# Nonlinear Disturbance Observer-Based Adaptive Sliding Mode Control for a Generic Hypersonic Vehicle 

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#### Abstract

In this paper, a new adaptive sliding mode control method is presented for the longitudinal model of a generic hypersonic vehicle subject to uncertainties and external disturbance. Firstly, an oriented-control model with mismatched uncertainties is built for a generic hypersonic vehicle. Secondly, the back-stepping technique is introduced to design a sliding mode controller with an adaptive law to adapt to the disturbance and uncertainty. Thirdly, a set of nonlinear disturbance observers are designed to estimate the lumped disturbance and compensate the sliding mode controller, and the stability of the proposed controller is analyzed by utilizing Lyapunov stability theory. Finally, simulation results show that the effectiveness of the proposed controller is validated by the nonlinear model and the proposed method exhibits promising robustness to mismatched uncertainties.


## 1. Introduction

Generic hypersonic vehicles (GHVs) provide a reliable way to enter space and attract worldwide attentions in recent years. As GHVs are sensitive to physical and aerodynamic parameter changes, a concernful task is to design an efficient control system that makes the flight of GHVs feasible.

Faced with the complexities of GHV dynamics, the design methods of the guidance and control system have attracted considerable interests [1-3]. However, modeling inaccuracies and various disturbance can lead to some adverse effects on the controller; it is a challenging problem to design the controller of GHVs [4].

Sliding mode control (SMC) is widely used in dealing with parameter uncertainties and external disturbances for the flight of GHVs [5-8]. Furthermore, back-stepping technique is also an effective way for the control system design, in which the virtual control input can be obtained at each step and the actual controller comes to being [9-12]. The combination of back-stepping method and dynamic surface method is used to design a robust controller. However, the problem of compute explosion in the back-stepping method have to be solved

Meanwhile, the adaptive control approach is applied to adapt to the parameter uncertainties as well as constraint on states and control inputs [12-20]. By selecting a proper adaptive law, the satisfied performance can be easily achieved.

It is well known that the disturbance observer is an efficient and active method to compensate the controller against uncertainties and external disturbances [21-24]. At present, the nonlinear disturbance observer (NDO) can be employed to design the controller of the GHV with matched disturbance and mismatched disturbance [24-29].

Motivated by the abovementioned researches, a new adaptive SMC strategy that consists of the adaptive control method, back-stepping method, and nonlinear disturbance observer method is proposed in this paper. In the proposed controller framework, a new nonlinear disturbance observer (NDO) is employed to estimate the lumped disturbances that are introduced into the sliding surface and virtual control input at each step to compensate the effects of disturbances. It is proved that the closed-loop system is asymptotically stable here. Finally, simulation results show that the proposed method has a good disturbance rejection performance without sacrificing the nominal control performance.

The key innovations are listed below:
(i) A new adaptive SMC method is proposed to meet the flight performance for the GHV with highly nonlinear and mismatched uncertainties.
(ii) A nonlinear disturbance observer is introduced into the control system to estimate the lumped uncertainties and external disturbance to compensate the sliding mode controller.
(iii) The compute explosion problem is solved in the back-stepping method by utilizing the adaptive controller.

## 2. Hypersonic Air Vehicle Model

2.1. Original Model. The longitudinal dynamics of a GHV can be described with a set of differential equations composed by velocity $V$ and the flight path angle $\gamma$, altitude $h$, angle of attack $\alpha$, and pitch rate $q$ [3].

$$
\begin{align*}
& \dot{V}=\frac{T \cos \alpha-D}{m}-\frac{\mu \sin \gamma}{r^{2}}+d_{1},  \tag{1}\\
& \dot{\gamma}=\frac{L+T \sin \alpha}{m V}-\frac{\mu-V^{2} r}{V r^{2}} \cos \gamma+d_{2},  \tag{2}\\
& \dot{h}=V \sin \gamma  \tag{3}\\
& \dot{\alpha}=q-\dot{\gamma}+d_{3}  \tag{4}\\
& \dot{q}=\frac{M_{y y}}{I_{y y}}+d_{4} \tag{5}
\end{align*}
$$

where $d_{i}(i=1,2,3,4)$ represents the lumped disturbances in (1), (2), (4), and (5), respectively. $m, I_{y y}$, and $\mu$ represent the mass of the vehicle, moment of inertia, and gravity constant, respectively. $L, D, T$, and $M_{y y}$ represent the lift force, the drag force, the thrust force, and the pitching moment, respectively. The $r$ is the radial distance from Earth's center. They can be described as

$$
\begin{align*}
L & =\frac{1}{2} \rho V^{2} S C_{L}, \\
T & =\frac{1}{2} \rho V^{2} S C_{T}, \\
D & =\frac{1}{2} \rho V^{2} S C_{D},  \tag{6}\\
M_{y y} & =\frac{1}{2} \rho V^{2} S \bar{c}\left[C_{M}(\alpha)+C_{M}\left(\delta_{e}\right)+C_{M}(q)\right], \\
r & =h+R_{\mathrm{E}},
\end{align*}
$$

where $C_{L}, C_{T}$, and $C_{D}$ represent the lift, thrust, and drag coefficients, respectively. $C_{M}(\alpha), C_{M}(q)$, and $C_{M}\left(\delta_{e}\right)$ represent the coefficients referred to as the angle of attack, pitch rate, and elevator deflection, respectively. The parameters $\rho, S, \bar{c}$, and $R_{\mathrm{E}}$ represent the air density, the reference area, the mean aerodynamic chord, and the radius of the earth, respectively.

The engine dynamics is modeled by a second-order system

$$
\begin{equation*}
\ddot{\beta}=-2 \zeta \omega_{n} \dot{\beta}-\omega_{n}^{2} \beta_{c} \tag{7}
\end{equation*}
$$

where $\beta$ is the throttle setting, and $\beta_{c}$ is the throttle setting command.

