due to edge effects, is found.

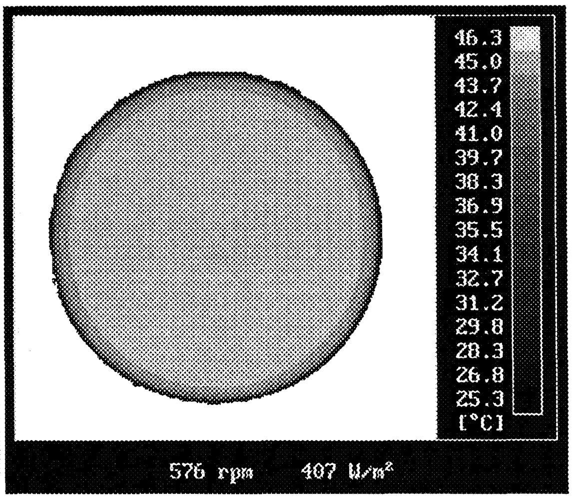

FIGURE 2 Thermogram of the 300*mm* disk at 576*rpm* and q_j = 407 W/m^2.

Fig. 3 is a thermal picture of the largest disk (450*mm*) recorded while it is rotating at 4390*rpm* and is subject to a heat flux of 871W/m^2. A relatively small (about 16% of the total surface) region around the disk center, where the wall temperature is constant, is clearly evident. On the basis of the previous discussion the flow is laminar there. In the outer zone the temperature decreases, first quickly in the transitional regime and then slowly in the turbulent one;

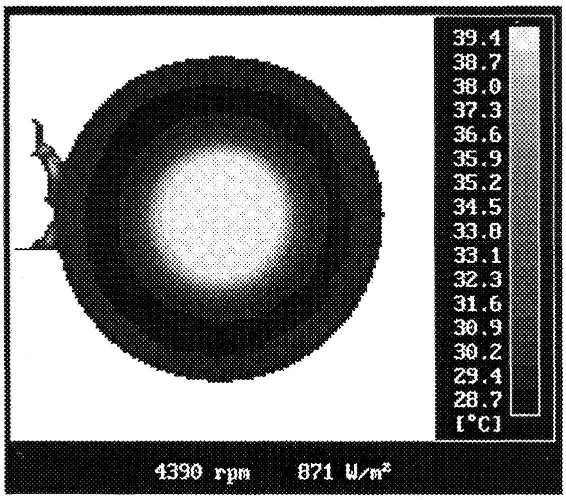

FIGURE 3 Thermogram of the 450*mm* disk at 4390*rpm* and q_j = 871W/m^2.

immediately after the temperature trend is reversed as T_w slowly begins to rise. Also in this case edge effects appear near the disk periphery.

In order to explain the temperature behavior in the turbulent regime, first consider that due to the turbulent correlation law [e.g. eq. (5)] the heat transfer coefficient is expected to increase as the local radius increases. As long as the adiabatic wall temperature distribution is uniform eq. (7) shows that the wall temperature must decrease along the radial direction. However, by examining the *cold* thermogram of Fig. 4, which is relative to the adiabatic wall temperature recorded at the same disk angular speed as that of Fig. 3, it should be noted that T_{aw} is practically constant (and equal to T_a) only within the circumference whose radius is about 60% of that of the disk. Afterwords T_{aw} experiences a significant increase (about 3°C over T_a near the disk edge). Since for the present experimental conditions $T_w - T_{aw}$ is of the same order of magnitude as $T_{aw} - T_a$, the increasing trend of the wall temperature shown in Fig. 3 is accordingly explained. It has to be explicitly pointed out that in the case of relatively high boundary heat fluxes q_j (i.e. high ΔT), the effect of the adiabatic wall temperature becomes negligible and T_w is monotonically decreasing towards the disk periphery. The thermogram of Fig. 5, showing the ΔT map, may be interpreted as an overall picture of the heat transfer coefficient surface distribution. The sequence of annular rings, which increasingly darkens towards the disk limb, proves the qualitative trend of the h coefficient.