In this paper, the aerodynamics and physical coefficients are simplified around the nominal cruising flight. The terms of $\Delta$ denote the parameter uncertainties.

$$
\begin{align*}
C_{L} & =0.6203 \alpha, \\
C_{D} & =0.6450 \alpha^{2}+0.0043378 \alpha+0.003772, \\
C_{T} & = \begin{cases}0.02576 \beta, & \text { if } \beta<1, \\
0.0224+0.00336 \beta, & \text { if } \beta>1,\end{cases} \\
C_{M}(\alpha) & =-0.035 \alpha^{2}+0.036617 \times\left(1+\Delta C_{m \alpha}\right) \alpha+5.3261 \times 10^{-6}, \\
C_{M}(q) & =\left(\frac{\bar{c}}{2 V}\right) q\left(-6.796 \alpha^{2}+0.3015 \alpha-0.2289\right), \\
C_{M}\left(\delta_{e}\right) & =c_{e}\left(\delta_{e}-\alpha\right), \\
m & =m_{0}(1+\Delta m), \\
I_{y y} & =I_{0}(1+\Delta I), \\
S & =S_{0}(1+\Delta S), \\
\bar{c} & =\bar{c}_{0}(1+\Delta \bar{c}), \\
\rho & =\rho_{0}(1+\Delta \rho), \\
c_{e} & =0.0292\left(1+\Delta c_{e}\right), \tag{8}
\end{align*}
$$

where $m_{0}=9375, I_{0}=7 \times 10^{6}, S_{0}=3603, \bar{c}=80$, and $\rho_{0}=0.24325 \times 10^{-4}$. The maximum value of the additive uncertainties is listed below.

$$
\begin{align*}
|\Delta m| & \leq \Delta_{m} \\
|\Delta I| & \leq \Delta_{I} \\
|\Delta S| & \leq \Delta_{S} \\
|\Delta \bar{c}| & \leq \Delta_{\bar{c}}  \tag{9}\\
|\Delta \rho| & \leq \Delta_{\rho} \\
\left|\Delta c_{e}\right| & \leq \Delta_{c_{e}} \\
\left|\Delta c_{M \alpha}\right| & \leq \Delta_{c_{M \alpha}}
\end{align*}
$$

The velocity is mainly related to throttle setting $\beta$ while the change of altitude is mainly related to the elevator deflection $\delta_{e}$.

### 2.2. Preparation and System Transformation

2.2.1. Preparation. The model of a GHV described by (1), (2), (3), (4), and (5) can be decoupled into two parts, which are velocity subsystem and altitude subsystem. In this paper, the flight path angle is set in a small area.

Assumption 1. The lumped disturbances are bounded and the maximum value is as follows:

$$
\begin{equation*}
\left|d_{i}\right| \leq \xi_{i}, \quad i=1,2,3,4 \tag{10}
\end{equation*}
$$

where $d_{i}$ represents the lumped disturbances in (1), (2), (4), and (5), respectively. $\xi_{i}>0$ is a known constant.
2.2.2. System Transformation. The variable states are chosen as $x_{1}=V, x_{2}=\gamma, x_{3}=\theta$, and $x_{4}=q$, where $\theta=\alpha+\gamma$.
(1) Velocity subsystem:

$$
\begin{align*}
& \dot{x}_{1}=f_{1}+g_{1} u_{1}+d_{1}, \\
& u_{1}=\beta,  \tag{11}\\
& y_{1}=x_{1} .
\end{align*}
$$

If $\beta>1$,

$$
\begin{align*}
& f_{1}=\frac{\mu \sin x_{3}}{r^{2}}-\frac{D}{m}+\frac{1}{2} \rho x_{1}^{2} S \times 0.0224 \frac{\cos \left(x_{4}-x_{3}\right)}{m}, \\
& g_{1}=\frac{1}{2} \rho x_{1}^{2} S \times 0.00336 \frac{\cos \left(x_{4}-x_{3}\right)}{m} . \tag{12}
\end{align*}
$$

Otherwise

$$
\begin{align*}
& f_{1}=-\frac{\mu \sin x_{3}}{r^{2}}-\frac{D}{m}  \tag{13}\\
& g_{1}=\frac{1}{2} \rho x_{1}^{2} S \times 0.02576 \frac{\cos \left(x_{4}-x_{3}\right)}{m} .
\end{align*}
$$

(2) Altitude subsystem: the tracking error is described by $\tilde{h}=h-h_{d}$. The altitude is denoted by $x_{2}$ as well as the altitude command is represented by $x_{2 d}$. Then, the derivative of altitude tracking error can be obtained as

$$
\begin{equation*}
\dot{\tilde{h}}=\dot{h}-\dot{h}_{d}=x_{1} \sin \gamma-\dot{h}_{d}=x_{1} \gamma+x_{1}(\sin \gamma-\gamma)-\dot{h}_{d} . \tag{14}
\end{equation*}
$$

The command referred to flight path angle is chosen as

$$
\begin{equation*}
\gamma_{d}=\frac{-k_{h} \tilde{h}-x_{1}(\sin \gamma-\gamma)+\dot{h}_{d}}{x_{1}} \tag{15}
\end{equation*}
$$

where the parameter $k_{h}$ donates the control gain.
$\dot{x}_{2}=f_{2}+g_{2} x_{3}+d_{2}$,
$\dot{x}_{3}=f_{3}+g_{3} x_{4}+d_{3}$,
$\dot{x}_{4}=f_{4}+g_{4} u_{4}+d_{4}$,
$u_{2}=\delta_{e}$,
$y_{2}=x_{2}$,
$f_{2}=-\frac{1}{2} \rho x_{1}^{2} S \times 0.6023 \frac{1}{m x_{1}} x_{2}-\frac{\mu-x_{1}^{2} r}{x_{1} r^{2}} \cos x_{2}$,

$$
\begin{align*}
& g_{1}=\frac{1}{2} \rho x_{1}^{2} S \times 0.6023 \frac{1}{m x_{1}}, \\
& f_{2}=0 \\
& g_{2}=1 \\
& f_{3}=\rho x_{1}^{2} S c \frac{\left[C_{M}\left(x_{3}-x_{2}\right)+C_{M}\left(x_{4}\right)-0.0292\left(x_{3}-x_{2}\right)\right]}{2 I_{y y}}, \tag{20}
\end{align*}
$$

$$
\begin{equation*}
g_{3}=\frac{1}{2} \rho x_{1}^{2} S \bar{c} \frac{0.0292}{I_{y y}} . \tag{21}
\end{equation*}
$$

## 3. Controller Design

And the composited controller, consisting of an adaptive back-stepping method and nonlinear disturbance observer, is designed for a GHV. The NDO is added into the controller for improving performance of the controller.
3.1. New Adaptive Sliding Mode Controller Design. The proposed controller utilizes the back-stepping method while the virtual control inputs can be obtained at each step. The designed adaptive law can compensate for the modeling uncertainties effectively.
3.1.1. Controller for the Velocity Subsystem. The velocity tracking error can be defined as

$$
\begin{equation*}
z_{1}=x_{1}-x_{1 d} . \tag{22}
\end{equation*}
$$

A new sliding mode is chosen as

$$
\begin{equation*}
s_{1}=\dot{z}_{1}+\frac{c_{1} z_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}+\widehat{d}_{1} \tag{23}
\end{equation*}
$$

where $\hat{d}_{1}$ represents the disturbance estimation of $d_{1}$.
The adaptive parameter is chosen as

$$
\begin{equation*}
\dot{\delta}_{1}=-\lambda_{1} \frac{c_{1}\left|z_{1}\right| \delta_{1} \eta_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}} \tag{24}
\end{equation*}
$$

When the differentiation of tracking error is taken into the dynamics, the equation can be obtained as

$$
\begin{align*}
\dot{s}_{1} & =\ddot{z}_{1}+\left[\frac{c_{1} z_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}\right]+\dot{\hat{d}}_{1} \\
& =\dot{f}_{1}+\left[g_{1} u_{1}\right]^{\prime}+\dot{d}_{1}-\ddot{x}_{1 d}+\left[\frac{c_{1} z_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}\right]+\dot{\vec{d}}_{1} \tag{25}
\end{align*}
$$

The command of throttle setting can be designed as

$$
\begin{align*}
u_{1}=-g_{1}^{-1} & {\left[\int k_{1,1} s_{1}+k_{1,2} \operatorname{sgn}\left(s_{1}\right) d t\right.}  \tag{26}\\
& \left.+f_{1}-\dot{x}_{1 d}+\frac{c_{1} z_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}+\widehat{d}_{1}\right]
\end{align*}
$$

where $k_{1,1}$ and $k_{1,2}$ represent the controller parameters, respectively, which determine the convergence rate of this subsystem.
3.1.2. Controller for Altitude Subsystem. The back-stepping method is used to design the controller for altitude subsystem.

Step 1 (the control input design for the flight path angle). The tracking error in this step can be defined as

$$
\begin{equation*}
z_{2}=x_{2}-x_{2 d} \tag{27}
\end{equation*}
$$

For making the tracking error converge to zero, the sliding mode surface can be designed as

$$
\begin{equation*}
s_{2}=\dot{z}_{2}+\frac{c_{2} z_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}+\hat{d}_{2} \tag{28}
\end{equation*}
$$

where $\hat{d}_{2}$ is the disturbance estimation of $d_{2}$.
The adaptive parameter is chosen as

$$
\begin{equation*}
\dot{\delta}_{2}=-\lambda_{2} \frac{c_{2}\left|z_{2}\right| \delta_{2} \eta_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}} \tag{29}
\end{equation*}
$$

If the differentiation is taken into the sliding mode, it can be obtained as

$$
\begin{align*}
\dot{s}_{2} & =\ddot{z}_{2}+\left[\frac{c_{2} z_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}\right]+\dot{\hat{d}}_{2}  \tag{30}\\
& =\dot{f}_{2}+\left[g_{2} x_{3}\right]^{\prime}+\dot{d}_{2}-\ddot{x}_{2 d}+\left[\frac{c_{2} z_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}\right]+\dot{\vec{d}}_{2}
\end{align*}
$$

It can be obtained as

$$
\begin{gather*}
x_{3 d}=-g_{2}^{-1}\left[\int k_{2,1} s_{2}+k_{2,2} \operatorname{sgn}\left(s_{2}\right) \mathrm{d} t+f_{2}\right.  \tag{31}\\
\left.-\dot{x}_{2 d}+\frac{c_{2} z_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}+\hat{d}_{2}\right] .
\end{gather*}
$$

Step 2 (the control input design for the pitching angle). The tracking error and sliding mode surface can be defined as follows:

$$
\begin{align*}
& z_{3}=x_{3}-x_{3 d} \\
& s_{3}=\dot{z}_{3}+\frac{c_{3} z_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}+\widehat{d}_{3} \tag{32}
\end{align*}
$$

where $\hat{d}_{3}$ is the disturbance estimation of $d_{3}$.
The adaptive parameter is chosen as

$$
\begin{equation*}
\dot{\delta}_{3}=-\lambda_{3} \frac{c_{3}\left|z_{3}\right| \delta_{3} \eta_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}} \tag{33}
\end{equation*}
$$

The equation can be transformed when the derivation of $s_{3}$ is taken into the dynamics

$$
\begin{align*}
\dot{s}_{3} & =\ddot{z}_{3}+\left[\frac{c_{3} z_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}\right]+\dot{\hat{d}}_{3} \\
& =\dot{f}_{3}+\left[g_{3} x_{4}\right]^{\prime}+\dot{d}_{3}-\ddot{x}_{3 d}+\left[\frac{c_{3} z_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}\right]+\dot{\vec{d}}_{3} \tag{34}
\end{align*}
$$

It can be obtained as

$$
\begin{align*}
x_{4 d}=-g_{3}^{-1}[ & \int k_{3,1} s_{3}+k_{3,2} \operatorname{sgn}\left(s_{3}\right) d t+f_{3} \\
& \left.-\dot{x}_{3 d}+\frac{c_{3} z_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}+\widehat{d}_{3}\right] \tag{35}
\end{align*}
$$

Step 3 (the control input design for the pitching rate angle). The tracking error $z_{4}$ and sliding mode surface $s_{4}$ in this step can be designed as follows:

$$
\begin{align*}
& z_{4}=x_{4}-x_{4 d} \\
& s_{4}=\dot{z}_{4}+\frac{c_{4} z_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}+\hat{d}_{4} \tag{36}
\end{align*}
$$

where $\widehat{d}_{4}$ is the disturbance estimation of $d_{4}$.
The adaptive parameter is chosen as

$$
\begin{equation*}
\dot{\delta}_{4}=-\lambda_{4} \frac{c_{4}\left|z_{4}\right| \delta_{4} \eta_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}} \tag{37}
\end{equation*}
$$