The temperature difference $T_{aw} - T_a$ for the unheated 150*mm* disk is plotted in Fig. 6 for angular speeds varying from 11,900 to 20,600*rpm*. Each temperature profile is the average of data relative to three

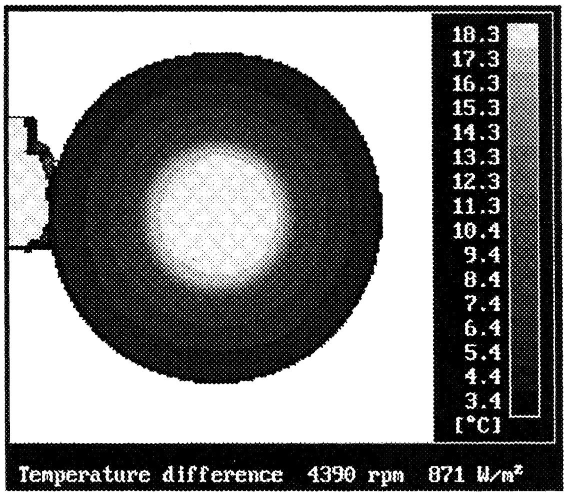

FIGURE 5 Temperature difference of the 450*mm* disk at 4390*rpm* and $q_j = 871 W/m^2$.

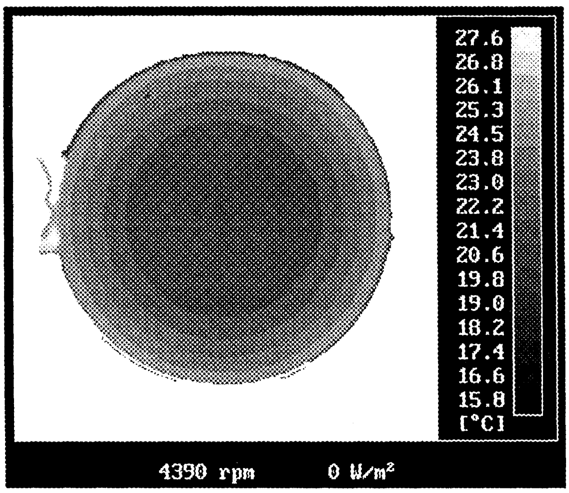

FIGURE 4 Thermogram of the 450*mm* disk at 4390*rpm* and $q_j = 0 W/m^2$.

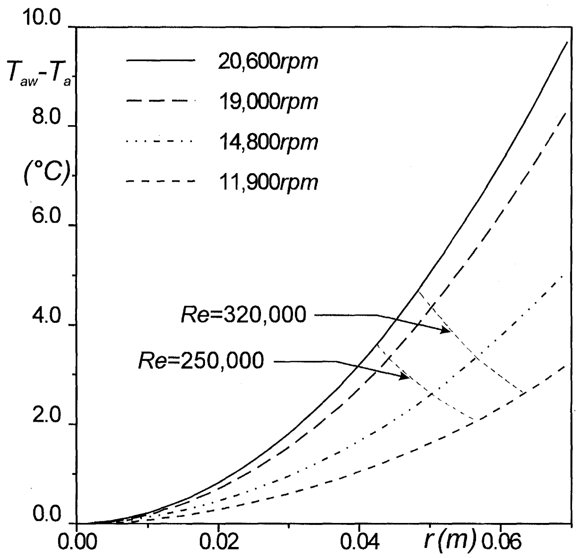

FIGURE 6 Temperature difference of the 150*mm* unheated disk for various angular speeds.

different tests which shows a very small spread (<1%). In the same figure the line corresponding to the onset of transitional and turbulent flows are also shown.

Since the recovery factor is nearly constant throughout the disk, the temperature profile is practically parabolic. In fact the recovery factor in the laminar regime, as computed from data of Fig. 6, is in accordance with the theoretical value indicated in the appendix which, for $Pr = 0.71$, gives $R = 0.891$. The recovery factor in the transitional and turbulent flow regimes is not very different since it varies from 0.886 to 0.894 which is very close to $\sqrt[3]{Pr} = 0.892$. It has to be pointed out however, that since the temperature differences are small and the resolution of the scanner is equal to 0.1°C, no definitive statement can be made for the laminar flow regime. Furthermore it must be recalled that the R values are computed by considering also the losses for thermal radiation which account for a correction of a few percent.