The equation can be transformed when the derivation of $z_{4}$ is taken into the tracking error.

$$
\begin{align*}
\dot{s}_{4} & =\ddot{z}_{4}+\left[\frac{c_{4} z_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}\right]+\dot{\hat{d}}_{4}  \tag{38}\\
& =\dot{f}_{4}+\left[g_{4} u_{2}\right]^{\prime}+\dot{d}_{4}-\ddot{x}_{4 d}+\left[\frac{c_{4} z_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}\right]+\dot{\hat{d}}_{4}
\end{align*}
$$

The slide mode controller can be obtained as follows:

$$
\begin{array}{r}
u_{2}=-g_{4}^{-1}\left[\int k_{4,1} s_{4}+k_{4,2} \operatorname{sgn}\left(s_{4}\right) d t+f_{4}\right.  \tag{39}\\
\left.-\dot{x}_{4 d}+\frac{c_{4} z_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}+\hat{d}_{4}\right]
\end{array}
$$

3.2. Nonlinear Disturbance Observer Design. Inspired by the works of Zhang et al. [26], Liu et al. [27], and Tian et al. [28], a nonlinear disturbance observer is designed as follows.
3.2.1. NDO for Velocity Subsystem. A nonlinear disturbance observer for (1) is designed as

$$
\begin{gather*}
{\left[\begin{array}{c}
\dot{\hat{x}}_{1} \\
\dot{\hat{d}}_{1}
\end{array}\right]=\left[\begin{array}{cc}
A_{12} & 1 \\
0 & 0
\end{array}\right]\left[\begin{array}{l}
\hat{x}_{1} \\
\hat{d}_{1}
\end{array}\right]+\left[\begin{array}{c}
A_{11} \\
0
\end{array}\right]\left(x_{3}-x_{2}\right)} \\
\\
+\left[\begin{array}{c}
B_{1} \\
0
\end{array}\right] u_{1}-\left[\begin{array}{l}
L_{11} \\
L_{12}
\end{array}\right]\left(\hat{x}_{1}-x_{1}\right) \\
A_{11}=-0.0043378 \frac{\rho x_{1}^{2} S}{2 m}, A_{12}=0  \tag{40}\\
B_{1}=0.00336 \beta_{c} \frac{\rho x_{1}^{2} S}{2 m}
\end{gather*}
$$

where $L_{11}=A_{11}+2 \Lambda_{1}, L_{12}=\Lambda_{1}^{2}$, and $\Lambda_{1}>0$.
3.2.2. NDO for Altitude Subsystem. Similarly, an NDO for (3) is obtained as

$$
\left[\begin{array}{c}
\dot{\hat{h}}  \tag{41}\\
\dot{\hat{d}}_{h}
\end{array}\right]=\left[\begin{array}{cc}
H_{11} & 1 \\
0 & 0
\end{array}\right]\left[\begin{array}{c}
\widehat{h} \\
\widehat{d}_{h}
\end{array}\right]-\left[\begin{array}{l}
H_{21} \\
H_{22}
\end{array}\right](\widehat{h}-h), \quad H_{11}=x_{c},
$$

where $H_{21}=H_{11}+2 \Lambda_{H}, H_{22}=\Lambda_{H}^{2}$, and $\Lambda_{H}>0$.
A nonlinear disturbance observer for (2) is designed as

$$
\begin{gather*}
{\left[\begin{array}{c}
\dot{\hat{x}}_{2} \\
\dot{\hat{d}}_{2}
\end{array}\right]=\left[\begin{array}{cc}
A_{22} & 1 \\
0 & 0
\end{array}\right]\left[\begin{array}{l}
\hat{x}_{2} \\
\hat{d}_{2}
\end{array}\right]+\left[\begin{array}{c}
A_{21} \\
0
\end{array}\right] x_{3}-\left[\begin{array}{l}
L_{21} \\
L_{22}
\end{array}\right]\left(\hat{x}_{2}-x 2\right),} \\
A_{21}=-0.6427 \frac{\rho x_{1}^{2} S}{2 m x_{1}}, A_{22}=0.6427 \frac{\rho x_{1}^{2} S}{2 m x_{1}} \tag{42}
\end{gather*}
$$

where $L_{21}=A_{21}+2 \Lambda_{2}, L_{22}=\Lambda_{2}^{2}$, and $\Lambda_{2}>0$.
A nonlinear disturbance observer for (4) is designed as

$$
\begin{array}{r}
{\left[\begin{array}{l}
\dot{\hat{x}}_{3} \\
\dot{\hat{d}}_{3}
\end{array}\right]=\left[\begin{array}{ll}
A_{31} & 1 \\
0 & 0
\end{array}\right]\left[\begin{array}{l}
\hat{x}_{3} \\
\hat{d}_{3}
\end{array}\right]+\left[\begin{array}{c}
A_{32} \\
0
\end{array}\right] x_{4}-\left[\begin{array}{l}
L_{31} \\
L_{32}
\end{array}\right]\left(\hat{x}_{3}-x_{3}\right)} \\
A_{31}=0, A_{32}=1 \tag{43}
\end{array}
$$

where $L_{31}=A_{31}+2 \Lambda_{3}, L_{32}=\Lambda_{3}^{2}$, and $\Lambda_{3}>0$.
Similarly, a nonlinear disturbance observer for (5) is designed as

$$
\begin{align*}
{\left[\begin{array}{l}
\dot{\hat{x}}_{4} \\
\dot{\vec{d}}_{4}
\end{array}\right]=} & {\left[\begin{array}{cc}
A_{42} & 1 \\
0 & 0
\end{array}\right]\left[\begin{array}{l}
\hat{x}_{4} \\
\hat{d}_{4}
\end{array}\right]+\left[\begin{array}{c}
A_{41} \\
0
\end{array}\right] x_{3}+\left[\begin{array}{l}
B_{2} \\
0
\end{array}\right] u_{2} } \\
& -\left[\begin{array}{l}
L_{41} \\
L_{42}
\end{array}\right]\left(\hat{x}_{4}-x_{4}\right), \\
& A_{41}=\frac{\rho x_{1}^{2} S \bar{c}}{2 I_{y y}}\left(0.036617-c_{e}\right), A_{42}=-0.2289 \frac{\rho x_{1} S \bar{c}^{2}}{4 I_{y y}}, \\
& B_{2}=\frac{c_{e} \rho x_{1}^{2} S \bar{c}}{2 I_{y y}}, \tag{44}
\end{align*}
$$

where $L_{41}=A_{41}+2 \Lambda_{4}, L_{42}=\Lambda_{4}^{2}$, and $\Lambda_{4}>0$.