Data obtained in laminar flow regime, with both the 300*mm* and 450*mm* in diameter disks rotating at 34 different angular speeds, are shown in Fig. 7 in terms of local Nusselt number for $4 < Re < 200{,}000$. The theoretical prediction of Millsaps and Pohlhausen [1952] is also reported. All points fall around a straight line in the log-log plane down to very low Reynolds numbers. Following the Wagner [1948] theory, a correlation of all the data in terms of equation (1) is made and the value of the constant results $a = 0.333$. This value looks to be very much in accordance with previous theoretical and experimental findings. It has to be stressed that the validity of (1) down to very low local Reynolds numbers has not been proved before.

To check the actual slope of data in the log-log plane, a generalization of Wagner [1948] relation is investigated, namely:

$$Nu = a' \cdot Re^b \tag{8}$$

A linear regression based on eq. (8) is applied to a sample of data, first by neglecting T_{aw} effects and then by taking them into account. In the former case the evaluated slope is $b = 0.488$, in the latter one $b = 0.499$. This finding demonstrates that the not very accurate slope shown by data of previous investigators may be attributed not only to the natural convection effects but also to the T_{aw} effect which was systematically neglected.

The results relative to the 450*mm* in diameter disk for Re ranging from 1,000 to 1,400,000, are shown in Fig. 8. While the almost sudden rise of Nu around $Re = 250{,}000$ is to be ascribed to the onset of transition, the second slope change, which appears at the right end of the figure, is be to attributed to the presence of fully turbulent flow. A linear regression of the data in the transitional range of Re from 260,000 to 320,000 yields:

$$Nu = 8.01 \cdot 10^{-14} \cdot Re^{2.8} \tag{9}$$

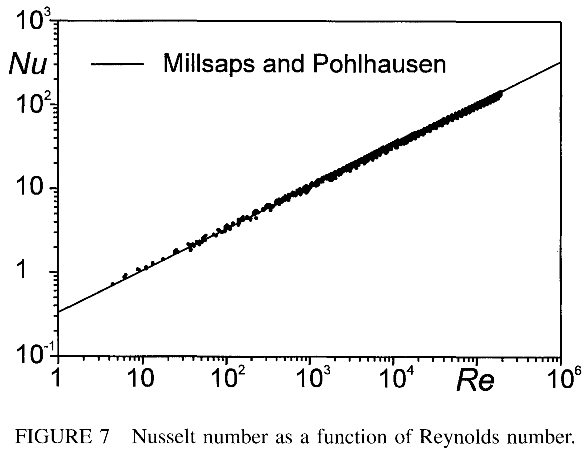

FIGURE 7 Nusselt number as a function of Reynolds number.

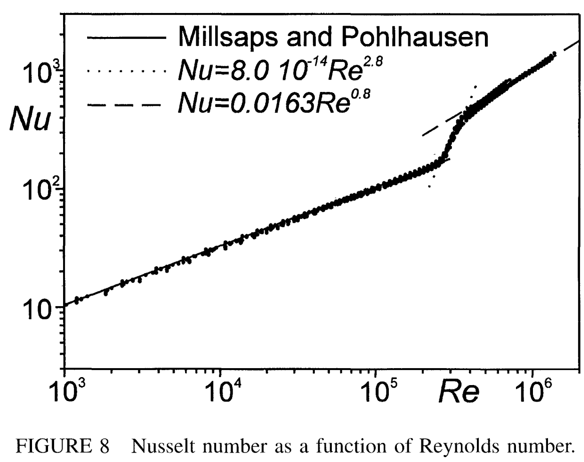

FIGURE 8 Nusselt number as a function of Reynolds number.

Present transitional results do not agree with those of Popiel and Boguslawski [1975] as far as both the transitional *Re* range and the regression slope are concerned. Apart from the remark that measurements in transitional flow regimes may be in general strongly affected by the environmental conditions of the actual experimental apparatus, it should be stressed that the results of Popiel and Boguslawski [1975] are relative to an isothermal boundary condition and are obtained by using a calorimetric device too large to achieve the fine spatial resolution exhibited by the data reported in their paper.

In the fully turbulent regime present data fits the relation:

$$Nu = 0.0163 \cdot Re^{0.8} \tag{10}$$

According to (5) the convective heat transfer coefficient is proportional to $2^{0.6}$. In the case of a constant heat flux boundary condition, ΔT is inversely proportional to h so that the coefficient n is equal to -0.6; therefore, for $Pr = .71$ eq. (5) reduces to the relationship:

$$Nu = 0.0184\, Re^{0.8} \tag{11}$$

which is quite in accordance with the present experimental data.

Finally it has to be said that the obtained experimental results are practically identical for both disks perpendicular to the direction of the gravitational force and disks parallel to this latter and facing downward; this behavior confirms that the neglecting of natural convection effects, for the tested rotational speeds, is correct.

CONCLUSIONS

Heat transfer measurements on a rotating disk are performed in the laminar, transitional and turbulent flow regimes by making use of the *heated-thin-foil* technique and by measuring temperature maps with an infrared scanning radiometer. The use of the radiometer is proved to be advantageous on account of its relatively good spatial resolution and thermal sensitivity and because it facilitates the making of measurements down to very low local Reynolds numbers.

Experimental data is correlated in terms of *Nu* and *Re* numbers, both based on the local radius and shows to be in accordance with theoretical predictions for a range of Reynolds numbers much wider than that obtained in previous studies. Moreover, in the authors' opinion, transitional results seem to be more reliable than the ones available in the literature. In particular, the onset of the transition to turbulence is found around $Re = 250{,}000$.

A theoretical discussion about the role played by the adiabatic wall temperature in the laminar regime is made. The analysis seems to be confirmed by experimental results. In the turbulent regime the fact that the adiabatic wall temperature rises towards the disk limb gives an explanation of the increase of the *heated* disk surface temperature in that region.

APPENDIX

A cylindirical coordinate system is assumed, as shown in Fig. 9. To reduce the Navier Stokes equations to a dimensionless ordinary differential system, von Kàrmàn [1921] imposed the pertinent variables to satisfy the relations:

$$\begin{aligned}
z &= (\nu/\omega)^{1/2}\, z^* \\
V_r &= r\omega F^*(z^*) \\
V_\theta &= r\omega G^*(z^*) \\
V_z &= (\nu\omega)^{1/2}\, H^*(z^*) \\
p &= \rho\nu\omega P^*(z^*)
\end{aligned} \tag{12}$$

so as to obtain following governing equations:

$$H^{*\prime} + 2\,F^* = 0$$

$$F^{*\prime\prime} - H^*\,F^{*\prime} - F^{*2} = -G^{*2}$$

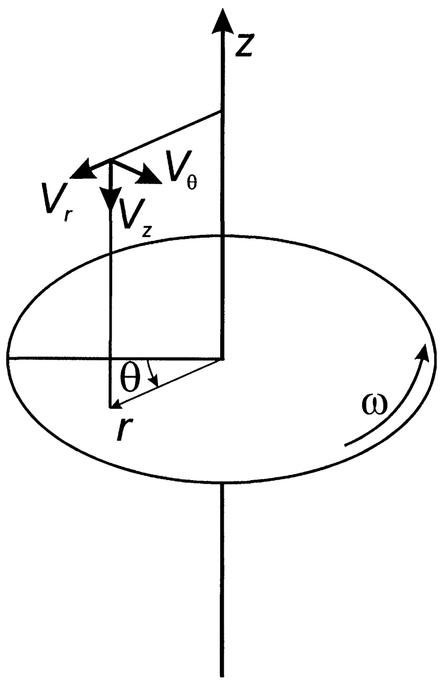

FIGURE 9 Sketch of the coordinate system.