## 4. Stability Analysis

4.1. Convergence of SMC. The stability of the closed control system is proved by the Lyapunov stability theory.

Firstly, a Lyapunov function is chosen as

$$
\begin{equation*}
F=F_{1}+F_{2}+F_{3}+F_{4}=\frac{1}{2} s_{1}^{2}+\frac{1}{2} s_{2}^{2}+\frac{1}{2} s_{3}^{2}+\frac{1}{2} s_{4}^{2} . \tag{45}
\end{equation*}
$$

The derivation of the Lyapunov function is obtained as

$$
\begin{equation*}
\dot{F}=\dot{F}_{1}+\dot{F}_{2}+\dot{F}_{3}+\dot{F}_{4}=s_{1} \dot{s}_{1}+s_{2} \dot{s}_{2}+s_{3} \dot{s}_{3}+s_{4} \dot{s}_{4} . \tag{46}
\end{equation*}
$$

Step 1 (stability analysis for the velocity subsystem). The derivation of the Lyapunov function for system (11) can be obtained as

$$
\begin{align*}
\dot{F}_{1}= & \left\{-k_{1,1} s_{1}-k_{1,2} \operatorname{sgn}\left(s_{1}\right)-\dot{f}_{1}+\ddot{x}_{1 d}-\left[\frac{c_{1} z_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}\right]^{\prime}-\hat{d}_{1}\right. \\
& \left.+\dot{f}_{1}+\dot{d}_{1}-\ddot{x}_{1 d}+\left[\frac{c_{1} z_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}\right]^{\prime}+\hat{d}_{1}\right\} s_{1}  \tag{47}\\
= & -k_{1,1} s_{1}^{2}-k_{1,2}\left|s_{1}\right|+\dot{d}_{1} s_{1} .
\end{align*}
$$

If the parameters of the controller (26) is chosen to meet $k_{1,2} \geq D_{1,2} \geq\left|\dot{d}_{1}\right|$, it can be obtained as

$$
\begin{equation*}
\dot{F}_{1} \leq-k_{1,1} s_{1}^{2} \leq 0 . \tag{48}
\end{equation*}
$$

Step 2 (stability analysis for the angles)
(a) Flight path angle:

$$
\begin{align*}
\dot{F}_{2}=s_{2} \dot{s}_{2}= & \left\{-k_{2,1} s_{2}-k_{2,2} \operatorname{sgn}\left(s_{2}\right)-\dot{f}_{2}+\ddot{x}_{2 d}-\left[\frac{c_{2} z_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}\right]^{\prime}\right. \\
& \left.-\hat{d}_{2}+\dot{f}_{2}+\dot{d}_{2}-\ddot{x}_{2 d}+\left[\frac{c_{2} z_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}\right]^{\prime}+\hat{d}_{2}\right\} s_{2} \\
= & -k_{2,1} s_{2}^{2}-k_{2,2}\left|s_{2}\right|+\dot{d}_{2} s_{2} . \tag{49}
\end{align*}
$$

If $k_{2,2} \geq D_{2,2} \geq\left|\dot{d}_{2}\right|$ is satisfied, it can be obtained as

$$
\begin{equation*}
\dot{F}_{2} \leq-k_{2,1} s_{2}^{2} \leq 0 \tag{50}
\end{equation*}
$$

(b) Pitching angle:

$$
\begin{align*}
\dot{F}_{3}= & \left\{-k_{3,1} s_{3}-k_{3,2} \operatorname{sgn}\left(s_{3}\right)-\dot{f}_{3}+\ddot{x}_{3 d}-\left[\frac{c_{3} z_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}\right]^{\prime}-\widehat{d}_{3}\right. \\
& \left.+\dot{f}_{3}+\dot{d}_{3}-\ddot{x}_{3 d}+\left[\frac{c_{3} z_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}\right]^{\prime}+\hat{d}_{3}\right\} s_{3} \\
= & -k_{3,1} s_{3}^{2}-k_{3,2}\left|s_{3}\right|+\dot{d}_{3} s_{3} . \tag{51}
\end{align*}
$$

If $k_{3,2} \geq D_{3,2} \geq\left|\dot{d}_{3}\right|$ is satisfied, it can be obtained as

$$
\begin{equation*}
\dot{F}_{3} \leq-k_{3,1} s_{3}^{2} \leq 0 . \tag{52}
\end{equation*}
$$

(c) Pitching rate:

$$
\begin{align*}
\dot{F}_{4}= & \left\{-k_{4,1} s_{4}-k_{4,2} \operatorname{sgn}\left(s_{4}\right)-\dot{f}_{4}+\ddot{x}_{4 d}-\left[\frac{c_{4} z_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}\right]^{\prime}-\widehat{d}_{4}\right. \\
& \left.+\dot{f}_{4}+\dot{d}_{4}-\ddot{x}_{4 d}+\left[\frac{c_{4} z_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}\right]^{\prime}+\widehat{d}_{4}\right\} s_{4} \\
= & -k_{4,1} s_{4}^{2}-k_{4,2}\left|s_{4}\right|+\dot{d}_{4} s_{4} . \tag{53}
\end{align*}
$$

If $k_{4,2} \geq D_{4,2} \geq\left|\dot{d}_{4}\right|$ is satisfied, it can be obtained as

$$
\begin{equation*}
\dot{F} \leq-k_{4,1} s_{4}^{2} \leq 0 \tag{54}
\end{equation*}
$$

The Lyapunov stability is proved.