$$G^{*''} - H^* G^{*'} + 2F^* G = 0$$

$$P^{*'} - H^{*''} - H^*H^{*'} = 0 \tag{13}$$

The boundary conditions reduce to:

$$F^*(0) = 0; HF^*(\infty) = 0; HG^*(0) = 1; HG^*(\infty) = 0; H^*(0) = 0; \tag{14}$$

Millsaps and Pohlhausen [1952] proposed, for the thermal field, the following relations:

$$T = (\nu\omega/c_p)\, T^*$$

$$T^* = Re\, S^*(z^*) + Q^*(z^*) + T^*_a \tag{15}$$

so that the governing equations of the thermal field are:

$$S^{*''} - Pr\, H^*S^{*'} + Pr\, H^{*'}S^* = Pr\,(F^{*'2} + G^{*'2})$$

$$Q^{*''} - Pr\, H^*Q^{*'} = -(4\, S^* + 12\, Pr\, F^{*2}) \tag{16}$$

They imposed a constant temperature boundary condition over the disk. Herein a constant heat flux and particularly the adiabatic wall condition, $\partial T/\partial z|_w = 0$ is imposed, which implies that on the disk surface:

$$S^{*'}(0) = 0; \quad Q^{*'}(0) = 0 \tag{17}$$

while at a large distance from the disk the boundary condition of the constant ambient temperature imposed by Millsaps and Pohlhausen still holds:

$$S^*(\infty) = 0; \quad Q^*(\infty) = 0 \tag{18}$$

The variation with r of the wall temperature is due only to S:

$$T^*_{aw} = Re^2\, S^*(0) + T^*_a \tag{19}$$

and, by returning to dimensional quantity, the following is obtained:

$$T_{aw} - T_a = (r\omega)^2/c_p S^*(0) \tag{20}$$

The recovery factor which is defined as:

$$R = \frac{T_{aw} - T_a}{\dfrac{(r\omega)^2}{2c_p}} \tag{21}$$

in the flow laminar regime turn out to be equal to $2 \cdot S(0)$.

Table 1 reports the recovery factor, obtained by solving the system of differential equations (13) and (16) with the boundary conditions (14), (17) and (18), as a function of the Prandtl number.

TABLE 1 Recovery Factor as a Function of *Pr*.

Pr	.5	.6	.7	.8	.9	1.0	1.2	1.4	1.6	1.8	2.0	3.0	4.0	5.0	7.0	10.0
R	0.799	0.844	0.887	0.927	0.964	1.00	1.066	1.128	1.184	1.237	1.287	1.502	1.677	1.828	2.082	2.389

NOMENCLATURE

a	Constant [Eq. (1)]
a'	Constant [Eq. (8)]
b	Constant [Eq. (8)]
c	Constant [Eq. (6)]
c_p	Specific heat at constant temperature
D	Disk diameter
F	Velocity component
G	Velocity component
H	Velocity component
h	Convective heat transfer coefficient
$\bar{h}$	Averaged convective heat transfer coefficient
k	Coefficient of thermal conductivity
n	Power-low exponent [Eq. (6)]
Nu	Nusselt number (hr/k)
$\overline{Nu}$	Averaged Nusselt number ($\bar{h}$D/2k)
P	Pressure
Pr	Prandtl number ($c_p\mu/k$)
Q	Temperature component
q	Heat flux
R	Recovery factor
r	Local radius
Re	Reynolds number ($\omega r^2/v$)
$\overline{Re}$	Averaged Reynolds number ($\omega D^2/4v$)
S	Temperature component
T	Temperature
ΔT	Temperature difference ($T_w - T_{aw}$)
V	Velocity
z	Axial coordinate
ω	Angular speed
v	Coefficient of kinematic viscosity
μ	Coefficient of viscosity
ρ	Density
θ	Azimuthal coordinate

Subscripts

a	Ambient
aw	Adiabatic wall conditions
j	Joule
r	Component along the r coordinate
ra	Radiative
w	Wall condition
z	Component along the z coordinate
θ	Component along the θ coordinate

Superscripts

*	Dimensionless quantity
′	Ordinary derivative

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