$$
\begin{align*}
F & =F_{1}+F_{2}+F_{3}+F_{4}=\frac{1}{2} s_{1}^{2}+\frac{1}{2} s_{2}^{2}+\frac{1}{2} s_{3}^{2}+\frac{1}{2} s_{4}^{2} \geq 0 \\
\dot{F} & =\dot{F}_{1}+\dot{F}_{2}+\dot{F}_{3}+\dot{F}_{4}=s_{1} \dot{s}_{1}+s_{2} \dot{s}_{2}+s_{3} \dot{s}_{3}+s_{4} \dot{s}_{4} \\
& \leq-k_{1,1} s_{1}^{2}-k_{2,1} s_{2}^{2}-k_{3,1} s_{3}^{2}-k_{4,1} s_{4}^{2} \tag{55}
\end{align*}
$$

4.2. Convergence of Tracking Error. The Lyapunov function is chosen as

$$
\begin{align*}
W= & \frac{1}{2} z_{1}^{2}+\frac{1}{2 \lambda_{1}} \delta_{1}^{2}+\frac{1}{2} \tilde{h}^{2}+\frac{1}{2} z_{3}^{2}+\frac{1}{2 \lambda_{2}} \delta_{2}^{2} \\
& +\frac{1}{2} z_{4}^{2}+\frac{1}{2 \lambda_{3}} \delta_{3}^{2}+\frac{1}{2} z_{5}^{2}+\frac{1}{2 \lambda_{4}} \delta_{4}^{2} \tag{56}
\end{align*}
$$

where $z_{1}, \tilde{h}, z_{3}, z_{4}$, and $z_{5}$ donate the tracking error, respectively.

The derivation of the Lyapunov function can be obtained as

$$
\begin{align*}
\dot{W}= & z_{1} \dot{z}_{1}+\frac{1}{\lambda_{1}} \delta_{1} \dot{\delta}_{1}+\tilde{h} \dot{\tilde{h}}+z_{3} \dot{z}_{3}+\frac{1}{\lambda_{2}} \delta_{2} \dot{\delta}_{2} \\
& +z_{4} \dot{z}_{4}+\frac{1}{2 \lambda_{3}} \delta_{3} \dot{\delta}_{3}+z_{5} \dot{z}_{5}+\frac{1}{2 \lambda_{4}} \delta_{4} \dot{\delta}_{4} \tag{57}
\end{align*}
$$

Step 1 (stability analysis for velocity). The Lyapunov function for system (11) is chosen as

$$
\begin{equation*}
W_{1}=\frac{1}{2} z_{1}^{2}+\frac{1}{2 \lambda_{1}} \delta_{1}^{2} \tag{58}
\end{equation*}
$$

The derivation of the Lyapunov function can be obtained as

$$
\begin{align*}
\dot{W}_{1} & =z_{1} \dot{z}_{1}+\frac{1}{\lambda_{1}} \delta_{1} \dot{\delta}_{1}=-\frac{c_{1} z_{1}^{2}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}-\widehat{d}_{1} z_{1}+\frac{1}{\lambda_{1}} \delta_{1} \dot{\delta}_{1} \\
& =-c_{1}\left|z_{1}\right|+\frac{c_{1}\left|z_{1}\right| \delta_{1}^{2} \eta_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}-\widehat{d}_{1} z_{1}-\frac{c_{1}\left|z_{1}\right| \delta_{1}^{2} \eta_{1}}{\left|z_{1}\right|+\delta_{1}^{2} \eta_{1}}  \tag{59}\\
& =-c_{1}\left|z_{1}\right|-\widehat{d}_{1} z_{1} .
\end{align*}
$$

The $c_{1}$ is chosen to satisfy the inequation

$$
\begin{equation*}
c_{1} \geq D_{1,1} \geq\left|\hat{d}_{1}\right| \tag{60}
\end{equation*}
$$

It can be obtained as

$$
\begin{equation*}
\dot{W}_{1}=-c_{1}\left|z_{1}\right|-\widehat{d}_{1} z_{1} \leq-\sigma_{1}\left|z_{1}\right| \tag{61}
\end{equation*}
$$

where $\sigma_{1}=c_{1}-D_{1,1}$ is a positive constant and $\dot{W}_{1} \leq 0$ is satisfied.

Step 2 (stability analysis for the altitude subsystem).
(a) Altitude:

$$
\begin{equation*}
W_{h}=\frac{1}{2} \tilde{h}^{2} . \tag{62}
\end{equation*}
$$

The derivation of $W_{h}$ can be obtained as

$$
\begin{align*}
& \dot{W}_{h}=\tilde{h}\left[x_{1} \frac{-k_{h} \tilde{h}-x_{1}\left(\sin x_{2}-x_{2}\right)+\dot{h}_{d}}{x_{1}} 1\right. \\
&\left.+x_{1}\left(\sin x_{2}-x_{2}\right)-\dot{h}_{d}\right] \tag{63}
\end{align*}
$$

$$
=-k_{h} \tilde{h}^{2}
$$

where $k_{h}$ is a positive constant and $\dot{W}_{h} \leq 0$ is satisfied.
(b) Flight path angle: the Lyapunov function is chosen as

$$
\begin{equation*}
W_{2}=\frac{1}{2} z_{2}^{2}+\frac{1}{2 \lambda_{2}} \delta_{2}^{2} \tag{64}
\end{equation*}
$$

The derivation of $W_{2}$ can be obtained as

$$
\begin{align*}
\dot{W}_{2} & =z_{2} \dot{z}_{2}+\frac{1}{\lambda_{2}} \delta_{2} \dot{\delta}_{2}=-\frac{c_{2} z_{2}^{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}-\hat{d}_{2} z_{2}+\frac{1}{\lambda_{2}} \delta_{2} \dot{\delta}_{2} \\
& =-c_{2}\left|z_{2}\right|+\frac{c_{2}\left|z_{2}\right| \delta_{2}^{2} \eta_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}-\hat{d}_{2} z_{2}-\frac{c_{2}\left|z_{2}\right| \delta_{2}^{2} \eta_{2}}{\left|z_{2}\right|+\delta_{2}^{2} \eta_{2}}  \tag{65}\\
& =-c_{2}\left|z_{2}\right|-\widehat{d}_{2} z_{2} .
\end{align*}
$$

The parameter $c_{2}$ is chosen to satisfy

$$
\begin{align*}
c_{2} & \geq D_{2,1} \geq\left|\widehat{d}_{2}\right|,  \tag{66}\\
\dot{W}_{2} & \leq-\sigma_{2}\left|z_{2}\right|
\end{align*}
$$

where $\sigma_{2}=c_{2}-D_{2,1}$ is a positive constant and $\dot{W}_{2} \leq 0$ is satisfied.
(c) Pitching angle: the Lyapunov function is chosen as

$$
\begin{equation*}
W_{3}=\frac{1}{2} z_{3}^{2}+\frac{1}{2 \lambda_{3}} \delta_{3}^{2} \tag{67}
\end{equation*}
$$

The derivation of $W_{3}$ can be obtained as

$$
\begin{align*}
\dot{W}_{3} & =z_{3} \dot{z}_{3}+\frac{1}{\lambda_{3}} \delta_{3} \dot{\delta}_{3}=-\frac{c_{3} z_{3}^{2}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}-\widehat{d}_{3} z_{3}+\frac{1}{\lambda_{3}} \delta_{3} \dot{\delta}_{3} \\
& =-c_{3}\left|z_{3}\right|+\frac{c_{3}\left|z_{3}\right| \delta_{3}^{2} \eta_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}-\widehat{d}_{3} z_{3}-\frac{c_{3}\left|z_{3}\right| \delta_{3}^{2} \eta_{3}}{\left|z_{3}\right|+\delta_{3}^{2} \eta_{3}}  \tag{68}\\
& =-c_{3}\left|z_{3}\right|-\widehat{d}_{3} z_{3} .
\end{align*}
$$

If the parameters $c_{3}$ and $\widehat{d}_{3}$ satisfy $c_{3} \geq D_{3,1} \geq\left|\hat{d}_{3}\right|$, it can be obtained as

$$
\begin{equation*}
\dot{W}_{3}=-c_{3}\left|z_{3}\right|-\widehat{d}_{3} z_{3} \leq-\sigma_{3}\left|z_{3}\right|, \tag{69}
\end{equation*}
$$

where $\sigma_{3}=c_{3}-D_{3,1}$ is a positive constant.
(d) Pitching rate: the Lyapunov function is chosen as

$$
\begin{equation*}
W_{4}=\frac{1}{2} z_{4}^{2}+\frac{1}{2 \lambda_{4}} \delta_{4}^{2} \tag{70}
\end{equation*}
$$

The derivation of $W_{4}$ can be obtained as

$$
\begin{align*}
\dot{W}_{4} & =z_{4} \dot{z}_{4}+\frac{1}{\lambda_{4}} \delta_{4} \dot{\delta}_{4}=-\frac{c_{4} z_{4}^{2}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}-\widehat{d}_{4} z_{4}+\frac{1}{\lambda_{4}} \delta_{4} \dot{\delta}_{4} \\
& =-c_{4}\left|z_{4}\right|+\frac{c_{4}\left|z_{4}\right| \delta_{4}^{2} \eta_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}-\widehat{d}_{4} z_{4}-\frac{c_{4}\left|z_{4}\right| \delta_{4}^{2} \eta_{4}}{\left|z_{4}\right|+\delta_{4}^{2} \eta_{4}}  \tag{71}\\
& =-c_{4}\left|z_{4}\right|-\widehat{d}_{4} z_{4} .
\end{align*}
$$

If the $c_{4}$ is chosen to satisfy $c_{4} \geq D_{4,1} \geq\left|\hat{d}_{4}\right|$, it can be obtained as

$$
\begin{equation*}
\dot{W}_{4} \leq-\sigma_{4}\left|z_{4}\right|, \tag{72}
\end{equation*}
$$

where $\sigma_{4}=c_{4}-D_{4,1}$ is a positive constant. Thus, $\dot{W}_{4} \leq 0$ is satisfied.

$$
\begin{align*}
W= & \frac{1}{2} z_{1}^{2}+\frac{1}{2 \lambda_{1}} \delta_{1}^{2}+\frac{1}{2} z_{2}^{2}+\frac{1}{2} z_{3}^{2}+\frac{1}{2 \lambda_{2}} \delta_{2}^{2} \\
& +\frac{1}{2} z_{4}^{2}+\frac{1}{2 \lambda_{3}} \delta_{3}^{2}+\frac{1}{2} z_{5}^{2}+\frac{1}{2 \lambda_{4}} \delta_{4}^{2} \\
\dot{W}= & z_{1} \dot{z}_{1}+\frac{1}{\lambda_{1}} \delta_{1} \dot{\delta}_{1}+z_{2} \dot{z}_{2}+z_{3} \dot{z}_{3}+\frac{1}{\lambda_{2}} \delta_{2} \dot{\delta}_{2}  \tag{73}\\
& +z_{4} \dot{z}_{4}+\frac{1}{2 \lambda_{3}} \delta_{3} \dot{\delta} 3+z_{5} \dot{z}_{5}+\frac{1}{2 \lambda_{4}} \delta_{4} \dot{\delta}_{4} \\
\leq & -\sigma_{1}\left|z_{1}\right|-k_{h} \tilde{h}^{2}-\sigma_{2}\left|z_{2}\right|-\sigma_{3}\left|z_{3}\right|-\sigma_{4}\left|z_{4}\right|
\end{align*}
$$

Then it can be obtained as

$$
\begin{align*}
& W \geq 0 \\
& \dot{W} \leq 0 \tag{74}
\end{align*}
$$

The convergence of tracking error is proved now.

## 5. Simulation

In this section, the effectiveness and performance of the developed controller are verified by simulations. The longitudinal model is considered under its cruise flight condition. The initial values are chosen as $V=15060 \mathrm{ft} / \mathrm{s}, h=110000 \mathrm{ft}$, $\gamma=0 \mathrm{rad}, q=0 \mathrm{rad} / \mathrm{s}$, and $\theta=0.01 \mathrm{rad}$, respectively.

The controller parameters are chosen as

$$
\begin{align*}
k_{h} & =2 \\
\lambda_{1} & =0.005 \\
c_{1} & =15.7 \\
\eta_{1} & =0.1 \\
\lambda_{2} & =0.3 \\
c_{2} & =0.5 \\
\eta_{2} & =0.007 \\
\lambda_{3} & =0.1 \\
c_{3} & =11.2  \tag{75}\\
\eta_{3} & =0.01 \\
\lambda_{4} & =0.05 \\
c_{4} & =8.7 \\
\eta_{4} & =0.01 \\
\Lambda_{1} & =2.6 \\
\Lambda_{H} & =2.2 \\
\Lambda_{2} & =3.1 \\
\Lambda_{3} & =3.1 \\
\Lambda_{4} & =3.3
\end{align*}
$$

The external disturbances are chosen to be $5 \times 10^{-4}$ $\cos (0.5 t), \quad 5 \times 10^{-4} \cos (0.5 t), \quad 1 \times 10^{-2} \cos (0.5 t)$, and $5 \times$ $10^{-3} \cos (0.5 t)$ for the system (11) and (16).

In this part, the square wave and step are applied in command generator, respectively.


Figure 1: Velocity command, response, and controller. The uncertainty terms are added into the system.


Figure 2: Altitude command, response, and controller. The uncertainty terms are added into the system.


Figure 3: Altitude command, response, and controller. The uncertainty terms and external disturbances are added into the system.


Figure 4: Lumped disturbances estimated by NDO.


$$
\begin{array}{ll}
- \text { - } & \text { Back-stepping } \\
- \text { - } & \text { Back-stepping with DOB } \\
\text {-... } & \text { Adaptive back-stepping }
\end{array}
$$

(a)


-     -         - Back-stepping
-     -         - Back-stepping with DOB
..... Adaptive back-stepping
(c)

(e)


$$
\begin{aligned}
& \text { - - - Back-stepping } \\
& \text {-- Back-stepping with DOB } \\
& \text { … Adaptive back-stepping }
\end{aligned}
$$

(h)


> -- - Back-stepping
> -- Back-stepping with DOB
> … Adaptive back-stepping
(f)


[^0](i)


-     -         - Back-stepping
. - - Back-stepping with DOB
..... Adaptive back-stepping
(g)

-     - Back-stepping
.-.- Back-stepping with DOB
...... Adaptive back-stepping
(j)

Figure 5: Velocity command, response, and controller. The uncertainty terms and external disturbances are added into the system.


Figure 6: Lumped disturbance estimated by NDO.

(a)


$$
\begin{aligned}
& ---\cdot \text { Back-stepping } \\
& \text {-- . . Back-stepping with DOB }
\end{aligned}
$$

(c)
-- Back-stepping
...- Back-stepping with DOB
...... Adaptive back-stepping
(e)


> --- Back-stepping
> -- Back-stepping with DOB
> $\cdots$ Adaptive back-stepping
(h)

--- Back-stepping

-     -         - Back-stepping with DOB
… Adaptive back-stepping
(f)

-     - Back-stepping
. . . Back-stepping with DOB
Adaptive back-stepping
(i)

-- Back-stepping
-.- Back-stepping with DOB
...... Adaptive back-stepping
(g)

-     - Back-stepping
-     - Back-stepping with DOB Adaptive back-stepping
(j)

Figure 7: Command, response, and controller. The uncertainty terms and external disturbances are added into the system.


Figure 8: Lumped disturbance estimated by nonlinear disturbance observer.

(a)


$$
\begin{aligned}
& --K=0.8 \\
& --K=1.0 \\
& \cdots \cdot K=1.2
\end{aligned}
$$


(b)


$$
--K=0.8
$$

$$
-K=1.0
$$

$\cdots \cdots K=1.2$
(c)


$$
\begin{aligned}
& --K=0.8 \\
& -K=1.0 \\
& \cdots \cdot K=1.2
\end{aligned}
$$

(e)


$$
\begin{aligned}
& --K=0.8 \\
& --K=1.0
\end{aligned}
$$

(h)


$$
\begin{aligned}
&---K= 0.8 \\
&--K= 1.0 \\
& \cdots \cdots K=1.2
\end{aligned}
$$

(f)


$$
\begin{aligned}
--K & =0.8 \\
--K & =1.0 \\
\cdots \cdots & =1.2
\end{aligned}
$$

(i)
(d)


$$
-K=0.8
$$

$$
\cdots K=1.0
$$

$$
\cdots \cdot K=1.2
$$

(g)

(j)

Figure 9: Altitude command, response, and controller. The uncertainty terms and external disturbance are added into the model.

(a)

$\begin{array}{ll}\text {--- } & K=0.8 \\ K=1.0\end{array}$
$K=1.0$
$K=1.2$


$$
\begin{array}{ll} 
& K=0.8 \\
\cdots- & K=1.0 \\
\cdots \cdots & K=1.2
\end{array}
$$

(b)

(d)
(c)

(e)

FIgure 10: Lumped disturbance estimated by nonlinear disturbance observer. The parameters deflection, uncertainty terms, and external disturbance are added into the model.

Case 1. The square wave is adopted to prove the effectiveness of controller. The uncertainties are added into this system. The simulation results are shown in Figures 1 and 2.

It is obtained from Figure 1 that the velocity can track the given command well while the altitude is stable. Meanwhile, as shown in Figure 2, the altitude can track the reference command well and the velocity is stable.

Case 2. The proposed controller is compared to the backstepping method [11] and back-stepping with NDO [29]. The external disturbances are added into the model at $t=$ 150 s. The simulation results are shown in Figures 3-8.

The performance of the controller is proved with the existence of uncertainties and external disturbances. At $t=$ 150 s , the external disturbances are taken into the system. Compared with other methods mentioned in Case 2, the performance of GHV under the proposed controller is better.

Case 3. In order to verify the effectiveness of the controller against parameter perturbation, the coefficients of deflection are chosen as $K=0.8, K=1.0$, and $K=1.2$ while the external disturbances are taken at $t=150 \mathrm{~s}$. The simulation results are shown in Figures 9 and 10.

The system under the proposed controller exhibits good performance against both positive and negative parameter perturbation.

## 6. Conclusion

A new adaptive sliding mode control method combined with the nonlinear disturbance observer is proposed to solve the tracking problem for the longitudinal model of a GHV. The compute explosion problem is solved by utilizing the new adaptive control algorithm. In addition, the proposed controller for a GHV model has achieved favorable results in terms of robustness without the cost of sacrificing the nominal control performance. Finally, the performance of the proposed control algorithm has been demonstrated by simulation results.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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